Community Evolution

Network and Community Evolution

- How does a **network** change over time?
- How does a **community** change over time?
- What properties do you expect to remain roughly constant?
- What properties do you expect to change?

How Networks Evolve?

Network Growth Patterns

1. Network Segmentation
2. Graph Densification
3. Diameter Shrinkage

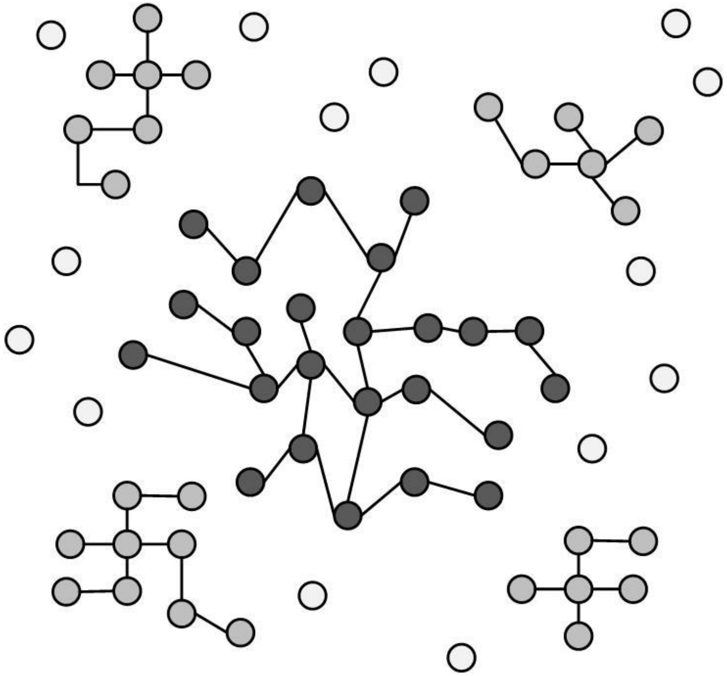
1. Network Segmentation

- Often, in evolving networks, segmentation takes place, where the large network is decomposed over time into three parts

1. **Giant Component**: As network connections stabilize, a giant component of nodes is formed, with a large proportion of network nodes and edges falling into this component.

2. **Stars**: These are isolated parts of the network that form star structures. A star is a tree with one internal node and n leaves.

3. **Singletons**: These are orphan nodes disconnected from all nodes in the network.



2. Graph Densification

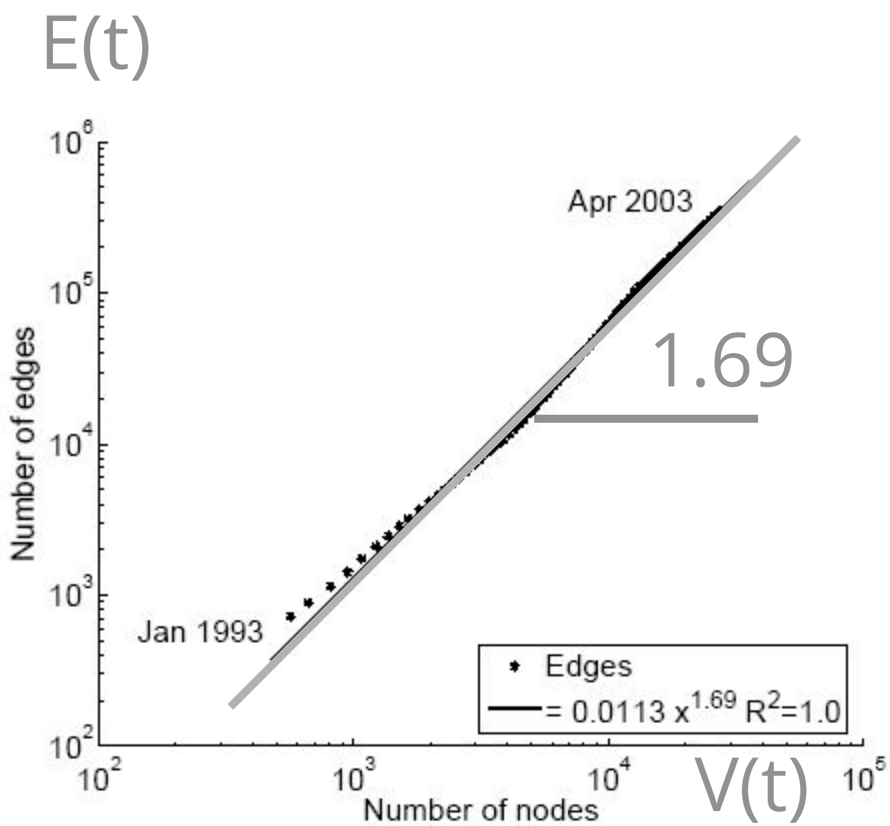
- The density of the graph increases as the network grows
  - The number of edges increases faster than the number of nodes does

