Assignment 1

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Solutions

Q.1) Geometric Image Formation

Answer a)

Given $f = 10$
point $P = (3,2,1) \Rightarrow$ (world point)

find coordinate of P when projecting onto image.

$$\tilde{p} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\tilde{p} = \begin{bmatrix} 30 \\ 20 \\ 1 \end{bmatrix}$$

or $\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z} \begin{bmatrix} 30 \\ 20 \end{bmatrix} = \begin{bmatrix} 30 \\ 20 \end{bmatrix}$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 30 \\ 20 \end{bmatrix}$$

Answer b)

Differenc → When the center of projection is ~~in front~~ in ~~front~~ behind of image plone, the image formed is inverted while when it is behind it won't.

~~Refer → We will The pinhole camera where the image plane is in front of the center of projection corresponds better to a physical camera model as it is easier to visualize~~

In pinhole camera where the image plone is behind the center of projection is better because it gives more accurate ~~result~~ image then the pinhole camera model where the image plone is in front of center of projection & it will give better corresponds to a physical pinhole camera model.

Answer c)

From the eqn of image formation:

$$\begin{bmatrix} u \\ v \end{bmatrix} = 1/z \begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

When the focal length (f) gets bigger the projection gets bigger & when the distance of to the object z gets bigger the projection gets smaller.

Answer d)

$$\left[\begin{matrix} 2D & \text{to} & 2DH \\ (x, y) & \Longleftrightarrow & (x, y, 1) \end{matrix} \right]$$

$(1,1) \longrightarrow (1,1,1)$

2D point — 2D H point

another corresponding point can be

$\longrightarrow (2,2,2)$

Answer e

2DH to 2D

$(x, y, w) \longrightarrow (x/w, y/w, 1)$

$(1, 1, 2) \longrightarrow (1/2, 1/2, 1)$

Answer f

$(x, y, 0) \Rightarrow$ is point at infinity represent direction.

Hence; $\boxed{(1,1,0) \text{ will represent direction \& it is a point at } \infty}$

Answer g

eq^n ①
$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z}\begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$
$u = fx/z$
$v = fy/z$

eq^n ②
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & f \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$\cancel{u = v/z}$
$u = fx, \; v = fy, \; w = fz$

from eq^n ① & eq^n ② we see homogenizing (u, v) makes it possible to write non-linear eq^n as linear eq^n in homogenous form.

Answer h

$$M = K[I][0]$$

Dimensions of $M = 3\times4$	Dim. of $I = 3\times3$
Dimensions of $K = 3\times3$	Dim of $0 = 3\times1$

where; $K = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & f \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Answer i)

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

(Projection matrix)

2DH will be.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix}}_{3\times 4} \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}}_{4\times 1}$$

(Point P in 3D)

$$2DH \Rightarrow \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1+4+9+4 \\ 5+12+21+8 \\ 1+4+3+2 \end{bmatrix} = \begin{bmatrix} 18 \\ 46 \\ 10 \end{bmatrix}$$

&

$$\boxed{2D \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u/w \\ v/w \end{bmatrix} = \begin{bmatrix} 9/5 \\ 23/5 \end{bmatrix}}$$

Q2) Modeling Transformations:

Answer a)

Coordinates after translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \underset{3\times 3}{\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

or

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

Answer b)

Coordinates on Scaling

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Answer c

Coordinates on rotation.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For $\theta = 45°$ & $(x, y) = (1, 1)$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}}$$

Answer d

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(in continuation)

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

For $\theta = 45^\circ$ & $(x, y) = (1, 1)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} \; \end{bmatrix}}$$

$\cancel{x'}$
$\cancel{y^*}$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2-\sqrt{2} \\ 1 \end{bmatrix}$$

or

$$\boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 2-\sqrt{2} \end{bmatrix}}$$

Answer e) Combined Matrix $P' = T R P$

first

second.

Answer f) $$M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

on comparing M with matrix → $\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

we can say that it is a scaled matrix & P will be scaled by $(3,2)$

Answer g) $$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

just like f if we compare M with

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

∴ P will be transformed/ translate by $(1,2)$

Answer e) Combined Matrix $P' = T(R\,P)$

first

second.

