Non-interactive proofs of proof-of-work

Anonymized submission

***Abstract*—Blockchain protocols such as bitcoin provide decentralized consensus mechanisms based on proof-of-work. In this work we introduce and instantiate a new primitive for blockchain protocols called Non-Interactive-Proofs-of-Proof-of-Work (NIPoPoWs) which can be adapted into existing PoW-based cryptocurrencies. Unlike a traditional blockchain client which must verify the entire linearly-growing chain of PoWs, clients based on NIPoPoWs require resources only logarithmic in the length of the blockchain. NIPoPoWs solve two important open questions for PoW based consensus protocols: The problem of constructing efficient transaction verification (SPV) clients and the problem of constructing efficient sidechain proofs. We provide a formal model for NIPoPoWs. We prove our construction is secure and establish its succinctness in a well defined model setting. We provide simulations and experimental data to measure concrete communication efficiency and security. We also present an attack against the only previously known (interactive) PoPoW protocol that showcases the difficulty of designing such protocols. Finally, we provide two ways that our NIPoPoWs can be adopted by existing blockchain protocols, first via a soft fork, and second via a new update mechanism that we term a "velvet fork" and enables to harness some of the performance benefits of NIPoPoWs even with a minority upgrade.**

***Index Terms*—bitcoin, blockchain, proof-of-work, succinct, sidechains**

1. Introduction

Cryptocurrencies such as Bitcoin [15] [16] and Ethereum [22] are peer-to-peer networks that maintain a globally consistent transaction ledger, using a consensus protocol based on proof-of-work (PoW) puzzles [2], [8]. Worker nodes called "miners" expend computational work in order to reach agreement on the state of the network. Clients on the network, such as mobile phone apps, must verify these PoW in order to determine the correct view of the network's state, as necessary to transmit and receive payments.

In this work we introduce, analyze and instantiate a new primitive, Non-Interactive Proofs of Proof-of-Work (NIPoPoWs), which can be adapted into existing cryptocurrencies to support more efficient clients. A traditional blockchain client must verify the entire chain of proofs-of-work, which grows linearly over time. On the contrary, clients based on NIPoPoWs require resources only logarithmic in the length of the blockchain.

Motivation. There are three trends that motivate the need for more efficient clients, specifically clients that can verify transactions without having to process the whole blockchain. First, while Bitcoin remains by far the largest and most widely used cryptocurrency, the ecosystem has become significantly more diverse. Today there are hundreds of competitors, commonly called "altcoins", and Bitcoin's dominance in the market is at an all time low [20].[1] We envision that our protocol will form the basis of an efficient multi-blockchain client, which could efficiently support payments using hundreds of separate cryptocurrencies. This requires optimizing the "SPV client" described in the original Bitcoin paper [15] which, while quite efficient, requires processing an amount of data growing linearly with the size of the chain $|\mathcal{C}|$.

Second, cryptocurrencies are increasingly used as components of larger systems (the cryptocurrency system is typically called a "blockchain" when its currency feature is not salient). As one example, a recent system called Catena [21] uses the bitcoin blockchain as an authenticated log; the browser embeds a Bitcoin client that uses the blockchain to validate HTTPS certificates. The bandwidth costs that are tolerable for a dedicated Bitcoin wallet are likely unacceptable in such embedded contexts. Our techniques can directly reduce the cost of Catena and similar systems.

Finally, there has been significant interest in the development of "cross-chain" applications, i.e. logical transactions that span multiple separate blockchains. Simple cross-chain transactions are feasible today: the most well-known is the atomic exchange [17] e.g. a trade of Bitcoins for Ether. However, more sophisticated applications could be enabled by our protocol, which would allow the blockchain of one cryptocurrency to embed a client of a separate cryptocurrency. This concept, initially popularized by a proposal by Back et al. [1], can be used to avoid a difficult upgrade process: a new blockchain with additional features, such as experimental opcodes, can be backed by deposits

1. Using market capitalization as a measure of economic size is debatable but it still indicates the direction of the evolving landscape of cryptocurrencies towards multiple offerings.

in the original Bitcoin currency, obviating the need to bootstrap new currencies from scratch. The ability to use sidechains as an upgrade and experimentation mechanism has been heralded as a key enabler of scalability in the Bitcoin ecosystem; however, the sidechains proposal is incomplete, as it does not provide a protocol that satisfies a natural security definition, i.e. where attacks succeed with only negligible probability, even though several attempts were made [10], [18]. Our work is the first to answer the open problem posed by Back et al. Sidechains can also be used as a mechanism to off-load some of the workload incurred by the bitcoin network by moving coins to dedicated sidechains, which can later be moved back.

These examples illustrate that our solution is a key component for two important pillars needed for next-generation blockchains: *interoperability* and *scalability*.

Our contributions. In summary, we make the following contributions in this paper. Our main technical contribution is the introduction and instantiation of a new cryptographic primitive called Non-Interactive Proofs of Proof of Work (NIPoPoW). We present a formal model and a provably secure instantiation of NIPoPoWs in the backbone model [11]. Our contribution builds on previous work of [13] who posed *interactive* proofs of proof-of-work, which, in turn, are based on previous discussion of such concepts in the bitcoin forums [14]. We in fact show an explicit attack against the construction of [13] that showcases the difficulty of designing such protocols. It follows that our construction is the first secure proof-of-proof-of-work. Furthermore, our solution has the additional property of being non-interactive, as the previous construction could be forced into interaction by an adversarial prover. Similar to previous work, we prove that the proofs are optimistically succinct meaning that, in honest conditions, the proofs are logarithmic in size. Improving previous work, we show that optimistic succinctness can be achieved for adversarially-generated proofs for a number of blockchain predicates that are high value use cases. Our definition fills the gap in terms of security modelling and design that existed in previous proposals, especially the notion of cumulative "Dynamic Member Multisignature" [1].

We provide concrete parameterization and empirical analysis focusing on showing the potential savings of our approach versus existing clients. Using actual data from the bitcoin blockchain, we quantify the actual savings of NIPoPoWs over the previous known techniques of constructing efficient SPV verifiers.

We describe two practical deployment paths for our NIPoPoWs that existing cryptocurrencies can adopt to support our efficient clients. Specifically, we first outline how NiPoPoW's can be deployed using either a "soft fork" or a "hard fork" upgrade procedure, both of which have been successfully used by existing cryptocurrencies [5]. We then present a less disruptive update mechanism that we term a "velvet fork" and which may be of independent interest. In a velvet fork update the clients that have "forked" from the original implementation continue to be fully compatible with un-updated clients. It follows that, in a velvet fork, the blockchain system can remain supported by a diverse software codebase indefinitely, while it can still enjoy, at least in proportion, some of the (efficiency in our case) benefits of the update without any of the security downsides.

2. Model and Definitions

We define our problem in the setting of a blockchain client which is completely stateless beyond the knowledge of a common reference string, the genesis block. Without loss of generality, this single reference block could be any stable checkpoint block, either hard-coded in the client or obtained through a previous interaction with the network. However, importantly, the client does not maintain a blockchain. The challenge is, then, to build a client that is able to communicate with the network and, without downloading the whole chain headers, is convinced in a secure manner about a certain predicate on the "main chain", e.g., whether a specific a transaction has been confirmed. We term such a client a *verifier* and we term *provers* the full nodes it connects to. The provers maintain a complete blockchain and can be thought of as full nodes. The verifier connects to multiple provers, some of which may be malicious, but at least one of which is honest. All of the provers provide a proof and the verifier has to decide based on the feedback received what is the correct value of the predicate in question.

The prover-verifier interaction is as follows: for a given predicate on the chain (e.g. "the transaction took place"), the prover computes the proof string and transmits it to the verifier. The verifier accepts or rejects the string. Note that in the interactive case the prover and verifier may engage in more than one round of message passing.

The entities on the blockchain network are of 3 kinds: (1) miners, who try to mine new blocks on top of the longest known blockchain and broadcast them as soon as they are discovered (For simplicity we assume that difficulty is constant and thus the "longest chain rule" sufficiently describes honst miner behavior); (2) full nodes, which maintain the longest blockchain without mining and also act as the provers in the network; (3) verifiers or stateless clients, which connect to provers and ask for proofs in regards to which blockchain is the largest. The verifiers attempt to determine the value of a predicate on these chains.

We model proof-of-work discovery attempts by using a random oracle [4] as in [11]. The network is synchronized into rounds, which correspond to moments in time. The predicates of interest in our context are predicates on the active blockchain. Some of the predicates are more suitable for succinct proofs than others. We focus our attention in what we call *reliable* predicates. These predicates have the exceptional property that all honest miners share their view of them in a way that is updated in a predictable manner, with a truth-value that persists as the blockchain grows (an example of an unreliable predicate is e.g., the least significant bit of the hash of last block).

In our setting, for a given predicate Q, several provers (including adversarial ones) will generate proofs claiming potentially different truth values for Q based on their claimed local longest chains. The verifier receives these proofs and accepts one of them proofs, determining the truth value of the predicate. We denote a *blockchain proof protocol* for a predicate Q as a pair (P, V) where P is the *prover* and V is the *verifier*. P is a PPT algorithm that is spawned by a full node when they wish to produce a proof, accepts as input a full chain $\mathcal{C}$ and produces a proof π as its output. V is a PPT algorithm which is spawned at some round, receives a pair of proofs (π_A, π_B) from both an honest party and the adversary and returns its decision $d \in \{T, F, \perp\}$ before the next round and terminates. The honest miners produce proofs for V using P, while the adversary produces proofs following some arbitrary strategy. Before we introduce the security properties for blockchain proof protocols we introduce some necessary notation for blockchains.

Blockchain addressing. Blockchains are finite block sequences obeying the *blockchain property* that for every block in the chain there exists a pointer to its previous block. A chain is *anchored* if its first block is *genesis*, denoted *Gen*.

For chain addressing we use Python brackets $\mathcal{C}[\cdot]$. A zero-based positive number in a bracket indicates the indexed block in the chain. A negative index indicates a block from the end. A range $\mathcal{C}[i : j]$ is a subarray starting from i (inclusive) to j (exclusive).

Given chains $\mathcal{C}_1, \mathcal{C}_2$ and blocks A, Z we concatenate them as $\mathcal{C}_1\mathcal{C}_2$ or $\mathcal{C}_1 A$. $\mathcal{C}_2[0]$ must point to $\mathcal{C}_1[-1]$ and A must point to $\mathcal{C}_1[-1]$. We denote $\mathcal{C}\{A : Z\}$ the subarray of the chain from A (inclusive) to Z (exclusive). We can omit blocks or indices from the range to take the chain to the beginning or end.

Helper functions for blockchains. The *id* function returns the id of a block given its data. This is done by applying the hash functions H and G: $\mathsf{id} = H(ctr, G(x, \mathsf{interlink}))$. Here, ctr is the nonce, x is the Merkle tree root of the transactions.However, note that block B maintains an *interlink* data structure instead of a previous block pointer. We will explain this further in section 3. *depth* is a function which, given a block, returns its distance from the genesis block.

Valid blocks satisfy the proof-of-work condition: $id \leq T$, where T is the mining target. Some blocks will achieve a lower id. If $id \leq \frac{T}{2^\mu}$ we say that the block is of level μ. All blocks are level 0. Blocks with level μ are *μ-superblocks*. By convention, for *Gen* we set $id = 0$ and $\mu = \infty$.

In this paper, we will extend blocks to contain multiple pointers to previous blocks. Certain blocks can be omitted from a chain, obtaining a subchain. as long as the blockchain property that each block must contain a pointer to its previous block in the sequence is maintained.

Blockchains are sequences, but it is still more convenient to use set notation for some operation. Specifically, $B \in \mathcal{C}, \emptyset, \mathcal{C}_1 \subseteq \mathcal{C}_2$ have the obvious meaning. $\mathcal{C}_1 \cup \mathcal{C}_2$ is the chain obtained by sorting the blocks contained in both $\mathcal{C}_1$ and $\mathcal{C}_2$ into a sequence (this may be not always defined). $\mathcal{C}_1 \cap \mathcal{C}_2$ is the chain $\{B : B \in \mathcal{C}_1 \land B \in \mathcal{C}_2\}$. In both cases, the blockchain property must be maintained. The lowest common ancestor is $\mathsf{LCA}(\mathcal{C}_1, \mathcal{C}_2) = (\mathcal{C}_1 \cap \mathcal{C}_2)[-1]$. If $\mathcal{C}_1[0] = \mathcal{C}_2[0]$ and $\mathcal{C}_1[-1] = \mathcal{C}_2[-1]$, we say the chains *span* the same block range.

Given $\mathcal{C}$ and level μ, the *upchain* $\mathcal{C}\uparrow^\mu$ is $\{B \in \mathcal{C} : level(B) \geq \mu\}$. A chain containing only μ-superblocks is a *μ-superchain.* Given chains $\mathcal{C}' \subseteq \mathcal{C}$, the *downchain* $\mathcal{C}'\downarrow^{\mathcal{C}}$ is $\mathcal{C}[\mathcal{C}'[0] : \mathcal{C}'[-1]]$. $\mathcal{C}$ is the *underlying chain* of $\mathcal{C}'$. The underlying chain is often implied by context, so we will simply write $\mathcal{C}'\downarrow$. The $\mathcal{C}\uparrow$ operator is absolute: $(\mathcal{C}\uparrow^\mu)^{\mu+i} = \mathcal{C}\uparrow^{\mu+i}$. Given a set of consecutive rounds S, we define $\mathcal{C}^S = \{B \in \mathcal{C} : B \text{ was generated during } S\}$.

Desired properties. We now define two desired properties of a non-interactive blockchain proof protocol, *succinctness* and *security*. For clarity, the properties are described in the execution model of [11] and can be easily ported to a more refined model e.g., [**?**].

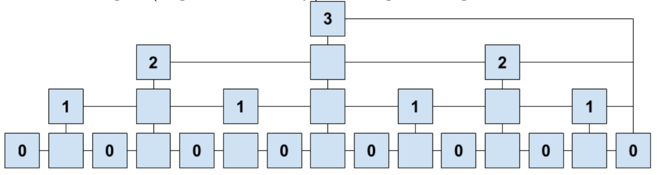
Definition 2.1. (Security) A *blockchain proof protocol* is *secure* if for all environments and for all PPT adversaries $\mathcal{A}$ the output of V on round r is the same as the evaluation of $Q(\mathcal{C})$ on all honest parties' chains $\mathcal{C}$ as long as $Q(\mathcal{C})$ is the same for all honest parties after round r.

Definition 2.2. (Succinctness) A *blockchain proof protocol* is *succinct* if the maximum proof size $|\pi|$ across all honest provers for a given round r is $O(polylog(r))$.

Based on these definitions, it is trivial to construct a secure but not succinct protocol: The prover provides the chain $\mathcal{C}$ itself as a proof and the verifier simply compares the proofs received by length: Since the longest proof is literally the longest chain, the protocol is trivially secure but not succinct. The challenge we will solve is to provide a non-interactive protocol which at the same time achieves security and succinctness over a large class of predicates.