$$E(t) \propto V(t)^{\alpha}$$

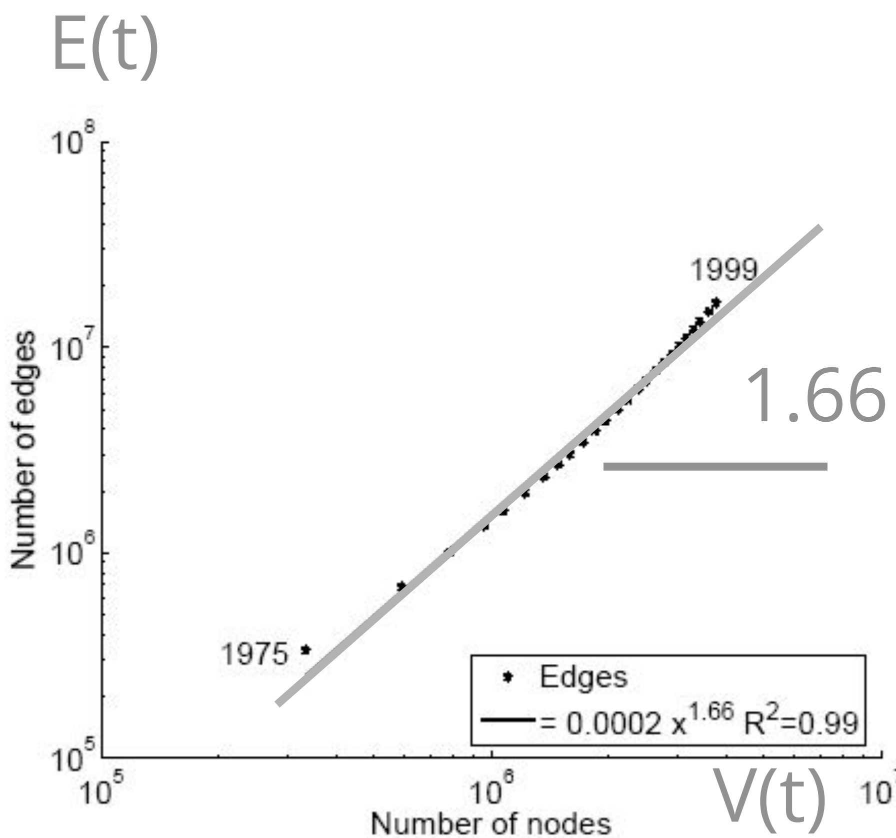
- Densification exponent: $1 \le \alpha \le 2$:
  - $\alpha = 1$: linear growth – constant out-degree
  - $\alpha = 2$: quadratic growth – clique

