Assignment - 5
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CS512 Computer Vision.

① Camera Caliberation I

Solution 1(a)

Given projection eqⁿ $p = MP$

or $p_i = M \underline{P_i} \rightarrow$ world point 3DH.

p_i ↑ image pt 2DH; M ↑ Projection matrix

explain forward projection, caliberation & reconstruction.

In forward projection our goal is to define a mathematical model to describe how 3D world points get projected into 2D pixel coordinates.

World coor.		Camera coor.		Film coor		Pixel coord.
U		X				
V	→	Y	→	x	→	u
W		Z		y		v

Calibration or Camera Calibration is the process of determining specific camera parameters in order to complete operations with specified performance measurements.
It can be defined as the technique of estimating the characteristics of camera.

Reconstruction or 3D reconstruction from multiple images is the creation of three-dimensional models from set of images. It is the reverse process of obtaining 2D images from 3D scenes. The essence of an image is a projection from a 3D scene onto a 2D plane, during which ~~the~~ process the depth is lost.

which is easiest & which is most difficult?

The easiest of all will be forward projection because it's easy to compute from 3D world points to 2D points.

whereas; the difficult of all is reconstruction as it is reverse process of obtaining 2D images from 3D & ~~the~~ we need depth knowledge for that.

Solution I b

6 Necessary input for camera calibration

Camera parameters include intrinsics, extrinsics & distortion coefficients.

To estimate camera parameters, we need to have 3D-world points & their corresponding 2D-image points.

from notes:

given corresponding points $\{p_i\}_{i=1}^{m} \longleftrightarrow \{\underline{P_i}\}_{i=1}^{m}$ (pixels; meters)

find parameters:

$M \rightarrow K^*, R^*, T^*$

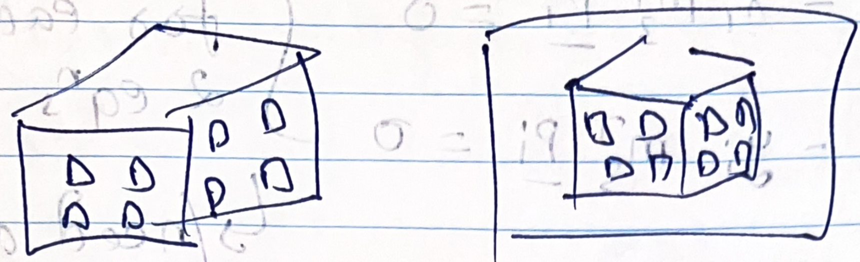
$\rightarrow R, T, f, k_u, k_v, u_0, v_0, \tan\theta$

Solution I c

Steps in non-coplanar calibration

Given: $\{p_i\}_{i=1}^{m} \longleftrightarrow \{\underline{P_i}\}_{i=1}^{m}$

find camera parameters:



$[P_i]$ $\quad$ $[p_i]$

3DH $\quad$ 2DH

Steps:

1. find projection matrix M
2. find parameters from M.

Estimating M

$$\underset{2DH}{p_i'} = \underset{3\times4}{M} \underset{3DH}{P_i}$$

$$p_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \qquad \underline{P_i} = \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_i' \\ y_i' \\ w_i' \end{bmatrix} = \begin{bmatrix} - m_1^T - \\ - m_2^T - \\ - m_3^T - \end{bmatrix} \underline{P_i}$$

$$x_i = \frac{x_i'}{w_i'} \qquad y_i = \frac{y_i'}{w_i'} \quad — (1)$$

$$\left.\begin{aligned} x_i' &= m_1^T \underline{P_i} \\ y_i' &= m_2^T \underline{P_i} \\ w_i' &= m_3^T \underline{P_i} \end{aligned}\right\} — (2)$$

from eq^n — (1) & (2)

or $\quad m_1^T \underline{P_i} - x_i m_3^T \underline{P_i} = 0$

$m_2^T \underline{P_i} - y_i m_3^T \underline{P_i} = 0$

— (3)

$$m_1^T \underline{P_i} - x_i m_3^T \underline{P_i} = 0$$

$$m_2^T \underline{P_i} - y_i m_3^T \underline{P_i} = 0$$

for each pt 2 $eq^n s$ with 12 unknowns

↳ (need atleast 6 point pairs)

for a single pt.

