Homework - III
Name: Akshay Jain
Subject: OSNA
SID: A20502846.

Solution ①

Independent Cascade Model (ICM) is a sender centric model of cascade. Each node has one chance to activate its neighbors.

In ICM nodes ~~as~~ that are active are senders & nodes that are being activated as receivers.

As per ICM Algorithm

↳ Node activated at time t, has one chance, at time step t+1, to activate its neighbors.

↳ if v is activated at time t

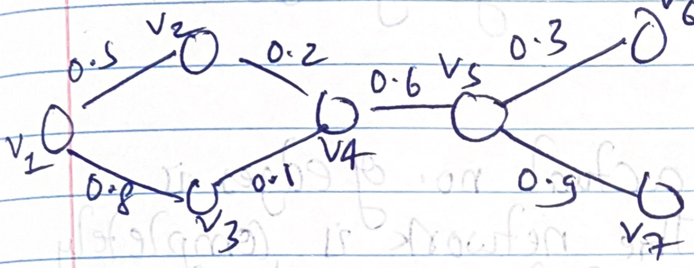
↳ for any neighbor w of v, there's a probability p_{vw} that node w gets activated at time t+1.

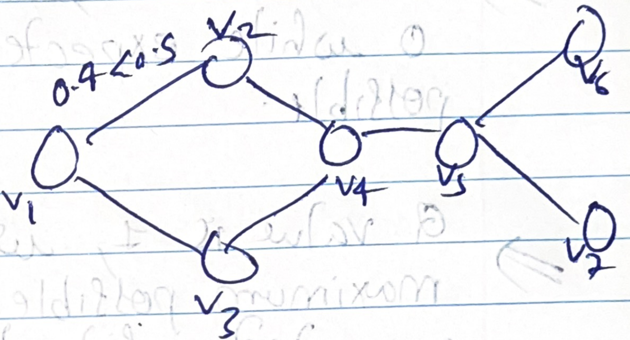
↳ Node v activated at time t has a single chance of activating its neighbors.

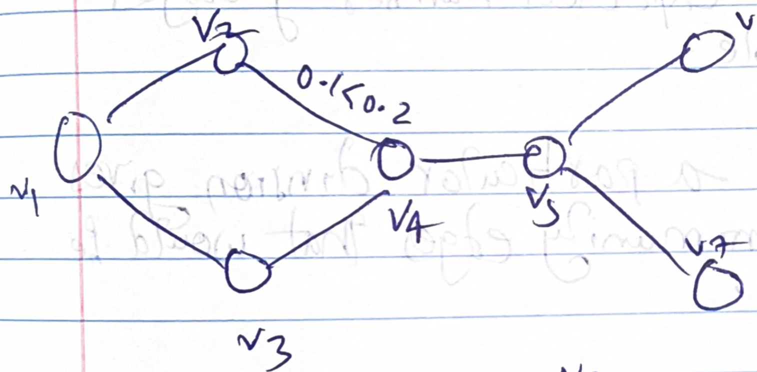
↳ activation can only happen at t+1.

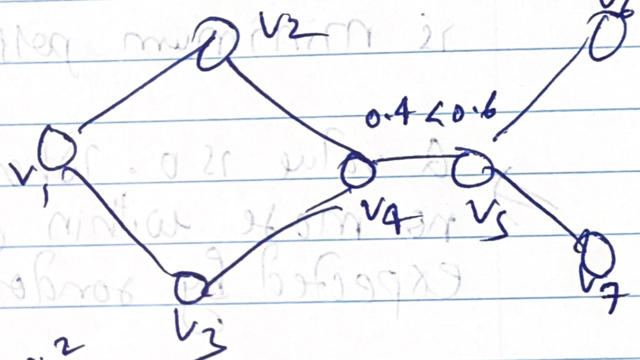
So, when the ICM stops running the algorithm can be considered as "Converged".
In ICM activation is a progressive process when nodes are changed from inactive state to active state but not in reverse order. It means the algorithm stops when there are no further activation of nodes.