Blockchain predicates of interest. Not all predicates are suitable for our purposes. In this section we characterize the class of blockchain predicates that will be of interest to us for constructing proofs. Given that we are in a decentralized setting it can be the case that for certain sensible blockchain predicate, there will be honest nodes that have not reached a conclusion about its value. For this reason we allow our predicates to have an undefined value $\perp$ before they take on a truth value of 0 or 1. We define a number of predicate properties and prove a simple but useful relation between them.

Figure 1. The hierarchical blockchain. Higher levels have achieved a lower target (higher difficulty) during mining.



Definition 2.3. (Monotonicity) A chain predicate $Q(\mathcal{C})$ is *monotonic* if for all chains $\mathcal{C}$ and for all blocks B we have that: $Q(\mathcal{C}) \neq \perp \Rightarrow Q(\mathcal{C}) = Q(\mathcal{C}B)$.

(Stability) Parameterized by $k \in \mathbb{N}$, a chain predicate Q has *stability* if its value only depends on $\mathcal{C}[: -k]$.

3. Consensus layer support

The interlink pointers data structure. In order to construct our protocols, as in [13], we take advantage of a data structure that will be stored in the header of a block. Observe that in a blockchain protocol execution it is expected half of the blocks will be of level 1, 1/4 of the blocks will be of level 2, 1/8 will be of level 3 and $1/2^{\mu}$ blocks will be of level μ. In expectation, the number of superblock levels of a chain $\mathcal{C}$ will be $\log(\mathcal{C})$. Figure 1 illustrates the blockchain superblocks starting from level 1 and going up to level 4 in case these blocks are distributed exactly according to expectation. Here, each level contains half the blocks of the level below.

The *interlink* data structure is proposed to be included in each block, replacing the existing pointer to the previous block with a list of pointers to a small number of previous blocks. For each block, this data structure contains a pointer to the most recent preceding block of every level μ. The algorithm for this construction is shown in Algorithm 1 and is borrowed from [13]. Genesis is of infinite level and hence a pointer to it is included in every block at the first available index within the interlink data structure. The interlink data structure turns the blockchain into a skiplist-like data structure. The number of pointers that need to be included per block is in expectation $\log(|\mathcal{C}|)$.

The updateInterlink algorithm accepts a block B', which already has an interlink data structure defined on it. The function evaluates the interlink data structure which needs to be included as part of the next block. It copies the existing interlink data structure and then modifies some of its entries from level 0 to $\mathsf{level}(B')$ to point to the block B'.

Algorithm 1 updateInterlink

```
1: function updateInterlink($B'$)
2:     interlink $\leftarrow B'$.interlink
3:     for $\mu = 0$ to $level(B')$ do
4:         interlink[$\mu$] $\leftarrow blockid(B')$
5:     end for
6:     return interlink
7: end function
```

Superchain quality. In order to argue formally about the security properties of blockchains that are equipped with the interlink data structure we will introduce a new concept of chain quality, inspired by the definition of this property in [11], called *superchain quality.* We first define a notion of "goodness" that bound the deviation of superblocks of a certain level from their expected mean. Using this we then define superchain quality.

Definition 3.1 (Locally good superchain)**.** A superchain $\mathcal{C}'$ of level μ with underlying chain $\mathcal{C}$ is said to be μ-locally-good with respect to security parameter δ, written $\mathsf{local\text{-}good}_{\delta}(\mathcal{C}', \mathcal{C}, \mu)$, if $|\mathcal{C}'| > (1-\delta)2^{-\mu}|\mathcal{C}|$.

Definition 3.2 (Superchain quality)**.** The (δ, m) superquality property Q^{μ}_{scq} of a chain $\mathcal{C}$ pertaining to level μ with security parameters $\delta \in \mathbb{R}$ and $m \in \mathbb{N}$ states that for all $m' \geq m$, it holds that $\mathsf{local\text{-}good}_{\delta}(C{\uparrow}^{\mu}[-m' :], C{\uparrow}^{\mu}[-m' :]{\downarrow}, \mu)$. That is, all suffixes that are sufficiently large are locally good.

While superchain quality guarantees that a superchain has a sufficient number of blocks w.r.t. the level 0 chain, we will also need to be able to compare it with other superchains. For this reason we introduce multilevel quality.

Definition 3.3 (Multilevel quality)**.** A μ-superchain $\mathcal{C}'$ is said to have *multilevel quality*, written $\text{multi-good}_{\delta,k_1}(\mathcal{C}, \mathcal{C}', \mu)$ with respect to an underlying chain $\mathcal{C} = \mathcal{C}'{\downarrow}$ with security parameters k_1, δ if for all $\mu' < \mu$ it holds that for any $\mathcal{C}^* \subseteq \mathcal{C}$, if $|\mathcal{C}^*{\uparrow}^{\mu'}| = k_1$, then $|\mathcal{C}^*{\uparrow}^{\mu}| \geq (1-\delta)2^{\mu-\mu'}|\mathcal{C}^*{\uparrow}^{\mu'}|$

Putting the above together we conclude with the notion of a *good* superchain.

Definition 3.4 (Good superchain)**.** A μ-superchain $\mathcal{C}'$ is said to be *good*, written $\text{good}_{\delta,k_1}(\mathcal{C}, \mathcal{C}', \mu)$, with respect to an underlying chain $\mathcal{C} = \mathcal{C}'{\downarrow}$ if it has both superquality and multilevel quality with parameters (δ, m).

It is not hard to see that the above good statistical properties are attained with overwhelming probability by all chains that are generated in optimistic environments, i.e. if no adversary tries to violate them.

Lemma 3.1 (Optimistic superchain distribution)**.** *For a level μ, and $0 < \delta < 0.5$, a chain $\mathcal{C}$ containing only honestly-generated blocks adopted by an honest party in an execution with random scheduling is (δ, m)-good at level μ with overwhelming probability in m.*

4. An attack against the PoPoW of [13]

In this section, we revisit the construction for interactive proofs of proof-of-work from [13] and its security. Their construction is the starting point for our non-interactive proofs of proof-of-work. We show that the construction is susceptible to a double-spending attack even in the case the adversary controls a minority of the hashing power.

We first show that a powerful attacker can break chain superquality with non-negligible probability and we construct a concrete attack based on this observation. Maintaining chain superquality is not in the original security model. However, we show how the property affects the security of the underlying proofs. We do this by presenting a concrete attack against their scheme that allows an adversary to double spend with non-negligible probability. This attack works under the assumption that the adversary has sufficient hashing power, which is still below 50%.

The interactive proofs of proof-of-work protocol. The interactive proof of proof-of-work of [13] works as follows. The verifier distinguishes between two candidate proofs (π_A, χ_A) and (π_B, χ_B). If $\pi_A = \pi_B$, the decision can be drawn immediately (without interaction) based on χ_A, χ_B. Otherwise, the verifier queries the two provers for their claimed anchored superchains $\pi_A{\uparrow}^\mu$ and $\pi_B{\uparrow}^\mu$ at a certain level μ. The verifier starts querying at the highest possible level μ and successively descends until level μ is sufficiently low such that block $b = LCA(\pi_A{\uparrow}^\mu, \pi_B{\uparrow}^\mu)$ is m blocks deep for at least one of the provers. More specifically, the querying stops at such μ when $max(|\pi_A{\uparrow}^\mu\{b:\}|, |\pi_B{\uparrow}^\mu\{b:\}|) \geq m$. Subsequently, the winner is designated as the prover with the most blocks after b at that level. More precisely, the winner is A if $|\pi_A{\uparrow}^\mu\{b:\}| \geq |\pi_B{\uparrow}^\mu\{b:\}|)$ and otherwise it is B. The communication overhead is reduced by only requesting blocks after the purported LCA.

Attacking chain superquality. Let us now construct an adversary to break this property at level μ. Suppose the adversary $\mathcal{A}$ controls a portion t/n of the hashing power. $\mathcal{A}$ works as follows. Assume she wants to attack the honest party B in order to have him adopt a chain $\mathcal{C}_B$ which has a bad distribution of superblocks, i.e. such that local goodness is violated in some sufficiently long subchain. She continuously determines the current chain $\mathcal{C}_B$ adopted by B. The adversary starts playing after $|\mathcal{C}_B| \geq 2$. If $level(\mathcal{C}_B[-1]) < \mu$, then $\mathcal{A}$ remains idle. However, if $level(\mathcal{C}_B[-1]) \geq \mu$, then $\mathcal{A}$ attempts to mine an adversarial block b on top of $\mathcal{C}_B[-2]$. If she is successful, then she attempts to mine another block b' on top of b. If she is successful, she broadcasts both b, b'. The adversarial mining continues until party B adopts a new chain, which can be due to two reasons: Either the adversary managed to successfully mine b, b' on top of $\mathcal{C}_B[-2]$ and succeeded in having B adopt it; or one of the honest parties was able to mine a block which was subsequently adopted by B. In either case, the adversary continues with the strategy by inspecting $\mathcal{C}[-1]$ and acting accordingly.

Assume now that an honestly-generated μ-superblock has been adopted by B at position $\mathcal{C}_B[i]$ at some round r. Let us now examine the probability that $\mathcal{C}_B[i]$ will remain a μ-superblock in the long run. Suppose $r' > r$ is the first round after r during which any block is generated. It is clear that $\mathcal{A}$ will succeed in this attack with non-negligible probability and cause B to abandon the μ-superblock from their adopted chain. Therefore, there will be some δ such that the adversary will be able to cause such variance with non-negligible probability in m. This suffices to show that superquality is harmed by this attack.

A double-spending attack. Extending the above attack, we modify the superquality attacker into an attacker that can cause a double spending attack in the proof of proof-of-work construction. As before, $\mathcal{A}$ targets the proofs generated by the honest party B by violating μ-superquality in B's adopted chain. $\mathcal{A}$ begins by remaining idle until a certain chosen block b. After block b is produced, $\mathcal{A}$ starts mining a secret chain which forks off from b akin to a selfish mining attacker [9]. The adversary performs a normal spending transaction on the honestly adopted blockchain and has it confirmed in the block immediately following block b. She also produces a double spending transaction which she secretly confirms in her secret chain in the block immediately following b.

$\mathcal{A}$ keeps extending their own secret chain as usual. However, whenever a μ-superblock is adopted by B, she temporarily pauses mining in her secret chain and devotes her mining power to harm the μ-superquality of B's adopted chain. Intuitively, for large enough μ, the time spent trying to harm superquality will be limited, because the probability of a μ-superblock occurring will be small. Therefore, the adversary's superchain quality will be larger than the honest parties' superchain quality (which will be harmed by the adversary) and therefore, even though the adversary's 0-level blockchain will be shorter than the honest parties' 0-level blockchain, the adversary's μ-superchain will be longer than the honest parties' μ-superchain.

The formal calculation of the probability of this attack succeeding is in the Appendix. However, we note here that, for appropriate choice of system parameters, the attack can be made to succeed with overwhelming probability.

5. Blockchain suffix proofs

We provide a concrete NIPoPoW construction which allows proving certain predicates Q of the chain $\mathcal{C}$. The construction is done in steps: First, we build a construction which is limited to a class of predicates easy to prove. Later, we extend this class of predicates by augmenting the construction.

Among the predicates which are stable, we now limit ourselves *suffix sensitive* predicates. These predicates are easier to prove, but at the same time allow us to use the construction as a scaffold for the more generic case.

Definition 5.1 (Suffix sensitivity)**.** A chain predicate Q is called k-suffix sensitive if for all chains $\mathcal{C}, \mathcal{C}'$ with $|\mathcal{C}| \geq k$ and $|\mathcal{C}'| \geq k$ such that $\mathcal{C}[-k:] = \mathcal{C}'[-k:]$ we have that $Q(\mathcal{C}) = Q(\mathcal{C}')$.

Given the suffix-sensitivity definition, we deduce that for a suffix-sensitive predicate Q there must be a quick way of deducing the value of $Q(\mathcal{C})$ by only examining the k-suffix of the chain.

Suffix-sensitive predicates allow proving that a recent transaction occurred in the longest chain and in a block buried under at most k blocks. Because the predicates we are interested in are stable, they must also pertain to properties that are independent of the last k blocks. Therefore, combining the two, the predicate can only describe something that is visible at the exact block $\mathcal{C}[-k-1]$ and that will remain true as the chain grows in perpetuity (due to monotonicity). These predicates make sense for blockchains such as Ethereum which maintain state [6] that is propagated and committed to in every block. One example is the existence of a particular Ethereum account.

5.1. Construction

We present a generic form of the verifier first and the prover afterwards. The generic form of the verifier works with any practical suffix proof protocol. Therefore, we describe the generic verifier first before we talk about the specific instantiation of our protocol. The generic verifier is given access to call a protocol-specific proof comparison operator $\leq_m$. We begin the description of our protocol by first illustrating the generic verifier. Next, we describe the prover specific to our protocol. Finally, we show the instantiation of the $\leq_m$ operator, which plugs into the generic verifier to make a concrete verifier for our protocol.

The generic verifier. The Verify function of our NIPoPoW construction is described in Algorithm 2. The verifier algorithm is parameterized by a chain predicate Q and security parameters k, m. k pertains to the amount of proof-of-work needed to bury a block so that it is believed to remain stable (e.g. $k = 6$). m is a security parameter pertaining to the prefix of the proof, which connects the Genesis block to the k-sized suffix. The verifier receives several proofs by different provers in a collection of proofs $\mathcal{P}$. Iterating over these proofs, it extracts the best.

Each proof is a chain. For honest provers, these are subchains of the adopted chain. Proofs consist of two parts, π and χ. $\pi\chi$ must be a valid chain. χ is the proof suffix; π is the prefix. We require $|\chi| = k$. For honest provers, χ is the last k blocks of the adopted chain, while π consists of a selected subset of blocks from the rest of their chain preceding χ. The method of choice of this subset will become clear soon.

The verifier compares the proofs provided to it by calling the $\geq_m$ operator, defined by the protocol. We will get to the operator's definition shortly. Proofs are checked for validity before comparison by ensuring $|\chi| = k$ and calling *validChain* which checks if $\pi\chi$ is an anchored blockchain.

Algorithm 2 The Verify algorithm for the NIPoPoW protocol

```
1: function $\mathsf{Verify}^{Q}_{m,k}(\mathcal{P})$
2:     $\tilde{\pi} \leftarrow (\text{Gen})$                ▷ Trivial anchored blockchain
3:     for $(\pi, \chi) \in \mathcal{P}$ do
4:         if $\mathsf{validChain}(\pi\chi) \wedge |\chi| = k \wedge \pi \geq_m \tilde{\pi}$ then
5:             $\tilde{\pi} \leftarrow \pi$
6:             $\tilde{\chi} \leftarrow \chi$                ▷ Update current best
7:         end if
8:     end for
9:     return $\tilde{Q}(\tilde{\chi})$
10: end function
```

The verifier loops through all candidate proofs π and extracts the best one $\tilde{\pi}$ using the $\geq_m$ operator for comparisons. $\tilde{\chi}$ is set to be the suffix associated with the best known prefix. While $\tilde{\chi}$ is needed for the final predicate evaluation, it is not used as part of any comparison, as it has the same size k for all proofs.

