# Neuroprothetik – Exercise 2: Mathematical Basics I

## 1 Plot slope fields and isocline

The following plots show the slope fields for $t \in [-5, 5]s$ and $V \in [-5, 5]V$, and the isocline for (-2, -1, 0, 1, 2)$^{V}/_{s}$ for the following differential equations:

$$\frac{dV}{dt} = 1 - V - t \tag{1}$$

$$\frac{dV}{dt} = sin(t) - \frac{1}{1.5}V \tag{2}$$

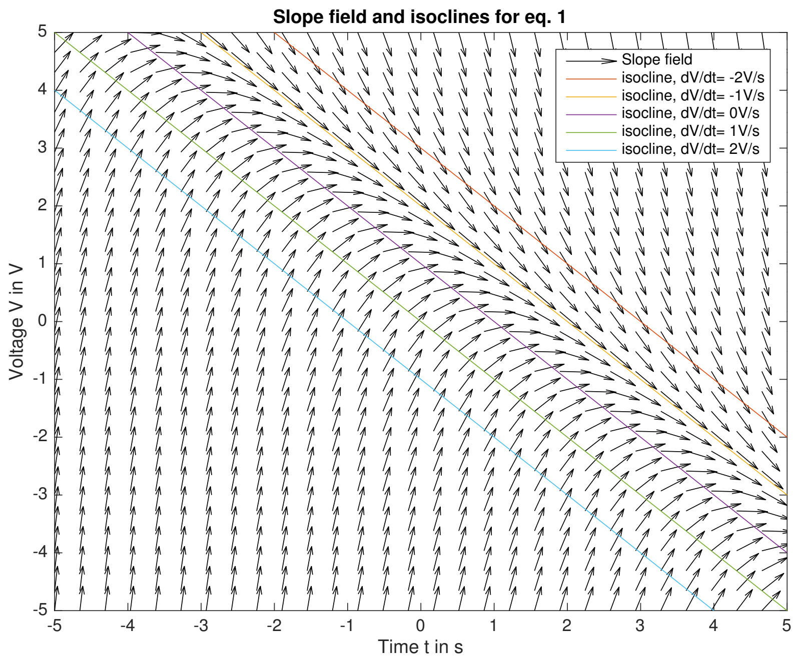


Figure 1: Slope field for equation 1 and its corresponding isoclines.

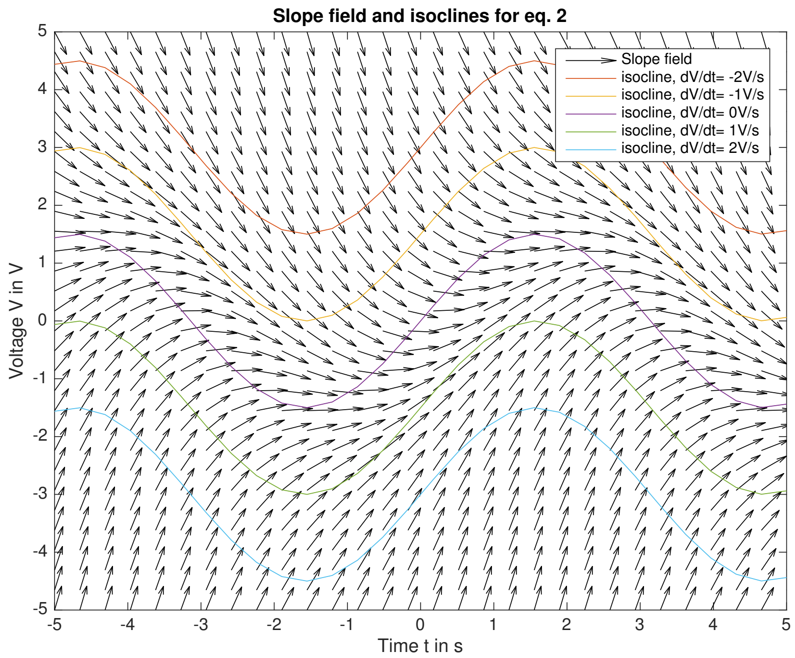


Figure 2: Slope field for equation 2 and its corresponding isoclines.

