MA7008

Financial Mathematics Coursework

Autumn 2023-24

Table of Contents

1. Portfolio Analysis: Historical Stock Prices, Company Descriptions, Expected Returns, Volatility, and Correlations

I initiated the project by conducting a comprehensive portfolio analysis, which involved retrieving historical stock prices of five meticulously chosen stocks from "Yahoo Finance." I took care to ensure that the data covered a substantial timeframe, ranging from January 1, 2023, to January 1, 2024, with the objective of achieving positive average returns. In addition to this, I provided succinct descriptions of each company and their respective business domains. Furthermore, I delved into the intricacies of calculating the expected return, volatility, and correlations among the asset returns, facilitating a meticulous financial assessment.

1.1. Companies' Description

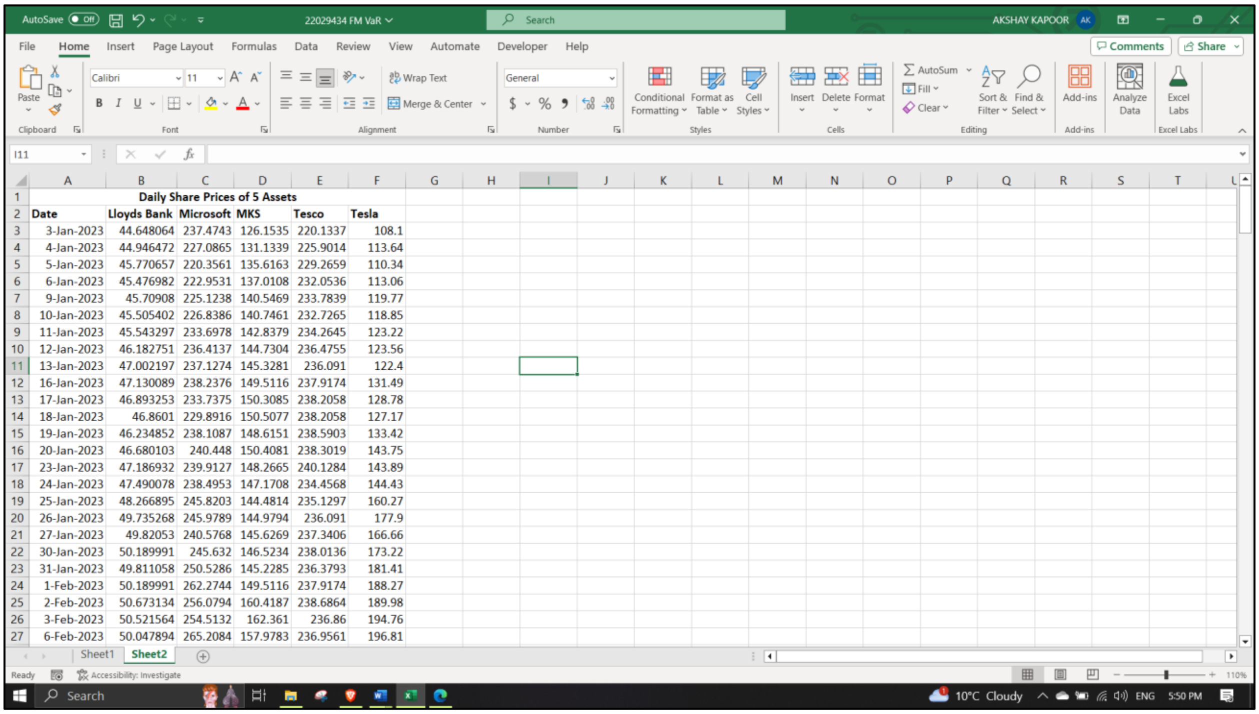
I. Lloyds Bank: Lloyds Banking Group is one of the biggest British financial institutions that deliver various banking and financial services, mainly for retail and commercial clients. One of the key brands is Lloyds Bank and it offers a suite of financial products and services which include business bank accounts, lending, transactional banking Insurance pensions among other things. The group caters to millions of customers in the UK and is dedicated to creating a positive impact on communities it operates. [1].

II. Microsoft: Microsoft was founded in 1975 and its mission remains to help empower people and businesses around the world by producing revolutionary technology. Headquartered in California and operating all over the globe, now more than 100 countries, this company is engaged in design as well as distribution of software programs hardware services for enhancing human lives. The generation of revenue is done through the development, licensing and support products software, hardware sales and online advertising. Microsoft products span across a wide range and include operating systems, application servers, productivity tools like Word processor or file creation tools such as OneNote, gaming consoles such as Xbox 360 plus other cloud-based solutions based on the internet delivery method that are provided in this company's portfolio. They also deliver services which use preconfigured data that can be turned into customizable reports for This company also offers consultancy, CSI and developer support services and training for computer system integrators. [2]

III. M&S: It is a well-known British company which provides millions of customers with worldwide high-quality, low-cost food, apparel, and homeware. It sells high-quality, low-cost food obtained from sustainable sources. The company is establishing a confident Clothing & Home business that provides current and fashionable items that are both affordable and of high quality. To do this, they are reorganizing our product lines, supporting their best-sellers, and emphasizing reliable daily value. they have also invited a carefully picked selection of guest businesses. Their International division is responsible for supplying our well-known clothes, food, and home items to over 32 million people worldwide. They provide a variety of financial services through M&S Bank, which is run by HSBC, to deliver a satisfying shopping experience. [3]

IV. Tesco: Tesco is a global general goods store with about 330,000 employees. [4] They run stores in a variety of sizes and product ranges, including big, small, dotcom exclusively, and one-stop. Tesco's principal operation is in the United Kingdom, where it is the largest private sector employer and the largest grocery retailer, with approximately 1,900 stores. [5] They strive to provide clients with inexpensive, nutritional, and sustainable food on a daily basis, allowing them to experience a higher quality of life and a simpler way of life. Tesco also has operations in nations such as the Czech Republic, Hungary, and Slovakia.

V. Tesla: Tesla Incorporated is a global company that focuses on electric cars and energy solutions. It operates in two segments: it focuses on automotive as well, energy production and storage of power and this company has working locations in United States , China along with providing its services worldwide. The Automotive industry has the production, sale as well as support of electric vehicles and associated services such after-sales assistance insurance, car finance. Energy Generating and Storage devises, produces, distributes devices to generate and store solar energy for residential users as well as commercial or industrial offices. 2017- Tesla Motors renamed itself and currently is based in Austin Texas, offering a wide range of solutions regarding the sustainable transportation sector as well as energy.



Daily Share Prices of 5 Assets					
Date	Lloyds Bank	Microsoft	MKS	Tesco	Tesla
3-Jan-2023	44.648064	237.4743	126.1535	220.1337	108.1
4-Jan-2023	44.946472	227.0865	131.1339	225.9014	113.64
5-Jan-2023	45.770657	220.3561	135.6163	229.2659	110.34
6-Jan-2023	45.476982	222.9531	137.0108	232.0536	113.06
9-Jan-2023	45.70908	225.1238	140.5469	233.7839	119.77
10-Jan-2023	45.505402	226.8386	140.7461	232.7265	118.85
11-Jan-2023	45.543297	233.6978	142.8379	234.2645	123.22
12-Jan-2023	46.182751	236.4137	144.7304	236.4755	123.56
13-Jan-2023	47.002197	237.1274	145.3281	236.091	122.4
16-Jan-2023	47.130089	238.2376	149.5116	237.9174	131.49
17-Jan-2023	46.893253	233.7375	150.3085	238.2058	128.78
18-Jan-2023	46.8601	229.8916	150.5077	238.2058	127.17
19-Jan-2023	46.234852	238.1087	148.6151	238.5903	133.42
20-Jan-2023	46.680103	240.448	150.4081	238.3019	143.75
23-Jan-2023	47.186932	239.9127	148.2665	240.1284	143.89
24-Jan-2023	47.490078	238.4953	147.1708	234.4568	144.43
25-Jan-2023	48.266895	245.8203	144.4814	235.1297	160.27
26-Jan-2023	49.735268	245.9789	144.9794	236.091	177.9
27-Jan-2023	49.82053	240.5768	145.6269	237.3406	166.66
30-Jan-2023	50.189991	245.632	146.5234	238.0136	173.22
31-Jan-2023	49.811058	250.5286	145.2285	236.3793	181.41
1-Feb-2023	50.189991	262.2744	149.5116	237.9174	188.27
2-Feb-2023	50.673134	256.0794	160.4187	238.6864	189.98
3-Feb-2023	50.521564	254.5132	162.361	236.86	194.76
6-Feb-2023	50.047894	265.2084	157.9783	236.9561	196.81