$$\underbrace{\begin{bmatrix} \underline{P_i}^T & 0 & -x_i \underline{P_i}^T \\ 0 & \underline{P_i}^T & -y_i \underline{P_i}^T \end{bmatrix}}_{2\times 12} \underbrace{\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}}_{12\times 1} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{2\times 1}$$

where $\underline{P_i} = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}_{4\times 1}$

$P_i^T = [X_i \quad Y_i \quad Z_i \quad 1]_{1\times 4}$

For m points

$$\underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{2m\times 12} \times \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{12\times 1} = \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{2m\times 1}$$

$\boxed{Ax = 0}$

To solve $\boxed{Ax = 0}$ using SVD

$$A = UDV^T$$

(Solution = column of V belonging to zero singular value.)

$$\hat{X} = \begin{bmatrix} \hat{m}_1 \\ \hat{m}_2 \\ \hat{m}_3 \end{bmatrix} \Rightarrow \hat{m} = \begin{bmatrix} - \hat{m}_1^T - \\ - \hat{m}_2^T - \\ - \hat{m}_3^T - \end{bmatrix}_{3\times 4}$$

Note:

$\Rightarrow$ Solution is not unique if $\boxed{A\hat{X} = 0}$

we also have $A(\rho\hat{X}) = 0 \Rightarrow \rho\hat{X}$ is a solution.

Or

Solution of $\hat{m}$ is not unique.

Hence, we need to find ρ

Find ρ so that:

$$\boxed{M = K^*[R^* | T^*] = \rho\hat{M}} \qquad \rho = ?$$

(find parameters & ρ from M)

finding params from M

$$K^*\left[R^* \mid T^*\right] = \rho \hat{M}$$

$$\left[K^*R^* \mid K^*T^*\right] = \rho \hat{M}$$

$$K^*R^* = \underbrace{\begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}}_{K^*} \underbrace{\begin{bmatrix} - r_1^T - \\ - r_2^T - \\ - r_3^T - \end{bmatrix}}_{R^*}$$

$$= \begin{bmatrix} \alpha_u r_1^T + s r_2^T + u_0 r_3^T \\ \alpha_v r_2^T + v_0 r_3^T \\ r_3^T \end{bmatrix}$$

$$\underbrace{\left[\begin{array}{c|c} \overbrace{\begin{matrix} \alpha_u r_1^T + s r_2^T + u_0 r_3^T \\ \alpha_v r_2^T + v_0 r_3^T \\ r_3^T \end{matrix}}^{K^*R^*} & K^*T^* \end{array}\right]}_{\text{unknowns}} = \rho \hat{M}$$

$$= \underbrace{\rho}_{\text{unknown}} \underbrace{\left[\begin{array}{c|c} \begin{matrix} - a_1^T - \\ - a_2^T - \\ - a_3^T - \end{matrix} & b \end{array}\right]}_{\text{known.}}$$

1) $\alpha_u r_1^T + s r_2^T + u_0 r_3^T = \rho a_1^T$

2) $\alpha_v r_2^T + v_0 r_3^T = \rho a_2^T$

3) $r_3^T = \rho a_3^T$

4) $K^* T^* = \rho b$

Knowns

a_1, a_2, a_3, b

unknowns

$r_1, r_2, r_3, T^* (K^*)$

$s, \alpha_u, \alpha_v, u_0, v_0, \rho$

To extract params using orthogonality of r_1, r_2, r_3

$r_1 \cdot r_2 = 0, \quad r_2 \cdot r_3 = 0, \quad r_1 \cdot r_3 = 0$

$r_1 \times r_2 = r_3, \quad r_2 \times r_3 = r_1, \quad r_3 \times r_1 = r_2$

Find unknown scale

$r_3^T = \rho a_3^T$

$[\ \underbrace{|r_3^T|}_{1} = |\rho|\, |a_3^T| \Rightarrow \boxed{|\rho| = 1/|a_3^T|}$ — ①

Find u_0:

$\rho a_1^T \cdot \rho a_3^T = (\alpha_u r_1^T + s r_2^T + u_0 r_3^T) \cdot (r_3^T)$