As per the example mentioned in the class content:

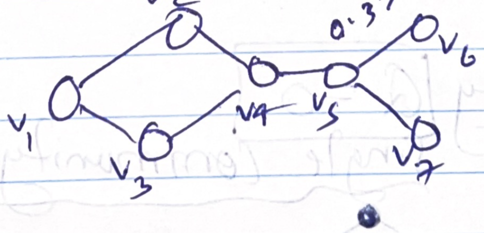






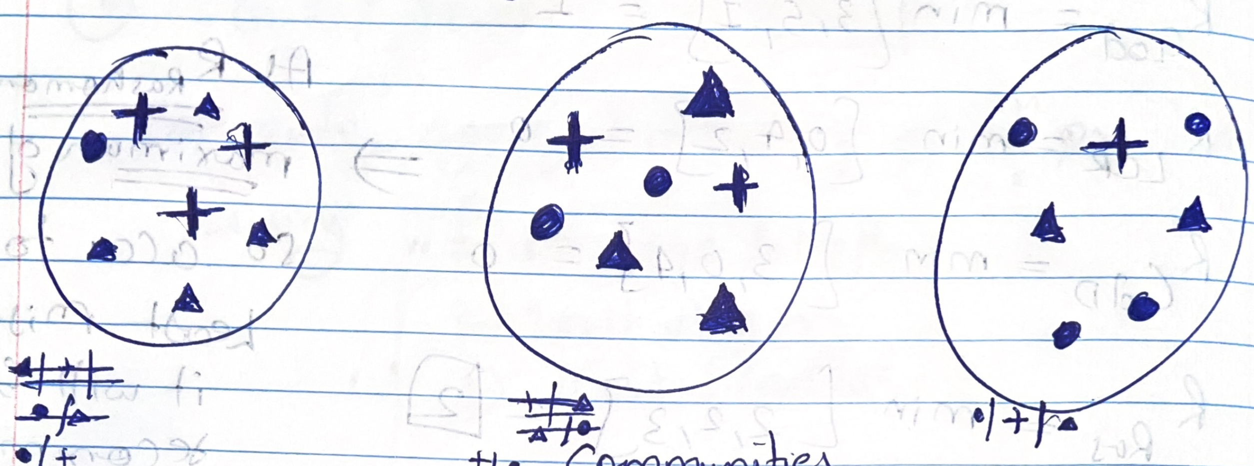


- $r_{1,2} = 0.4$
- $r_{1,3} = 0.9$
- $r_{2,4} = 0.1$
- $r_{3,4} = 0.9$
- $r_{4,5} = 0.4$
- $r_{5,6} = 0.2$
- $r_{5,7} = 1.0$



In the above process, ICM stops at node 6 no further nodes. Hence, Converged.

② [Community Analysis]



+• Communities

$$TP = {}^3C_2 + {}^4C_2 + {}^3C_2 + {}^2C_2 + {}^2C_2 + {}^4C_2 + {}^2C_2$$

$$= \boxed{21}$$

$$FN = (3\times2) + (3\times1) + (4\times3) + (4\times2) + (1\times2) + (1\times4) + (2\times1) + (2\times4) + (3\times2)$$

$$= \boxed{51}$$

$$FP = (4\times3) + (4\times1) + (3\times1) + (2\times3) + (2\times2) + (3\times2) + (4\times2) + (4\times1) + (2\times2)$$

$$= \boxed{49}$$

$$TN = (3\times5) + (3\times6) + (4\times4) + (4\times5) + (1\times5) + (1\times3) + (2\times6) + (3\times5) + (2\times3)$$

$$TN = \boxed{110}$$

Precision
① ~~Purity~~ & Recall

~~Prec~~ $\text{Precision} = \frac{21}{21+49} = \boxed{0.3}$

$\text{Recall} = \frac{21}{21+51} = \boxed{0.29}$

② F-measure.

$\text{F-measure} = \frac{2PR}{P+R} = \frac{2 \times (0.3) \times (0.29)}{(0.3 + 0.29)}$

$= \boxed{0.295}$

③ Purity.

$$\text{Purity} = 1/N \sum_{i=1}^{K} \max_{j} |C_i \cap L_j|$$

$N = 8 + 7 + 7 = 22$

$\therefore \text{Purity} = (4+3+4)/22$

$\text{Purity} = (11/22) = \boxed{0.5}$

④ **NMI (Normalized MI)**

$$NMI = \frac{MI}{\sqrt{H(L)}\ \sqrt{H(H)}}$$

$$NMI = \frac{\sum_{h \in H} \sum_{l \in L} n_{h,l} \log \frac{n \cdot n_{h,l}}{n_h, n_l}}{\sqrt{\left(\sum_{h \in H} n_h \log \frac{n_h}{n}\right)\left(\sum_{l \in L} n_l \log \frac{n_l}{n}\right)}}$$

where L & H are labels & found communities.

& n_h & n_l are the no. of data pts in comm. h & with label l resp.