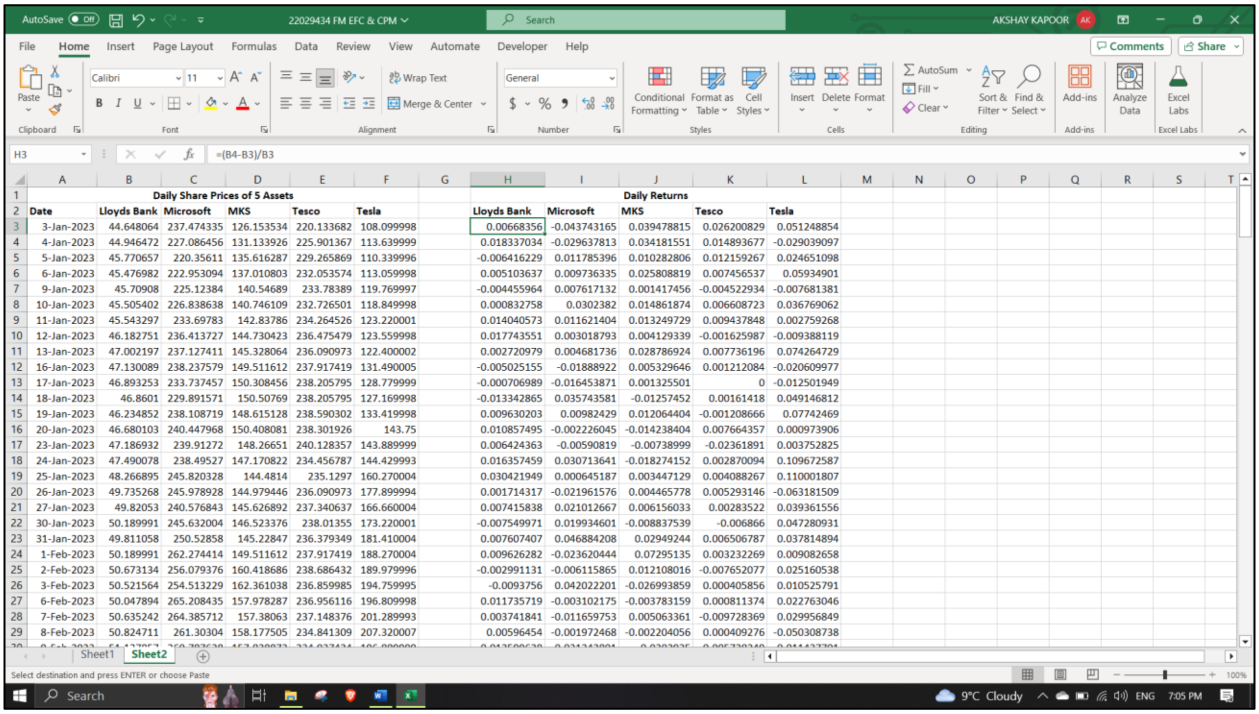
Figure 1: Screenshot of Dataset Used

1.2. Calculating Expected Return and Volatility

To compute the daily expected return of the assets, I established a section under the heading "Daily Returns." I designated each company's name as the title for its respective column. Commencing with cell H3, I employed the formula =(B4-B3)/B3 to determine the return of the first asset, where:

- **B4** represents the price of the asset on the current day.
- **B3** represents the price of the asset on the previous day.

Subsequently, utilizing the dragging feature in MS Excel, I systematically calculated the daily returns for all five stocks. This method facilitated the efficient computation of daily expected returns for the entire portfolio, providing a comprehensive overview of the assets' performance on a daily basis as shown in Figure 2.



H3 =(B4-B3)/B3

	Daily Share Prices of 5 Assets						Daily Returns				
2	Date	Lloyds Bank	Microsoft	MKS	Tesco	Tesla	Lloyds Bank	Microsoft	MKS	Tesco	Tesla
3	3-Jan-2023	44.648064	237.474335	126.153534	220.133682	108.099998	0.00668356	-0.043743165	0.039478815	0.026200829	0.051248854
4	4-Jan-2023	44.946472	227.086456	131.133926	225.901367	113.639999	0.018337034	-0.029637813	0.034181551	0.014893677	-0.029039097
5	5-Jan-2023	45.770657	220.35611	135.616287	229.265869	110.339996	-0.006416229	0.011785396	0.010282806	0.012159267	0.024651098
6	6-Jan-2023	45.476982	222.953094	137.010803	232.053574	113.059998	0.005103637	0.009736335	0.025808819	0.007456537	0.05934901
7	9-Jan-2023	45.70908	225.12384	140.54689	233.78389	119.769997	-0.004455964	0.007617132	0.001417456	-0.004522934	-0.007681381
8	10-Jan-2023	45.505402	226.838638	140.746109	232.726501	118.849998	0.000832758	0.0302382	0.014861874	0.006608723	0.036769062
9	11-Jan-2023	45.543297	233.69783	142.83786	234.264526	123.220001	0.014040573	0.011621404	0.013249729	0.009437848	0.002759268
10	12-Jan-2023	46.182751	236.413727	144.730423	236.475479	123.559998	0.017743551	0.003018793	0.004129339	-0.001625987	-0.009388119
11	13-Jan-2023	47.002197	237.127411	145.328064	236.090973	122.400002	0.002720979	0.004681736	0.028786924	0.007736196	0.074264729
12	16-Jan-2023	47.130089	238.237579	149.511612	237.917419	131.490005	-0.005025155	-0.01888922	0.005329646	0.001212084	-0.020609977
13	17-Jan-2023	46.893253	233.737457	150.308456	238.205795	128.779999	-0.000706989	-0.016453871	0.001325501	0	-0.012501949
14	18-Jan-2023	46.8601	229.891571	150.50769	238.205795	127.169998	-0.013342865	0.035743581	-0.01257452	0.00161418	0.049146812
15	19-Jan-2023	46.234852	238.108719	148.615128	238.590302	133.419998	0.009630203	0.00982429	0.012064404	-0.001208666	0.07742469
16	20-Jan-2023	46.680103	240.447968	150.408081	238.301926	143.75	0.010857495	-0.002226045	-0.014238404	0.007664357	0.000973906
17	23-Jan-2023	47.186932	239.91272	148.26651	240.128357	143.889999	0.006424363	-0.00590819	-0.00738999	-0.02361891	0.003752825
18	24-Jan-2023	47.490078	238.49527	147.170822	234.456787	144.429993	0.016357459	0.030713641	-0.018274152	0.002870094	0.109672587
19	25-Jan-2023	48.266895	245.820328	144.4814	235.1297	160.270004	0.030421949	0.000645187	0.003447129	0.004088267	0.110001807
20	26-Jan-2023	49.735268	245.978928	144.979446	236.090973	177.899994	0.001714317	-0.021961576	0.004465778	0.005293146	-0.063181509
21	27-Jan-2023	49.82053	240.576843	145.626892	237.340637	166.660004	0.007415838	0.021012667	0.006156033	0.00283522	0.039361556
22	30-Jan-2023	50.189991	245.632004	146.523376	238.01355	173.220001	-0.007549971	0.019934601	-0.008837539	-0.006866	0.047280931
23	31-Jan-2023	49.811058	250.52858	145.22847	236.379349	181.410004	0.007607407	0.046884208	0.02949244	0.006506787	0.037814894
24	1-Feb-2023	50.189991	262.274414	149.511612	237.917419	188.270004	0.009626282	-0.023620444	0.07295135	0.003232269	0.009082658
25	2-Feb-2023	50.673134	256.079376	160.418686	238.686432	189.979996	-0.002991131	-0.006115865	0.012108016	-0.007652077	0.025160538
26	3-Feb-2023	50.521564	254.513229	162.361038	236.859985	194.759995	-0.0093756	0.042022201	-0.026993859	0.000405856	0.010525791
27	6-Feb-2023	50.047894	265.208435	157.978287	236.956116	196.809998	0.011735719	-0.003102175	-0.003783159	0.000811374	0.022763046
28	7-Feb-2023	50.635242	264.385712	157.38063	237.148376	201.289993	0.003741841	-0.011659753	0.005063361	-0.009728369	0.029956849
29	8-Feb-2023	50.824711	261.30304	158.177505	234.841309	207.320007	0.00596454	-0.001972468	-0.002204056	0.000409276	-0.050308738

Figure 2: Computing Daily Returns

I determined annualized expected returns for each asset using the "AVERAGE" function in MS Excel, calculating the mean of daily returns with a multiplication factor of 252 to align with the financial convention of a 252-day trading year, excluding holidays. The formula applied is “=AVERAGE(H3:H252)*252” for Lloyds Bank stock. In order to get annualized volatility, I used Excel's "STDEV.P" function for the standard deviation of daily returns, multiplying the result by the square root of 252. This adjustment accounts for the fact that standard deviation is the square root of variance. The Excel formula utilized is “=STDEV.P(H3:H252)*SQRT(252).” Utilizing the dragging feature, I replicated the aforementioned calculations for all five stocks. The output of these computations is visually presented in Figure 3.

Mean & Std Dev. (Annualized)		
	Mean	**SD**
Lloyds Bank	0.091	0.221
Microsoft	0.480	0.251
MKS	0.821	0.301
Tesco	0.294	0.166
Tesla	0.938	0.526

Figure 3: Expected Return and Volatility

1.3. Calculating Correlations

To ascertain the correlations between various asset returns, I utilized the "Data Analysis" package available under the "Data" Tab in MS Excel. Specifically, I employed the "Correlation" tool within this package. For the input range, I selected the daily returns of the assets. The outcome of this analysis manifested as a correlation matrix, prominently displayed in Figure 4.