$= 0 + 0 + u_0 \cdot 1 = u_0$

$\boxed{u_0 = |\rho|^2 a_1 \cdot a_3}$ — ②

Similarly; $\boxed{v_0 = |\rho|^2 a_2 \cdot a_3}$ — 3

finding α_v:

$$\rho a_2 \cdot \rho a_2 = (\alpha_v r_2^T + v_0 r_3^T) \cdot (\alpha_v r_2^T + v_0 r_3^T)$$

$$= \alpha_v^2 + v_0^2$$

$$\boxed{\alpha_v = \sqrt{\rho^2 a_1 \cdot a_2 - v_0^2}} \quad \text{— 4}$$

Similarly for other parameters using the equations.

$$\boxed{S = \frac{1}{\alpha_v} \rho^4 (a_1 \times a_3) \cdot (a_2 \times a_3)} \quad \text{— 5}$$

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1 — $\boxed{\text{Finding } \rho \text{ (sign)}}$

$\rho = \epsilon |\rho|$

$$\cdot \; K^* T^* = \rho b = \epsilon |\rho| b$$

$$[K^* \cdot T^* \underset{\downarrow}{]}_z = \epsilon |\rho| b_z$$

+ve ⟸ positive object in front of camera

$$\therefore \boxed{\epsilon = \text{sign}(b_z)} \quad — ⑥$$

Finally finding T^*

Build K^* from recovered intrinsic params.

$$K^* T^* = \epsilon |\rho| b$$

$$\boxed{T^* = (K^*)^{-1} \epsilon |\rho| b}$$

~~Now~~ Degenerate Configuration

↳ to recover m we need to solve:

$$\begin{bmatrix} \underline{P}_1^T & 0 & \cdots & -x_1 \underline{P}_1^T \\ 0 & \underline{P}_1^T & & -y_1 \underline{P}_1^T \\ \vdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

when $\underline{P}_i$ are all on some plane π $\forall P_i$

we have $\pi^T \underline{P}_i = 0$

(a, b, c, d) — $(x, y, z, 1)$

normal, distance.

→ when $\underline{P_i}$ are on the same plane π ∴
there are many possible solutions

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \alpha \begin{bmatrix} \pi \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ \pi \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 \\ 0 \\ \pi \end{bmatrix}$$

eg:

$$\begin{bmatrix} \underline{P_1}^T & 0 & -x_1 \underline{P_1}^T \\ 0 & \underline{P_1}^T & -y_1 \underline{P_1}^T \\ \vdots & & \end{bmatrix} \begin{bmatrix} \pi \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

on solving $\boxed{\underline{P_1}^T \pi = 0}$

Assessing Quality of fit

$$E = 1/m \sum_i \left(\left| x_i - \frac{m_1^T \underline{P_i}}{m_3^T \underline{P_i}} \right|^2 + \left| y_i - \frac{m_2^T \underline{P_i}}{m_3^T \underline{P_i}} \right|^2 \right)$$

distance b/w known & predicted positions.

eg) $(7.3, 12.1, 15.3) \Leftrightarrow (5,3)$ known

$(6,2)$ predicted

error

Recovering R & T (from R^* & T^*)

$\rightarrow R^* = R^T \Rightarrow \boxed{R = (R^*)^T}$

$\rightarrow T^* = -R^T T$

$T = -R T^*$

$\boxed{T = -(R^*)^T T^*}$

Solution (1) (2)

As $\underset{2DH}{P_i} = M \underset{3DH}{P_i}$

given $P_i = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \Rightarrow$ So, $P_i = \begin{bmatrix} 18/7 \\ 2 \\ 1 \end{bmatrix}$

2D image coord.

$\boxed{(18/7, 2)}$

$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$$P_i = \underset{3\times4}{\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}} = \begin{bmatrix} 1+4+9+4 \\ 1+0+9+4 \\ 1+2+3+1 \end{bmatrix}$$

$$P_i = \begin{bmatrix} 18 \\ 14 \\ 7 \end{bmatrix} \text{ or } = \begin{bmatrix} 18/7 \\ 2 \\ 1 \end{bmatrix}$$

Solution 1 (e)

$$\underset{\substack{3DH \\ (\text{world})}}{\underline{P_i}} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \qquad \underset{\substack{2DH \\ \text{image}}}{p_i} = \begin{bmatrix} 100 \\ 200 \\ 1 \end{bmatrix} = \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

for a single point (we have)

$$\begin{bmatrix} \underline{P_i}^T & 0 & -x_i \underline{P_i}^T \\ 0 & \underline{P_i}^T & -y_i \underline{P_i}^T \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Downarrow$$