$E(t)$ and $V(t)$ are numbers of edges and nodes respectively at time t

Densification in Real Networks



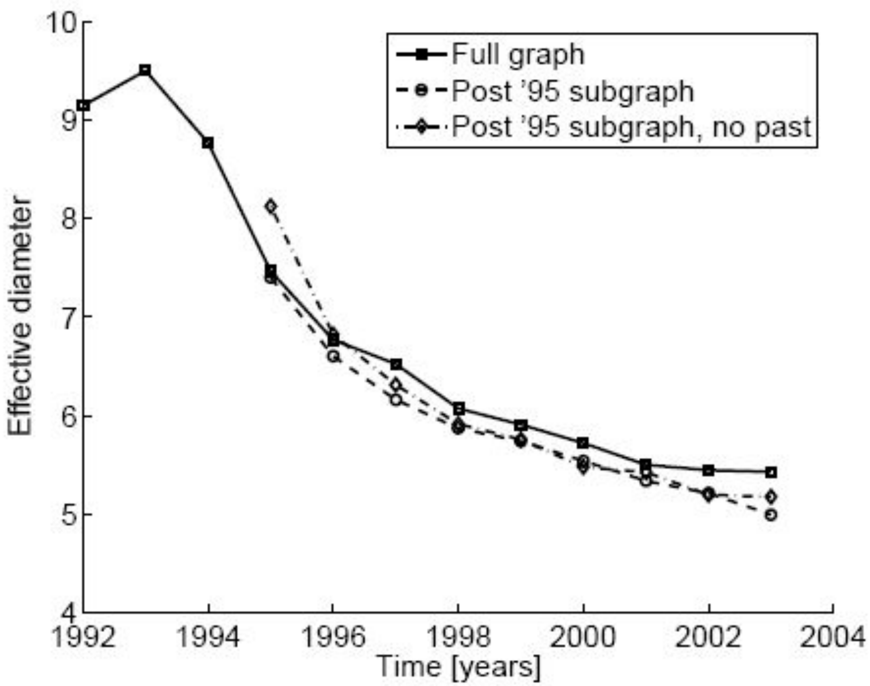
Physics Citations



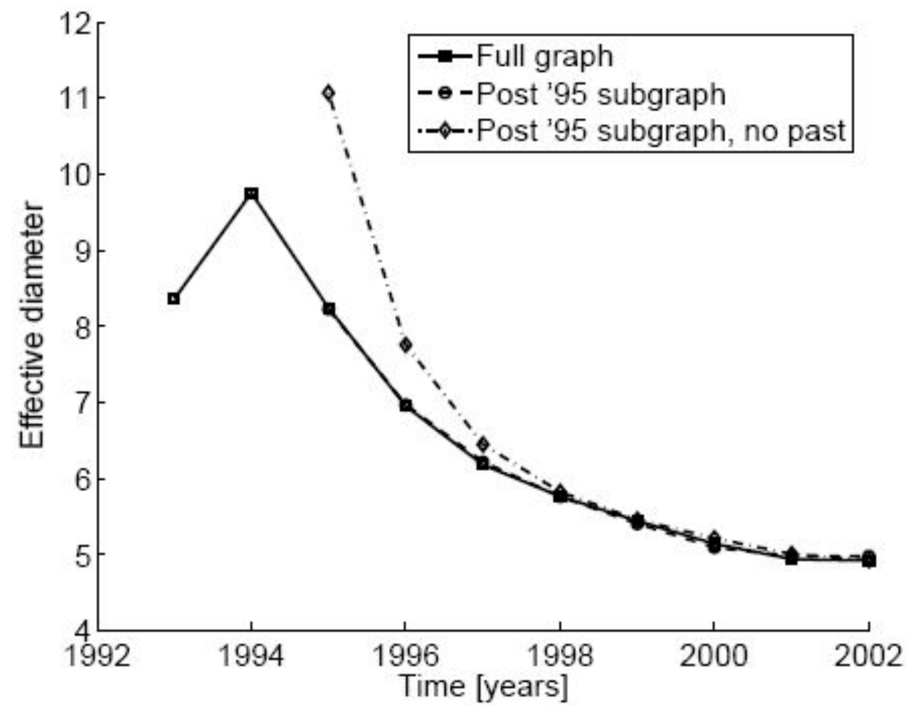
Patent Citations

3. Diameter Shrinking

- In networks diameter shrinks over time



ArXiv citation graph

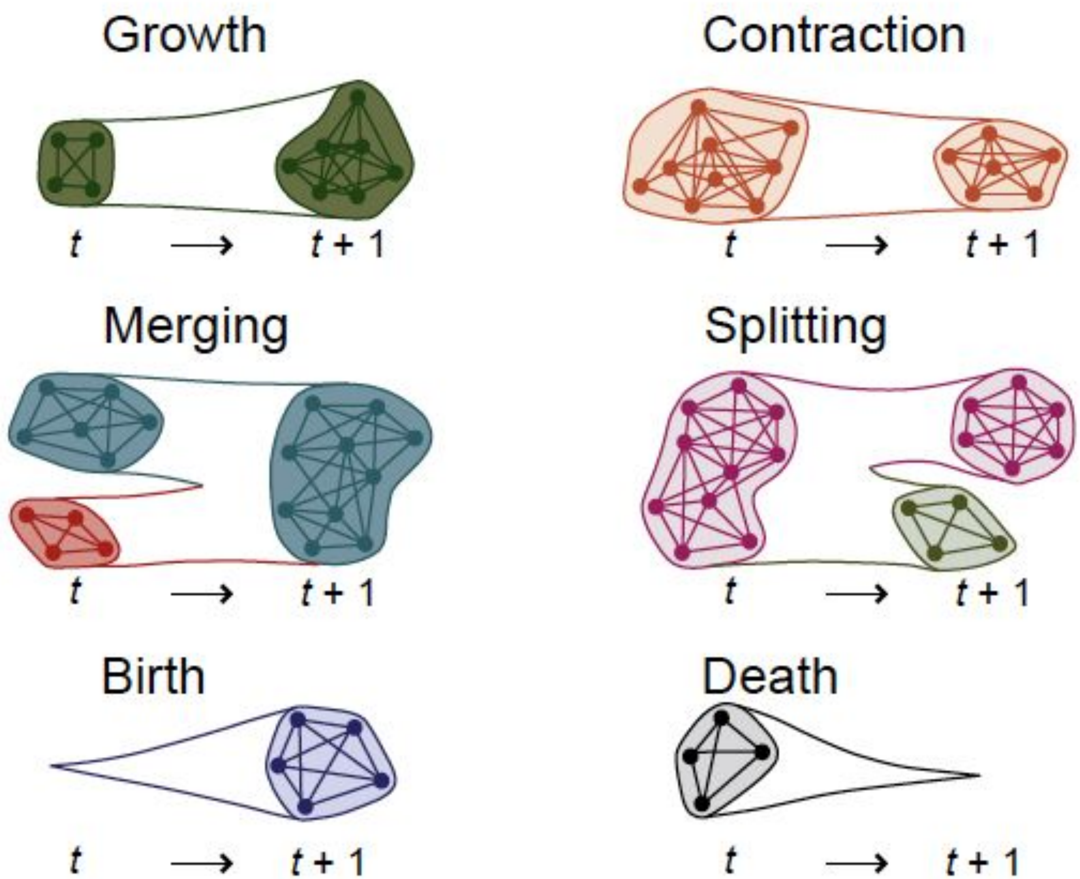


Affiliation Network

How Communities Evolve?

Community Evolution

- Communities also expand, shrink, or dissolve in dynamic networks

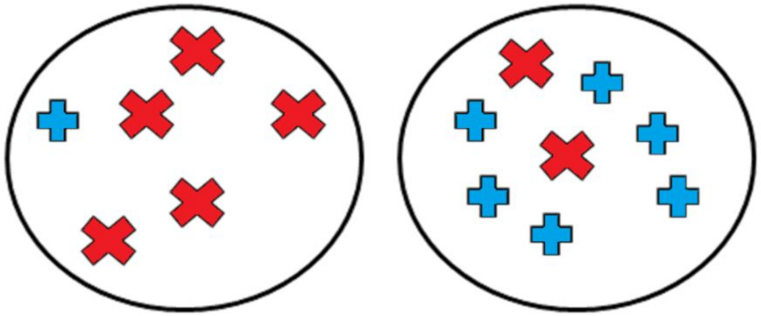


Community Evaluation

Evaluating the Communities

We are given objects of two different kinds (+, ×)

- **The perfect community:** all objects inside the community are of the same type



- **Evaluation with ground truth**
- **Evaluation without ground truth**

Evaluation with Ground Truth

- When ground truth is available
  - We have partial knowledge of what communities should look like
  - We are given the correct community (clustering) assignments

- **Measures**
  - Precision and Recall, or F-Measure
  - Purity
  - Normalized Mutual Information (NMI)

Precision and Recall

$$Precision = \frac{Relevant\ and\ retrieved}{Retrieved}$$

$$Recall = \frac{Relevant\ and\ retrieved}{Relevant}$$

$$P = \frac{TP}{TP + FP}$$

$$R = \frac{TP}{TP + FN}$$

True Positive (TP) :

- When similar members are assigned to the same communities
- A **correct** decision.