Assets	Lloyds Bank	Microsoft	MKS	Tesco	Tesla
Lloyds Bank	1.00	0.02	0.29	0.27	0.15
Microsoft	0.02	1.00	-0.09	-0.05	0.32
MKS	0.29	-0.09	1.00	0.27	-0.04
Tesco	0.27	-0.05	0.27	1.00	-0.02
Tesla	0.15	0.32	-0.04	-0.02	1.00

Figure 4: Correlations between Assets Returns

2. Portfolio Optimization: A Solver Approach to Achieve Target Returns and Efficient Frontier

To commence the portfolio optimization process, I initiated by computing the variance-covariance matrix. This was accomplished through the utilization of the “Data Analysis” package available under the "Data" Tab in MS Excel. Specifically, I employed the “Covariance” tool within this package, selecting the daily asset returns as the input range. The result was a comprehensive variance-covariance matrix. To annualize the covariance, I multiplied each element in the matrix by 252. The obtained matrix is visually represented in Figure 5.

Assets	Lloyds Bank	Microsoft	MKS	Tesco	Tesla
Lloyds Bank	0.049	0.001	0.020	0.010	0.017
Microsoft	0.001	0.063	-0.007	-0.002	0.042
MKS	0.020	-0.007	0.090	0.028	-0.002
Tesco	0.010	-0.002	0.014	0.028	-0.002
Tesla	0.017	0.042	-0.006	-0.002	0.277

Figure 5: Variance-Covariance Matrix

Subsequently, for the initial portfolio optimization, I adhered to an equally weighted distribution, assigning a weight of 0.2 to each asset. This allocation aligns with the default constraint that mandates the sum of weights for all assets in the portfolio to be equal to one. For this equally weighted portfolio, I proceeded to compute various performance measures, which are detailed as follows:

3. **Expected Return**: to calculate it for equally weighted portfolio I used “=MMULT(TRANSPOSE(O14:O18),(O3:O7))” formula where:
    - “**O14:O18**” represents weights of the assets.
    - “**O3:O7**” annual returns of the assets.
4. **Variance** : to calculate it for I used “=MMULT(MMULT(TRANSPOSE(O14:O18),S3:W7),O14:O18)” formula where:
    - “**O14:O18**” represents weights of the assets
    - **“S3:W7”** is the variance-covariance matrix shown in Figure 5.
5. **Standard Deviation:** To calculate the standard deviation, I used “=SQRT(O23)” where:

- **"O23"** represents variance of the equally weighted portfolio.

6. **Sharpe Ratio:** To calculate the sharpe ratio I utilized "=(O22-O9)/O24" formula where:
   - **"O22"** is the expected return of equally-weighted portfolio.
   - **"O9"** is the risk-free rate.
   - **"O24"** is the standard deviation of equally-weighted portfolio.

The equally-weighted portfolio obtained is shown in Figure 6:

Assets	Weights
Lloyds Bank	0.2
Microsoft	0.2
MKS	0.2
Tesco	0.2
Tesla	0.2
Sum	1
Expected Return	0.525
Variance	0.028
Std Dev	0.167
Sharpe Ratio	3.053

Figure 6: Equally-Weighted Portfolio

On moving forward, I computed minimum variance portfolio using the "Solver Package" present under "Data" tab. For achieving these following steps were taken:

i. In the "Set Objective" textbox I selected the cell containing standard deviation of equally-weighted portfolio and selecting the "Min" dialogue box.

ii. In the "By changing variable cells" I selected the range of cells holding the weights of equally-weighted portfolio

iii. In the "Subject to constraints" I made sum of weights equal to one and then unchecked the "Make unconstrained Variables Non-Negative" dialogue box to keep the adjusted weights positive. All the Solver Parameters are shown in Figure 7. Minimum Variance portfolio, thus obtained is shown in Figure 8.

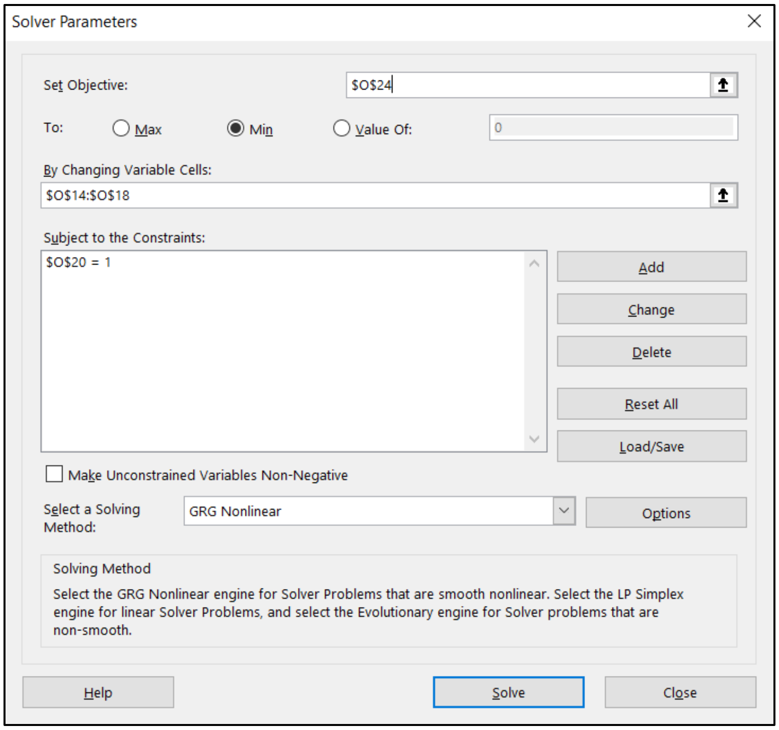


Figure 7: Solver Parameters To Get Minimum-Variance Portfolio

Assets	Weights
Lloyds Bank	0.201
Microsoft	0.263
MKS	0.040
Tesco	0.488
Tesla	0.008
Sum	1
Expected Return	0.329
Variance	0.016
Std Dev	0.125
Sharpe Ratio	2.500

Figure 8: Minimum Variance Portfolio

After this I computed the weights for the other two portfolios in which I set the following parameters:

i. In the “Set Objective” textbox I selected the cell containing expected return of equally-weighted portfolio and selecting the “Max” dialogue box.
ii. In the “By changing variable cells” I selected the range of cells holding the weights of equally-weighted portfolio.
iii. In the “Subject to constraints” I made sum of weights equal to one and added another constraint which was setting standard deviation of equally weighted portfolio to 0.14 for portfolio A and 0.18 for portfolio B then unchecked the “Make unconstrained Variables Non-Negative” dialogue box to keep the adjusted weights positive. All the Solver Parameters are shown in Figure 9. While Portfolio A and Portfolio B are shown in Figure 10.

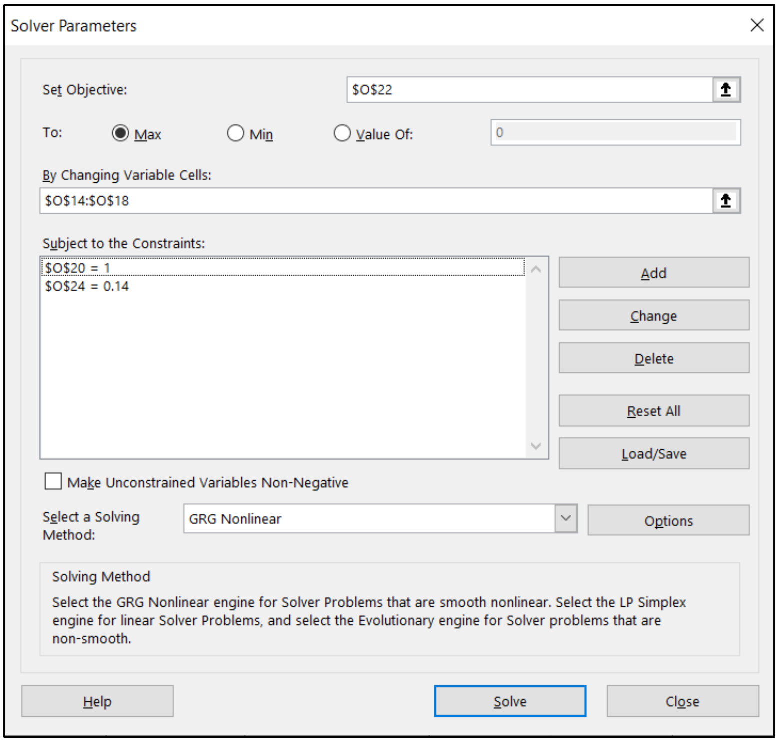


Figure 9: Solver Parameters for Portfolio A

Portfolio A	
Assets	**Weights**
Lloyds Bank	0.018369332
Microsoft	0.298262906
MKS	0.225496738
Tesco	0.393733538
Tesla	0.064137486
Sum	1
Expected Return	0.505885775
Variance	0.019600021
Std Dev	0.140000076
Sharpe Ratio	3.506325049