Answer f) $$m = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

on comparing m with matrix $\Rightarrow \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

we can say that it is a scaled matrix & P will be scaled by (3, 2)

Answer g) $$m = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

just like f if we compare m with

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ P will be transformed/ translate by (1, 2)

Answer h)

$$m = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

transformation matrix (m') which will reverse the effect:

$$\boxed{m' = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

Answer (i)

Matrix $m = R(45)\ T(1,2)$

As $\left(m^{-1} = T^{-1}_{(1,2)} R^{-1}_{(45)}\right)$

or

$\therefore\ m^{-1} = T(-1,-2)\ R^{T}$

Note $\left(\text{using } R^{-1} T^{-1}(t) = R^{T} T(-t)\right)$

Answer j)

Given: $V = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ find V^T?

$V \cdot V^T = 0$ $\begin{bmatrix} 1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$x + 3y = 0$

for $y = 1$, $x = -3$ So, $\boxed{V^T = \begin{bmatrix} -3 \\ 1 \end{bmatrix}}$

Answer (k) $u = (1,3)$

$v = (2,5)$

$$\text{Proj}_v u = \left(\frac{u \cdot v}{|v|^2}\right) v$$

$$= \left(\frac{1 \cdot 2 + 3 \cdot 5}{\left(\sqrt{4+25}\right)^2}\right)(2,5)$$

$$= \frac{17}{29}(2,5) = \left(\frac{34}{29}, \frac{85}{29}\right)$$

Q3) General Camera Model:

Answer a) By having a general projection matrix it becomes easy to relate 3D point in world coordinates to 2D point in image coordinate.

Answer b) Let $p^{(w)}$ is a point in world coordinates & $p^{(c)}$ point in camera coordinates.

$$p^{(c)} = M_{c \leftarrow w} \, p^{(w)}$$

where M = Transformation matrix,
R, T = Rotation, Translation parameters.

$$M_{c \to w} = \left(T(t) \, R\right)^{-1}$$

$$= R^{-1} \, T^{-1}_{(t)}$$

$$\boxed{M_{c \to w} = R^{T} \, T(-t)}$$

Answer c)

$$R = \begin{bmatrix} \hat{x_c} & \hat{y_c} & \hat{z_c} \end{bmatrix}$$

Answer d)

$$M = \begin{bmatrix} R^* & T^* \\ 0 & 1 \end{bmatrix} \qquad R^* = R^T, \quad T^* = -R^T t.$$

where R^* = is the transformation matrix responsible for rotation that aligns the two coordinate systems (world & camera)

T^* is the translation vector b/w relative positions of the origins.

Answer e) let $M_{i \leftarrow c}$ be the transformation matrix that will convert camera coordinate to image coordinates:

$$M_{i \leftarrow c} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

for $u_0, v_0 = (512, 512)$

$$M_{i \leftarrow c} = \begin{bmatrix} k_u & 0 & 512 \\ 0 & k_v & 512 \\ 0 & 0 & 1 \end{bmatrix}$$

Answer f) $M = K^* [R^* | T^*]$

K^* is an ~~ext~~ intrinsic parameter having K_u, K_v, u_0, v_0, f.

&

R^*, T^* are external parameters responsible for rotation & translation of the world & camera coordinate systems.

Answer g) The skew parameter 's' is added in camera model so that the x & y axes are not perpendicular. So to nullify this effect & hence increase its accuracy we ~~use~~ add skew parameter.

Answer h) If we consider radial lens distortion, the lines bend more near the edges than the center of the lens. Due to the straight lines in the image real world appears to be curved in the image.

Answer i) → The weak perspective model first projects points to the ~~reference~~ reference plane using orthogonal projection & then projects to the image plane using a projective transformation.

→ Due to the ~~depth~~ this, The depth variation in the scene is smaller compared to the distance from the camera.

→ Affine camera is same as the weak perspective camera model, the only difference is that this is a computational model & is more mathematically tracable using different affine transformations.

