

Q1 # Sample $(x_1, x_2, x_3, \dots, x_n)$ mean = θ_1 & Var = θ_2
(Normal distribution)

$$L = \prod f(x_i | \theta)$$
$$L(\theta_1, \theta_2 | x_1, x_2, \dots, x_n) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

take log both sides

$$\ln L(\theta_1, \theta_2 | x_1, x_2, \dots, x_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^n (x_i - \theta_1)^2$$

differentiate wrt θ_1 & θ_2 & then equate to 0

for θ_1 :

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2 | x_1, x_2, \dots, x_n) = \frac{1}{\theta_2} \sum_{i=1}^n (x_i - \theta_1) = 0$$

$$\sum_{i=1}^n (x_i - \hat{\theta}_1) = 0$$
$$\hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^n x_i$$

MLE for θ_1 is sample mean

for θ_2 :

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2 | x_1, x_2, \dots, x_n) = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^n (x_i - \hat{\theta}_1)^2 = 0$$

$$-\frac{n}{2\theta_2} + \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^n (x_i - \hat{\theta}_1)^2 = 0$$

$$\frac{n}{2\hat{\theta}_2} = \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^n (x_i - \hat{\theta}_1)^2$$

$$\hat{\theta}_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \hat{\theta}_1)^2$$

MLE for θ_2 is sample variance.

Q2

Bernoulli distribution
parameter $\rightarrow \theta \in \theta = (0, 1)$ unknown
 $\rightarrow m$ (known +ve \mathbb{Z})

likelihood of function is \rightarrow

$$L(\theta | x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i = x_i | \theta)$$

Since x_i follows ~~bernoulli~~ bernoulli distribution

$$P(x_i = x_i | \theta) = \theta^{x_i} (1-\theta)^{m-x_i} \quad \text{for each } i$$

Taking log on both sides

$$\begin{aligned} \ln L(\theta | x_1, x_2, \dots, x_n) &= \sum_{i=1}^n \ln(\theta^{x_i} (1-\theta)^{m-x_i}) \\ &= \sum_{i=1}^n (x_i \ln \theta + (m-x_i) \ln(1-\theta)) \end{aligned}$$

differentiate wrt θ

$$\frac{\partial}{\partial \theta} (\ln L(\theta | x_1, x_2, \dots, x_n)) = 0$$

$$\sum_{i=1}^n \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^n \frac{x_i}{\theta} = mn - \sum_{i=1}^n \frac{x_i}{1-\theta}$$

$$\theta = \frac{\sum x_i}{nm}$$

$$\hat{\theta}_{MLE} = \frac{\sum_{i=1}^n x_i}{n \cdot m}$$