& n=22 (no. of nodes)

$$NMI = \frac{4\log\left(\frac{22\times4}{8\times9}\right) + 3\log\left(\frac{22\times3}{8\times6}\right) + 1\log\left(\frac{22\times1}{8\times7}\right) + 3\log\left(\frac{22\times3}{7\times9}\right) + 2\log\left(\frac{22\times2}{7\times6}\right) + \log\left(\frac{22\times2}{7\times7}\right) + 2\log\left(\frac{22\times2}{7\times9}\right) + 1\log\left(\frac{22\times1}{7\times6}\right) + 4\log\left(\frac{22\times4}{7\times7}\right)}{\sqrt{\left(8\log(8/22) + 7\log(7/22) + 7\log(7/22)\right)\left(9\log(9/22) + 6\log(6/22) + 7\log(7/22)\right)}}$$

$$NMI = \frac{\left(\qquad \right)}{\sqrt{\left(8\log(8/22) + 7\log(7/22) + 7\log(7/22)\right)\left(9\log(9/22) + 6\log(6/22) + 7\log(7/22)\right)}}$$

$\cancel{NMI = 0.11}$

$\boxed{NMI = 0.07}$ (with log base 2)

Solution ③ [Influ. & Homophily]

$$Q = \frac{1}{2m} \sum_{ij} \left[A_{ij} - \frac{d_i d_j}{2m} \right] \delta(c_i, c_j)$$

the range for modularity Q is [-1, 1].

⇒ Q value is -1, when actual number of edges is 0 while expected number of edges is maximum possible.

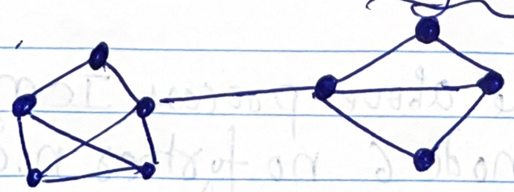
⇒ Q. value is 1, when actual no. of edges is maximum possible i.e. the network is completely connected while the expected number of edges is minimum possible.

⇒ Q value is 0. When a particular division gives no more within community edges that would be expected by random.

Examples

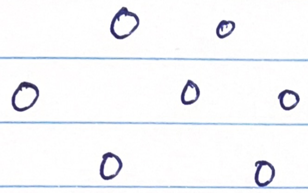
⇒ When modularity $\boxed{Q = 0}$ example can be single community

↳



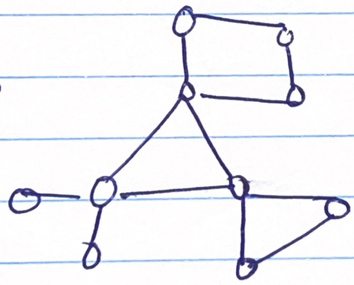
$\Rightarrow$ $\underline{\underline{Q = -1}}$ (No edge)

Example
↳



$\Rightarrow$ $\underline{\underline{Q = 1}}$ or Modularity = 1 (modular network)

example ↳



Ans (4)

Solution (5)

Recommendation

(b) $G = [\text{Newton}, \text{Ein}, \text{Gauss}]$

Compute aggregated rating of all products using
1) Average Satisfa.
2) Least Misery
3) Most Pleasure.

Averag. Satisf.

$R_{God} = \frac{3+5+1}{3} = 9/3 = \boxed{3}$, $R_{LCR} = \frac{0+4+2}{3} = \boxed{2}$

$R_{CaD} = \boxed{7/3}$, $R_{Rosh} = \boxed{7/3}$,

$R_{Lveb} = \boxed{8/3}$

As the Aver. Satis. of God $\boxed{R_{God} = 3}$ is maximum it will be the first recommended product.

Least Misery

$R_{God} = \min[3, 5, 1] = 1$

$R_{LCR} = \min[0, 4, 2] = 0$

$R_{CaD} = \min[3, 0, 4] = 0$

$R_{Ros} = \min[2, 2, 3] = \boxed{2}$

$R_{Lveb} = \min[4, 3, 1] = 1$

$\Rightarrow$ As $R_{Rashomon}$ is maximum of all so acc to Least misery it will be recommended first.

Most Pleasure

$R_{God} = \max\{3, 5, 1\} = \boxed{5}$

$R_{LCR} = \max\{0, 4, 2\} = 4$

$R_{CaD} = \max[3, 0, 4] = 4$

$R_{Ros} = \max\{2, 2, 3\} = 3$

$R_{Lveb} = \max[4, 3, 1] = 4$

As $\boxed{R_{God} \text{ is}}$ maximum using Most Pleasure strategy it will be recommended