After the end of the for loop, the verifier will have determined the best proof $(\tilde{\pi}, \tilde{\chi})$. We will later prove that this proof will necessarily belong to an honest prover with overwhelming probability. Since the proof has been generated by an honest prover, it is associated with an underlying honestly adopted chain $\mathcal{C}$. The verifier then extracts the value of the predicate Q on the underlying chain.

The concrete prover. The NIPoPoW honest prover construction is shown in Algorithm 3. The honest prover is supplied with an honestly adopted chain $\mathcal{C}$ and security parameters m, k, δ and returns proof $\pi\chi$, which is a chain. The suffix χ is the last k blocks of $\mathcal{C}$. The prefix π is constructed by selecting various blocks from $\mathcal{C}[:-k]$ and adding them to π, which consists of a number of blocks for every level μ. At the highest possible level at which at least m blocks, all these blocks are presented. Then, inductively, for every superchain of level μ that is included in the proof, the suffix of length m is taken. Then the underlying superchain of level $\mu - 1$ spanning the same blocks as that suffix is also included, until level 0 is reached. This underlying superchain will have $2m$ blocks in expectation and always at least m blocks.

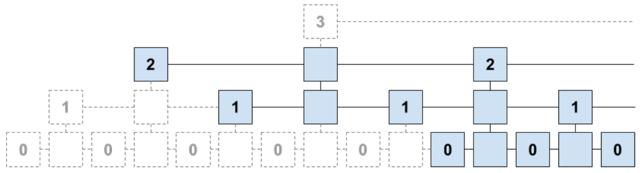
Algorithm 3 The Prove algorithm for the NIPoPoW

```
1: function $\mathsf{Prove}_{m,k,\delta}(\mathcal{C})$
2:     $B \leftarrow \mathcal{C}[0]$                                   ▷ Genesis
3:     for $\mu = |\mathcal{C}[-k].\mathsf{interlink}|$ down to 0 do
4:         $\alpha \leftarrow \mathcal{C}[: -k]\{B :\}{\uparrow^\mu}$
5:         $\pi \leftarrow \pi \cup \alpha$
6:         if $\text{good}_{\delta,m}(\mathcal{C}, \alpha, \mu)$ then
7:             $B \leftarrow \alpha[-m]$
8:         end if
9:     end for
10:    $\chi \leftarrow \mathcal{C}[-k :]$
11:    return $\pi\chi$
12: end function
```

When we take a μ-superchain and are interested in its last m blocks, we fill the same range of blocks with blocks from the superchain of level $\mu - 1$ below. All the μ-superblocks which are within this m blocks range will also be $(\mu-1)$-superblocks and so we do not want to keep them in the proof twice. Note that no check is necessary to make sure the top-most level has at least m blocks, even though the verifier requires this. The reason is the following: Assume the blockchain has at least m blocks in total. Then when a superchain of level μ has less than m blocks in total, these blocks will all be necessarily included into the proof by a lower-level superchain $\mu - i$ for some $i > 0$. Therefore, it does not hurt to add them to π earlier.

Figure 2 contains an example proof constructed for parameters $m = k = 3$. The top superchain level which contains at least m blocks is level $\mu = 3$. For the m-sized suffix of that level, 5 blocks of superblock level 2 are included for support spanning the same range. For the last 3 blocks of the level 2 superchain, blocks of level 1 are included for support.

Figure 2. NIPoPoW prefix π for $m = 3$ on a good chain.



The concrete verifier. The $\geq_m$ operator which performs the comparison of proofs is presented in Algorithm 4. It takes proofs π_A and π_B and returns true if the first proof is winning, or false if the second is winning. It first computes the LCA b between the proofs. As parties A and B agree that the blockchain is the same up to block b, arguments will then be taken for the diverging chains after b. The best possible argument from each player's proof is extracted by calling the best-arg$_m$ function. To find the best argument of a proof π given b, best-arg$_m$ collects all the μ indices which point to superblock levels that contain valid arguments after block b. Argument validity requires that there are at least m μ-superblocks following block b, which is captured by the comparison $|\pi{\uparrow^\mu} \{b :\}| \geq m$. 0 is always considered a valid level, regardless of how many blocks are present there. These level indices are collected into set M. For each of these levels, the score of their respective argument is evaluated by weighting the number of blocks by the level as $2^\mu|\pi{\uparrow^\mu} \{b :\}|$. The highest possible score across all levels is returned. Once the score of the best argument of both A and B is known, they are directly compared and the winner returned. An advantage is given to the adversary in case of a tie by using the $\geq$ operator favouring A.

Algorithm 4 The algorithm implementation for the $\geq$ operator to compare two proofs in the NIPoPoW protocol parameterized with security parameter m. Returns True if the underlying chain of player A is deemed longer than the underlying chain of player B

```
1: function $\mathsf{best\text{-}arg}_m(\pi, b)$
2:     $M \leftarrow \{\mu : |\pi{\uparrow^\mu} \{b :\}| \geq m\} \cup \{0\}$
3:     return $\max_{\mu \in M}\{2^\mu|\pi{\uparrow^\mu} \{b :\}|\}$
4: end function
5: operator $\pi_A \geq_m \pi_B$
6:     $b \leftarrow (\pi_{\mathcal{A}} \cap \pi_B)[-1]$                     ▷ LCA
7:     return $\mathsf{best\text{-}arg}_m(\pi_A, b) \geq \mathsf{best\text{-}arg}_m(\pi_B, b)$
8: end operator
```

5.2. Security

Proving the security of the scheme requires building an involved technical argument. It is included in the Appendix.

Theorem 5.1. *The non-interactive proofs-of-proof-of-work construction for k-stable suffix-sensitive predicates is secure with overwhelming probability in κ.*

Because of the centrality of this theorem to our construction's importance, we will here give a high-level overview of the proof. Suppose an adversary produces a proof $\pi_{\mathcal{A}}$ and an honest party produces a proof π_B such that the two proofs cause the predicate Q to evaluate to different values, while at the same time all honest parties have agreed that the correct value is the one obtained by π_B. Because of bitcoin's security, $\mathcal{A}$ will be unable to make these claims for an actual underlying 0-level chain. We now argue that the operator $\leq_m$ will signal in favour of the honest parties.

Suppose b is the LCA block between $\pi_{\mathcal{A}}$ and π_B. If the chain forks at b, there can be no more adversarial blocks after b than honest blocks after b, provided there are at least k honest blocks (due to the Common Prefix property). We will now argue that, further, there can be no more disjoint $\mu_{\mathcal{A}}$-level superblocks than honest μ_B-level superblocks after b.

To see this, let b be an honest block generated at some round r_1 and let the honest proof have been generated at some round r_3. Then take the sequence of consecutive rounds $S = (r_1, \cdots, r_3)$. Because the verifier requires at least m blocks from each of the provers, the adversary must have m $\mu_{\mathcal{A}}$-superblocks in $\pi_{\mathcal{A}}\{b :\}$ which are not in $\pi_B\{b :\}$. Therefore, using a negative binomial argument, we see that $|S|$ must be long; intuitively, it takes a long time to produce a lot of blocks $|\pi_{\mathcal{A}}\{b :\}|$. Given that $|S|$ is long and that the honest parties have more mining power, they must have been able to produce a longer $\pi_B\{b :\}$ argument (of course, this comparison will have the superchain lengths weighted by $2^{\mu_{\mathcal{A}}}, 2^{\mu_B}$ respectively). To prove this, we use a binomial argument; intuitively, given a long time $|S|$, a lot of μ_B-superblocks $|\pi_B\{b :\}|$ will have been honestly produced.

We therefore have a fixed value for the length of the adversarial argument, a negative binomial random variable for the number of rounds, and a binomial random variable for the length of the honest argument. By taking the expectations of the above random variables and applying a Chernoff bound, we see that the actual values will be close to their means with overwhelming probability, completing the proof.

Remark (Variance attacks). The critical issue addressed by this security proof is to avoid Bahack-style attack [3] where the adversary constructs "lucky" high-difficulty superblocks without filling in the underlying proof-of-work in the lower levels. Observe that, while setting $m = 1$ "preserves" the proof-of-work in the sense that expectations remain the same, the probability of an adversarial attack becomes approximately proportional to the adversary power if the adversary follows a suitable strategy (for a description of such a strategy, see the parameterization section). With higher values of m, the probability of an adversarial attack drops exponentially, even though they maintain constant computational power, and hence satisfy a strong notion of security. This is due to the application of the Chernoff bound.

5.3. Succinctness

We now prove that our construction produces succinct proofs.

Observe that full succinctness in the standard honest majority model is impossible to prove for our construction. To see why, recall that an adversary with sufficiently large mining power can significantly harm superquality as described in Subsection 4. By causing this harm in superquality of a sufficiently low level μ, for example $\mu = 3$, the adversary can cause the honest prover to digress in their proofs from high-level superchains down to low-level superchains, causing a linear proof to be produced.

Having established security in the general case of the standard honest majority model, we now concentrate our succinctness claims to the particular "optimistic" case where the adversary does not use their (minority) computational power or network power. Therefore, the superquality of the chain must be the same as a fully honestly-generated chain generated with no network adversary. Last, for now, we will not allow the adversary to produce any proofs; that is, all proofs consumed by the verifier are honestly-generated. We will lift this last assumption shortly.

Theorem 5.2 (Number of levels). *The number of superblock levels which have at least m blocks are at most* $\log(|S|)$*, where S is the set of all blocks produced, with overwhelming probability in m.*

The above theorem establishes that the number of superchains is small. What remains to be shown is that few blocks will be included at each superchain level.

Theorem 5.3 (Large upchain expansion). *Let $\mathcal{C}$ be an honestly generated chain and let $\mathcal{C}' = \mathcal{C}\uparrow^{\mu-1}[i : i{+}\ell]$ with $\ell \geq 4m$. Then $|\mathcal{C}'\uparrow^{\mu}| \geq m$ with overwhelming probability in m.*

Corollary 5.3.1 (Small downchain support). *Assume an honestly generated chain $\mathcal{C}$ and let $\mathcal{C}' = \mathcal{C}\uparrow^{\mu}[i : i{+}m]$. Then $|\mathcal{C}'{\downarrow\!\!\uparrow}^{\mu-1}| \leq 4m$ with overwhelming probability in m.*

This last theorem establishes the fact that the support of blocks needed under the m-sized chain suffix to move from one level to the one below is small. Based on this, we can establish our theorem on succinctness:

Theorem 5.4 (Optimistic succinctness). *Non-interactive proofs-of-proof-of-work produced by honest provers in the optimistic case are succinct with the number of blocks bounded by* $4m\log(|\mathcal{C}|)$*, with overwhelming probability in m.*

Certificates of badness. We proved security in the general case, but succinctness only in the optimistic case. Examine now the optimistic case extended with the adversary's ability to provide proofs. It is then still possible for that adversary to produce large dummy (incorrect) proofs that use up the resources of a verifier with logarithmic time. A verifier knowing ahead of time that a proof will be large cannot conclude that the prover providing it is malicious, as an attack on superquality of the blockchain could be taking place (and we want to maintain security in general), requiring an honest prover to provide long proofs.

To avoid the above undesirable scenario, we offer a generalization of our above construction. Our extended construction allows the verifier to stop processing input early, in a streaming fashion, thereby only requiring logarithmic communication complexity. To achieve this, observe that honest proofs need to be large only if there is a violation of *goodness*. However, goodness is not harmed when the chain is not under attack by the adversarial computational power or network. As such, we require the prover to produce a *certificate of badness* in case there is a violation of *goodness* in the blockchain. This certificate will always be logarithmic in size and must be sent prior to the rest of the proof by the prover to the verifier. Because the certificate will be logarithmic in size even in the case of an adversarial attack on the chain, the honest verifier can stop processing the certificate after a logarithmic time bound. If the certificate is claimed to be longer, the honest verifier can reject early by deciding that the prover is adversarial. Looking at the certificate, the honest verifier determines whether there is a possibility for a lack of goodness in the underlying chain. If there's no adversarial computational power in use, the certificate is impossible to produce.

The certificates of badness are produced simply. First, the honest verifier finds the maximum level max-μ at which there are at least m max-μ-superblocks and includes it in the certificate. Then, because there is a violation of goodness there must exist two levels $\mu < \mu'$ such that $2^{\mu}|\mathcal{C}{\uparrow}^{\mu}| > (1+\delta)2^{\mu'}|\mathcal{C}{\uparrow}^{\mu'}|$ in some part $\mathcal{C}$ of the honestly adopted chain. But $\mu' - \mu \leq$ max-μ. Therefore, there must exist two adjacent levels $\mu_1 < \mu_2$ which break goodness but with error parameter $(1+\delta)^{1/\text{max-}\mu}$. In particular, it will hold that $2^{\mu_1}|\mathcal{C}{\uparrow}^{\mu_1}| > (1+\delta)^{1/\text{max-}\mu}2^{\mu_2}|\mathcal{C}{\uparrow}^{\mu_2}|$. This condition is direct for the prover to find and trivial for the verifier to check and completes the construction.

6. Blockchain Infix proofs

6.1. Construction

The previous proofs we constructed proved suffix. The most useful class of proofs allow proving more general predicates that can depend on multiple blocks even buried deep within the blockchain.

We extend our prover to support this generalization. The generalized prover for *infix proofs* allows proving any predicate $Q(\mathcal{C})$ that depends on a number of blocks that can appear anywhere within the chain (except the k suffix for stability). These blocks constitute a *subset* $\mathcal{C}'$ of blocks which may not necessarily be a stand-alone blockchain. This allows proving powerful statements such as, for example, whether a transaction took place.

Definition 6.1 (Infix sensitivity)**.** A chain predicate $Q_{\ell,d,k}$ is k-infix sensitive if it can be written in the form

$$Q_{\ell,d,k}(\mathcal{C}) = \begin{cases} \text{undefined, if } |\mathcal{C}| < \ell, \text{ otherwise:} \\ \text{true, if } \exists \mathcal{C}' \subseteq \mathcal{C}[:-k] : |\mathcal{C}'| \leq d \wedge P(\mathcal{C}') \\ \text{false, otherwise} \end{cases}$$

Where P is an arbitrary predicate. Note that $\mathcal{C}'$ is a blockset and may not necessarily be a blockchain.

Similarly to suffix-sensitive predicates, infix-sensitive predicates Q can be evaluated by a short evaluation. In particular, here, because of the form of infix-sensitive predicates, to evaluate $Q(\mathcal{C})$ it suffices to pass to Q any short blockset $\mathcal{C}'' \subseteq \mathcal{C}$ such that there is a $\mathcal{C}' \subseteq \mathcal{C}''$ which satisfies the condition that $|\mathcal{C}'| \leq k \wedge P(\mathcal{C}')$. The $|\mathcal{C}| < \ell$ portion can be determined by examining the block number.

