Neuroprothetik – Exercise 1: Introduction

1 Generate a Signal

The following Matlab code generates a signal following eq. 1 and takes into account following input arguments: array of frequencies in Hz, array of amplitudes, signal duration in s, sampling rate in Hz.

$$f(t) = A_0 + \sum_{i=1}^{n} A_i \cdot sin(2\pi F_i \cdot t) \tag{1}$$

1.1 Plot the signal

a)

See code.

b)

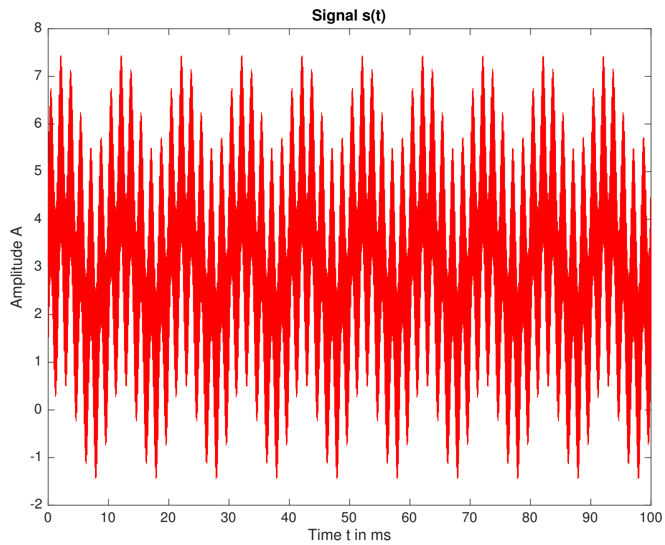

Figure 1: Sinusoidal signal s(t) consisting of superposed frequencies 100 Hz, 600 Hz and 9 kHz with amplitudes 1, 1.5 and 2 as well as an offset of 3 at a sampling rate of 100 kHz.

2 Calculate the Spectrum

a)

See code.

2.1 Plot the Spectrum

a)

In the following plots you see the amplitude spectrum for the different sampling rates (100 kHz, 20 kHz, 10 kHz).

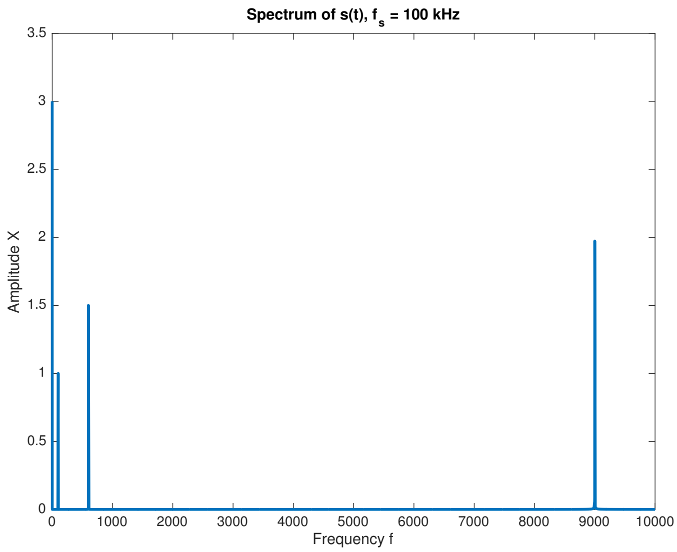

Figure 2: Amplitude spectrum for signal from section 1, with sampling rate of 100 kHz.