Portfolio B	
Assets	**Weights**
Lloyds Bank	-0.177792185
Microsoft	0.336685624
MKS	0.424574881
Tesco	0.292323486
Tesla	0.124208194
Sum	1
Expected Return	0.696511873
Variance	0.032400042
Std Dev	0.180000117
Sharpe Ratio	3.786174619

Figure 10: Portfolio A & Portfolio B

To construct the efficient frontier curve, I required a combination of expected return and volatilities. To achieve this, I created a linear combination of Portfolio A and Portfolio B (shown in Figure 12), computing the following parameters:

i. Covariance of Portfolio A and Portfolio B: To calculate the covariance of both portfolios I used the formula "=MMULT(MMULT(TRANSPOSE(W14:W18),(S3:W7)),Z14:Z18)" in which:

a. "**W14:W18**" is the range of weights of Portfolio A.

b. “**S3:W7**” is the variance-covariance matrix of the returns of assets.
c. “**Z14:Z18**” is the range of weights of Portfolio B.

ii. **Expected Return**: Formula used for this is “=S31*W22+(1-S31)*Z22” in which:
a. “**S31**”: is the weight of Portfolio A which is assigned the value as 0.4
b. “**W22**” is expected return of Portfolio A.
c. “**(1-S31)**” is the weight of Portfolio B.
d. “**Z22**” is the expected return of Portfolio B.

iii. **Variance:** Formula used for this is “=S31^2*W23+(1-S31)^2*Z23+2*S31*(1-S31)*S30” in which:
a. “**S31**”: is the weight of Portfolio A which is assigned the value as 0.4
b. “**W23**” is Variance of Portfolio A.
c. “**(1-S31)**” is the weight of Portfolio B.
d. “**Z23**” is variance of Portfolio B.
e. **“S30”** is the covariance of Portfolio A and Portfolio B.

This formula of variance is taken from the formula to calculate the variance of two portfolio which is as shown in :

- Portfolio variance = $w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2}$

Where:

- w_1 = the portfolio weight of the first asset
- w_2 = the portfolio weight of the second asset
- σ_1 = the standard deviation of the first asset
- σ_2 = the standard deviation of the second asset
- $Cov_{1,2}$ = the co-variance of the two assets, which can thus be expressed as $p_{(1,2)}\sigma_1\sigma_2$, where $p_{(1,2)}$ is the correlation co-efficient between the two assets

Figure 11: Formula to calculate variance of two assets

Linear Combination of Portfolio A & B	
Parameters	**Values**
Covariance	0.023
Port. A	0.400
Exp. Return	0.620
Std. Dev	0.161
VAR	0.026

Figure 12: Linear Combination of Portfolio A & Portfolio B

To obtain the values necessary for plotting the efficient frontier curve—specifically, the volatility and expected return—I created a new table with columns labeled "Iterations," "Risk," and "Expected Return." The "Iterations" column ranged from -2 to +2, with the first row containing the standard deviation and expected return values resulting from the linear combination of Portfolio A and Portfolio B.

To extract the values, I utilized the "What-If Analysis" feature in the "Data" tab. I selected the "Data Table" option, and in the "Column Input Cell," I designated cell "S31," which denotes the weight of Portfolio A (0.4, as shown in Figure 12). Upon completion, I obtained the necessary set of values, as illustrated in Figure 13. These values serve as crucial input data for constructing the efficient frontier curve.

Iterations	Volatility	Expected Return
-2	0.160815572	0.620261434
-1.9	0.29880403	1.05870146
-1.8	0.291935224	1.03963885
-1.7	0.285108225	1.02057624
-1.6	0.278326111	1.00151363
-1.5	0.271592243	0.982451021
-1.4	0.264910302	0.963388411
-1.3	0.258284317	0.944325801
-1.2	0.251718707	0.925263191
-1.1	0.245218321	0.906200581
-1	0.238788488	0.887137971
-0.9	0.23243506	0.868075362
-0.8	0.226164479	0.849012752
-0.7	0.219983828	0.829950142
-0.6	0.213900903	0.810887532
-0.5	0.207924282	0.791824922
-0.4	0.202063397	0.772762312
-0.3	0.196328614	0.753699703
-0.2	0.190731307	0.734637093
-0.1	0.185283937	0.715574483
6.38378E-16	0.180000117	0.696511873
0.1	0.174894669	0.677449263
0.2	0.169983668	0.658386653
0.3	0.165284447	0.639324044
0.4	0.160815572	0.620261434
0.5	0.156596765	0.601198824
0.6	0.152648761	0.582136214
0.7	0.148993088	0.563073604
0.8	0.14565176	0.544010994
0.9	0.142646868	0.524948384
1	0.140000076	0.505885775
1.1	0.137732032	0.486823165
1.2	0.135861705	0.467760555
1.3	0.134405698	0.448697945
1.4	0.133377582	0.429635335
1.5	0.132787295	0.410572725
1.6	0.132640684	0.391510116
1.7	0.132939216	0.372447506
1.8	0.133679909	0.353384896
1.9	0.134855477	0.334322286
2	0.136454681	0.315259676

Figure 13: Coordinates of EFC

Using the above coordinates efficient frontier curve shown in Figure 14 was obtained.

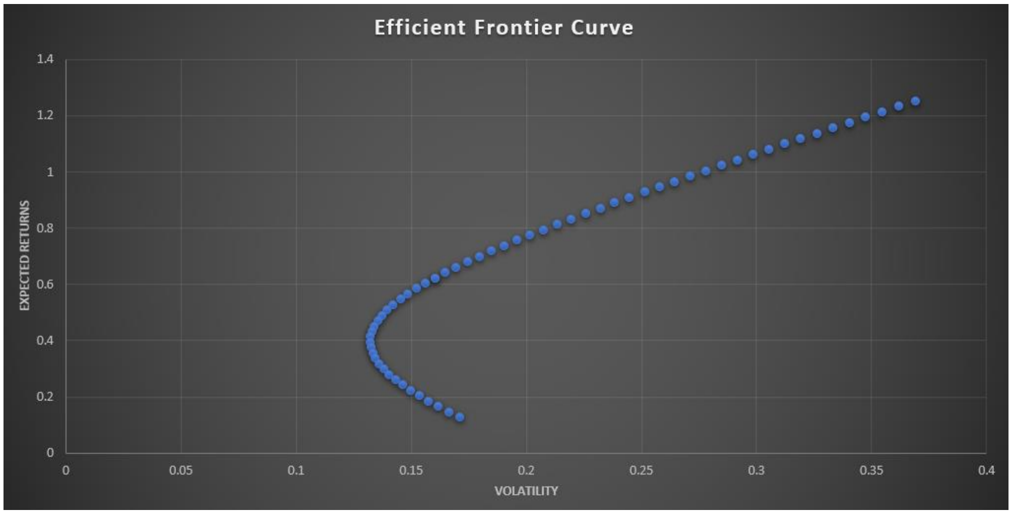


Figure 14: Efficient Frontier Curve

3. Sharpe Ratios and Capital Market Line

In order to calculate the sharpe ratios of the calculated coordinates (in Figure 13) I used "=(P38-O9)/O38" in excel in which:

- "**P38**": is Expected Return
- **"O9"** is Risk Free Rate, which is 1.5% in this case
- **"O38"** is Standard Deviation

The resulting sharpe ratios are shown in Figure 15 and the optimal portfolio is highlighted. It is the optimal portfolio because it has the highest sharpe ratio among all coordinates.

Iterations	Volatility	Expected Return	Sharpe Ratios
-2	0.305711827	1.07776407	3.476359032
-1.9	0.29880403	1.05870146	3.49292966
-1.8	0.291935224	1.03963885	3.509815764
-1.7	0.285108225	1.02057624	3.526998355
-1.6	0.278326111	1.00151363	3.544452328
-1.5	0.271592243	0.982451021	3.562145257
-1.4	0.264910302	0.963388411	3.580035976
-1.3	0.258284317	0.944325801	3.598072902
-1.2	0.251718707	0.925263191	3.616192069
-1.1	0.245218321	0.906200581	3.634314825
-1	0.238788488	0.887137971	3.652345136
-0.9	0.23243506	0.868075362	3.670166455
-0.8	0.226164479	0.849012752	3.68763811
-0.7	0.219983828	0.829950142	3.704591148
-0.6	0.213900903	0.810887532	3.720823616
-0.5	0.207924282	0.791824922	3.736095251
-0.4	0.202063397	0.772762312	3.750121613
-0.3	0.196328614	0.753699703	3.762567707
-0.2	0.190731307	0.734637093	3.773041262
-0.1	0.185283937	0.715574483	3.781085901
1.52656E-15	0.180000117	0.696511873	3.786174619
0.1	0.174894669	0.677449263	3.787704141
0.2	0.169983668	0.658386653	3.78499099
0.3	0.165284447	0.639324044	3.777270361
0.4	0.160815572	0.620261434	3.76369916
0.5	0.156596765	0.601198824	3.74336484
0.6	0.152648761	0.582136214	3.715301793
0.7	0.148993088	0.563073604	3.678516983
0.8	0.14565176	0.544010994	3.632026106
0.9	0.142646868	0.524948384	3.57490068
1	0.140000076	0.505885775	3.506325049
1.1	0.137732032	0.486823165	3.425660374
1.2	0.135861705	0.467760555	3.332510484
1.3	0.134405698	0.448697945	3.226782431
1.4	0.133377582	0.429635335	3.108733338
1.5	0.132787295	0.410572725	2.978995274
1.6	0.132640684	0.391510116	2.838571877
1.7	0.132939216	0.372447506	2.688804077
1.8	0.133679909	0.353384896	2.531307055
1.9	0.134855477	0.334322286	2.367885188
2	0.136454681	0.315259676	2.200435143

Figure 15:Sharpe Ratios Of Calculated Coordinates

To illustrate the Capital Market Line, I structured a table encompassing three columns: weights of investment in the optimal portfolio, volatility, and expected return. The default weights considered were zero, one, and two.