## 2 Differential equations of a simple cell model

To derive the differential equation for the equivalent circuit of a leaky integrate and fire neuron, we use Kirchhoff's law:

$$\begin{aligned} 0 &= I_c + I_{R_l} + I_{ex} \\ 0 &= C \cdot \frac{du}{dt} + \frac{u}{R_l} + I_{ex} \\ \Rightarrow \frac{du}{dt} &= -\frac{1}{C}\left(\frac{u}{R_l} + I_{ex}\right) \end{aligned} \tag{3}$$

With $I_{ex} = I_{max} \cdot sin(t)$ this results in:

$$\Rightarrow \frac{du}{dt} = -\frac{1}{C}\left(\frac{u}{R_l} + I_{max} \cdot sin(t)\right) \tag{4}$$

**Plot the slope fields:**

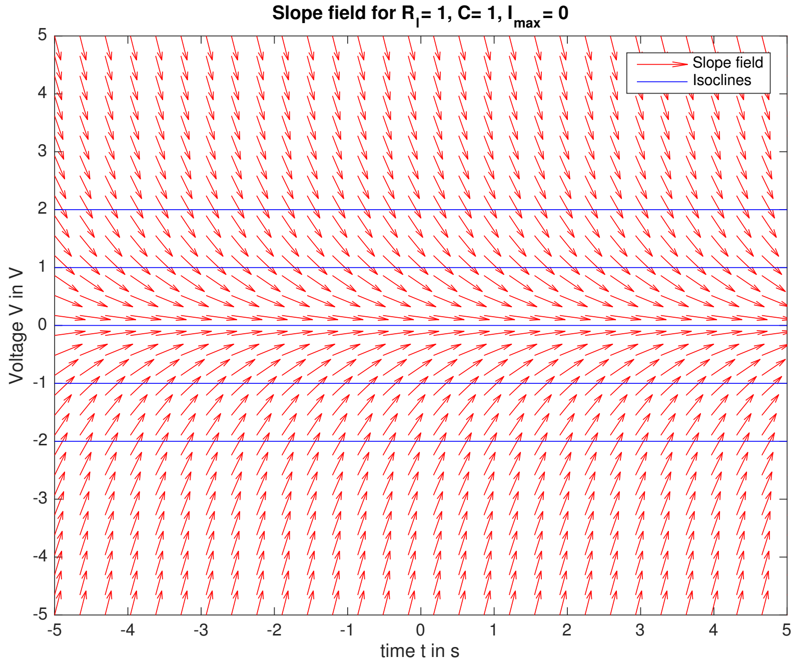


Figure 3: Slope field and isoclines of *equation 4* for $R_l = 1\Omega$, $C = 1\text{F}$, $I_{max} = 0\text{A}$.

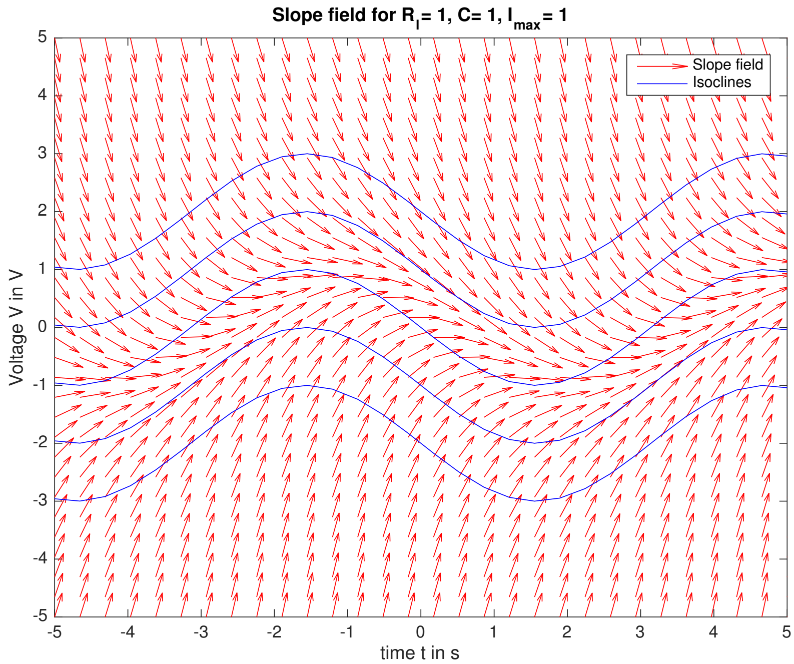


Figure 4: Slope field and isoclines of *equation 4* for $R_l = 1\Omega$, $C = 1\text{F}$, $I_{max} = 1\text{A}$.