The construction of these proofs is shown in Algorithm 6. The infix prover accepts two parameters: The chain $\mathcal{C}$ which is the full blockchain and $\mathcal{C}'$ which is a sub-blockset of the blockchain whose blocks are of interest for the predicate in question. The prover calls the previous suffix prover to produce a proof as usual. Then, having the prefix π and suffix χ of the suffix proof in hand, the infix prover adds a few auxiliary blocks to the prefix π. The prover ensures that these auxiliary blocks form a chain with the rest of the proof π. Such auxiliary blocks are collected as follows: For every block B of the subchain $\mathcal{C}'$, the immediate previous (E') and next (E) blocks in π are found. Then, a chain of blocks R which connects E back to B' is found by the algorithm followDown. If E' is of level μ, there can be no other μ-superblock between E' and B', otherwise it would have been included in π. Therefore, B' already contains a pointer to E' in its interlink, completing the chain.

Algorithm 5 The followDown function which produces the necessary blocks to connect a superblock hi to a preceeding regular block lo.

```
1: function followDown($hi$, $lo$, depth)
2:     $B \leftarrow hi$
3:     $aux \leftarrow [\,]$
4:     $\mu \leftarrow level(hi)$
5:     while $B \neq lo$ do
6:         $B' \leftarrow \mathsf{blockById}[B.\text{interlink}[\mu]]$
7:         if $\mathsf{depth}[B'] < \mathsf{depth}[lo]$ then
8:             $\mu \leftarrow \mu - 1$
9:         else
10:            $aux \leftarrow aux \cup \{B\}$
11:            $B \leftarrow B'$
12:        end if
13:    end while
14:    return $aux$
15: end function
```

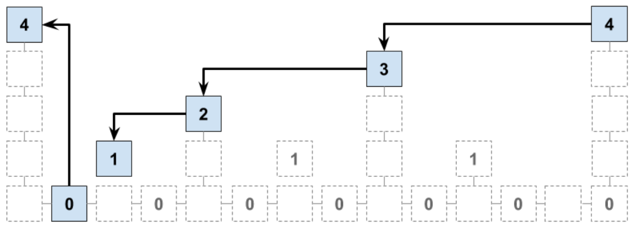
The way to connect a superblock to a previous lower-level block is implemented in Algorithm 5. Block B' cannot be of higher or equal level than E, otherwise it would be equal to E and the followDown algorithm would return. The algorithm proceeds as follows: Starting at block $hi = E$, it tries to follow a pointer to as far as possible. If following the pointer surpasses $lo = B'$, then the following is aborted and a lower level is tried, which will cause a smaller step within the skiplist. If a pointer was followed without surpassing B', the operation continues from the new block, until eventually B' will be reached, which concludes the algorithm.

Algorithm 6 The Prove algorithm for infix proofs

```
 1: function $\mathsf{ProveInfix}_{m,k}(\mathcal{C}, \mathcal{C}', \text{depth})$
 2:     $(\pi, \chi) \leftarrow \mathsf{Prove}_{m,k}(\mathcal{C})$
 3:     for $B' \in \mathcal{C}'$ do
 4:         for $E \in \pi$ do
 5:             if $\mathsf{depth}[E] \geq \mathsf{depth}[B']$ then
 6:                 $R \leftarrow \mathsf{followDown}(E, B', \mathsf{depth})$
 7:                 $aux \leftarrow aux \cup R$
 8:                 break
 9:             end if
10:             $E' \leftarrow E$
11:         end for
12:     end for
13:     return $(aux \cup \pi, \chi)$
14: end function
```

An example of the output of followDown is shown in Figure 3. This is a portion of the proof shown at the point where the superblock levels are at level 4. A descend to level 0 was necessary so that a regular block would be included in the chain. The level 0 block can jump immediately back up to level 4 because it has a high-level pointer.

Figure 3. An infix proof descend. Only blue blocks are included in the proof. Blue blocks of level 4 are part of π, while the blue blocks of level 1 and 3 are produced by followDown to get to the block of level 0 which is part of $\mathcal{C}'$.



The verification algorithm must then be modified as in 7.

Algorithm 7 The verify algorithm for the NIPoPoW infix protocol

```
 1: function $\mathsf{verify\text{-}infx}^{Q}_{m,k}(\mathcal{P})$
 2:     $\mathsf{blockById} \leftarrow \emptyset$        ▷ Initialize empty hashmap
 3:     for $(\pi, \chi) \in \mathcal{P}$ do
 4:         for $B \in \pi$ do
 5:             $\mathsf{blockById}[B.\mathsf{id}] \leftarrow B$
 6:         end for
 7:     end for
 8:     $\tilde{\pi} \leftarrow$ best $\pi \in \mathcal{P}$ according to suffix verifier
 9:     return $Q(\mathsf{ancestors}(\tilde{\pi}[-1], \mathsf{blockById}))$
10: end function
```

The algorithm works by calling the old verifier. It also maintains a blockDAG collecting blocks from all proofs (it is a DAG because *interlink* can be adversarially defined). This DAG is maintained in the $\mathsf{blockById}$ hashmap. Using it, $\mathsf{ancestors}$ uses simple graph search to extract the set of ancestors of a block present. In the final predicate evaluation, the set of ancestors of the best blockchain tip is passed to the predicate. The ancestors are included to avoid an adversary who presents an honest chain but skips the blocks of interest. The verifier must also check that $|\mathcal{C}| > \ell$.

6.2. Security

Theorem 6.1. *The infix NIPoPoW construction is secure for all infix-sensitive stable predicates Q, except with negligible probability in κ.*

The proof is included in the Appendix.

6.3. Succinctness

As long as the number of blocks on which the predicate depends is polylogarithmic ($< d$) with respect to the chain length, our proofs remain succinct. Specifically, the proof size for the suffix has exactly the same size. Then the part of the proof that is of interest is the output of the followDown algorithm. However, notice that this algorithm will on average produce as many blocks as the difference of levels between B' and E, which is at most logarithmic in the chain size. Hence the proof sizes will be in expectation $(m + |\mathcal{C}'|)\log(|\mathcal{C}|)$, which remains succinct if $|\mathcal{C}'| \in O(polylog(|\mathcal{C}|))$.

7. Gradual Deployment Paths

Our construction requires an upgrade to the consensus layer. We envision that new cryptocurrencies will adopt our construction in order to support efficient light clients. However, existing cryptocurrencies could also benefit by adopting our construction as an upgrade. In this section we outline several possible upgrade paths. We also contribute a novel upgrade approach, a "velvet fork," which allows for gradual deployment without harming unupgraded miners.

7.1. Hard Forks and Soft Forks

The obvious way to upgrade a cryptocoin to support our protocol is a hard fork: the block header is modified to include the interlink structure, and the validation rules modified to require that new blocks (after a "flag day") contain a correctly-formed interlink hash.

Hard forks are not "forward compatible," and are best avoided. A soft fork construction requires including the interlink not in the block header, but in the coinbase transaction. It is enough to only store a hash of the interlink structure. The only requirement for the PoPoWs to work is that the PoW commits to all the pointers within interlink so that the adversary cannot cause a chain reorg. If we take that route, then each PoPoW will be required to present not only the block header, but also a proof-of-inclusion path within the Merkle tree of transactions proving that the coinbase transaction is indeed part of the block. Once that is established, the coinbase data can be presented, and the verifier will thereby know that the hash of the interlink data structure is correct. Given that in Bitcoin implementation there is a block size limit, observe that including such proofs-of-inclusion will only increase the PoPoW sizes by a constant factor per block, allowing for the communication complexity to remain polylogarithmic.

7.2. Velvet Forks

We now describe a novel upgrade path that avoids the need for a fork at all. The key idea is that clients can make use of our scheme, even if only some blocks in the blockchain include the interlink structure. Given that intuitively these changes constitute rule modifications to the consensus layer, we call this technique a *velvet fork*.

We require upgraded miners to include the interlink data structure in the form of a new Merkle tree root hash in their coinbase data, similar to a soft fork. An unupgraded miner will ignore this data as comments. We further require the upgraded miners to accept all previously accepted blocks, regardless of whether they have included the interlink data structure or not. Even if the interlink data structure is included and contains invalid data, we require the upgraded miners to accept their containing blocks. Malformed interlink data could be simply of the wrong format, or the pointers could be pointing to superblocks of incorrect levels. Furthermore, the pointers could be pointing to superblocks of the correct level, but not to the most recent block. By requiring upgraded miners to accept all such blocks, we do not modify the set of accepted blocks. Therefore, the upgrade is simply a "recommendation" for miners and not an actual change in the consensus rules. The velvet fork has the effect that blocks produced by either upgraded or unupgraded clients are valid for either. Hence, the blockchain is never forked. Only the codebase is upgraded, and the data on the blockchain is interpreted differently.

The reason this can work is because provers and verifiers of our protocol can check the validity of the claims of miners who make false interlink chain claims. An upgraded prover can check whether a block contains correct interlink data and use it. If a block does not contain correct interlink data, the prover can opt not to use those pointers in their proofs. The Verifier verifies all claims of the prover, so adversarial miners cannot cause harm by including invalid data. The one thing the Verifier cannot verify in terms of interlink claims is whether the claimed superblock of a given level is the most recent previous superblock of that level. However, an adversarial prover cannot make use of that to construct winning proofs, as they are only able to present shorter chains in that case.

The velvet prover works as usual, but additionally maintains a *realLink* data structure, which stores the *correct* interlink for each block. Whenever a new winning chain is received from the network, the prover checks it for blocks that it hasn't seen before. For those blocks, it maintains its own realLink data structure which it updates accordingly to make sure it is correct regardless of what the interlink data structure of the received block claims.

The velvet $\mathcal{C}\uparrow$ operator is implemented identically as before, except that instead of following the interlink pointer blindly it now calls the helper function *followUp*, shown in Algorithm 8. It accepts block B and level μ and creates a connection from B back to the most recent preceding μ-superblock, by following the interlink pointer if it is correct. Otherwise, it follows the previd link which is available in all blocks, and tries to follow the interlink pointer again from there. Finally, the velvet prover simply applies the velvet $\mathcal{C}\uparrow$ operator and includes the auxiliary connecting nodes within the final proof. No changes in the verifier are needed.

Algorithm 8 followUp produces the blocks to connect two superblocks in velvet forks.

```
function followUp($B$, $\mu$, realLink, blockById)
    aux ← [$B$]
    while $B \neq$ Gen do
        if $B$.interlink[$\mu$] = realLink[id($B$)][$\mu$] then
            $id \leftarrow B$.interlink[$\mu$]
        else                                    ▷ Invalid interlink
            $id \leftarrow B$.interlink[0]
        end if
        $B \leftarrow$ blockById[$id$]
        aux ← aux ∪ {$B$}
        if $level(B) = \mu$ then
            return $B$, aux
        end if
    end while
    return $B$, aux
end function
```

Velvet NIPoPoWs preserve security. Additionally, if a constant minority of miners has upgraded their nodes, then succinctness is also preserved, as there is only a constant factor penalty as proven in the following theorem.

Theorem 7.1. *Velvet non-interactive proofs-of-proof-of-work on honest chains by honest provers remain succinct as long as a constant percentage g of miners has upgraded, with overwhelming probability.*

The reason we were able to upgrade using a velvet fork was because the changes we made were helpful but verifiable by those looking at the chain. We would

Table 1. Size of NIPoPoWs applied to Bitcoin today ($\approx$450k blocks) for various values of m, setting $k = 6$.

m	NIPoPoW size	Blocks	Hashes
6	70 kB	108	1440
15	146 kB	231	2925
30	270 kB	426	5400
50	412 kB	656	8250
100	750 kB	1206	15000
127	952 kB	1530	19050

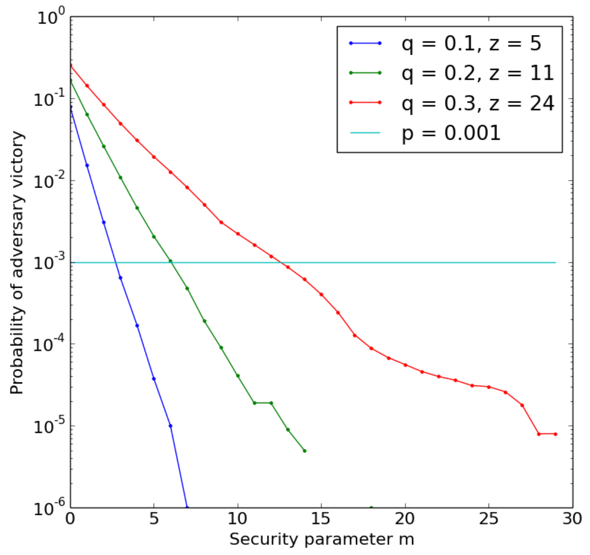
not have been able to pull off this upgrade without modifications to the consensus layer in the sense that the interlink data structure could not have been maintained somewhere independently of the blockchain: It is critical that the proof-of-work commits to the interlink data structure. Interestingly, the interlink data structure does not need to be part of coinbase and can be produced and included in regular transactions by users (such as OP_RETURN transactions). Thus, the miners can be completely oblivious to it, while users and provers make use of the feature. Interested users regularly create transactions containing the most recent interlink pointers so that they are included in the next block. If the transaction makes it to the next block, it can be used by the prover who keeps track of these. Otherwise, if it becomes part of a subsequent block, in which case some of the pointers it contains are invalid, it can be ignored or only partially used.

8. Implementation & Parameters

To determine concrete values for security parameter m, we focus on a particular adversarial strategy and analyze its probability of success. This adversarial strategy is only one possible strategy; an advanced adversary can perform more sophisticated attacks. However, the fact that this attack is reasonable and possible to simulate allows us to extract specific values for m. The attack is an extension of the stochastic processes described in [15] and [19].

The experiment works as follows: m is fixed and some adversarial computational power percentage q of the total network computational power is chosen. k is chosen based on q according to Nakamoto [15]. The number of blocks y during which parallel mining will occur is also fixed. The experiment begins with the adversary and honest parties sharing a common blockchain which ends in block B. After B is mined, the adversary starts mining in secret and in parallel with the honest parties on his own private fork on top of B. She ignores the honest chain, so that the two chains remain disjoint after B. As soon as y blocks have been mined in total, the adversary attempts a double spend by creating two conflicting transactions which are committed to an honest block and an adversarial block respectively on top of each current chain. Finally, the adversary mines k blocks on top of the double spending transaction within her private chain. After these k blocks have been mined, she

Figure 4. Simulation results for a private mining attacker with k according to Nakamoto and parallel mining parameter $y = 100$. Probabilities in logarithmic scale. The horizontal line indicates the threshold probability of [15] is indicated by the horizontal line.



publishes her private chain in an attempt to overcome the honest chain.

We measure the probability of success of this attack. We experiment with various values of m for $y = 100$, indicating 100 blocks of secret parallel mining. We make the assumption that honest party communication is perfect and immediate. We ran $1,000,000$ Monte Carlo executions [2] of the experiment for each value of m from 1 to 30. We ran the simulation for values of computational power percentage $q = 0.1$, $q = 0.2$ and $q = 0.3$. The results are plotted in Figure 4.