- A weight of zero signifies the risk and expected return if an investor invests solely in the risk-free rate.
- A weight of one provides the expected return and risk of the optimal portfolio.
- A weight of two represents the scenario where an investor invests twice the amount in the optimal portfolio.

Formula used to calculate volatility and expected return on Capital Market Line are as follows:

i. **Volatility** – "=V31*O69" where,
   a. "**V31**" is the weight of Optimal Portfolio.
   b. "**O69**" is the volatility of Optimal Portfolio.
ii. Expected Return – "=V31*P69+(1-V31)*O9" where,
   a. "**V31**" is the weight of Optimal Portfolio.
   b. "**P69**" is the Expected Return of Optimal Portfolio.
   c. "**O9**" is the Risk-Free Rate.

Coordinates Of Capital Market Line		
Weight	**Volatility**	**Exp. Return**
0	0	0.015
1	0.174895	0.677449263
2	0.349789	1.339898526

Figure 16: Coordinates of Capital Market Line

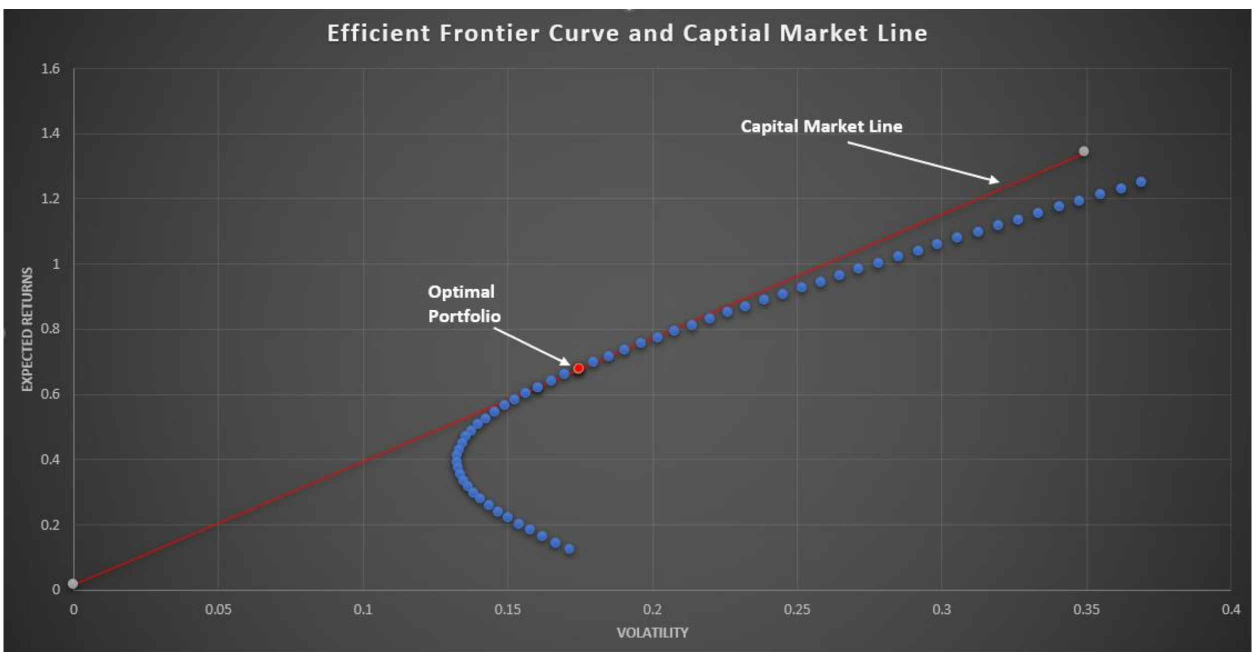


Figure 17: Efficient Frontier Curve with Capital Market Line

The equation of capital market line is given by the formula: $\mathbf{R_p} = \mathbf{r_f} + \frac{\mathbf{R_T} - \mathbf{r_f}}{\boldsymbol{\sigma_T}} \boldsymbol{\sigma_p}$

Where, $\mathbf{R_p}$- represents the portfolio return,

$\mathbf{r_f}$- represents the risk-free rate,

$\mathbf{R_T}$ - represents the market return,

$\boldsymbol{\sigma_T}$ - represents the standard deviation of market returns, and

$\boldsymbol{\sigma_p}$ - represents the standard deviation of portfolio returns.

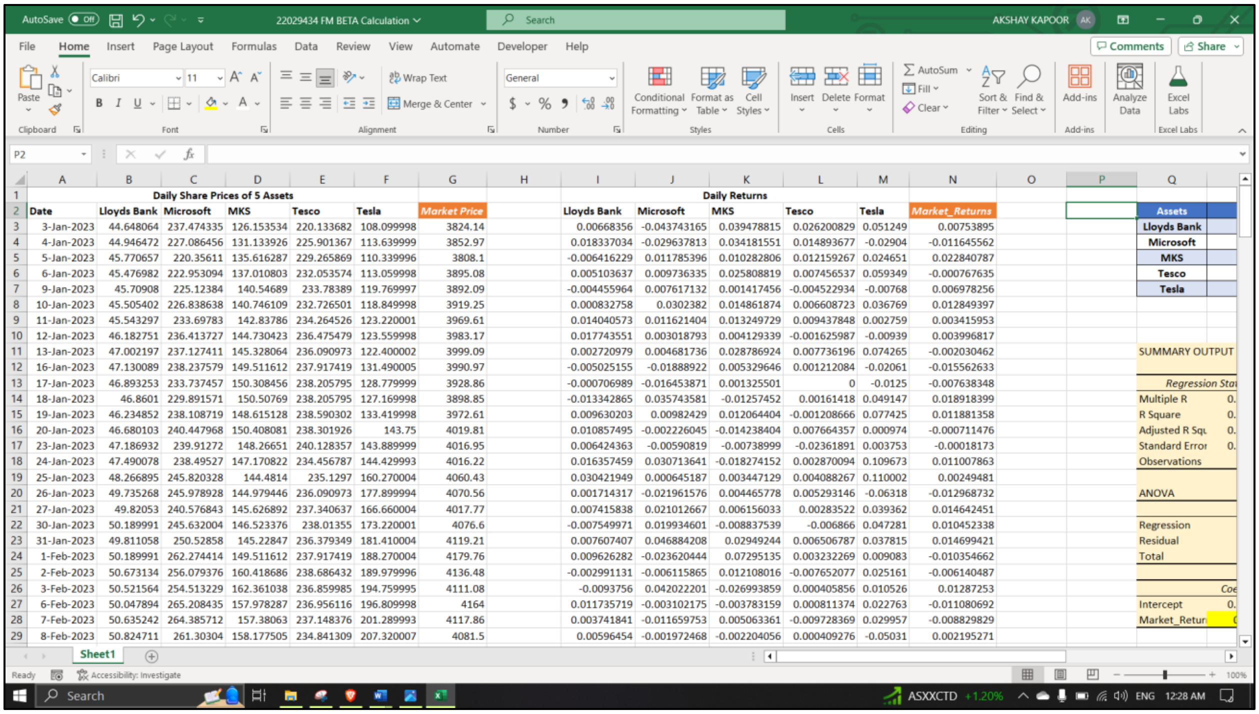
For this portfolio equation is $\mathbf{R_p}$ = 0.015 + 3.79$\boldsymbol{\sigma_p}$

Economic Significance of Capital Market Line – On the Capital Market Line (CML), portfolios to the left of the market portfolio combine risky assets with a risk-free investment, essentially representing a blend of market exposure and a secure, risk-free return from instruments like government bonds. Conversely, portfolios to the right of the market portfolio involve short selling, where assets are sold short to finance the purchase of others, often using borrowed funds at the risk-free rate. The slope of the CML is termed the risk premium, signifying the additional return per unit of market risk. A steeper slope indicates a higher premium for taking on additional market risk. Short selling, denoted by negative weights, allows investors to profit from falling asset prices but introduces heightened risk. Overall, the CML provides a visual representation of risk-return trade-offs and different portfolio strategies based on investors' risk preferences and market expectations.