$\underline{P_i}^T = \begin{bmatrix} 1 & 2 & 3 & 1 \end{bmatrix}$ & $(x_i, y_i) = (100, 200)$

$x_i P_i^T = \begin{bmatrix} 100 & 200 & 300 & 100 \end{bmatrix}$ $\quad y_i P_i^T = \begin{bmatrix} 200 & 400 & 600 & 200 \end{bmatrix}$

(first 2 lines of matrix)

$$= \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & -100 & -200 & -300 & -100 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 1 & -200 & -400 & -600 & -200 \end{bmatrix}$$

$\boxed{2 \times 12}$

Solution I (f)

From the eqns

$$m_1^T \underline{P_i} - x_i m_3^T \underline{P_i} = 0$$
$$m_2^T \underline{P_i} - y_i m_3^T \underline{P_i} = 0$$

for each point 2 eqns with 12 unknowns

So, we need at least 6 point pairs

~~SVD~~ In order to get the solution we decompose the projection matrix using SVD & take the last column of V belonging to zero singular value of this matrix.

Solution (1) (h)

Quality of projection matrix M can be estimated using Error values.

Given $[P_i]_{i=1}^{m} \leftrightarrow [P_i]_{i=1}^{m}$ & estimated $M = \begin{bmatrix} - m_1^T - \\ - m_2^T - \\ - m_3^T - \end{bmatrix}$

$$\boxed{E = \frac{1}{m} \sum \left(\left\| x_i - \frac{m_1^T P_i}{m_3^T P_i} \right\|^2 + \left\| y_i - \frac{m_2^T P_i}{m_3^T P_i} \right\|^2 \right)}$$

↳ distance b/w known & predicted positions

for eg

$(7.3, 12.1, 15.3) \leftrightarrow (5,3)$ known

↳ $(6,2)$ predicted using M

Error

Solution (1) (i) Principal of planar camera calibration

In planar camera calibration we use several images to estimate 2D homography or projective map(H). Between calibration plane & image.

~~Solution (1)(ii) So, using~~

Once we get the required Homography & other parameters we can estimate intrinsic & extrinsic parameters for view of interest.

$$H = [h_1, h_2, h_3] = \begin{bmatrix} h_{11} & h_{21} & h_{31} \\ h_{12} & h_{22} & h_{32} \\ h_{13} & h_{23} & h_{33} \end{bmatrix}$$

Difference

Non-Planar Calibration	Planar Calibration
↳ 3D reference object based calibration.	↳ 2D plane based calibration.
↳ Camera Calibration is performed by observing a calibration object whose geomtry in 3D space is known with very good precision. Calibration object usually consists of 2 or 3 planes orthogonal ~~planes~~ to each other	↳ In planar Calibration we use several images to estimate 2D Homography or projective map (H).
↳ $\underset{\substack{\text{image point}\\ \text{2DH}}}{P_i} = \underset{\substack{\text{Projective matrix}\\ \boxed{3\times4}}}{M} \underset{\substack{\text{world pt}\\ \text{3DH}}}{P_i}$ So, Projective Matrix is of size 3×4	↳ ~~In No~~ $\underset{2DH}{P_i'} = \underset{3\times3}{M} \underset{2DH}{P_i}$ In Non-planar M is of size 3×4 whereas in planar M is of size 3×3

Solution ① (i)

Difference b/w H & M

Projection Matrix (M)

$$P_i = \underset{\boxed{3\times4}}{M}\, \underline{P_i}$$

image pts 2DH ; world pt 3DH

Homography (H)

$$P' = \underset{\boxed{3\times3}}{M}\, \underline{P_i}$$

or Homography

2DH ; 2DH

~~or H~~

So we can say that the difference b/w M & H is that M is of size $\boxed{3\times4}$ & H is of size $\boxed{3\times3}$.