Based on this data, we conclude that $m = 5$ is sufficient to achieve a 0.001 probability of failure against an adversary with 10% mining power. To secure against an adversary with more than 30% mining power, a choice of $m = 15$ is needed.

9. Applications & Evaluation

Multi-blockchain wallets. An application of our technique is an efficient multi-cryptocoin client. Consider a merchant who wishes to accept payments in any cryptocoin, not just the popular ones. She could install an SPV client for each known cryptocoin. This approach would entail downloading the header chain for each cryptocoin, and periodically syncing up by fetching any newly generated block headers. An alternative would be to use an online service supporting multiple currencies,

2. The URL to the GitHub repository of this MIT-licensed experiment has been redacted for anonymity and will be provided in the proceedings version.

Table 2. Cost of header chains for all active PoW-based cryptocoins (collected from coinwarz.com)

Hash	Coins	Size today	Growth rate
Scrypt	44	4.3 GB	5.5 MB / day
SHA-256	15	1.4 GB	937.0 kB / day
X11	5	581.1 MB	556.3 kB / day
Quark	3	647.9 MB	518.4 kB / day
CryptoNight	2	199.0 MB	115.2 kB / day
EtHash	2	588.6 MB	921.6 kB / day
Groestl	2	300.3 MB	184.2 kB / day
NeoScrypt	2	310.6 MB	153.6 kB / day
Others	5	266.2 MB	311.1 kB / day
Total	80	8.5 GB	9.2 MB / day

but this introduces reliance on a third party (e.g. Jaxx and Coinomi rely on third party networks).

A NIPoPoW-based client maintains a most recent k-stable block hash for each of its supported cryptocoins. Each time a payment is received, the client connects to peers on the corresponding network and asks for a NIPoPoW proof relative to the most recently stored block hash. For popular cryptocoins, the NIPoPoW-based client downloads nearly every block header, like an ordinary SPV client; for cryptocoins used infrequently, the NIPoPoW-based client can skip over most blocks.

Simulation. We simulated the resources savings resulting from the use of a NIPoPoW-based client. We model the arrival of payments in each cryptocoin as a Poisson process and assume that the market cap of a cryptocoin is a proxy for usage. Currently, a total of 731 cryptocurrencies are listed on coin market directories[3]. We narrow our focus to the 80 cryptocurrencies that have their own PoW blockchains (i.e., no PoS) with a market cap of over USD $100,000.

In Table 2 we show aggregate statistics about these 80 cryptocurrencies, grouped according to the their PoW puzzle. While the entire chain in Bitcoin only amounts to 40 MB, taken together, the 80 cryptocurrencies comprise 10 GB of proofs-of-work, and generate 10 MB more each day. In Table 3 we show the resulting bandwidth costs from simulating a period of 60 days with $m = 24, k = 6$, with varying rates of payments received.

For the naïve SPV client, the total bandwidth cost is dominated by fetching the entire chain of headers, which the NIPoPoW client does not do. The marginal cost for naïve SPV depends on the number of blocks generated each day, as well as the transaction inclusion proofs associated with each payment. The NIPoPoW based client provides the most savings when the number of transactions per day is smallest, reducing the necessary bandwidth per day (not including the initial sync up) by 90%.

Certificate Transparency. In Catena [21] the Bitcoin blockchain is used as an equivocation-resistant public log in which to publish SSL certificate commitments. The client is based on the BitcoinJ library, and therefore

3. https://coinmarketcap.com/

Table 3. Simulated bandwidth of multi-blockchain clients after two months (Averaged over 10 trials each)

Daily	Naive SPV		NIPoPoW		
tx	Total	(Daily)	Total	(Daily)	Savings
100	5.5 GB	(5.5 MB)	31.7 MB	(507 kB)	99% (91%)
500	5.5 GB	(5.7 MB)	68.2 MB	(1.1 MB)	99% (81%)
1000	5.5 GB	(6.0 MB)	99.1 MB	(1.6 MB)	98% (73%)
3000	5.6 GB	(7.0 MB)	192 MB	(3.1 MB)	97% (56%)

requires downloading the entire chain. Catena could therefore immediately be improved using NIPoPoWs. The Catena authors anticipate needing to launch a dedicated Header Relay Network [21] to accommodate the extra bandwidth demands from new Catena clients. A variant based on NIPoPoWs could obviate this, since it eliminates the need to bootstrap new clients by transmitting the entire 40MB header chain. Second, the steady state cost of operating a Catena client depends on how frequently certificate digests are published. For example, one usage scenario cited by Catena [21] is Keybase, a service which publishes certificate digests every 6 hours. During a 6 hour period, Bitcoin would generate 6 kilobytes of headers, whereas a NIPoPoW proof covering the same range would require less than half this size. The savings would increase further if Catena were implemented using any PoW blockchain with more frequent blocks.

Sidechains. It is widely known that Bitcoin faces significant scaling hurdles [7], but upgrading Bitcoin is notoriously difficult. A widely anticipated solution is to treat the Bitcoin blockchain as a host for "sidechains," which are proof-of-work blockchains separate from Bitcoin, but that can be backed by Bitcoin deposits. Our efficient NIPoPoW construction solves an open problem posed by Back et al., and thus enables this vision.

The idea behind sidechains is to implement an SPV verifier for one blockchain (the "sidechain") as a smart contract within another blockchain (the "host chain"). An (inefficient) implementation of this idea, called BTCRelay [22], is already running on Ethereum. It is an Ethereum smart contract allowing users to submit Bitcoin PoW, which it validates and stores. This allows Ethereum smart contracts to condition their behavior based on Bitcoin transactions. The downside is that every Bitcoin header must be stored. Currently the cost of Ethereum blockchain storage 7 cents per kB. If Bitcoin were upgraded with the interlink data structure, the BTCRelay functionality could be provided cheaply.

The anticipated use of sidechains is to implement a virtual asset on the sidechain backed by currency on the host chain (a two-way peg) [1]. This is accomplished by defining two new transaction types: "deposit" transactions lock coins on the host chain and create new assets on the side chain; "withdrawal" transactions do the opposite. Deposit transactions committed on the host chain are delivered to the sidechain using an SPV proof; vice versa for withdrawal transactions. To confirm transaction inclusion, NiPoPoW can be used with an infix predicate — the predicate claims that a transaction occurs at a particular height. Lest the host chain receives a fraudulent NiPoPoW withdrawal, the host chain must wait for a period of time during which honest parties may submit a better proof. Succinctness is important for sidechains, since the proofs must be included in the host chain and are thus costly.

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Algorithm 9 The backbone protocol

```
1: $\mathcal{C} \leftarrow \varepsilon$
2: $st \leftarrow \varepsilon$
3: $round \leftarrow 1$
4: while TRUE do
5:     $\tilde{\mathcal{C}} \leftarrow \mathsf{maxvalid}(\mathcal{C}, \text{any chain } \mathcal{C}' \text{ found in } \textsc{Receive}())$
6:     if INPUT() contains READ then
7:         write $R(\mathcal{C})$ to OUTPUT()
8:     end if
9:     $\langle st, x\rangle \leftarrow I(st, \tilde{\mathcal{C}}, round, \textsc{Input}(), \textsc{Receive}())$
10:    $\mathcal{C}_{\mathsf{new}} \leftarrow \mathsf{pow}(x, \tilde{\mathcal{C}})$
11:    if $\mathcal{C} \neq \mathcal{C}_{\mathsf{new}}$ then
12:        $\mathcal{C} \leftarrow \mathcal{C}_{\mathsf{new}}$
13:        DIFFUSE($broadcast$)
14:    else
15:        DIFFUSE($\perp$)
16:    end if
17:    $round \leftarrow round + 1$
18: end while
```

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Appendix

In this section, we illustrate the detailed formalisms to integrate our proof systems with the backbone model [11].

The backbone protocol of a miner node is shown in Algorithm 9. The honest miner maintains the longest chain from the network and tries to mine on top of it. In Algorithm 10, we illustrate our new entity for the model, the full node or *prover*. While the full node is not mining, it maintains a state with the longest chain from the network. Furthermore, whenever it is asked to prove a predicate Q about its local chain $\mathcal{C}$, it calls a Prove function to provide the proof. We leave this Prove function undefined here, as it is part of the concrete protocol construction. On the other hand, in Algorithm 11 we illustrate another new entity to the model, the generic *verifier*, which is stateless, but receives proofs from the provers on the network (via the environment) and takes a decision about a predicate on the blockchain it believes to be the longest.

Algorithm 10 Our generic honest Prover entity addition to the backbone model. It is augmented with a *proof generating function* $\mathsf{Prove}^Q(\cdot)$ parameterized by a predicate Q

```
1: $\mathcal{C} \leftarrow \varepsilon$
2: while TRUE do
3:     $\tilde{\mathcal{C}} \leftarrow \mathsf{maxvalid}(\mathcal{C}, \text{any chain } \mathcal{C}' \text{ found in } \textsc{Receive}())$
4:     if INPUT() contains READPROOF then
5:         $\pi \leftarrow \mathsf{Prove}^Q(\mathcal{C})$
6:         DIFFUSE($\pi$)
7:     else
8:         DIFFUSE($\perp$)
9:     end if
10: end while
```

Algorithm 11 The generic proof verifier augmented with a *proof verification function* $\mathsf{Verify}^Q(\cdot)$ parameterized by a predicate Q

```
1: $\Pi \leftarrow$ all proofs found in RECEIVE()
2: if $\Pi \neq \emptyset$ then
3:     $\mathcal{C} \leftarrow \mathsf{Verify}^Q(\Pi)$
4:     write $\mathcal{C}$ to OUTPUT()
5: end if
6: if INPUT() contains CHALLENGE then
7:     DIFFUSE(READPROOF)
8: end if
```

The prover which additionally maintains the *blockById*, *depth* and *realLink* data structures is illustrated in Algorithm 12.

We now proceed to calculate the attack probabilities precisely and in formal detail. We simplify the above attack to ease the formal probabilistic analysis. The attack is parameterized by two parameters r and μ which are picked by the adversary. μ will be the superblock level at which the adversary will attempt to produce a proof longer than the honest proof. The modified attack works precisely as follows: Without loss of generality, we fix block b to be the Genesis block. The adversary always mines on the secret chain which forks off from genesis, unless a *superblock generation event* occurs. If a superblock generation event occurs, then the adversary pauses mining on the secret chain and attempts a *block suppression attack* on the honest chain. The adversary devotes exactly r rounds to this suppression attack and then resumes mining on their secret chain. Our goal is to show that, despite this simplification (of fixing r) which is harmful to the adversary, the probability of a successful attack is non-negligible for certain (reasonable) values of the protocol parameters.

Algorithm 12 The modified prover that allows for a velvet fork.

```
1: $\mathcal{C} \leftarrow [\mathsf{Gen}]$
2: $\mathsf{realLink} \leftarrow \{\}$
3: $\mathsf{blockById} \leftarrow \{\}$
4: $\mathsf{genId} \leftarrow \mathsf{id}(\mathsf{Gen})$
5: $\mathsf{blockById}[\mathsf{genId}] \leftarrow \mathsf{Gen}$
6: $\mathsf{realLink}[\mathsf{genId}] \leftarrow [\mathsf{Gen}]$
7: while True do
8:     $\tilde{\mathcal{C}} \leftarrow \mathsf{maxvalid}(\mathcal{C}, \text{any chain } \mathcal{C}' \text{ found in } \textsc{Receive}())$
9:     $\mathsf{new} \leftarrow []$
10:    for $i = |\mathcal{C}| - 1$ down to 0 do
11:        if $\mathcal{C}[i] \in \mathsf{realLink}$ then
12:            $B' \leftarrow \mathcal{C}[i]$
13:            $\mathsf{interlink} \leftarrow \mathsf{realLink}[B']$
14:            break
15:        end if
16:        $\mathsf{new.append}(\mathcal{C}[i])$
17:    end for
18:    for $B \in \mathsf{new}$ do
19:        for $\mu = 0$ to $level(B')$ do
20:            $\mathsf{interlink}[\mu] \leftarrow \mathsf{blockid}(B')$
21:        end for
22:        $B' \leftarrow B$
23:        $\mathsf{blockById}[blockid(B)] \leftarrow B$
24:        $\mathsf{realLink}[B] \leftarrow \mathsf{interlink}$
25:    end for
26:    if Input() contains ReadProof then
27:        $\tilde{\Pi} \leftarrow \mathsf{Prove}'_q(\mathcal{C}, \mathsf{realLink}, \mathsf{blockById})$
28:        $\textsc{Diffuse}(\tilde{\Pi})$
29:    else
30:        $\textsc{Diffuse}(\perp)$
31:    end if
32: end while
```

A superblock generation event is detected by the adversary by monitoring the network. Whenever an honest party diffuses an honestly-generated μ-superblock at the end of a given round r_1, then the superblock generation event will have occurred and the adversary will starting devoting their mining power to block suppression starting from the next round.

A block suppression attack works as follows. Let B be the honestly generated μ-superblock which was diffused at the end of the previous round. If the round was not uniquely successful, let B be any of the diffused honestly-generated μ-superblocks. Let B be the tip of an honest chain $\mathcal{C}_B$. The adversary first mines on top of $\mathcal{C}_B[-2]$. If she is successful in mining a block B', then she continues extending the chain ending at B' (to mine B'' and so on). The value r is fixed, and so the adversary devotes exactly r rounds to this whole process; the adversary will keep mining on top of $\mathcal{C}_B[-2]$ (or one of the adversarially-generated extensions of it) for exactly r rounds, regardless of whether B' or B'' have been found. At the same time, the honest parties will be mining on top of B (or a competing block in the case of a non-uniquely successful round). Again, further successful block diffusion by the honest parties shall not affect that the adversary is going to spend exactly r rounds for suppression.

Having laid out the attack scenario precisely, we are ready to prove that it will succeed with non-negligible probability. In fact, as we will see, perhaps surprisingly, it will succeed with overwhelming probability for the right choice of protocol values.

Theorem A.1 (Double-spending attack)**.** *There exist parameters p, n, t, q, μ, δ, with $t \leq (1-\delta)(n-t)$, and a double spending attack against KLS PoPoW that succeeds with overwhelming probability.*

We now discuss the manner in which the interlink data structure is hashed for coinbase inclusion. A Merkle tree is used to hash the interlink data structure into a single hash [13]. NIPoPoWs form a chain of various levels which can omit blocks. For each block in the proof, only a single pointer needs to be presented to convince the Verifier. Near genesis, the pointers needed correspond to high levels; near the tip, the pointers to low levels. In the construction of the proof, the highest superchain with at least m blocks is included, and assume it is of level μ. The level $\mu - 1$ superchain is fully included and has an expected number of $2m$ blocks. Since all μ-superblocks are also $(\mu-1)$-superblocks, they only need to be counted once, for $\mu - 1$. Among the expected $2m$ $(\mu-1)$-superblocks, the last m will be supported by level $\mu - 2$. As before, since $(\mu-1)$-superblocks are $(\mu-2)$-superblocks, in expectation only m $(\mu-1)$-superblocks are counted. The argument continues inductively, until $2m$ 0-blocks are included in expectation immediately before the χ suffix. This gives an estimation on the proof size: a total of $m(\log(|\mathcal{C}|)-\log(m))$ blocks in expectation, m at each of the $\mu - 1$ levels and $2m$ 0-blocks.

