Assignment 3

Name: Akshay Jain
Student ID: A20502846
CS512 Computer Vision

Line Detection

Ans (I) a) On using slope & y-intercept to find a & b in $y = ax + b$ we obtain values in the range of $a \in [-\infty, \infty]$ & $b \in [-\infty, \infty]$ that's a problem as values with infinite ~~a & b~~ values of parameters are not possible. Therefore the implicit line equation is used instead.

Ans (I) b) Given $\theta = 45°$, $d = 10$

$$ax + by + c = 0$$

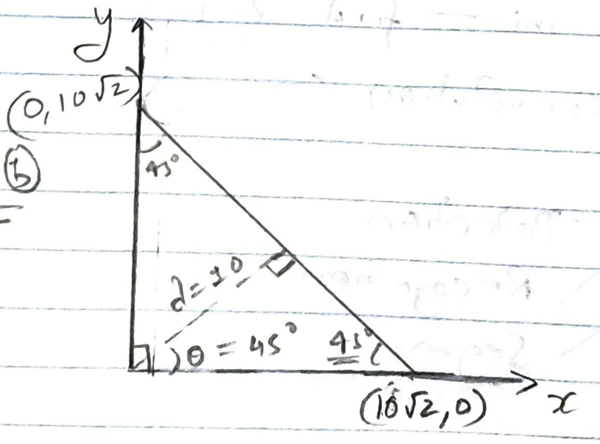
or $(\cos\theta)x + (\sin\theta)y - d = 0$

$$x/\sqrt{2} + y/\sqrt{2} = 10$$

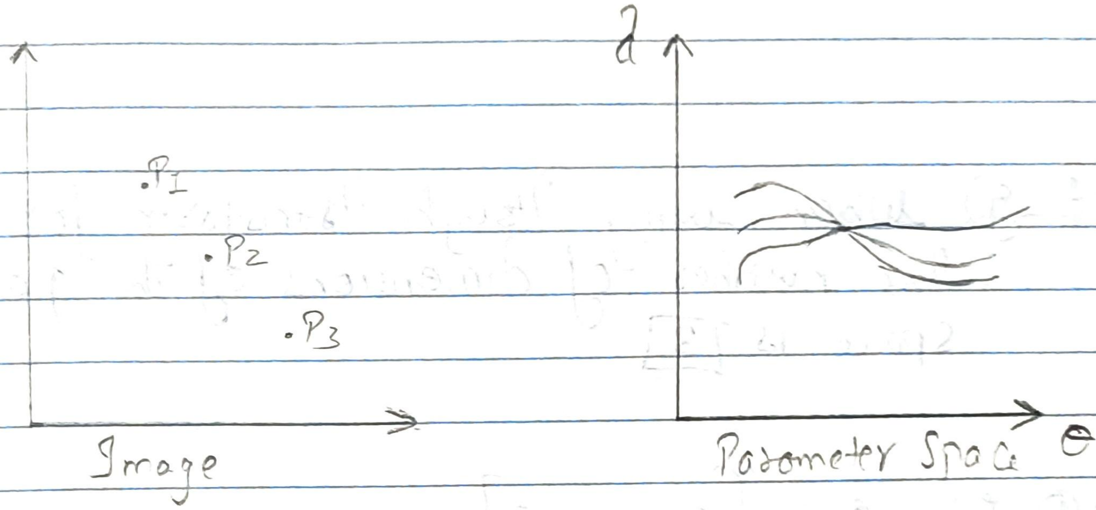
$$x + y = 10\sqrt{2}$$

fig.

Ans (I) (b)



Ans① c) When using the polar representation of lines the vote of each point in the image looks like sinusoidal curve in the parameter plane.



Ans① d) Using equation : $\boxed{x\cos\theta + y\sin\theta - d = 0}$

where; θ = angle

d is distance from the origin.

we use different lines which intersect at a point, the distance & angle of that point represent the line.

Ans① e) ~~Smaller bin~~

⇒ Larger ~~bin~~ the bin size it is more efficient & faster but it is less accurate.

⇒ Smaller bin size is slower than larger one and it gives no intersection.

