

# Big O Notation (to measure time complexity)

## Order of time complexity (lower to higher)

- 1
- ↓
- \*  $\log(n)$  (2)
- ↓
- 3  $\sqrt{n}$
- ↓
- \*  $n$  (4)
- ↓
- 5  $n \log n$
- ↓
- 6  $n^2$
- ↓
- 7  $2^n$
- ↓
- 8  $n!$  Highest

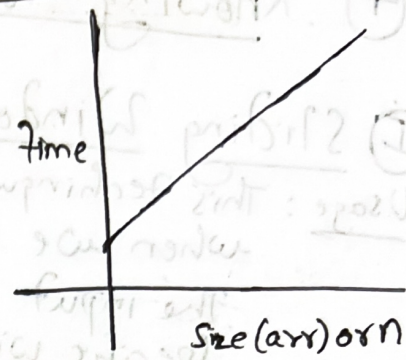
ex I for  $n=4$ ,  $\log(n)$  or  $\log_2 4 = 2$   
 $\log(n)$  (half of 'n')

## Rules for finding time complexity

II  $O(n)$  time =  $a * n + b$

\* (1) keep fastest growing term

↓  
time =  $a * n$



\* (2) Drop Constants : ie. (drop 'a' so we are left with 'n' only)

↓  
time =  $O(n)$

So, the Big O for time =  $a * n + b$  is  
 order of  $n$

## Other examples of $O(n)$

I def get-squared num (nums):  
 squared\_nums = []  
 for n in nums:  
 squared\_nums.append(n\*n)  
 return squared\_nums

Big O  
 $O(n)$

Console:  
 nums = [2, 5, 8, 9]  
 get-squared-nums (nums)  
 # return [4, 25, 64, 81]

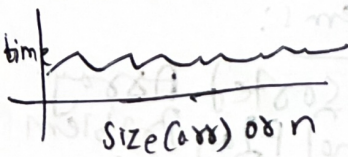
Reason for  $O(n)$

Because the time it takes for iterating this program is proportional to no. of computation it is doing.  
 let say the input array size is 4 it will do 4 iterations if the array is 4 million it will take 4 million

## II $O(1)$ Complexity

Size(arr) → 100 → 0.22 milli seconds.  
 Size(arr) → 1000 → 0.23 milli seconds.

Here, for Size(arr) function the order of complexity is  $O(1)$  i.e. when increase the size of input the time is almost the same.



time =  $a$   
 1. Keep fastest growing term  
 2. Drop constants.  
 ↓  
 time =  $O(1)$



### III $O(n^2)$

ex:  $nums = [3, 6, 2, 4, 3, 6, 8, 9]$   
 for i in range(len(nums)):  
   for j in range(i+1, len(nums)):

    if  $nums[i] == nums[j]$ :  
       print( $nums[i] + \text{" is a duplicate"}$ )  
       break.

$O(n^2)$

Reason

Here, we are running two for loops & comparing num to find duplicate.

### Note 2 Block problem

$Block \rightarrow n$   
 $nums = [3, 6, 2, 4, 3, 6, 8, 9]$  duplicate = No  
 for i in range(len(nums)):  
   for j in range(i+1, len(nums)):  
     if  $nums[i] == nums[j]$ :  
       duplicate =  $nums[i]$   
       break

Block II  $\rightarrow n$  iter  
 for i in range(len(nums)):  
   if num(i) == duplicate:  
     print(i) (log me base 2)  
     total he

Linear eqn of time of above fn.

is:  $time = a \cdot n^2 + b$   $\rightarrow O(n^2)$   
 after applying 2 rules  $\rightarrow O(n^2)$

For Block I & II represent  $time = a \cdot n^2 + b \cdot n + c$   
 apply 2 rules  $O(n^2)$

twice of  $\log n$

### IV Measuring Space Complexity

4 | 9 | 15 | 21 | 34 | 57 | 68 | 91

ways of finding 68.

1. for i in range(len(nums)):  
   if  $nums[i] == 68$ :  
     print(i)

$O(n)$

It might be good for less no.s but let say if we want to apply it for million nums might be very time consuming.

THERE IS A BETTER WAY  $\rightarrow$  BINARY SEARCH

### BINARY SEARCH

Steps  $\rightarrow$  First find middle element & compare with 68.  
 if is less you discard the left side array.

Iteration 1 =  $n/2$  (middle element)

4 | 9 | 15 | 21 | 34 | 57 | 68 | 91  
 discard

Iteration 2 =  $(n/2)/2 = n/2^2$  (again find middle element)

34 | 57 | 68 | 91  
 discard

Iteration 3 =  $(n/2^2)/2 = n/2^3$  i.e.  $8/8 = 1$

68 | 91

Conclusion: Using Binary Search we found ans in 3 iterations instead of n which is 7 in above ex.

for k  
 Iteration k =  $(n/2^k)$

### Note

Iteration  $k = n/2^k$  (to get the ans size to 1)

$1 = n/2^k$  (1 for worst case scenario)

$n = 2^k$

$\log_2 n = \log_2 2^k = k \log_2 2$   
 $\downarrow$   
 $k$

no. of iteration  $\rightarrow k = \log(n) \rightarrow O(\log n)$

Hence, the Complexity of Binary search is  $O(\log n)$

$k = O(\log n) \Rightarrow \log_2(8)$   
 $\rightarrow \log_2(2^3)$   
 $\rightarrow 3 \log_2 2$   
 $\rightarrow 3$  iterations.

### Example of $O(1)$

def find-first-pe(prices, eps, index)  
   pe = prices[index]/eps[index]  
   return pe

Reason of  $O(1)$  pe function is a constant function as does matter whatever index we go the time execution will go to remain constant hence  $O(1)$ .