Q 4) Color & photometric image formation:

Answer a) Difference b/w Surface radiance & image radiance:

Surface Radiance: it is the power of light per surface area reflected from surface.

Image Radiance (Irradiance): it is the power of light per surface area received at each point.

Answer b) Radiosity equation relating surface radiance & image radiance

$$E(P) = L(P) \frac{\pi}{4} \left(\frac{d}{f}\right)^2 (\cos\alpha)^4$$

where; $E(P)$ = image radiance

$L(P)$ = surface radiance.

d = diameter

f = focal length.

Answer c) 'Albedo' is a measure of the proportion of incident light or radiation that is reflected by a surface.

Albedo varies b/w 0 & 1. A value of '0' means the surface is a "perfect absorber" that absorbs all incident energy & value of '1' means the surface is a 'perfect reflector' that reflects all incoming energy.

Answer d) The RGB color model is based on the theory that all the visible colors can be created using red, green, & blue. These colors are known as primary addictives because, when combined in equal amount they produce white. When 2 or 3 of them are combined in diff. amount, other colors are produced.

For ex: ① combining red & green in equal amounts creates yellow;

② green & blue creates cyan

③ red & blue creates magenta

(1)

These particles formulate create the CMYK (cyan, magenta, yellow black) colors used in printing.

Eg F

Changing the amount of red, green, & blue, you can produce a nearly endless arrays of colors. When one of these primary additives is not present, you get black.

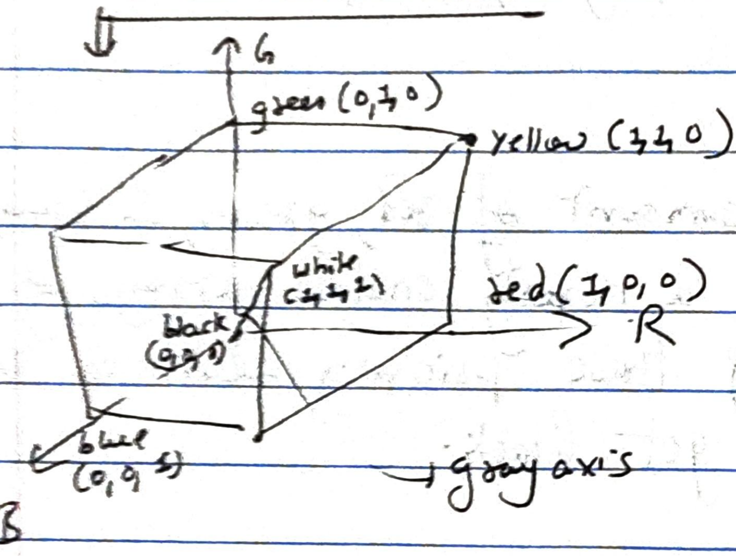
eg) For Yellow   R = 255, G = 255, B = 0

Note: An RGB color is expressed as a series of three numbers known as a hexadecimal triplet; each numbers corresponds to a red, green or blue value in that order; ranging from (0-255) eg)

rgb (255, 255, 255) produces white.

Answer(e)

RGB color cube



The diagonal line that connects (0,0,0) & (1,1,1) corresponds to all gray colors b/w white & black, which is also known as 'Gray Axis'.

Answer(f) RGB colors are mapped to real world by using CIE tables, where in people are shown colors and asked to map them by turning different values of RGB. ~~If they could not match~~

Answer g) In the XYZ color space, Y corresponds to relative luminance, Y also carries color information related to the eye's "m" (yellow-green) response. X & Z carry additional information about how the cones in the human eye responds to light waves of varying frequencies.

Answer h)
Lab color is a more accurate color space. It uses three values (L, a, and b) to specify colors. RGB & CMYK color spaces specify a color by telling a device how much of each color is needed. Lab color works more like the human eye.

It specifics a color using a 3-axis system. The a-axis (green to red), b-axis (blue to yellow) & Lightness axis.

The best thing about Lab color is that it's device independent. That means that it's easier to achieve exactly the same color across different media. It's mainly used in plastics, automotive & textile industry.

eg) If your company wants to put their logo on a cup, t-shirt/banner it is good idea to go Lab color.