Organizing interlink into a Merkle tree of $\log(|\mathcal{C}|)$ items, a proof-of-inclusion is provided in $\log\log(|\mathcal{C}|)$ space; 0-level pointers need not be included in it, but the genesis block does. The root of the tree can be proved to have been included in the block header in $\log(|\overline{x}|)$ using the standard Merkle tree of transactions. This makes the proof size require $\log(|\overline{x}|) + \log\log(|\mathcal{C}|)$ hashes per block for a total of $m(\log(|\mathcal{C}|) - \log(m))(\log(|\overline{x}|) + \log\log(|\mathcal{C}|))$ hashes. In addition, $m(\log(|\mathcal{C}|) - \log(m))$ headers and coinbase transactions are needed. Concretely, given that currently in bitcoin $|\mathcal{C}| = 464,185$ and $|\overline{x}| = 2000$, we have $\log(|\mathcal{C}|) = 18, \log\log(|\mathcal{C}|) = 5, \log(|\overline{x}|) = 11$. For the k-suffix, only k headers are needed. We set $k = 6$ and see that headers are 80 bytes and hashes 32 bytes. For the k-suffix as well as the $2m$ 0-blocks in π, neither coinbase data nor prev ids are needed, limiting header size to 48 bytes. The root and leaves of the pointers tree can be omitted from coinbase when transmitting the proof. In fact, no block ids need to be transmitted. From these observations, we estimate our scheme's proof sizes for various parameterizations of m in Table 1.

Assume there are t adversarial parties and n total parties, each with q proof-of-work random oracle queries per round. We will call a query to the random oracle *μ-successful* if the random oracle returns a value h such that $h \leq 2^{-\mu}T$.

In order to prove our construction secure, we define the boolean random variables X^{μ}_r, Y^{μ}_r and Z^{μ}_r. For each round r and for each query index j and for each k adversarial party index (out of t), define these random variables as follows. If at round i an honest party obtains a PoW with $id < 2^{-\mu}T$, then $X^{\mu}_r = 1$, otherwise $X^{\mu}_r = 0$. If at round r exactly one honest party obtains a PoW with $id < 2^{-\mu}T$, then $Y^{\mu}_r = 1$, otherwise $Y^{\mu}_r = 0$. Regarding the adversary, if at round r the j-th query of the k-th corrupted party is μ-successful, then $Z^{\mu}_{ijk} = 1$, otherwise $Z^{\mu}_{ijk} = 0$. Further define $Z^{\mu}_r = \sum_{k=1}^{t} \sum_{j=1}^{q} Z^{\mu}_{ijk}$. For a set of rounds S, let $X^{\mu}(S) = \sum_{r \in S} X_r$ and similarly define $Y^{\mu}(S), Z^{\mu}(S)$.

We now define a *typical execution.*

Definition A.1. (Typical execution) An execution of the protocol is (ϵ, η)-*typical* if:

Block counts don't deviate. For all $\mu \geq 0$ and any set S of consecutive rounds with $|S| \geq 2^{\mu}\eta\kappa$, we have:

- $(1-\epsilon)E[X^{\mu}(S)] < X^{\mu}(S) < (1+\epsilon)E[X^{\mu}(S)]$ and $(1-\epsilon)E[Y^{\mu}(S)] < Y^{\mu}(s)$.
- $Z^{\mu}(S) < (1+\epsilon)E[Z^{\mu}(S)]$.

Round count doesn't deviate. Let S be a set of consecutive rounds such that $Z^{\mu}(S) \geq k$ for some security parameter k. Then we have that $|S| \geq (1-\epsilon)2^{\mu}\frac{k}{pqt}$ with overwhelming probability in k.

Chain regularity. No insertions, no copies, and no predictions [11] have occurred.

Theorem A.2 (Typicality)**.** *Executions are (ϵ, η)-typical with overwhelming probability in κ.*

The following lemma is at the heart of the security proof that will follow.

Lemma A.3. *Suppose S is a set of consecutive rounds $r_1 \dots r_2$ and $\mathcal{C}_B$ is a chain adopted by an honest party at round r_2 of a typical execution. Let $\mathcal{C}^S_B = \{b \in \mathcal{C}_B :$ b was generated during $S\}$. Let $\mu_{\mathcal{A}}, \mu_B \in \mathbb{N}$. Suppose $\mathcal{C}^S_B{\uparrow}^{\mu_B}$ is good. Suppose $\mathcal{C}'_{\mathcal{A}}$ is a $\mu_{\mathcal{A}}$-superchain containing only adversarially generated blocks generated during S and suppose that $|\mathcal{C}'_{\mathcal{A}}| \geq k$. Then:*

$$2^{\mu_{\mathcal{A}}}|\mathcal{C}'_{\mathcal{A}}| < \frac{1}{3}2^{\mu_B}|\mathcal{C}^S_B{\uparrow}^{\mu_B}|$$

Definition A.2 (Adequate level of honest proof)**.** Let π be an honestly generated proof constructed upon some adopted chain $\mathcal{C}$ and let $b \in \pi$.

Then μ' is defined as follows:

$$\mu' = \max\{\mu : |\pi\{b:\}{\uparrow}^{\mu}| \geq \max(m+1, (1-\delta)2^{-\mu}|\pi\{b:\}{\uparrow}^{\mu}{\downarrow}|)\}$$

We call μ' the *adequate* level of proof π with respect to block b with security parameters δ and m. Note that the adequate level of a proof is a function of both the proof π and the chosen block b.

Lemma A.4. *Let π be some honest proof generated with security parameters δ, m. Let $\mathcal{C}$ be the underlying chain, $b \in \mathcal{C}$ be any block and μ' be the adequate level of the proof with respect to b and the same security parameters. Then $\mathcal{C}\{b:\}{\uparrow}^{\mu'} = \pi\{b:\}{\uparrow}^{\mu'}$.*

Suppose the verifier evaluates $\pi_{\mathcal{A}} \geq \pi_B$ in a protocol interaction where B is honest and assume during the comparison that the compared level of the honest party is μ_B. Let $b = \mathsf{LCA}(\pi_{\mathcal{A}}, \pi_B)$ and let μ'_B be the adequate level of π_B with respect to b. Then $\mu'_B \geq \mu_B$.

Theorem 5.1. *The non-interactive proofs-of-proof-of-work construction for k-stable suffix-sensitive predicates is secure with overwhelming probability in κ.*

Proof. We will prove security by contradiction. Let $m = k_1 + k_2 + k_3$ and let k_1, k_2, k_3 be polynomial functions of the security parameter κ. Let Q be a k-stable suffix sensitive chain predicate. Assume the NIPoPoWs construction with parameters m, k is not secure with respect to Q. Then, during an execution at some round r_3, $Q(\mathcal{C})$ is defined and has the same value for all honest miners. Assume the execution is typical. The verifier V communicates with at least two provers, $\mathcal{A}$, the adversary, and B, an honest prover. The verifier receives the proofs $\pi_{\mathcal{A}}, \pi_B$ from the adversary and the honest prover respectively. Because B is honest, π_B will be a proof constructed based on an underlying blockchain $\mathcal{C}_B$ (with $\pi_B \subseteq \mathcal{C}_B$) which is the adopted blockchain of B during round r_3 at which π_B was generated. Furthermore, $\pi_{\mathcal{A}}$ will have been generated at some round $r'_3 \leq r_3$.

The verifier then outputs $\neg Q(\mathcal{C}_B)$, and so $\mathsf{Verify}^Q_{m,k} = \neg Q(\mathcal{C}_B)$. Thus it is necessary that $\pi_{\mathcal{A}} \geq \pi_B$, otherwise, because Q is suffix sensitive, Verify^Q would have returned $Q(\mathcal{C}_B)$. We will now show that $\pi_{\mathcal{A}} \geq \pi_B$ is a negligible event.

Let $b = \mathsf{LCA}(\pi_{\mathcal{A}}, \pi_B)$ and let b^* be the most recent honestly generated block in $\mathcal{C}_B$ preceeding b (and note that b^* necessarily exists because Genesis was honestly generated). let the levels of comparison decided by the verifier be $\mu_{\mathcal{A}}$ and μ_B respectively.

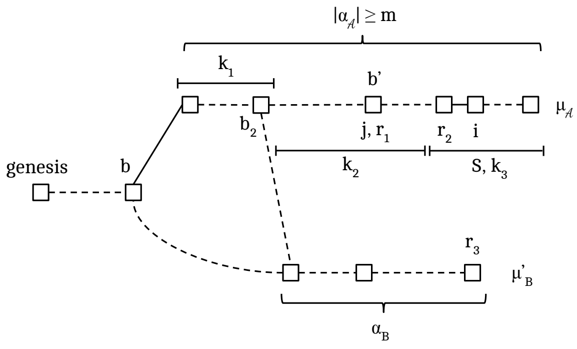
Let μ'_B be the adequate level of proof π_B with respect to block b. Call $\alpha_{\mathcal{A}} = \pi_{\mathcal{A}}{\uparrow}^{\mu_{\mathcal{A}}}\{b:\}$, $\alpha'_B = \pi_B{\uparrow}^{\mu'_B}\{b:\}$.

We will proceed in the proof by showing three successive claims: First, $\alpha_{\mathcal{A}}$ and $\alpha'_B{\downarrow}$ are mostly disjoint. Second, $\alpha_{\mathcal{A}}$ contains mostly adversarially-generated blocks. And third, the adversary is able to produce this $\alpha_{\mathcal{A}}$ with negligible probability.

Claim 1a: If $\mu'_B \leq \mu_{\mathcal{A}}$ then $\alpha_{\mathcal{A}}[1:]$ and $\alpha_B[1:]{\downarrow}$ are completely disjoint.

Applying Lemma A.4 to $\mathcal{C}_B\{b:\}{\uparrow}^{\mu'_B}$ we see that $\mathcal{C}_B\{b:\}{\uparrow}^{\mu'_B} = \pi_B{\uparrow}^{\mu'_B}\{b:\}$ and so indeed $\pi_B{\uparrow}^{\mu'_B}\{b:\}[1:] \cap \pi_{\mathcal{A}}{\uparrow}^{\mu_{\mathcal{A}}}\{b:\}[1:] = \emptyset$.

Figure 5. Two competing proofs at different levels.



Claim 1b: If $\mu_{\mathcal{A}} < \mu'_B$ then $|\alpha_{\mathcal{A}}[1:] \cap \alpha_B\!\downarrow [1:]| \leq 2^{\mu'_B - \mu_{\mathcal{A}}} k_1$.

First, observe that, because the adversary is winning, therefore $|\alpha_{\mathcal{A}}| > 2^{\mu'_B - \mu_{\mathcal{A}}} m$.

Suppose for contradiction that $|\alpha_{\mathcal{A}}[1:] \cap \alpha_B\!\downarrow [1:]| > 2^{\mu'_B - \mu_{\mathcal{A}}} k_1$. This means that there are indices $1 \leq i < j$ such that $|\mathcal{C}_B\!\uparrow^{\mu_{\mathcal{A}}} [i:j]| > 2^{\mu'_B - \mu_{\mathcal{A}}} k_1$ but $|\mathcal{C}_B\!\uparrow^{\mu_{\mathcal{A}}} [i:j]\!\downarrow\,\uparrow^{\mu'_B}| = 0$. But this contradicts the goodness of $\mathcal{C}_B\!\uparrow^{\mu'_B}$. Therefore there are more than $2^{\mu'_B - \mu_{\mathcal{A}}}(k_2 + k_3)$ blocks in $\alpha_{\mathcal{A}}$ that are not in α_B, and clearly also more than $k_2 + k_3$ blocks.

From Claim 1a and Claim 1b, we conclude that there are at least $k_2 + k_3$ blocks after block b in $\alpha_{\mathcal{A}}$ which do not exist in α_B. We now set $b_2 = \mathsf{LCA}(\mathcal{C}_B, \alpha_{\mathcal{A}})$.

Claim 2: At least k_3 superblocks of $\alpha_{\mathcal{A}}$ are adversarially generated.

We will show this by showing that $\alpha_{\mathcal{A}}[k_2 + 1:]$ contains no honestly mined blocks. By contradiction, assume that the block $\alpha_{\mathcal{A}}[i]$ for some $i \geq k_1 + k_2 + 1$ was honestly generated. But this means that an honest party had adopted the chain $\alpha_{\mathcal{A}}[i-1]$ at some round $r_2 \leq r_3$. Because of the way the honest parties adopt chains, this means that the superchain $\alpha_{\mathcal{A}}[:i-1]$ has an underlying properly constructed 0-level anchored chain $\mathcal{C}_{\mathcal{A}}$ such that $\mathcal{C}_{\mathcal{A}} \subseteq \alpha_{\mathcal{A}}[:i-1]$. Let j be the index of block b_2 within $\mathcal{C}_{\mathcal{A}}$. As $\alpha_{\mathcal{A}} \subseteq \mathcal{C}_{\mathcal{A}}$, observe that $|\mathcal{C}_{\mathcal{A}}[j+1:]| > i - 1 \geq k_2 + k_1$. Therefore $\mathcal{C}_{\mathcal{A}}[:-(k_2 + k_1)] \not\preceq \mathcal{C}_B$. But $\mathcal{C}_{\mathcal{A}}$ was adopted by an honest party at round r_2 which is prior to round r_3 during which $\mathcal{C}_B$ was adopted by an honest party. This contradicts the Common Prefix [11] property with parameter k_2. It follows that with overwhelming probability in k_2, the $k_3 = m - k_2 - k_1$ last blocks of the adversarial proof have been adversarially mined.

Claim 3: $\mathcal{A}$ is able to produce a $\alpha_{\mathcal{A}}$ that wins against α_B with negligible probability.

Let b' be the latest honestly generated block in $\alpha_{\mathcal{A}}$, or b^* if no honest block exists in $\alpha_{\mathcal{A}}$. Let r_1 be the round during which b^* was generated. Let j be the index of block b^*. Consider now the set S of consecutive rounds $r_1 \ldots r_3$. Every block in $\alpha_{\mathcal{A}}[-k_3:]$ has been adversarially generated during S and $|\alpha_{\mathcal{A}}[-k_3:]| = k_3$. $\mathcal{C}_B$ is a chain adopted by an honest party at round r_3 and filtering the blocks by the rounds during which they were generated to obtain $\mathcal{C}_B^S$, we see that $\mathcal{C}_B^S = \mathcal{C}_B\{b^*:\}$. But chain $\mathcal{C}_B^S\!\uparrow^{\mu'_B}$ is good with respect to $\mathcal{C}_B^S$. Applying Lemma A.3, we obtain that with overwhelming probability

$$2^{\mu_{\mathcal{A}}}|\alpha_{\mathcal{A}}\{b':\}| < \frac{1}{3} 2^{\mu'_B} |\mathcal{C}_B^S\!\uparrow \mu'_B|$$

But $|\alpha_B| \geq |\mathcal{C}_B^S\!\uparrow \mu'_B|$ and $|\alpha_{\mathcal{A}}\{b':\}| \geq |\alpha_{\mathcal{A}}| - k_2$, therefore:

$$2^{\mu_{\mathcal{A}}}|\alpha_{\mathcal{A}}| - k_2 < \frac{1}{3} 2^{\mu'_B}|\alpha_B|$$

But $|\alpha_{\mathcal{A}}| - k_2 \geq k_3$, therefore $\frac{1}{3}2^{\mu'_B}|\alpha_B| > k_3$ and so $2^{\mu'_B}|\alpha_B| > 3k_3$

Taking $k_2 = k_3$, we obtain:

$$2^{\mu_{\mathcal{A}}}|\alpha_{\mathcal{A}}| < \frac{1}{3} 3k_3 + k_3 = 2k_3 < 2^{\mu'_B}|\alpha_B|$$

But this contradicts the fact that $\pi_{\mathcal{A}} \geq \pi_A$, and so the claim is proven.