True Negative (TN) :

- When dissimilar members are assigned to different communities
- A **correct** decision

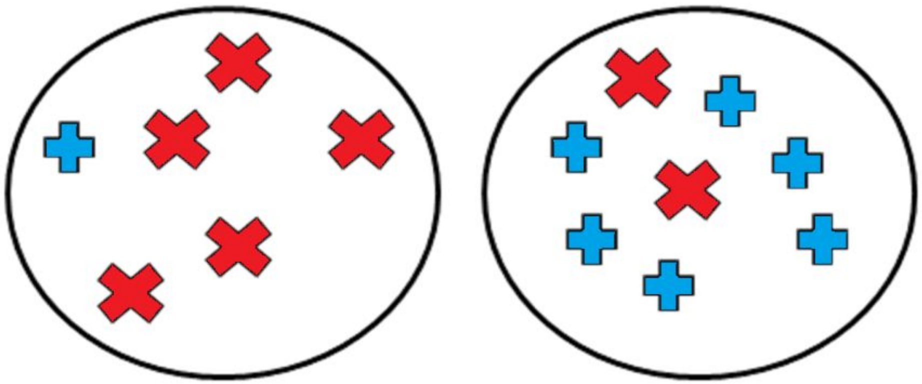
False Negative (FN) :

- When similar members are assigned to different communities
- An **incorrect** decision

False Positive (FP) :

- When dissimilar members are assigned to the same communities
- An **incorrect** decision

Precision and Recall: Example



$$
\begin{aligned}
TP &= \binom{5}{2} + \binom{6}{2} + \binom{2}{2} = 26, \\
FP &= (5 \times 1) + (6 \times 2) = 17, \\
FN &= (5 \times 2) + (6 \times 1) = 16, \\
TN &= (6 \times 5) + (2 \times 1) = 32.
\end{aligned}
$$

$$P = \frac{26}{26+17} = 0.60$$

$$R = \frac{26}{26+16} = 0.61$$

F-Measure

Either P or R measures one aspect of the performance,

- To integrate them into one measure, we can use the harmonic mean of precision of recall

$$F = 2 \cdot \frac{P \cdot R}{P + R}$$

For the example earlier,

$$F = 2 \times \frac{0.6 \times 0.61}{0.6 + 0.61} = 0.60$$

Purity

We can assume the majority of a community represents the community

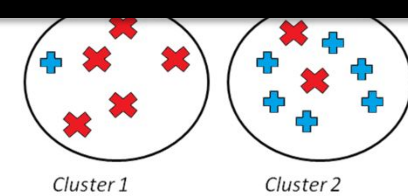
– We use the label of the majority against the label of each member to evaluate the communities

Purity can be easily **tampered** by

- Points being singleton communities (of size 1); or by
- Very large communities

$$Purity = \frac{1}{N}\sum_{i=1} \max_j |C_i \cap L_j|$$

- k: the number of communities
- N: total number of nodes,
- L_j: the set of instances with label j in all communities
- C_i: the set of members in community i



$purity\ is: \frac{6+5}{14} = 0.78$

Mutual Information

- **Mutual information (MI).** The amount of information that two random variables share.
  - By knowing one of the variables, it measures the amount of uncertainty reduced regarding the others

$$MI = I(H, L) = \sum_{h \in H} \sum_{l \in L} \frac{n_{h,l}}{n} \log \frac{n \cdot n_{h,l}}{n_h n_l}$$

- L and H are labels and found communities;
- n_h and n_l are the number of data points in community h and with label l, respectively;
- $n_{h,l}$ is the number of nodes in community h and with label l; and n is the number of nodes

Normalizing Mutual Information (NMI)

- Mutual information (MI) is unbounded
- To address this issue, we can normalize MI
- How? We know that

$$MI \le min(H(L), H(H)),$$
$$(MI)^2 \le H(H)H(L).$$
$$MI \le \sqrt{H(H)}\sqrt{H(L)}.$$