Assumption used for (H)

To get a 2D project map (H) we assume:

$$[\underset{\sim}{P_i}]_Z = 0 \iff \underset{\sim}{P_i} = (x_i, y_i, 0)$$

2) Camera Calibration - 2

$\boxed{\text{Solution ② ⓐ}}$

for single point we have:

where P_i 3D world

$$\begin{bmatrix} P_i^T & 0 & -x_i P_i^T \\ 0 & P_i^T & -y_i P_i^T \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$P_i^T = \begin{bmatrix} 3 & 4 & 5 & 1 \end{bmatrix}$ & $(x_i, y_i) = (1, 2)$

$x_i P_i^T = \begin{bmatrix} 3 & 4 & 5 & 1 \end{bmatrix}$, $y_i P_i^T = \begin{bmatrix} 6 & 8 & 10 & 2 \end{bmatrix}$

(first 2 lines of matrix rows) →

$$= \begin{bmatrix} 3 & 4 & 5 & 1 & 0 & 0 & 0 & 0 & -3 & -4 & -5 & -1 \\ 0 & 0 & 0 & 0 & 3 & 4 & 5 & 1 & -6 & -8 & -10 & -2 \end{bmatrix}$$

$\boxed{\text{Solution (2) (b)}}$

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix} \quad \text{find } u_0, v_0 ?$$

$$u_0 = |\rho|^2 a_1 \cdot a_3 \quad , \quad v_0 = |\rho|^2 a_2 \cdot a_3 \quad \text{--- (1)}$$

$$\rho = 1/|a_3|$$

$$M = \left[\begin{array}{c|c} - a_1^T - & \\ - a_2^T - & b \\ - a_3^T - & \end{array}\right]$$

$$a_1^T = [1 \ 2 \ 3] \cancel{4} \qquad a_3^T = [3 \ 4 \ 5] \cancel{6}$$

$$a_2^T = [2 \ 3 \ 4] \cancel{5} \quad \text{--- (2)}$$

from eq^n (1)

$$u_0 = \frac{1}{|a_3|^2}(a_1 \cdot a_3) \qquad v_0 = \frac{1}{|a_3|^2}(a_2 \cdot a_3)$$

$$a_1 \cdot a_3 = \begin{bmatrix} 3 \\ 8 \\ 15 \\ \cancel{24} \end{bmatrix} , \quad a_2 \cdot a_3 = \begin{bmatrix} 6 \\ 12 \\ 20 \\ \cancel{30} \end{bmatrix}$$

$$|a_3|^2 = \left(\sqrt{9+16+25}\right)^2 = (\sqrt{50})^2 \quad \therefore (|a_3|^2 = 50)$$

$26/50, \ 38/50$

$$u_0 = \frac{1}{50}\begin{bmatrix} 3 \\ 8 \\ 15 \end{bmatrix}, \quad v_0 = \frac{1}{50}\begin{bmatrix} 6 \\ 12 \\ 20 \end{bmatrix}.$$

or ~~on~~ adding $(u_0, v_0) = (26/50, 38/50)$

$\boxed{\text{Solution (2) (c)}}$

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix} \begin{array}{l} \to m_1^T{}_{1\times4} \\ \to m_2^T{}_{1\times4} \\ \to m_3^T{}_{1\times4} \end{array} \quad (x_i, y_i) = (1, 2)$$

$$\underline{P_i} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix}_{4\times1}$$

$$E = \frac{1}{m} \sum_i \left(\left| x_i - \frac{m_1^T P_i}{m_3^T P_i} \right|^2 + \left| y_i - \frac{m_2^T P_i}{m_3^T P_i} \right|^2 \right)$$

$$m_1^T \underline{P_i} = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1\times4} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix}_{4\times1} = [3 + 8 + 15 + 4]$$

$$= [11 + 19]$$

$$= [30]$$

$$m_2^T \underline{P_i} = \begin{bmatrix} 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix} = [6 + 12 + 20 + 5]$$

$$= [18 + 25] = [43]$$

$$m_3^T P_i = [3\ 4\ 5\ 6] \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix}$$

$$= [9 + 16 + 25 + 6]$$

$$= [56]$$

$$E = \frac{1}{m} \sum \left(\left\| 1 - \frac{30}{56} \right\|^2 + \left\| 2 - \frac{43}{56} \right\|^2 \right)$$

$$= \frac{1}{m} \sum \left(\left(\frac{26}{56}\right)^2 + \left(\frac{\overset{(69)}{\underline{(56)2 - 43}}}{56}\right)^2 \right)$$

$$E = \frac{1}{m} \left[\frac{(676 + 4761)}{(56)^2} \right] \rightarrow (3136)$$

as no. of points is 1 $(m = 1)$

$$\boxed{E = \left[\frac{5437}{3136}\right] = \underline{\underline{1.733}}}$$

Projection error of Matrix m

Solution 2 a

~~Rotation w.r.t. world~~

R_w w.r.t camera $= I + Q$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^* = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$R = (R^*)^T = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = -(R^*)^T T^* = - \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -2 \\ -3 \\ -1 \end{bmatrix}$$

