vol equation model eg 7,8points

Module 6:

Non Linear Time Series

Introduction:

- It comprises of set of methods that extract dynamic info. about the succession values in a dataset
- In linear time series, each data point X_t can be viewed as a linear combination of past or future values or differenc
- In non-linear time series there is no straight line or direct relationship b/w independent and dependent.
- There are two types of volatility models for non-linear TS ARCH and GARCH
- The application of non-linear TS - in finance industry as many asset prices are conditional heteroskedstic
- These models are related to economic forecasting and measuring volatility (uncertain)

Autoregressive (previous data) conditional (depending on value +se or -se) heteroskedastic (ARCH):
↳ variance constant in homoskedasticity but it will never be constant.

- AR - The current value can be expressed as a func of prev previous value
- Conditional - variance is based on past error or any external factor
- Heteroskedasticity - this implies that the series displays unusual variance (varing variance)

Because if value of yest is low today will be low.

Need of ARCH model?

- AR model are used for univariate TS data ie stationary (AR), has a trend (ARIMA) has a seasonal component (SARIMA) but these model donot model the data with a change in variance over time.
- Error term in stochastic processes generating TS were homoskedastic ie constant variance (stationary model)

- There are some TS where variance changes consistently over time, which is called as ↑se or ↓se volatility
- If change in variance can be co-related overtime then it can be modelled using ARCH model.

* When to apply ARCH model ? (we donot work with actual value but variance)
- Fit possible model to the data - used to model the expected variance on the residual after another AR model has been used.
- Consider residual ε_t, by squaring the residual and by examining the correlogram ensure that mean of residual is zero.

ARCH model of Order Unity:
A TS ε_t is given at each time interval by,

$$\varepsilon_t = w_t * \sigma_t \text{ (unit time)}$$

where, no info to relate with data
$w_t \rightarrow$ white noise with zero mean and unit variance

$$\text{Var}(\varepsilon_t) = \sigma_t^2 = \underbrace{\alpha_0}_{\text{mean(const)}} + \underbrace{\alpha_1}_{\text{const}} \underbrace{\sigma_{t-1}^2}_{\text{volatility (lagged square error)}}$$

ε_t is on ARCH model of order unity denoted by ARCH(1)
As, we are modelling error today as a function of error yesterday,

$$\varepsilon_t = w_t \sqrt{\alpha_0 + \alpha_1 \underbrace{\varepsilon_{t-1}^2}_{\text{error from yesterday}}}$$

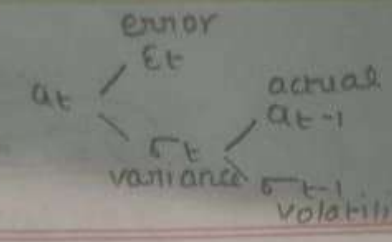
∵ because if value of yest is low today will be low.

Similarly, ARCH(2) can be written as,

$$\varepsilon_t = w_t \sqrt{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2}$$

Generalised equation of ARCH(p)

$$\varepsilon_t = w_t \sqrt{\alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2}$$



Generalised Autoregressive Conditional Heteroskedastic (GARCH)

- The problem with ARCH model is that if the data contains spike (bursty - sudden ↑se or ↓se) then ARCH model is unable to resolve the problem to which a GARCH model can be applied.

GARCH (1,1)

$$a_t = \varepsilon_t \sqrt{\alpha + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2}$$

a_t: actual value; ε_t: actual error of current time period; α: mean; α_1: const; a_{t-1}^2: past value; β_1: const; σ_{t-1}^2: past volatility

GARCH (2,2) (past value, past volatility)

$$a_t = \varepsilon_t \sqrt{\alpha + \alpha_1 a_{t-1}^2 + \alpha_2 a_{t-2}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2}$$

- GARCH process provides a more real world context than other model when predicting the prices of rates for finance data.
- GARCH aims to ~~min~~ minimize errors in forecasting by accounting for errors in prior forecasting and enhancing accuracy of ongoing prediction.