- $H(.)$ is the entropy function

$$H(L) = -\sum_{l \in L} \frac{n_l}{n} \log \frac{n_l}{n}$$
$$H(H) = -\sum_{h \in H} \frac{n_h}{n} \log \frac{n_h}{n}.$$

Normalized Mutual Information

Normalized Mutual Information

$$NMI = \frac{MI}{\sqrt{H(L)}\sqrt{H(H)}}.$$

$$NMI = \frac{\sum_{h\in H}\sum_{l\in L} n_{h,l}\log\frac{n\cdot n_{h,l}}{n_h n_l}}{\sqrt{(\sum_{h\in H} n_h \log\frac{n_h}{n})(\sum_{l\in L} n_l \log\frac{n_l}{n})}}.$$

We can also define it as

Note that $MI < 1/2(H(H) + H(L$

$$NMI = \frac{I(H;L)}{\frac{1}{2}(H(L) + H(H))}$$

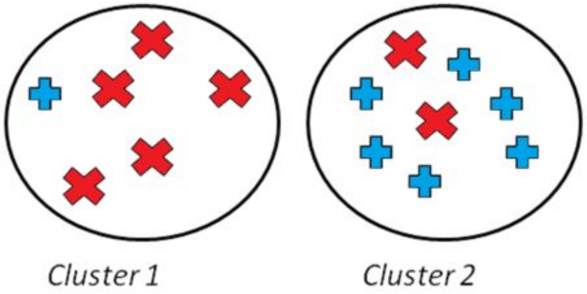
Normalized Mutual Information

$$NMI = \frac{\sum_{h,l} n_{h,l} \log \frac{n \cdot n_{h,l}}{n_h n_l}}{\sqrt{(\sum_h n_h \log \frac{n_h}{n})(\sum_l n_l \log \frac{n_l}{n})}}$$

- where l and h are known (with labels) and found communities, respectively
- n_h and n_l are the number of members in the community h and l, respectively,
- $n_{h,l}$ is the number of members in community h and labeled l,
- n is the size of the dataset

- **NMI** values close to one indicate high similarity between communities found and labels
- Values close to zero indicate high dissimilarity between them

Normalized Mutual Information: Example



Found communities (H)

– [1,1,1,1,1,1, 2,2,2,2,2,2,2,2]

Actual Labels (L)

– [2,1,1,1,1,1, 2,2,2,2,2,2,1,1]

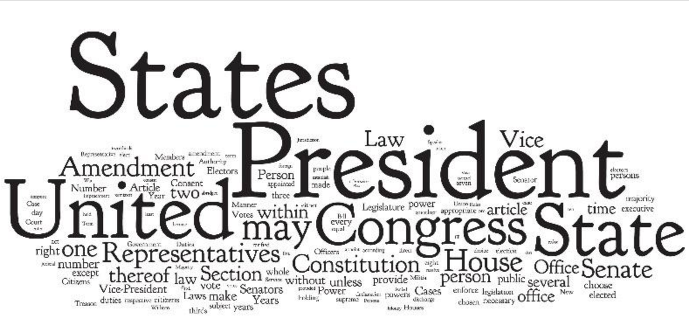
$n = 14$

	$\mathbf{n}_h$
h=1	6
h=2	8

	$\mathbf{n}_l$
	7
	7

$\mathbf{n}_{h,l}$		
h=1	5	1
h=2	2	6

Evaluation without Ground Truth



(a) U.S . Constitution

(b) Sports

- **Evaluation with Semantics**
  - A simple way of analyzing detected communities is to analyze other attributes (posts, profile information, content generated, etc.) of community members to see if there is a coherency among community members
  - The coherency is often checked via human subjects.
    - Or through labor markets: Amazon Mechanical Turk
  - To help analyze these communities, one can use word frequencies. By generating a list of frequent keywords for each community, human subjects determine whether these keywords represent a coherent topic.

- **Evaluation Using Clustering Quality Measures**
  - Use clustering quality measures (SSE)
  - Use more than two community detection algorithms and compare the results and pick the algorithm with better quality measure