Solution 2 e

$(x, y) = (1, 2)$

3D world pts $= (3, 4, 0)$

$$\underbrace{\begin{bmatrix} p_1^{*T} & 0^T & -x_1 \underline{p}_1^{*T} \\ 0 & \underline{p}_1^{*T} & -y_1 p_1^{*T} \end{bmatrix}}_{2\times 9} \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}}_{9\times 1} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{2\times 1}$$

$$\underbrace{p'}_{2DH} = \underbrace{H}_{3\times 3} \underbrace{\underline{p}_i^{*}}_{2DH}$$

$$p' = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad \underline{p}_i^{*} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 & -3 & -4 & 0 \\ 0 & 0 & 0 & 3 & 4 & 0 & -6 & -8 & 0 \end{bmatrix}$$

$\boxed{\text{First 2 rows}}$ $\Downarrow$

$$= \begin{bmatrix} 3 & 4 & 0 & 0 & 0 & 0 & -3 & -4 & 0 \\ 0 & 0 & 0 & 3 & 4 & 0 & -6 & -8 & 0 \end{bmatrix}_{2\times 9}$$

③ "Multiple view geometry I"

Solution ③ ⓐ

There are 2 main strategies for solving correspondence problem: sparse & dense.

While sparse feature based methods are often used for estimating the fundamental matrix by matching a small set of sophisticated optimised interest points dense energy based methods marks the state of the art in optical flow computation.

Main difference

Sparse correspondence	Dense correspondence
① Find feature points	① Instead of feature points compare all ~~pts~~ patches.
② Find local characteristics	② Instead of distance b/w feature vectors measure correlation or SSD.
③ Find corresponding pts having similar features.	

Advantages & Disadvantages

1. Advantages of sparse → it can handle large disparities.
1. Disadv. of sparse → there might be chance that it miss some of the feature because of localisation or
1. Advantages of dense → it is good for small disparities.
1. Disadvantage of dense → it has many points to match

Solution ③ ⑥

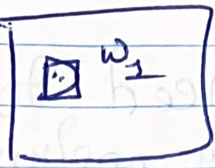


image 1

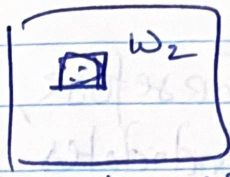


image 2

NCC ⇔

$$\boxed{\psi(w_1, w_2) = \sum_i \frac{(w_1(x_i, y_i) - \mu_{w_1})(w_2(x_i, y_i) - \mu_{w_2})}{\sigma_{w_1} \sigma_{w_2}}}$$

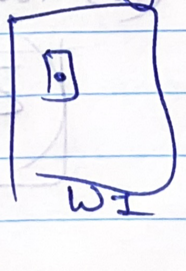
SSD

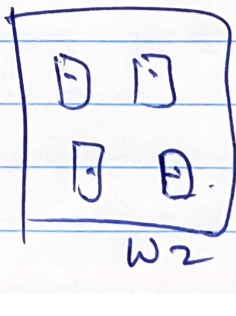
$$\boxed{\psi(w_1, w_2) = \sum_i \left(\frac{(w_1(x_i, y_i) - \mu_{w_1})}{\sigma_{w_1}} - \frac{(w_2(x_i, y_i) - \mu_{w_2})}{\sigma_{w_2}} \right)^2}$$

Using above formula for NCC & SSD we can get the similarity score for point matching.

So, if correlation (NCC) is high good similarity else poor similarity. whereas if negative of SSD is high good correspondence.

If we take entire image as a search space there are high chances of getting error because of many similar points in w_1 & w_2.

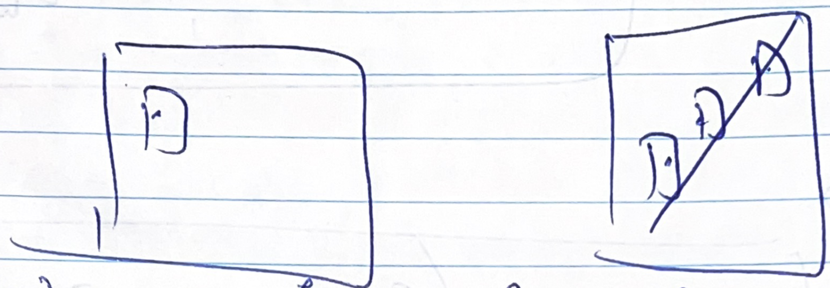

W1


W2

Therefore, we need to reduce number of candidates ~~for~~ by applying some constraints.