Therefore we have proven that $2^{\mu'_B}|\pi_B\!\uparrow^{\mu'_B}| > 2^{\mu_{\mathcal{A}}}|\pi_{\mathcal{A}}^{\mu_{\mathcal{A}}}|$.

From the definition of μ_B, we know that $2^{\mu_B}|\pi_B\!\uparrow^{\mu_B}| \geq 2^{\mu'_B}|\pi_B\!\uparrow^{\mu'_B}|$, and therefore we conclude that $2^{\mu_B}|\pi_B\!\uparrow^{\mu_B}| > 2^{\mu_{\mathcal{A}}}|\pi_{\mathcal{A}}\!\uparrow^{\mu_{\mathcal{A}}}|$, which completes the proof. □

Suppose the verifier evaluates $\pi_{\mathcal{A}} \geq \pi_B$ in a protocol interaction where B is honest and assume during the comparison that the compared level of the honest party is μ_B. Let $b = \mathsf{LCA}(\pi_{\mathcal{A}}, \pi_B)$ and let μ'_B be the adequate level of π_B with respect to b. Then $\mu'_B \geq \mu_B$. [Local goodness] Assume chain $\mathcal{C}$ contains only honestly-generated blocks and has been adopted by an honest party in an execution with random network scheduling. Then for all levels μ, for all constant $\delta > 0$, all continuous subchains $\mathcal{C}' = \mathcal{C}[i:j]$ with $|\mathcal{C}'| \geq m$ are locally good, $\mathsf{local\text{-}good}_\delta(\mathcal{C}', \mathcal{C}, \mu)$, with overwhelming probability in m.

Proof. **(Sketch)** Observing that for each honestly generated block the probability of being a μ-superblock for any level μ follows an independent Bernoulli distribution, we can apply a Chernoff bound to show that the number of superblocks within a chain will be close to its expectation, which is what is required for local goodness. □

Suppose the verifier evaluates $\pi_{\mathcal{A}} \geq \pi_B$ in a protocol interaction where B is honest and assume during the comparison that the compared level of the honest party is μ_B. Let $b = \mathsf{LCA}(\pi_{\mathcal{A}}, \pi_B)$ and let μ'_B be the adequate level of π_B with respect to b. Then $\mu'_B \geq \mu_B$. [Multilevel quality] For all levels μ, for all constant $0 < \delta \leq 0.5$, a chain $\mathcal{C}$ containing only honestly-generated blocks adopted by an honest party in an execution with random scheduling has (δ, k_1) multilevel quality at level μ with overwhelming probability in k_1.

Proof. **(Sketch)** This proof is identical to the one of Lemma A. □

Suppose the verifier evaluates $\pi_{\mathcal{A}} \geq \pi_B$ in a protocol interaction where B is honest and assume during the comparison that the compared level of the honest party is μ_B. Let $b = \mathsf{LCA}(\pi_{\mathcal{A}}, \pi_B)$ and let μ'_B be the adequate level of π_B with respect to b. Then $\mu'_B \geq \mu_B$. [Superquality] For all levels μ, for all constant $\delta > 0$, a chain $\mathcal{C}$ containing only honestly-generated blocks adopted by an honest party in an execution with random network scheduling has (δ, m)-superquality at level μ with overwhelming probability in m.

Proof. Let $\mathcal{C}' = \mathcal{C}{\uparrow}^{\mu}$ and let $\mathcal{C}^* = \mathcal{C}'[-m' :]$ for some $m' \geq m$. Then let $B \in \mathcal{C}^*{\downarrow}$ and let X_B be the random variable equal to 1 if *level*$(B) \geq \mu$ and 0 otherwise. $\{X_B : B \in \mathcal{C}^*\}$ are mutually independent Bernoulli random variables with expectation $E(X_B) = 2^{-\mu}|\mathcal{C}^*{\downarrow}\ |$. Let $X = \sum_{B \in \mathcal{C}^*{\downarrow}} X_B$. Then X follows a Binomial distribution with parameters $(m', 2^{-\mu})$ and note that $|\mathcal{C}^*| = X$. Then $\mathbb{E}(|\mathcal{C}^*{\downarrow}\ |) = 2^{-\mu}|\mathcal{C}^*|$. Applying a Chernoff bound on $|\mathcal{C}^*{\downarrow}\ |$ we obtain that:

$$\Pr[|\mathcal{C}^*{\downarrow}\ | \leq (1-\delta)2^{-\mu}|\mathcal{C}^*{\downarrow}] \leq \exp(-\delta^2 2^{-\mu-1}|\mathcal{C}^*|)$$

□

Proof. Let $\mathcal{C}' = \mathcal{C}{\uparrow}^{\mu}$ and let $\mathcal{C}^* = \mathcal{C}'[-m' :]$ for some $m' \geq m$. Then let $B \in \mathcal{C}^*{\downarrow}$ and let X_B be the random variable equal to 1 if *level*$(B) \geq \mu$ and 0 otherwise. $\{X_B : B \in \mathcal{C}^*\}$ are mutually independent Bernoulli random variables with expectation $E(X_B) = 2^{-\mu}|\mathcal{C}^*{\downarrow}\ |$. Let $X = \sum_{B \in \mathcal{C}^*{\downarrow}} X_B$. Then X follows a Binomial distribution with parameters $(m', 2^{-\mu})$ and note that $|\mathcal{C}^*| = X$. Then $\mathbb{E}(|\mathcal{C}^*{\downarrow}\ |) = 2^{-\mu}|\mathcal{C}^*|$. Applying a Chernoff bound on $|\mathcal{C}^*{\downarrow}\ |$ we obtain that:

$$\Pr[|\mathcal{C}^*{\downarrow}\ | \leq (1-\delta)2^{-\mu}|\mathcal{C}^*{\downarrow}] \leq \exp(-\delta^2 2^{-\mu-1}|\mathcal{C}^*|)$$

□

Theorem A.1 (Double-spending attack)**.** *There exist parameters p, n, t, q, μ, δ, with $t \leq (1-\delta)(n-t)$, and a double spending attack against KLS PoPoW that succeeds with overwhelming probability.*

Proof. Recall that in the backbone notation n denotes the total number of parties, t denotes the number of adversarial parties, q denotes the number of the random oracle queries allowed per party per round and p is the probability that one random oracle query will be successful and remember that $p = T/2^{\kappa}$ where T is the mining target and κ is the security parameter (or hash function bit count). Then f denotes the probability that a given round is successful and we have that $f = 1 - (1-p)^{q(n-t)}$. Recall further that a requirement of the backbone protocol is that the honest majority assumption is satisfied, that is that $t \leq (1-\delta)(n-t)$ were $\delta \geq 2f + 3\epsilon$, where $\epsilon \in (0, 1)$ is an arbitrary small constant describing the quality of the concentration of the random variables.

Denote $\alpha_{\mathcal{A}}$ the secret chain generated by the adversary and α_B the honest chain belonging to any honest party. We will show that for certain protocol values we have that $\Pr[|\alpha_{\mathcal{A}}{\uparrow}^{\mu}\ | \geq |\alpha_B{\uparrow}^{\mu}\ |]$ is overwhelming.

Assume that, to the adversary's harm and to simplify the analysis, the adversary plays at beginning of every round and does not perform adversarial scheduling. At the beginning of the round when it is the adversary's turn to play, she has access to the blocks diffused during the previous round by the honest parties.

First, observe that at the beginning of each round, the adversary finds herself in one of two different situations: Either she has been forced into an r-round-long period of suppression, or she is not in that period. If she is within that period, she blindly performs the suppression attack without regard for the state of the world. If she is not within that period, then she must initially observe the blocks diffused at the end of the previous round by the honest parties. Call these rounds during which the diffused data must be examined by the adversary *decision rounds*. Let there be ω decision rounds in total. In each such decision round, it is possible that the adversary discovers a diffused μ-superblock and therefore decides that a suppression attack must be performed starting with the current round. Call these rounds during which this discovery is made by the adversary *migration rounds*. Let there be y migration rounds in total. The adversary devotes the migration round to performing the suppression attack as well as $r-1$ non-migration rounds after the migration round. Call these rounds, including the migration round, *suppression rounds*. In the rest of the decision rounds, the adversary will not find any μ-superblocks diffused. Call these *secret chain rounds*; these are rounds where the adversary devotes her queries to mining on the secret chain. Let there be x secret chain rounds. If the adversary devotes ω decision rounds to the attack in total, then clearly we have that $\omega = x+y$. If the total number of rounds during which the attack is running is s then we also have that $s = x + ry$, because for each migration round there are $r-1$ non-decision rounds that follow.

We will analyze the honest and adversarial superchain lengths with respect to ω, which roughly corresponds to time (because note that $\omega \geq s/r$, and so ω is proportional to the number of rounds).

Let us calculate the probability p_{SB} ("superblock probability") that a decision round ends up being a migration round. Ignoring the negligible event that there will be random oracle collisions, we have that $p_{SB} = (n-t)qp2^{-\mu}$.

Given this, note that the decision taken at the beginning of each decision round follows independent

Bernoulli distributions with probability p_{SB}. Denote z_i the indicator random variable indicating whether the decision round was a migration round. Therefore we can readily calculate the expectations for the random variables x and y, as $x = \omega - y$, $y = \sum_{i=1}^{\omega} z_i$. We have $E[x] = (1 - p_{SB})\omega$ and $E[y] = p_{SB}\omega$. Applying a Chernoff bound to the random variables x and y, we observe that they will attain values close to their mean for large ω and in particular $\Pr[y \geq (1+\delta)E[y]] \leq \exp(-\frac{\delta^2}{3}E[y])$ and similarly $\Pr[x \leq (1-\delta)E[x]] \leq \exp(-\frac{\delta^2}{2}E[x])$, which are negligible in ω.

Given that there will be x secret chain rounds, we observe that the random variable indicating the length of the secret adversarial superchain follows the binomial distribution with xtq repetitions and probability $p2^{-\mu}$. We can now calculate the expected secret chain length as $E[|\alpha_{\mathcal{A}}{\uparrow}^{\mu}|] = xtqp2^{-\mu}$. Observe that in this probability we have given the adversary the intelligence to continue using her random oracle queries during a round even after a block has been found during a round and not to wait for the next round. Applying a Chernoff bound, we obtain that $\Pr[|\alpha_{\mathcal{A}}{\uparrow}^{\mu}| \leq (1-\delta)E[|\alpha_{\mathcal{A}}{\uparrow}^{\mu}|]] \leq exp(-\frac{\delta^2}{2}E[|\alpha_{\mathcal{A}}{\uparrow}^{\mu}|])$, which is negligible in ω (because we know that with overwhelming probability $x > (1-\delta)(1-p_{SB})\omega$).

It remains to calculate the behavior of the honest superchain.

Suppose that a migration round occurs during which at least one superblock B is diffused. We will now calculate the probability p_{sup} that the adversary is able to suppress that block after r rounds by performing the suppression attack and cause all honest parties to adopt a chain not containing B.

One way for this to occur is if the adversary has generated exactly 2 shallow blocks (blocks which are not μ-superblocks) after exactly r rounds and the honest parties having generated exactly 0 blocks after exactly r rounds. This provides a lower bound for the probability, which is sufficient for our purposes. Call ADV-WIN the event where the adversary has generated exactly 2 shallow blocks after exactly r rounds since the diffusion of B and call HON-LOSE the event where the honest parties have generated exactly 0 blocks after exactly r rounds since the diffusion of B.

The number of blocks generated by the adversary after the diffusion of B follows the binomial distribution with r repetitions and probability p_{LB}, where p_{LB} denotes the probability that the adversary is able to produce a shallow block ("low block probability") during a single round. We have that $p_{LB} = tqp(1 - 2^{-\mu})$. To evaluate Pr[ADV-WIN], we evaluate the binomial distribution for 2 successes to obtain $\Pr[\text{ADV-WIN}] = \frac{r(r-1)}{2}p_{LB}^2(1-p_{LB})^{r-2}$. The number of blocks generated by the honest parties after the diffusion of B follows the binomial distribution with r repetitions and probability f. To evaluate Pr[HON-LOSE], we evaluate the binomial distribution for 0 successes to obtain $\Pr[\text{HON-LOSE}] = (1-f)^r$. Note that this is an upper bound in the probability, in particular because there can be multiple blocks during a non-uniquely successful round during which a μ-superblock was generated.

Then observe that the two events ADV-WIN and HON-LOSE are independent and therefore $p_{sup} = \Pr[\text{ADV-WIN}]\Pr[\text{HON-LOSE}] = \frac{r(r-1)}{2}p_{LB}^2(1-p_{LB})^{r-2}(1-f)^r$.

Now that we have evaluated p_{sup}, we will calculate the honest chain length in two chunks: The superblocks generated and adopted by the honest parties during secret chain rounds, $\mathcal{C}_1$, and the superblocks generated and adopted by the honest parties during suppression rounds, $\mathcal{C}_2$ (and note that these sets of blocks are not blockchains on their own).

$|\mathcal{C}_1|$ is a random variable following the binomial distribution with $s(n-t)q$ repetitions and probability $p2^{-\mu}(1-p_{sup})$. In the evaluation of this distribution, we give the honest parties the liberty to belong to a mining pool and share mining information within a round, an assumption which only makes matters for the adversary worse. We can now calculate the expected length of $\mathcal{C}_1$ to find $E[|\mathcal{C}_1|] = s(n-t)qp2^{-\mu}(1-p_{sup})$. Applying a Chernoff bound, we find that $\Pr[|\mathcal{C}_1| \geq (1+\delta)E[|\mathcal{C}_1|]] \leq exp(-\frac{\delta^2}{3}E[|\mathcal{C}_1|])$, which is negligible in s.