4. Calculating Beta Using Linear Regression

To calculate asset betas, I downloaded Standard & Poor's 500 historical prices and computed daily returns(as shown in Figure 18). Following steps were taken to calculate beta:

i. Used "Data Analysis" under "Data" Tab.
ii. Selected "Regression" tool.
iii. Set Y-range as daily returns of assets, X-range as S&P 500 returns.
iv. Retrieved beta from the coefficient of "Market Return" in the output (shown in Figure 19).
v. Repeated for all assets.
vi. Values of beata for all five assets is shown in Figure 20.



Daily Share Prices of 5 Assets

Date	Lloyds Bank	Microsoft	MKS	Tesco	Tesla	Market Price
3-Jan-2023	44.648064	237.474335	126.153534	220.133682	108.099998	3824.14
4-Jan-2023	44.946472	227.086456	131.133926	225.901367	113.639999	3852.97
5-Jan-2023	45.770657	220.35611	135.616287	229.265869	110.339996	3808.1
6-Jan-2023	45.476982	222.953094	137.010803	232.053574	113.059998	3895.08
9-Jan-2023	45.70908	225.12384	140.54689	233.78389	119.769997	3892.09
10-Jan-2023	45.505402	226.838638	140.746109	232.726501	118.849998	3919.25
11-Jan-2023	45.543297	233.69783	142.83786	234.264526	123.220001	3969.61
12-Jan-2023	46.182751	236.413727	144.730423	236.475479	123.559998	3983.17
13-Jan-2023	47.002197	237.127411	145.328064	236.090973	122.400002	3999.09
16-Jan-2023	47.130089	238.237579	149.511612	237.917419	131.490005	3990.97
17-Jan-2023	46.893253	233.737457	150.308456	238.205795	128.779999	3928.86
18-Jan-2023	46.8601	229.891571	150.50769	238.205795	127.169998	3898.85
19-Jan-2023	46.234852	238.108719	148.615128	238.590302	133.419998	3972.61
20-Jan-2023	46.680103	240.447968	150.408081	238.301926	143.75	4019.81
23-Jan-2023	47.186932	239.91272	148.26651	240.128357	143.889999	4016.95
24-Jan-2023	47.490078	238.49527	147.170822	234.456787	144.429993	4016.22
25-Jan-2023	48.266895	245.820328	144.4814	235.1297	160.270004	4060.43
26-Jan-2023	49.735268	245.978928	144.979446	236.090973	177.899994	4070.56
27-Jan-2023	49.82053	240.576843	145.626892	237.340637	166.660004	4017.77
30-Jan-2023	50.189991	245.632004	146.523376	238.01355	173.220001	4076.6
31-Jan-2023	49.811058	250.52858	145.22847	236.379349	181.410004	4119.21
1-Feb-2023	50.189991	262.274414	149.511612	237.917419	188.270004	4179.76
2-Feb-2023	50.673134	256.079376	160.418686	238.686432	189.979996	4136.48
3-Feb-2023	50.521564	254.513229	162.361038	236.859985	194.759995	4111.08
6-Feb-2023	50.047894	265.208435	157.978287	236.956116	196.809998	4164
7-Feb-2023	50.635242	264.385712	157.38063	237.148376	201.289993	4117.86
8-Feb-2023	50.824711	261.30304	158.177505	234.841309	207.320007	4081.5

Daily Returns

Lloyds Bank	Microsoft	MKS	Tesco	Tesla	Market_Returns
0.00668356	-0.043743165	0.039478815	0.026200829	0.051249	0.00753895
0.018337034	-0.029637813	0.034181551	0.014893677	-0.02904	-0.011645562
-0.006416229	0.011785396	0.010282806	0.012159267	0.024651	0.022840787
0.005103637	0.009736335	0.025808819	0.007456537	0.059349	-0.000767635
-0.004455964	0.007617132	0.001417456	-0.004522934	-0.00768	0.006978256
0.000832758	0.0302382	0.014861874	0.006608723	0.036769	0.012849397
0.014040573	0.011621404	0.013249729	0.009437848	0.002759	0.003415953
0.017743551	0.003018793	0.004129339	-0.001625987	-0.00939	0.003996817
0.002720979	0.004681736	0.028786924	0.007736196	0.074265	-0.002030462
-0.005025155	-0.01888922	0.005329646	0.001212084	-0.02061	-0.015562633
-0.000706989	-0.016453871	0.001325501	0	-0.0125	-0.007638348
-0.013342865	0.035743581	-0.01257452	0.00161418	0.049147	0.018918399
0.009630203	0.00982429	0.012064404	-0.001208666	0.077425	0.011881358
0.010857495	-0.002226045	-0.014238404	0.007664357	0.000974	-0.000711476
0.006424363	-0.00590819	-0.00738999	-0.02361891	0.003753	-0.00018173
0.016357459	0.030713641	-0.018274152	0.002870094	0.109673	0.011007863
0.030421949	0.000645187	0.003447129	0.004088267	0.110002	0.00249481
0.001714317	-0.021961576	0.004465778	0.005293146	-0.06318	-0.012968732
0.007415838	0.021012667	0.006156033	0.00283522	0.039362	0.014642451
-0.007549971	0.019934601	-0.008837539	-0.006866	0.047281	0.010452338
0.007607407	0.046884208	0.02949244	0.006506787	0.037815	0.014699421
0.009626282	-0.023620444	0.07295135	0.003232269	0.009083	-0.010354662
-0.002991131	-0.006115865	0.012108016	-0.007652077	0.025161	-0.006140487
-0.0093756	0.042022201	-0.026993859	0.000405856	0.010526	0.01287253
0.011735719	-0.003102175	-0.003783159	0.000811374	0.022763	-0.011080692
0.003741841	-0.011659753	0.005063361	-0.009728369	0.029957	-0.008829829
0.00596454	-0.001972468	-0.002204056	0.000409276	-0.05031	0.002195271

Figure 18: Market Returns Calculations

SUMMARY OUTPUT Lloyds Bank

Regression Statistics	
Multiple R	0.177560201
R Square	0.031527625
Adjusted R Squ	0.027622494
Standard Error	0.013726878
Observations	250

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.001521245	0.001521245	8.073385676	0.004865404
Residual	248	0.04672994	0.000188427		
Total	249	0.048251186			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.000123601	0.00087221	0.141710195	0.88742397	-0.001594282	0.001841484	-0.00159428	0.001841484
Market_Retur	0.29349808	0.103294552	2.841370387	0.004865404	0.090051646	0.496944514	0.090051646	0.496944514

Figure 19: Output of Regression Analysis on Lloyds Bank Stock

Assets	*Beta*
Lloyds Bank	0.3
Microsoft	1.2
MKS	0.0
Tesco	0.0
Tesla	2.2

Figure 20: Value of Beta for all Assets in Portfolio

Following are the significance of the values of beta obtained:

Assets	Beta Value	Significance
Lloyds Bank	0.3	A beta value that is less than 1.0 means that the security is theoretically less volatile than the market. Including this stock in a portfolio makes it less risky than the same portfolio without the stock
Microsoft	1.2	The asset price is theoretically more volatile than the market. This indicates that adding the stock to a portfolio will increase the portfolio's risk but may also increase its expected return.
M&S	0	Asset price is uncorrelated to the market.
Tesco	0	Asset price is uncorrelated to the market.
Tesla	2.2	Asset price is more volatile than that of market as a whole. This indicates that adding the stock to a portfolio will increase the portfolio's risk but may also increase its expected return.

5. Value At Risk (VaR)

The delta-normal method was employed to compute the Value at Risk (VaR) for the portfolio. The standard normal form (Z-Stat) corresponding to a 95% confidence interval was determined for the given case. The formula utilized for calculating Z-Stat is "=NORM.S.INV(1-0.95)." Subsequently, the delta normal VaR was computed using the formula "=-(O22+S16*O24)," where:

- **"O22"** represents the Expected Return of the portfolio.
- **"S16"** denotes the Z-Stat.
- **"O24"** stands for the Volatility of the portfolio.

To quantify the amount at risk, the calculated delta normal VaR was multiplied by the total value of the portfolio, which, for the purposes of this analysis, was hypothetically considered to be 2,000,000 USD. Results are shown in Figure 21.

Confidence Interval	Z-stat	Delta_Normal VAR(%)	VCV VAR($)
90%	-1.281551566	-0.310835309	$(621,670.62)
95%	-1.644853627	-0.250161062	$(500,322.12)
99%	-2.326347874	-0.136346265	$(272,692.53)

Figure 21: Value at Risk

To assess the individual contribution of each asset to the estimated value at risk, delta normal values were computed by considering the expected return and risk associated with each asset. To determine the value of contribution by each asset, the formula "=U22*S12*O14" was employed, where:

- "**U22**" signifies the Delta Normal Value.
- "**S12**" represents the Total Value of the portfolio.
- "**O14**" denotes the weight of the asset.

The application of this formula yielded the contribution by each asset in the value at risk for the portfolio, as illustrated in the Figure 22.

