The Hodgkin & Huxley Model

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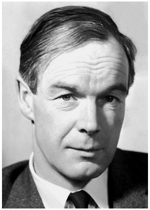
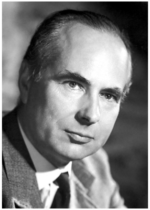
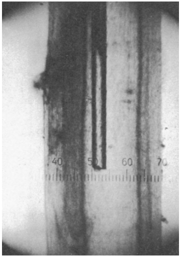


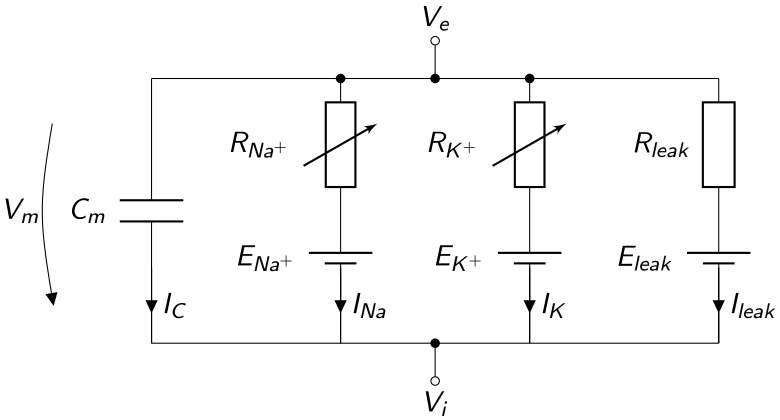


- Alan Hodgkin and Andrew Huxley
- 1937 - 1952 Work on the giant axon of the squid.
- Awarded with the 1963 Nobel Price in Medicine and Physiology.



Main findings:

- Independent sodium (Na^+) and potassium (K^+) currents.
- Currents depend on time (t) and membrane voltage (V_m).



$$0 = C\frac{dV}{dt} + I_{Na^+} + I_{K^+} + I_{leak}$$

The channels are opend and closed by so called gating variables.

- Variabel n for Potassium
- Variables h and m for Sodium

$$
\begin{aligned}
I_{Na^+} &= \hat{g}_{Na^+} m^3 h (V - E_{Na^+}) \\
I_{K^+} &= \hat{g}_{K^+} n^4 (V - E_{K^+})
\end{aligned}
$$

Where $m, n, h \in [0, 1]$ and where $\hat{g}$ is the channel conductivity when all are open.

The Gating Variables

Channel opening described by a two state process

$$\text{closed} \quad \underset{\beta(V_m)}{\overset{\alpha(V_m)}{\rightleftharpoons}} \quad \text{open}$$

The proportion of open channels ($x \in [0, 1]$) can be given by the following DGL:

$$\begin{aligned}\frac{dx_i}{dt} &= \alpha(1 - x_i) - \beta x_i \\ &= (\alpha + \beta)\left(\frac{\alpha}{\alpha + \beta} - x_i\right) = \frac{1}{\tau_x}(x_{i,\infty} - x_i)\end{aligned}$$

$$\frac{dx_i}{dt} = (\alpha + \beta)\left(\frac{\alpha}{\alpha + \beta} - x_i\right) = \frac{1}{\tau_x}(x_{i,\infty} - x_i)$$

- τ_x is the timeconstant of an exponential function.
- $x_{i,\infty}$ is the steady state value of the function.
- both τ_x and $x_{i,\infty}$ are functions depending on V_m.

The Rate Equations

α and β are described by Bolzmann functions.

$$\alpha_m = \frac{2.5 - 0.1V}{e^{(2.5-0.1V)} - 1} \qquad \beta_m = 4e^{-V/18}$$

$$\alpha_n = \frac{0.1 - 0.01V}{e^{(1-0.1V)} - 1} \qquad \beta_n = 0.125e^{-V/80}$$

$$\alpha_h = 0.07e^{-V/20} \qquad \beta_h = \frac{1}{e^{(3-0.1V)} + 1}$$

We have a system of differential equations to solve. First the equation for the membrane potential.

$$0 = C\frac{dV}{dt} + I_{Na^+} + I_{K^+} + I_{leak}$$

And then three equations for the gating variables.

$$\frac{dm}{dt} = \frac{1}{\tau_m}(m_\infty - m)$$
$$\frac{dn}{dt} = \frac{1}{\tau_n}(n_\infty - n)$$
$$\frac{dh}{dt} = \frac{1}{\tau_h}(h_\infty - h)$$

Implementation

We use the exponential method for the gating variables and the forward euler for the membrane voltage. All values are initialized with there steady state values.
For each timestep we first calculate the new gating values. These can then be used to calculate the ionic currents with which we can then calculate a new membrane potential