For example, we can reduce the number of candidates to a line.

~~Add text~~



~~Steps to reduce search space to a line:~~

First step is to construct epipolar plane. then we take intersection of this epipolar plane with the right image to find the right epipolar line. The right correspondence point of a point in the left image can be found on this right epipolar line.

Solution 3 c

left pt → (100, 200)

right pt → (103, 200)

$$f = 10, \quad \text{baseline} = 100$$

$$Z = f\frac{T}{d}$$

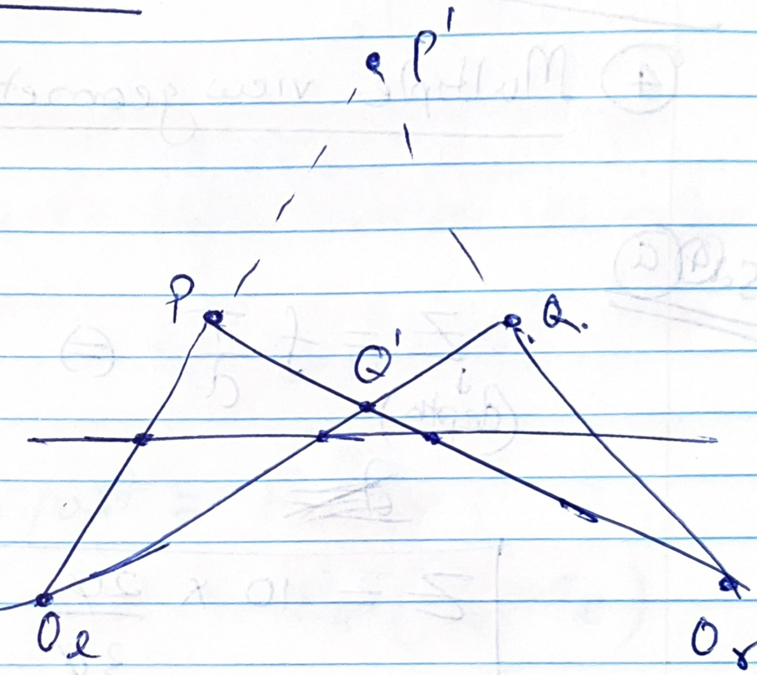
$$= 10 \frac{100}{(103-100)} = \frac{10 \times 100}{3}$$

$$\boxed{Z = \left(\frac{1000}{3}\right)}$$

Sol. 3 d **Ambiguity problem**

Correct P, Q.

incorrect P', Q'



When some points of the left image are wrongly matched to the points in the right image it causes problem during reconstruction in the form of outliers which can be dealt with using RANSAC.

Here, P, Q are correct & P', Q' are incorrect or wrongly matched.

Solution 3 e

Expression of rotation & translation of right camera w.r.t left camera can be found using : $R_l^T T_l^{-I} T_r R_r$

Rotation part $= R_l^T R_r$

Trans. part $= R_l^T (T_r - T_l)$

~~Solution 3 c~~

4 Multiple view geometry - 2

Sol. 4 a

$Z = f \frac{T}{d}$ $\Leftrightarrow$

(depth)

$$\boxed{Z = 10 \times \frac{2\cancel{0}}{3\cancel{0}} = 20/3 \text{ mm}}$$

Sol. 4 b

from Lect 22

$A \to [1\ 2\ 3]$

$B \to [2\ 3\ 4]$

if $A = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$ then $[A]_x = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$

Required matrix $A_x = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$

Sol. 4 c

$P_r^T F P_e$

given $F = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

$$P_r^T F P_e = [1\ 2\ 1] \begin{bmatrix} & & \\ & F & \\ & & \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= [8\ 12\ 16] \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = 68$$

Solⁿ 4 d

Respective row from the given pts

$(x_e, y_e) = (1, 2)$

$(x_r, y_r) = (2, 3)$

$[2\ 4\ 2\ 3\ 6\ 3\ 1\ 2\ 1]$