Finally, some additional μ-superblocks could have been generated by the honest parties while the adversary is spending r rounds attempting to suppress a previous μ-superblock. These μ-superblocks will be adopted in the case the adversary fails to suppress the previous μ-superblock. As the adversary does not devote any decision rounds to these new μ-superblocks, they will never be suppressed if the previous μ-superblock is not suppressed. We collect these in the set $\mathcal{C}_2$. To calculate $|\mathcal{C}_2|$, observe that the number of unsuppressed μ-superblocks which caused an adversarial suppression period is $|\mathcal{C}_1|$. For each of these blocks, the honest parties spend r rounds attempting to form further μ-superblocks on top. The total number of such attemps is $r|\mathcal{C}_1|$. Therefore, the number of further honestly generated μ-superblocks attained during the $|\mathcal{C}_1|$ different r-round periods follows a binomial distribution with $|\mathcal{C}_1|rq(n-t)$ repetitions and probability $p2^{-\mu}$. Here we allow the honest parties to use repeated queries within a round even after a shallow success and to work in a pool to obtain an upper bound for the expectation. Therefore $E[|\mathcal{C}_2|] = |\mathcal{C}_1|rq(n-t)p2^{-\mu}$ and applying a Chernoff bound we obtain that $\Pr[|\mathcal{C}_2| \geq (1+\delta)E[|\mathcal{C}_2|]] \leq exp(-\frac{\delta}{3}E[|\mathcal{C}_2|])$, which is negligible in s and has a quadratic error term. We deduce that $|\mathcal{C}_2|$ will have a very small length compared to the rest of the honest chain, as it is a vanishing term in μ.

Concluding the calculation of the adversarial superchain, we get $E[|\alpha_B{\uparrow}^{\mu}|] = E[|\mathcal{C}_1|] + E[|\mathcal{C}_2|]$.

Finally, it remains to show that there exist values $p, n, t, q, r, \mu, \delta$ such that a $E[|\alpha_{\mathcal{A}}{\uparrow}^{\mu}|] \geq (1+\delta)E[|\alpha_B{\uparrow}^{\mu}|]$.

Using the values $p = 10^{-5}, q = 1, n = 1000, t =$

$489, \mu = 25, r = 200$, we observe that the honest majority assumption is preserved. Replacing these values into the expectations formulae above, we obtain $E[|\alpha_{\mathcal{A}}{\uparrow}^{\mu}] \approx 1.457 * 10^{-10} * \omega$ and $E[|\alpha_B{\uparrow}^{\mu}] \approx 1.424 * 10^{-10} * \omega$, which result to a constant gap δ. Because the adversarial chain grows linearly in ω, the adversary only has to wait sufficient rounds for obtaining m blocks to create a valid proof. Therefore, for these values, the adversary will be able to generate a convincing PoPoW at some level μ which is longer than the honest parties' proof, even when the adversary does not have a longer underlying blockchain. □

Theorem A.5 (Typicality)**.** *Executions are (ϵ, η)-typical with overwhelming probability in κ.*

Proof. **Block counts and regularity.** For the blocks count and regularity, we refer the reader to [11] for the full proof.

Round count. First, observe that $Z^{\mu}_{ijk} \sim \mathsf{Bern}(2^{-\mu}p)$ and these are jointly independent. Therefore $Z^{\mu}_S \sim \mathsf{Bin}(tq|S|, 2^{-\mu}p)$ and $|S| \sim \mathsf{NB}(Z_S, 2^{-\mu}p)$. So $\mathbb{E}(|S|) = 2^{\mu}\frac{Z_S}{pqt}$. Applying a tail bound to the negative binomial distribution, we obtain that $\Pr[|S| < (1-\epsilon)\mathbb{E}(|S|)] \in \Omega(\epsilon^2 m)$. □

Suppose the verifier evaluates $\pi_{\mathcal{A}} \geq \pi_B$ in a protocol interaction where B is honest and assume during the comparison that the compared level of the honest party is μ_B. Let $b = \mathsf{LCA}(\pi_{\mathcal{A}}, \pi_B)$ and let μ'_B be the adequate level of π_B with respect to b. Then $\mu'_B \geq \mu_B$. Suppose S is a set of consecutive rounds $r_1 \dots r_2$ and $\mathcal{C}_B$ is a chain adopted by an honest party at round r_2 of a typical execution. Let $\mathcal{C}^S_B = \{b \in \mathcal{C}_B : b \text{ was generated during } S\}$. Let $\mu_{\mathcal{A}}, \mu_B \in \mathbb{N}$. Suppose $\mathcal{C}^S_B{\uparrow}^{\mu_B}$ is good. Suppose $\mathcal{C}'_{\mathcal{A}}$ is a $\mu_{\mathcal{A}}$-superchain containing only adversarially generated blocks generated during S and suppose that $|\mathcal{C}'_{\mathcal{A}}| \geq k$. Then:

$$2^{\mu_{\mathcal{A}}}|\mathcal{C}'_{\mathcal{A}}| < \frac{1}{3}2^{\mu_B}|\mathcal{C}^S_B{\uparrow}^{\mu_B}|$$

Proof. From $|\mathcal{C}'_{\mathcal{A}}| \geq k$ we know that $Z_S \geq k$. Applying Theorem A.5, we conclude that $|S| \geq (1-\epsilon')2^{\mu_{\mathcal{A}}}\frac{1}{pqt}|\mathcal{C}'_{\mathcal{A}}|$.

Applying the chain growth theorem [11] we obtain that $|\mathcal{C}^S_B| \geq (1-\epsilon)f|S|$. But from the goodness of $\mathcal{C}^S_B{\uparrow}^{\mu_B}$, we know that $|\mathcal{C}^S_B{\uparrow}^{\mu_B}| \geq (1-\delta)2^{-\mu_B}|\mathcal{C}^S_B|$. Therefore $|\mathcal{C}^S_B{\uparrow}^{\mu_B}| \geq 2^{-\mu_B}(1-\delta)(1-\epsilon)f(1-\epsilon')2^{\mu_{\mathcal{A}}}\frac{1}{pqt}|\mathcal{C}'_{\mathcal{A}}|$ and so:

$$2^{\mu_{\mathcal{A}}}|\mathcal{C}'_{\mathcal{A}}| < \frac{pqt}{(1-\delta)(1-\epsilon')(1-\epsilon)f}2^{\mu_B}|\mathcal{C}^S_B{\uparrow}^{\mu_B}|$$

□

Suppose the verifier evaluates $\pi_{\mathcal{A}} \geq \pi_B$ in a protocol interaction where B is honest and assume during the comparison that the compared level of the honest party is μ_B. Let $b = \mathsf{LCA}(\pi_{\mathcal{A}}, \pi_B)$ and let μ'_B be the adequate level of π_B with respect to b. Then $\mu'_B \geq \mu_B$. Let π be some honest proof generated with security parameters δ, m. Let $\mathcal{C}$ be the underlying chain, $b \in \mathcal{C}$ be any block and μ' be the adequate level of the proof with respect to b and the same security parameters.

Then $\mathcal{C}\{b:\}{\uparrow}^{\mu'} = \pi\{b:\}{\uparrow}^{\mu'}$.

Proof. $\pi\{b:\}{\uparrow}^{\mu'} \subseteq \mathcal{C}\{b:\}{\uparrow}^{\mu'}$ is trivial.

For the converse, we will show that for all $\mu^* > \mu'$, we have that in the iteration of the Prove `for` loop with $\mu = \mu^*$, the block stored in variable B preceeds block b in $\mathcal{C}$.

Suppose $\mu = \mu^*$ is the first `for` loop iteration during which the property is violated. Clearly this cannot be the first iteration, as there $B = \mathcal{C}[0]$ and Genesis preceeds all blocks, including itself. By the induction hypothesis we see that during the iteration $\mu = \mu^* + 1$, block B preceeded block b. But from the definition of μ' we know that μ' is the highest level for which $|\pi\{b:\}{\uparrow}^{\mu'}[1:]| \geq \max(m, (1-\delta)2^{-\mu'}|\pi\{b:\}{\uparrow}^{\mu'}[1:]{\downarrow}|)$.

Hence, this property cannot hold for $\mu^* > \mu'$ and therefore $|\pi_B\{b:\}{\uparrow}^{\mu^*_B}[1:]| < m$ or $\neg\mathsf{local\text{-}good}_{\delta}(\pi\{b:\}{\uparrow}\mu^*[1:], \mathcal{C}, \mu^*)$.

In case $\mathsf{local\text{-}good}$ is violated, variable B remains unmodified and the induction step holds. If $\mathsf{local\text{-}good}$ is not violated, then $|\pi\{b:\}{\uparrow}^{\mu^*}[1:]| < m$ and so $\pi{\uparrow}^{\mu^*}[-m]$ preceeds b, and so we are done. □

Theorem 5.2 (Number of levels)**.** *The number of superblock levels which have at least m blocks are at most* $\log(|S|)$*, where S is the set of all blocks produced, with overwhelming probability in m.*

Proof. Let S be the set of all blocks successfully produced by the honest parties or the adversary. Because each block id is generated by the random oracle, the probability that it is less than $T2^{-\mu}$ is $2^{-\mu}$. These are independent Bernoulli trials. For each block $B \in S$, define $X^{\mu}_B \in \{0,1\}$ to be the random variable indicating whether the block belongs to superblock level μ and let D_{μ} indicate their sum, which is a Binomial distribution with parameters $(|S|, 2^{-\mu})$ and expected value $E[D_{\mu}] = |S|2^{-\mu}$.

For level μ to exist in any valid proof, at least m blocks of level μ must have been produced by the honest parties or the adversary. We will now show that m blocks of level $\mu = \log(|S|)$ are produced with negligible probability in m.

As all of the X^{μ} are independent, we can apply a Binomial Chernoff bound to the probability of the sum. Therefore we have

$\Pr[D_{\mu} \geq (1+\Delta)E[D_{\mu}]] \leq \exp(-\frac{\Delta^2}{2+\Delta}E[D_{\mu}])$. But for this μ we have that $E[D_{\mu}] = 1$. Therefore $\Pr[D_{\mu} \geq 1+\Delta] \leq \exp(-\frac{\Delta^2}{2+\Delta})$. Requiring $1+\Delta = m$, we get $\Pr[D_{\mu} \geq m] \leq \exp(-\frac{(m-1)^2}{m+1})$, which is negligible in m. □

Theorem 5.3 (Large upchain expansion)**.** *Let $\mathcal{C}$ be an honestly generated chain and let $\mathcal{C}' = \mathcal{C}{\uparrow}^{\mu-1}[i : i+\ell]$ with*

$\ell \geq 4m$. Then $|\mathcal{C}'{\uparrow}^{\mu}| \geq m$ with overwhelming probability in m.

Proof. Assume the $(\mu - 1)$-level superchain has $4m$ blocks. Because each block of level $\mu - 1$ was generated as a query to the random oracle, it constitutes an independent Bernoulli trial and the number of blocks in level μ, namely $\pi{\uparrow}^{\mu}$, is a Binomial distribution with parameters $(4m, 1/2)$. Clearly $\Pr[|\pi{\uparrow}^{\mu}| = m] \leq \Pr[|\pi{\uparrow}^{\mu}| \leq m]$. Observing that $E[\pi{\uparrow}^{\mu}] = 2m$ and applying a Chernoff bound, we get $\Pr[|\pi{\uparrow}^{\mu}| \leq (1-\frac{1}{2})2m] \leq \exp(-\frac{(1/2)^2}{2}2m)$ which is negligible in m.

This probability bounds the probability of fewer than m blocks occurring in the μ level restriction of $(\mu - 1)$-level superchains of more than $4m$ blocks. □

Corollary 5.3.1 (Small downchain support)**.** *Assume an honestly generated chain $\mathcal{C}$ and let $\mathcal{C}' = \mathcal{C}{\uparrow}^{\mu}[i : i+m]$. Then $|\mathcal{C}'{\Downarrow}{\uparrow}^{\mu-1}| \leq 4m$ with overwhelming probability in m.*

Proof. Assume the $(\mu - 1)$-level superchain had at least $4m$ blocks. Then by Theorem 5.3 it follows that more than m blocks exist in level μ with overwhelming probability in m, which is a contradiction. □

Theorem 5.4 (Optimistic succinctness)**.** *Non-interactive proofs-of-proof-of-work produced by honest provers in the optimistic case are succinct with the number of blocks bounded by $4m\log(|\mathcal{C}|)$, with overwhelming probability in m.*

Proof. Assume $\mathcal{C}$ is the honest parties chain. From Theorem 5.2, the number of levels in the NIPoPoW is at most $\log(|\mathcal{C}|)$ with overwhelming probability in m.

First, observe that the count of blocks in the highest level will be less than $4m$ from Theorem 5.3; otherwise a higher superblock level would exist.

From Corollary 3.1, we know that at all levels μ the chain will be good. Therefore, for each μ superchain $\mathcal{C}$ the supporting $(\mu - 1)$-superchain will only need to span the m-long suffix of the μ-superchain above.

Then for the m-long suffix of each superchain of level μ, the supporting superchain of level $\mu - 1$ will have at most $4m$ blocks from Corollary 5.3.1.

Therefore the size of the proof is $4m\log(|\mathcal{C}|)$, which is succinct. □

Theorem 6.1. *The infix NIPoPoW construction is secure for all infix-sensitive stable predicates Q, except with negligible probability in κ.*

Proof. Assume a typical execution. To prove the security of the construction, it suffices to show that if all honest verifiers agree on the value of $Q(\mathcal{C})$, then the verifier will output the same value. Assume that all verifiers agree on the value v.

By Theorem 5.1 and because the evaluation of $\tilde{\pi}$ is identical in the suffix-sensitive and in the infix-sensitive case, we deduce that $b = \tilde{\pi}[-1]$ will be an honestly adopted block. Furthermore, due to the Common Prefix property of backbone, b will belong to all honest parties' chains and in the same position, as it is buried under $|\tilde{\chi}| = k$ blocks.

By assumption, at least one honest party B has provided an honest NIPoPoW π_B. Let the chain adopted by that honest party during the round in which the NIPoPoW was produced be $\mathcal{C}$. Because the predicate Q is infix-sensitive and stable, this means that $\exists \mathcal{C}' \subseteq \mathcal{C}[: -k] : P_v(\mathcal{C}')$. Due to B being honest, $\mathcal{C}' \subseteq \pi_B$. Let $S = \mathsf{ancestors}(b)$ be the ancestors evaluated by the verifier. Clearly $\mathcal{C}' \subseteq S$ and therefore $Q(S) = Q(\mathcal{C}') = v$. □

Theorem 7.1. *Velvet non-interactive proofs-of-proof-of-work on honest chains by honest provers remain succinct as long as a constant percentage g of miners has upgraded, with overwhelming probability.*

Proof. From Theorem 5.4 we know that the proofs π contain only a $O(polylog(m))$ amount of blocks. For each of these blocks, the velvet client needs to include a followUp tail of blocks. Assume a percentage $0 < g \leq 1$ of miners have upgraded with NIPoPoW support. Then the question of whether each block in the honest chain is upgraded follows a Bernoulli distribution. If the velvet proof were to be larger than Δ times the soft fork proof in the number of blocks included, then this would require at least one of the followUp tails to include at least Δ sequential unupgraded blocks. But since the upgrade status of each block is independent, the probability of this occurring is g^{Δ}, which is negligible in Δ. □