Ans ① f) Voting can be improved if we know the normal so we can skip calculating at a range of 0° to 180° for θ & take (*calculating)
$\theta \in [\theta_{max}, \theta_{min}]$

Ans ① g) When using Hough transform for circles the number of dimensions of the parameter space is $\boxed{3}$

Ans ① h) $\theta \in [45°, 135°]$
line equation : $x\cos\theta + y\sin\theta = d$

or $\dfrac{x\cos\theta}{\sin\theta} + y\dfrac{\cancel{\sin\theta}}{\cancel{\sin\theta}} = \dfrac{d}{\sin\theta}$

$$\boxed{y = -\frac{\cos\theta}{\sin\theta}x + \frac{d}{\sin\theta}}$$

As $x \in [0, n]$ & $\theta \in [45°, 135°]$

if $\underline{x=0}$ & $\underline{\theta = 45°}$

$\left(y = \dfrac{d}{1/\sqrt{2}}\right)$ or $\boxed{y = \sqrt{2}d}$

if $x=0$ & $\theta = 135°$

$y = -\dfrac{\cos 135°}{\sin 135°}(0) + d/\sin(135°)$ or $y = \dfrac{d}{1/\sqrt{2}}$

continue Ans (1) (h)

$$\boxed{y = \sqrt{2}d}$$

$$\boxed{\text{Pixel point} = (0, \sqrt{2}d)}$$

Ans (1) (i)

$$x\cos\theta + y\sin\theta = d$$

on dividing with $\cos\theta$ & rearranging.

$$\boxed{x = -\frac{\sin\theta}{\cos\theta} y + \frac{d}{\cos\theta}}$$

As ~~$y \in m$~~ $y \in [0, m]$ & $\theta \in [-45°, 45°]$

if $y = 0$ & $\theta = -45°$

$$x = -\frac{\sin 45°}{\cos 45°}(0) + d/(1/\sqrt{2})$$

$$\boxed{x = \sqrt{2}d}$$

if $y = 0$ & $\theta = 45°$

$$x = d/(1/\sqrt{2}) = \sqrt{2}d$$

$$\boxed{\text{Pixel point } (\sqrt{2}d, 0)}$$

② Model fitting I

Ans ② ⓐ Disadvantage of using $y = ax + b$ for line fitting:

1) Geometric Distance & real point are not minimized.
2) Hence, it will result in non accurate fitting.
3) Lines with higher slopes cannot be fitted accurately.

Ans ② ⓑ Given:

Normal $(1, 2)$, distance $= 2$ from origin.

$l^T x = 0$ (Note: l is a 3×1 vector)

So, $\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} [x \ y \ 1]$

or; $x + 2y - 2 = 0$

$$\boxed{l = [1, 2, -2]}$$

Ans ② ⓒ We use implicit line execution to minimize geometric distance.

$l^T x = 0$, where all points 'x' should be on the line l.

Objective function to be minimized:

$$\boxed{E(l) = \sum_{i=1}^{n} (l^T x_i)^2}$$

$$= l^T \underbrace{\left(\sum_{i=1}^{n} x_i x_i^T \right)}_{C} l$$

$$E(l) = T^T C l$$

$$C = \sum_{i=1}^{n} (x_i x_i^T)$$

$$l^* = \underset{l}{\operatorname{argmin}} (E(l))$$

$$\Delta E(l) = 0$$

$$Cl = 0$$

where,

$$\boxed{C = \begin{pmatrix} \sum x_i^2 & 0 & 0 \\ 0 & \sum y_i^2 & 0 \\ 0 & 0 & \sum n \end{pmatrix}}$$

Ans ② (d) Given points : $\{(0,1), (1,3), (2,6)\}$

$$S = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i & \sum x_i \\ \sum x_i y_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & n \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 15 & 3 \\ 15 & 46 & 10 \\ 3 & 10 & 3 \end{bmatrix}$$

Ans ② (e) Implicit Equation (conic curves) is given by :

$\cancel{ax^2 + bxy^2 +}$

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

Constraint for ellipse:

$$b^2 - 4ac < 0$$

Ans ② (f) Equation for fitting elipse :

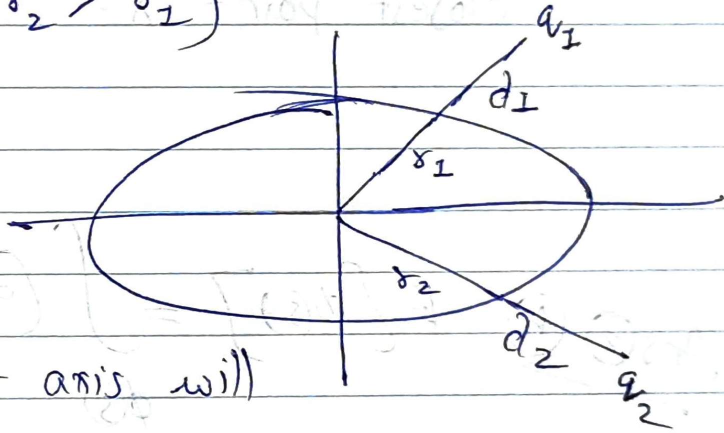
$$\boxed{l^T x_i = 0}$$

when x_i is off the elipse : $q_i = l^T x_i$

algebric distance $\boxed{q_i = \frac{d_i}{d_i + r_i}}$

$d_1 \approx d_2$

but $q_1 > q_2$ (as $r_2 > r_1$)