Now, we add another constant $D = 2\text{A}$ to the differential equation. As the unit of $D$ is *Ampere*, we will add this constant to the current $I_{ex}$. Equation 4 now looks as following:

$$\Rightarrow \frac{du}{dt} = -\frac{1}{C}\left(\frac{u}{R_l} + I_{max} \cdot sin(t) + D\right) \tag{5}$$

This results in the following slope fields and theirs corresponding isoclines:

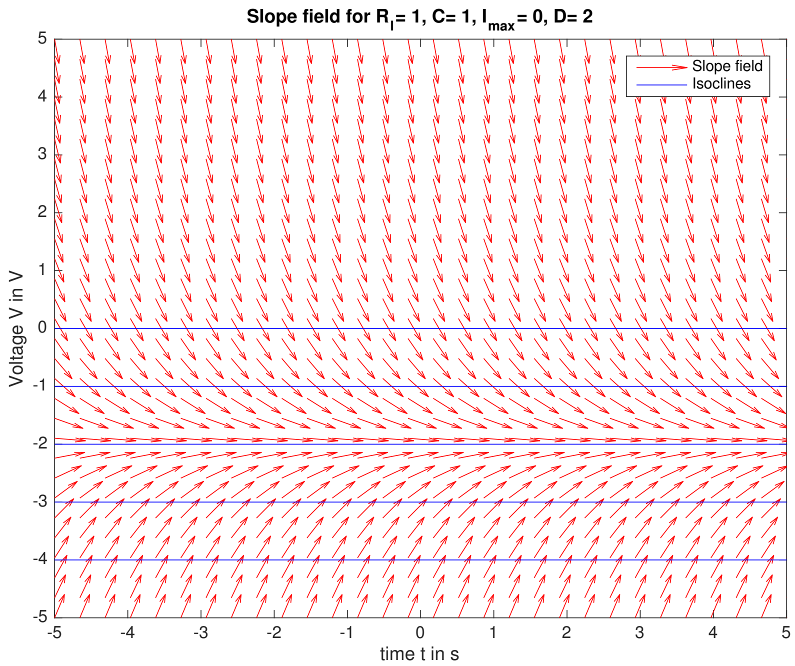


Figure 5: Slope field and isoclines of *equation 5* for $R_l = 1\Omega$, $C = 1\text{F}$, $I_{max} = 0\text{A}$ and $D = 2\text{A}$.

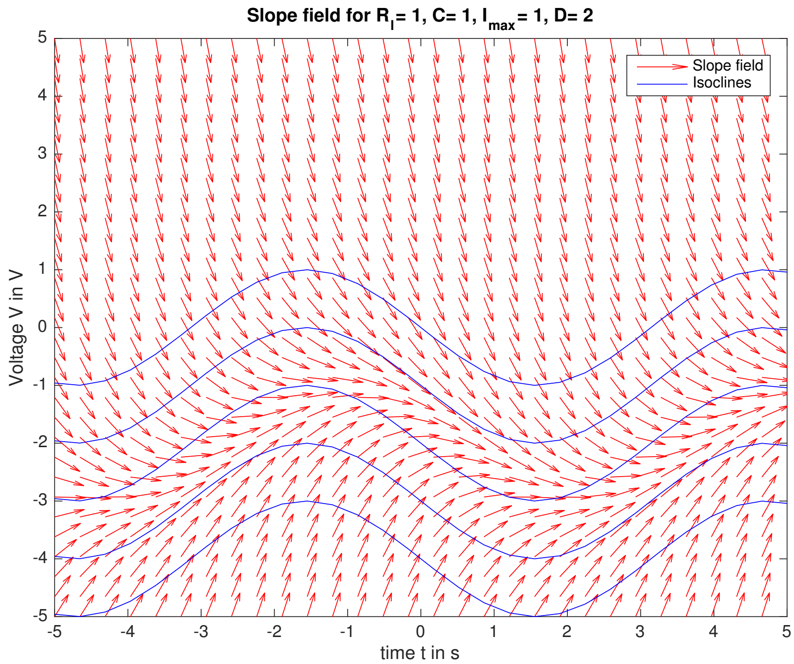


Figure 6: Slope field and isoclines of *equation 5* for $R_l = 1\Omega$, $C = 1\text{F}$, $I_{max} = 1\text{A}$ and $D = 2\text{A}$.