Assets	Confidence Interval	Z-stat	Delta_Normal VAR(%)	VCV VAR($)
Lloyds Bank	95%	-1.644853627	-0.271524836	$ (108,609.93)
Microsoft	95%	-1.644853627	0.067920408	$ 27,168.16
MKS	95%	-1.644853627	0.326763289	$ 130,705.32
Tesco	95%	-1.644853627	0.019816028	$ 7,926.41
Tesla	95%	-1.644853627	0.072501034	$ 29,000.41

Figure 22: Value at Risk for each Asset

6. Asset Volatility Estimation: ARCH/GARCH in R

To estimate the volatility of an asset, RStudio was utilized. The volatility estimation involved a series of steps and analyses performed using the rugarch package in RStudio. The key procedures are outlined below:

Step 1: To facilitate the process, the necessary libraries were imported, as illustrated in Figure 23. This step ensured that the essential tools and functions required for subsequent operations were readily available and accessible.

```
# Libraries
library(quantmod)
library(xts)
library(PerformanceAnalytics)
library(rugarch)
```

Figure 23: Importing Required Libraries in RStudio

Step 2: Stock market data was retrieved for the symbol "TSLA" from January 1, 2023, to January 1, 2024, using the "getSymbols" function. This function likely fetches historical stock price data for the specified time range. Subsequently, "chartSeries" function was employed to generate a financial chart for the "TSLA" stock, visually displaying its price movements and other relevant information over the specified time period. Code which is used is shown in Figure 24 and output is shown in Figure 25.

```
# Tesla daily prices
getSymbols("TSLA",from = "2023-01-01",to = "2024-01-01")
chartSeries(TSLA)
```

Figure 24:Retrieving the Stock Prices for Tesla

Figure 25: Financial Chart for Tesla

The output reveals a considerable level of volatility in stock prices throughout the observed time period. Such erratic fluctuations pose a challenge for analysis using conventional time-series methods. The inherent complexity and variability in the data make it imperative to explore alternative analytical approaches to gain a deeper understanding of the underlying patterns and trends.

Step 3: To analyze stock prices, daily returns of "TSLA" closing prices were computed using the 'CalculateReturns' function, and the results were stored in the variable "return," as depicted in Figure 26. This transformation to daily returns enhances the data's suitability for certain analytical methods.

Subsequently, histograms and charts were generated for the calculated returns. The 'hist' function created a basic histogram, while 'chart.histogram' improved it by overlaying density and normal distribution curves. This visualization aids in understanding the distribution of returns, with added insights from density and normal curves. The corresponding curve is shown in Figure 27

```
# Daily returns
return <- CalculateReturns(TSLA$TSLA.Close)
return <- return[-1]
hist(return)
chart.Histogram(return,
                methods = c('add.density', 'add.normal'),
                colorset = c('blue', 'green', 'red'))
chartSeries(return)
```

Figure 26: Calculating Daily Returns and Plotting Histogram

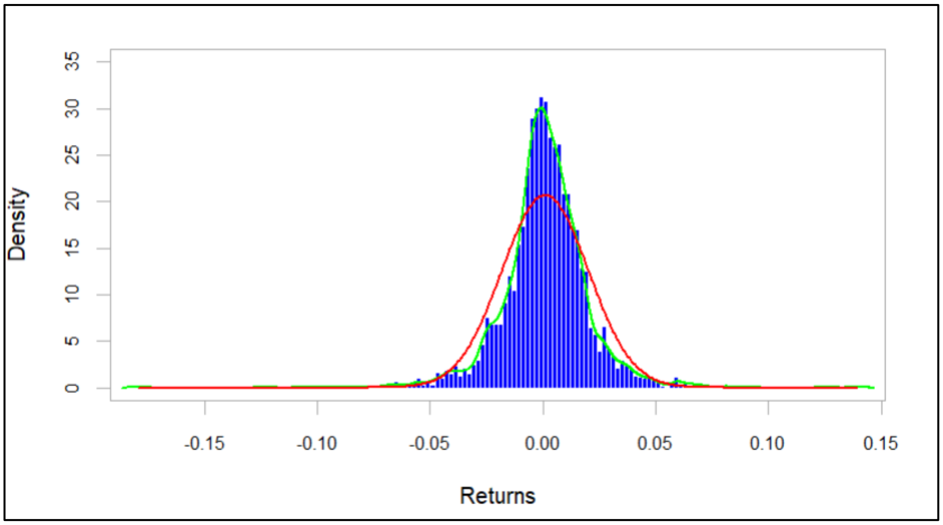


Figure 27: Histogram of Returns

The presented histogram of returns displays a symmetric distribution, with daily returns predominantly clustered around zero. However, there are instances where returns spike notably, reaching levels of 5% to 10%. Notably, the curve for returns surpasses the normal distribution curve, and the thickness of the green lines on the tails indicates occurrences of higher returns compared to the normal distribution. This observation suggests a degree of volatility and outliers in the data, highlighting the need for robust risk management strategies.

Step 4: In order to model financial time-series data of Tesla stock I started with univariate GARCH Model. A GARCH specification, namely 's', using 'ugarchspec' function was created with a constant mean model (ARMA order of (0,0)), a squared GARCH variance model (model = "sGARCH"), and a normal distribution for the innovations. Subsequently, a GARCH model (ugarchfit) named 'm' was estimated using the provided return data and the previously defined specification 's'. This process involves fitting the GARCH model to the observed return series to estimate the parameters of the model. Code for the same is shown in Figure 28 and summary of model is shown in Figure 29 and Figure 30.

```
# 1. sGARCH model with contant mean
s <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                variance.model = list(model = "sGARCH"),
                distribution.model = 'norm')
m <- ugarchfit(data = return, spec = s)
```

Figure 28: Standard GARCH Model

```
> m

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001836    0.000276   6.6592        0
omega   0.000014    0.000001  10.0031        0
alpha1  0.105545    0.001219  86.5745        0
beta1   0.854392    0.009939  85.9641        0

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001836    0.000363   5.0645 0.000000
omega   0.000014    0.000004   3.5477 0.000389
alpha1  0.105545    0.019804   5.3294 0.000000
beta1   0.854392    0.015308  55.8148 0.000000

LogLikelihood : 7994.752

Information Criteria
------------------------------------

Akaike       -5.2936
Bayes        -5.2857
Shibata      -5.2936
Hannan-Quinn -5.2908
```

Figure 29: Summary of Model m(Part-a)

```
Weighted ARCH LM Tests
------------------------------------
             Statistic Shape Scale P-Value
ARCH Lag[3]     0.2744 0.500 2.000  0.6004
ARCH Lag[5]     2.4827 1.440 1.667  0.3741
ARCH Lag[7]     2.8325 2.315 1.543  0.5457

Nyblom stability test
------------------------------------
Joint Statistic:  11.706
Individual Statistics:
mu     0.1434
omega  2.6664
alpha1 0.7241
beta1  0.7183

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.07 1.24 1.6
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias            1.207 0.2275
Negative Sign Bias   1.268 0.2048
Positive Sign Bias   0.534 0.5934
Joint Effect        10.167 0.0172  **


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     119.3    1.531e-16
2    30     130.8    7.063e-15
3    40     138.5    4.622e-13
4    50     144.7    2.212e-11
```

Figure 30: Summary of Model m(Part-b)

From Figure 29, it is interesting to note that for this model all four optimal parameters have p-value less than 0.05 which means all of them are statistically significant so these parameters should be there in the model. So, equation of GARCH mean at time t can be written as:

$$\mathbf{R_t} = \mu + \varepsilon_t = \mathbf{0.001836} + \varepsilon_t$$

And equation of GARCH variance at time can be written a:

$$\sigma_t^2 = \omega + a\epsilon_{t-1}^2 + b\sigma_{t-1}^2 = \mathbf{0.000014} + \mathbf{0.105549}\epsilon_{t-1}^2 + \mathbf{0.85438}\sigma_{t-1}^2$$

Additional noteworthy information from the output in Figure 30 is the "Goodness-of-Fit Test." The observed p-values, all below 5%, lead to the rejection of the null hypothesis. This suggests that the chosen model for residuals, in this case, the normal distribution, may not be an optimal choice. The results indicate room for improvement in the model, urging a reassessment or modification to enhance its accuracy and reliability.