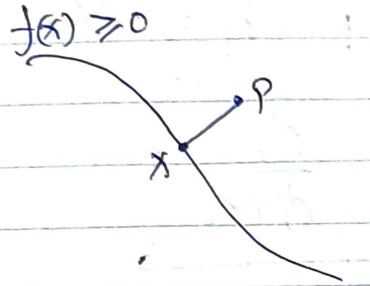
$$\frac{d_1}{d_1 + r_1} > \frac{d_2}{d_2 + r_2}$$

So points close to short axis will affect more to fitting.

Ans ② (g) $d(P, f) = |P - x^*|$

$\nearrow$ geometric distance

x^* : closest point.



$$= \frac{|f(P)|}{|\nabla f(x^*)|} \leftarrow \text{algebraic dist.}$$

So, the problem over here is what should be the closest point 'x'.

Ans ② (h) $$E[\phi(s)] = \int_{\phi(s)} \left(\alpha(s) E_{continuity} + \beta(s) E_{curvature} + \gamma(s) E_{image} \right) ds$$

where: $\alpha(s)$, $\beta(s)$, $\gamma(s)$ are coefficient of different energy terms.

$\alpha E_{continuity} + \beta E_{curvature} \rightarrow$ internal parameter.

$\gamma E_{image} \rightarrow$ external parameter.

(i) Continuity of Discrete Curve.

$$E_{contin.} = \left| \frac{d\phi}{ds} \right|^2 = \left| P_{i+1} - P_i \right|^2$$

(distance b/w neighbouring pt)

(ii) Curvature of Discrete Curve.

$$E_{curv} = \left|\frac{d^2\phi}{ds^2}\right|^2 = \left|(P_{i+1} - P_i) - (P_i - P_{i-1})\right|^2$$

$$= \left|P_{i+1} - 2P_i + P_{i-1}\right|^2$$

Ans (2) (i) The continuity of active contours will be $|P_{i+1} - P_i| - d$ to allow for sharp corners.

③ Model fitting 2

Ans ③ ⓐ Given vector : (1,2) & (3,4)

$$C = \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i y_i \\ \Sigma x_i y_i & \Sigma y_i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 2+12 \\ 2+12 & 20 \end{bmatrix} = \begin{bmatrix} 10 & 14 \\ 14 & 20 \end{bmatrix}$$

Ans ③ ⓑ Given points (10,10) & (20,20)

Line eq^n : $y = ax + b$

$$\boxed{X = A^{-1} b}$$

where, $X = \begin{bmatrix} a \\ b \end{bmatrix}$

$A = \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i \\ \Sigma x_i & n \end{bmatrix}$

$b = \begin{bmatrix} \Sigma x_i y_i \\ \Sigma y_i \end{bmatrix}$

$$A = \begin{bmatrix} 500 & 30 \\ 30 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/50 & -3/10 \\ -3/10 & 5 \end{bmatrix} \qquad b = \begin{bmatrix} 500 \\ 30 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 1/50 & -3/10 \\ -3/10 & 5 \end{bmatrix} \begin{bmatrix} 500 \\ 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$ Hence, $a = 1$ & $b = 0$

So; $y = ax + b$ will become:

$y = x$ (for $a = 1$ & $b = 0$)

or $\boxed{x - y = 0}$

Ans ③ ⓒ Given line coefficients = (1, 2, 3)

To find y-intercept when $x = 2$?

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$x + 2y + 3 = 0$

For $\boxed{x = 2}$

$2 + 2y + 3 = 0$

$\boxed{y = -2.5}$ $(x, y) \Rightarrow (2, -2.5)$

Ans (3) (d) Given point : (1, 1)

$\theta = 0$

$d = x\cos\theta + y\sin\theta$

$d = 1\cos(0^\circ) + 1\sin(0^\circ)$

$d = 1$

for $\theta = 0$ & for point (1, 1)

$\boxed{\text{vote in parameter space is } d = 1}$

Ans (3) (e) Points [(1,2), (3,4)]

$$S = \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i y_i & \Sigma x_i \\ \Sigma x_i y_i & \Sigma y_i^2 & \Sigma y_i^2 \\ \Sigma x_i & \Sigma y_i & n \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 14 & 4 \\ 14 & 20 & 6 \\ 4 & 6 & 2 \end{bmatrix}$$

Ans③ (f) $f(P) = 1$

$\nabla f(P) = 2$

$|f(P)|$ gives us the algebraic distance of point from the implicit curve f.