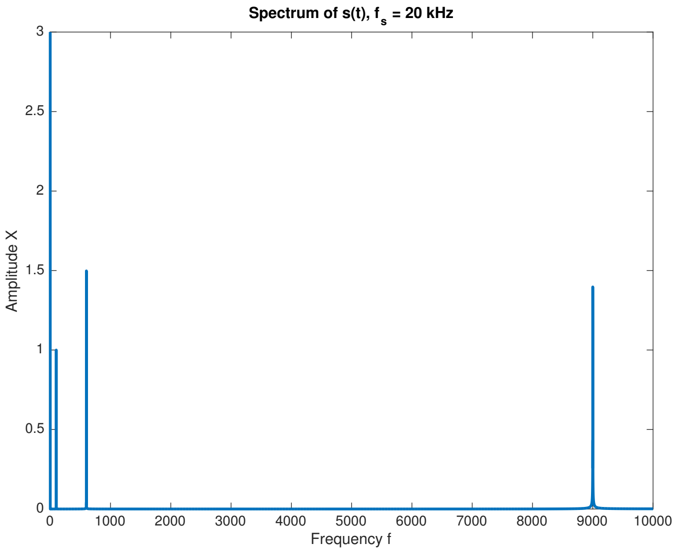

Figure 3: Amplitude spectrum for signal from section 1, with sampling rate of 20 kHz.

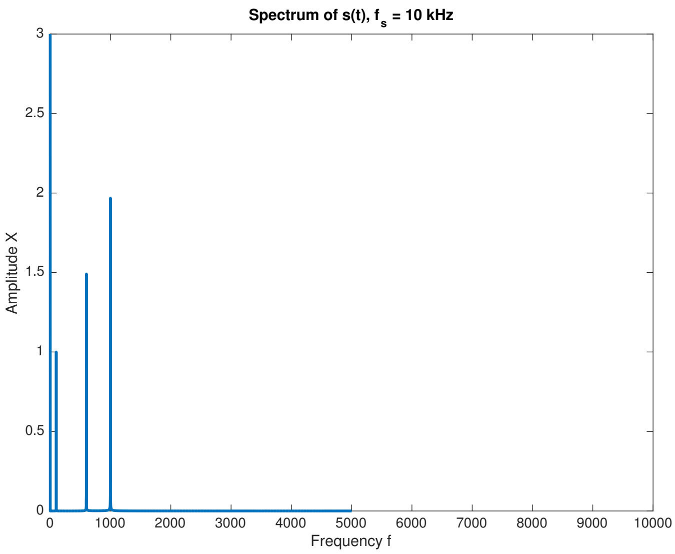

Figure 4: Amplitude spectrum for signal from section 1, with sampling rate of 10 kHz.

b)

For *figures 1* and *2*: the peak at $f = 0$ Hz is the offset with the correct amplitude $A = 3$. The next peak from the left is at $f = 100$ Hz (1^{st} frequency component of the signal) with its amplitude $A = 1$. At $f = 600$ Hz is the 2^{nd} frequency component of the signal with $A = 1.5$. The last peak at $f = 9$ kHz with an amplitude of $A = 2$ is the 3^{rd} frequency component of the signal.
For *figure 3*, there is no peak at $f = 9$ kHz, but an additional peak at $f = 1$ kHz. That peak was shifted from the *left-side spectrum*, as the FFT spectrum initially was symmetric, but only by $f_s = 10$ kHz, thus resulting in $f = 1$ kHz ($(10 - 9)$ kHz $= 1$ kHz), see also following paragraph).
The last spectrum (the one that was calculated based on the signal sampled at a rate of 10 kHz) violates the Nyquist-Shannon sampling theorem. The Nyquist-Shannon sampling theorem states that for a given bandlimit B of a signal, the sampling frequency f_s needs to be at least twice the bandlimit:

$$2 \cdot B < f_s \tag{2}$$

This condition is **not fulfilled**, as the maximum frequency in signal $s(t)$ is 9 kHz, i.e.:

$$2 \cdot 9 \text{ kHz} = 18 \text{ kHz} < 10 \text{ kHz} \tag{3}$$

Therefore, aliasing artifacts occur in the amplitude spectrum of signal $s_3(t)$ between the two main spikes at 100 Hz and 600 Hz. The maximum admissible frequency in any signal $s(t)$ must not exceed $f_s/2$ for proper sampling and perfect reconstruction of the signal.

Edit (11/05/2016): The necessary