Step 5: as it is clear from the model m that normal distribution is not an ideal choice for residuals this time I used GARCH with Skew Student-t Distribution. The code for same is shown in Figure 31.

```r
# 2. GARCH with sstd
s <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                variance.model = list(model = "sGARCH"),
                distribution.model = 'sstd')
m1 <- ugarchfit(data = return, spec = s)
```

Figure 31: GARCH Model with Student-t Distribution

```
Conditional Variance Dynamics
-----------------------------------
GARCH Model      : sGARCH(1,1)
Mean Model       : ARFIMA(0,0,0)
Distribution     : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001480    0.000279   5.3109 0.000000
omega   0.000006    0.000004   1.6959 0.089906
alpha1  0.085036    0.014276   5.9568 0.000000
beta1   0.902291    0.018798  48.0000 0.000000
skew    1.005530    0.025139  39.9986 0.000000
shape   4.752778    0.436673  10.8841 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001480    0.000271  5.45327 0.000000
omega   0.000006    0.000009  0.68476 0.493498
alpha1  0.085036    0.018977  4.48103 0.000007
beta1   0.902291    0.037398 24.12651 0.000000
skew    1.005530    0.024643 40.80407 0.000000
shape   4.752778    0.608406  7.81185 0.000000

LogLikelihood : 8135.742

Information Criteria
------------------------------------

Akaike       -5.3857
Bayes        -5.3738
Shibata      -5.3857
Hannan-Quinn -5.3814
```

Figure 32: Summary of Model m1 (Part A)

```
             Statistic Shape Scale P-Value
ARCH Lag[3]     0.1532 0.500 2.000  0.6955
ARCH Lag[5]     1.5861 1.440 1.667  0.5700
ARCH Lag[7]     1.8544 2.315 1.543  0.7481

Nyblom stability test
------------------------------------
Joint Statistic:  3.4839
Individual Statistics:
mu     0.12760
omega  0.75082
alpha1 1.70491
beta1  1.70865
skew   0.05767
shape  2.42582

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.49 1.68 2.12
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           1.3822 0.16702
Negative Sign Bias  1.1429 0.25316
Positive Sign Bias  0.5915 0.55424
Joint Effect       11.1419 0.01098  **


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     22.09       0.2796
2    30     37.99       0.1226
3    40     49.39       0.1230
4    50     49.88       0.4381


Elapsed time : 1.019448
```

Figure 33: Summary of Model m1(Part B)

As depicted in Figure 33, the p-values for the "Goodness-of-Fit Test" exceed the 5% threshold. Consequently, the null hypothesis cannot be rejected, leading to the conclusion that the model with skewed t-distribution is a more fitting choice. This assertion is further supported by additional evidence, such as the lower AIC value for model m1 (shown in Figure 32) compared to that of model m (shown in Figure 29). These findings collectively indicate the superiority of the model with skewed t-distribution.

Step 6: In an effort to enhance the model, the GJR-GARCH Model (Generalized Jump Robust GARCH) was explored. This model is particularly designed to address substantial jumps in the volatility of stock prices. The corresponding code for this endeavor is detailed in Figure 34, outlining the steps taken to implement the GJR-GARCH Model for improved volatility modeling.

```r
# 3. GJR-GARCH
s <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                variance.model = list(model = "gjrGARCH"),
                distribution.model = 'sstd')
m2 <- ugarchfit(data = return, spec = s)
```

Figure 34: GJR-GARCH Model

```
> m2

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001308    0.000264   4.9492    1e-06
omega   0.000010    0.000001  10.3481    0e+00
alpha1  0.022802    0.004044   5.6385    0e+00
beta1   0.873061    0.011001  79.3632    0e+00
gamma1  0.163845    0.023679   6.9195    0e+00
skew    1.001581    0.025054  39.9762    0e+00
shape   5.188518    0.437218  11.8671    0e+00

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001308    0.000282   4.6324 0.000004
omega   0.000010    0.000001   8.1916 0.000000
alpha1  0.022802    0.008655   2.6346 0.008424
beta1   0.873061    0.012203  71.5453 0.000000
gamma1  0.163845    0.027334   5.9942 0.000000
skew    1.001581    0.024872  40.2702 0.000000
shape   5.188518    0.514089  10.0926 0.000000

LogLikelihood : 8166.682

Information Criteria
------------------------------------

Akaike       -5.4056
Bayes        -5.3916
Shibata      -5.4056
Hannan-Quinn -5.4005
```

Figure 35: Summary of Model m2

A notable observation from the results is that all seven coefficients of model GJR-GARCH demonstrate statistical significance, with p-values below 0.05. Furthermore, the AIC value of model GJR-GARCH is smaller than that of model Standard GARCH with SSTD distribution, signaling that the performance of this model stands out as the best among the three fitted models thus far. These findings collectively affirm the robustness and efficacy of the implemented model.

Step 7: In the concluding phase, the volatility for the next 20 days was forecasted using the selected final model, which is the GJR-GARCH Model. The process of forecasting is outlined in Figure 36, and the resulting output is displayed in Figure 37. This final step enables anticipation and preparation for potential volatility in the stock prices over the specified future period.

```
#forecasting volatility for the next 20 days using GJR-GARCH.
f <- ugarchforecast(fitORspec = m2, n.ahead = 20)
plot(sigma(f))
```

Figure 36: Forecasting volatility for next 20 days

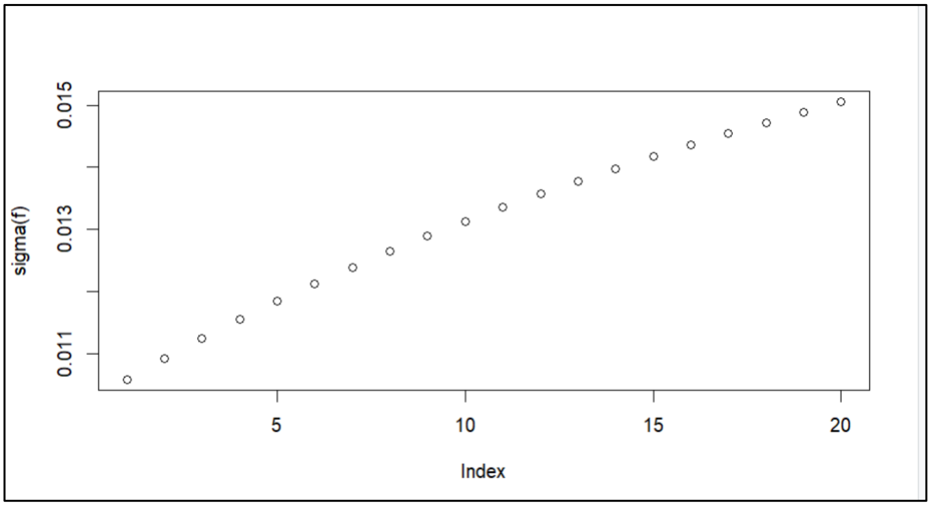


Figure 37: Output of Forecast

The forecast shown in Figure 37 clearly states that the volatility is going to be increased in next 20 days.

7. Summary of Findings

In exploring the historical performance of selected stocks, I've uncovered valuable insights to guide potential investors in building an efficient portfolio. I crunched the numbers to calculate the expected return and volatility for each company, giving me a sense of how much investor can potentially gain and how much risk is associated with each investment. Moreover, I explored how these stocks move in relation to each other, unveiling correlations that are crucial for a diversified portfolio. Using excel Solver function, I optimized the portfolio to find the right mix of assets which in this case comes out to be 31% in Microsoft, 43% in M&S, 13% in Tesco, 12% in Tesla and rest in Lloyds Bank. This minimizes risk for a given target return. By repeating this process for different target returns, I charted the efficient frontier, illustrating the trade-off between risk and return. To further guide investors, I calculated Sharpe ratios for a range of portfolio returns and volatilities. By factoring in a risk-free investment with a guaranteed return, we drew the Capital Market Line, shedding light on the optimal risk-return combinations and the economic significance of this line in making investment decisions. Employing linear regression analysis, I determined the beta for each asset, offering insights into their sensitivity to market movements. Assessing the Value at Risk which comes out to be 500,322 USD for portfolio with an investment of 2,000,000 USD, I quantified the potential losses at a 5% confidence level and discussed how each asset contributes to this estimate. Delving deeper, I estimated the volatility of a single asset using advanced statistical models like ARCH/GARCH. I identified the best model, providing a robust explanation for our choice. In a nutshell, my findings suggest that constructing an efficient portfolio involves careful consideration of expected returns, volatility, and inter-asset correlations. The efficient frontier provides a visual guide for investors to strike a balance between risk and reward. The Sharpe ratios and Capital Market Line enhance decision-making by factoring in risk-free options. Additionally, understanding beta values and VaR contributes to a comprehensive risk management strategy.

Ultimately, investors should weigh these factors against their own risk tolerance and investment goals when selecting the best efficient portfolio. Keep an eye on how each asset contributes to overall risk and stay informed about market conditions to make informed decisions in the dynamic world of investments.

References

[1] Lloyds Banking Group, "About us," Lloyds Banking Group, [Online]. Available: https://www.lloydsbankinggroup.com/who-we-are/group-overview.html. [Accessed 7 Jan 2024].

[2] Microsoft Corporation, "2012 Annual Report," Microsoft Corporation, [Online]. Available: https://www.microsoft.com/investor/reports/ar12/financial-review/business-description/index.html. [Accessed 7 jan 2024].

[3] M&S, "Our Business," M&S, [Online]. Available: https://corporate.marksandspencer.com/about-us/our-businesses#:~:text=M%26S%20is%20a%20leading%20British,to%20millions%20of%20customers%20globally.. [Accessed 7 Jan 2024].

[4] Tesco PLC, "About," Tesco, [Online]. Available: https://www.tescoplc.com/about/. [Accessed 7 January 2024].

[5] "Tesco plc - Company Profile, Information, Business Description, History, Background Information on Tesco plc," References for Business, [Online]. Available: https://www.referenceforbusiness.com/history2/56/Tesco-plc.html. [Accessed 7 January 2024].