So, for given data, algebraic distance is $\boxed{1}$

Ans③ (g) Geometric distance

$f(P) = 1$ & $\nabla f(P) = 2$

$$d(P,f) = \frac{|f(P)|}{|\nabla f(P)|} = 1/2$$

$$d(P,f) = 0.5$$

$\therefore$ geometric distance of point from implicit curve f is $\boxed{0.5}$

Ans (3) (h) Given $P_1 = (1,2)$ & $P_2 = (2,3)$ &

$P_3 = (3,4)$

$$E_{contin.} = |P_{i+1} - P_i|^2$$

$$= |P_2 - P_1|^2$$

$$= \sqrt{(3-2)^2 + (4-3)^2}$$

$$= \boxed{2}$$

$$E_{\cancel{cons}\,curv.} = |P_{i+1} - 2P_i + P_{i-1}|^2$$

$$= |P_3 - 2P_2 + P_1|^2$$

$$= |(3,4) - 2(2,3) + (1,2)|^2$$

$$= |(4,6) - (4,6)|^2$$

$$\boxed{E_{\cancel{cons}\,curv.} = 0}$$

Ans (3) (i) We have to set value of $\beta = 0$

when $|P_{i+1} - 2P_i + P_{i-1}| > z$

to allow tight fitting or to allow precise corners.

(4) Robust Estimation

Ans (4) (a) Outliers are extreme values that deviate from other observations on data, they may indicate a variability in a measurement, experimental errors or a novelty.

Problems with outliers is that it influences the model fitting value estimates; the outliers do not fit the right model we need to detect the outliers to make the model better.

Ans (4) (b) $\boxed{E(\theta) = \sum \rho_\sigma (d(x_i; \theta))}$

Note: (In robust estimation

$\rho_\sigma(x) = x^2$)

~~Whereas,~~

In standard least squares objective function outliers will have higher values & influence the model more. while in robust estimation σ a will lower the influence of outliers on the model.

Ans 4 (c) Germon - McClure function

$$\rho_\sigma(x) = \frac{x^2}{x^2+\sigma^2}$$

It will lower the influence of outliers on model.

$\Rightarrow$ If σ is bigger it will include more points.

$\Rightarrow$ If σ is small it will include fewer points.

Advantages: ↳ Reduce effect of outliers
↳ Capture the error

Ans 4 (d) $x = 1$ & $\sigma = 1$

$$\rho_\sigma(x) = \frac{1^2}{1^2+1^2} = \underline{0.5}$$

$$E(\theta) = \sum (0.5)(d(x_i;\theta))$$

Ans ④ (e) Random Sample Consensus (RANSAC)

Perform multiple requirements
Choose best results.
Use small set in hope that atleast one set will not have outliers.

Repeat k times:
→ Draw n points uniformly at random.
(with replacement)
→ Fit a model to points.
→ Find inliers set
→ Recompute model if atleast 'd' inliers.

Choose Best Solution:
The number of points drawn at each ~~ent~~ attempt should be small. As we need to minimize points to fit the model.
If size is big chances of having an outliers in it increases.

Ans 4 (f) Parameters of RANSC

n = number of points drawn at each evolution.
d = minimum number of points needed to estimate model.
k = number of trials.
t = distance threshold to identify inliers.

$$\boxed{w = \frac{\#\text{ inliers}}{\#\text{ points}}}$$

Propability that all k experiments failed :

$$(1-P) = (1-w^n)^k$$

$$\log(1-P) = k \log(1-w^n)$$

$$\boxed{k = \frac{\log(1-P)}{\log(1-w^n)}}$$

Ans 4 (g) Given $P = 0.99$
$w = 0.9$
(assume $n = 1$)

$$\therefore k = \frac{\log(1-0.9)}{\log(1-(0.9)^1)} = \frac{\log(0.01)}{\log(0.1)}$$

$$k = -2/_{-1} \approx 2$$

$$\boxed{\boxed{k=2}}$$

$\therefore$ number of experiments needed to be performed in RANSAC algorithm is $\boxed{2}$.