$$p_{cost} = \min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} P_i$$

$$p_{time} = \min \sum_{i \in I} \sum_{j \in J} p_{ij} x_{ij} + \sum_{i \in I} P_i$$

$$makespan = \min \left(\max \left(f_i \right) - \min \left(s_i \right) \right) + \sum_{i \in I} P_i$$

s.t. $s_i \geq r_i \quad \forall i \in I$

$s_i \leq d_i - \sum_{j \in J} p_{ij} x_{ij} \quad \forall i \in I$

$s_i \leq s_k - st_{ik} - \sum_{j \in J} p_{ij} x_{ij},$
if order i precedes order k on the same machine

$$P_i = \begin{cases} 1000, & \text{if } s_i \geq d_i - \sum_{j \in J} p_{ij} x_{ij} \quad \forall i \in I \\ 0, & otherwise \end{cases}$$

where,

P_i = the Penalty incurred by order i exceeding its due date.

$$x_{ij} = \begin{cases} 1, & \text{if order i is assigned to machine j} \\ 0, & \text{otherwise} \end{cases}$$

f_i = finish time of order i

$$p_{cost} = \min \sum_{i \in I} \sum_{j \in J} c(i,j) m(i,j)$$

$$p_{time} = \min \sum_{i \in I} \sum_{j \in J} p(i,j) m(i,j)$$

$$makespan = \min \left(\max \left(f_i \right) - \min \left(s_i \right) \right)$$

subject to:

$$f_i = s_i + \sum_{j \in J} p(i,j) m(i,j) \tag{1}$$

$$\sum_{j \in J} m(i,j) = 1 \tag{2}$$

$$f(i) \leq d(i) \tag{3}$$

$$f(i) + adj(i,ip) * setup(i,ip) \leq s_{ip} + U * (1 - B(i,ip)) \tag{4}$$

$$1 + s(ip) \leq f(i) + adj(i,ip) * setup(i,ip) + U * B(i,ip) \tag{5}$$

$$\sum_{j \in J} m(i,j) id(j) - \sum_{j \in J} m(ip,j) id(j) \leq U * (1 - A(i,ip)) \tag{6}$$

$$1 + \sum_{j \in J} m(ip,j) id(j) \leq U * A(i,ip) + \sum_{j \in J} m(i,j) id(j) \tag{7}$$

$$1 \text{ if not same as}(i,ip) \leq B(ip,i) + B(i,ip) + U * (1 - (A(i,ip) + A(ip,i) - 1)) \tag{8}$$

$$(B(i,ip) + (A(i,ip) + A(ip,i) - 1)) - 1 \leq U * Z(i,ip) \tag{9}$$

$$1 \leq (B(i,ip) + (A(i,ip) + A(ip,i) - 1)) - 1 + U * (1 - Z(i,ip)) \tag{10}$$

$$C(i) = \sum_{ip \in I} Z(i, ip) - Z(i, i) \tag{11}$$

$$C(i) - C(ip) \leq U * Y(i, ip) \tag{12}$$

$$1 \leq C(i) - C(ip) + U * (1 - Y(i, ip)) \tag{13}$$

$$2 \leq U * Yp(i, ip) + C(i) - C(ip) \tag{14}$$

$$C(i) - C(ip) \leq 1 + U * (1 - Yp(i, ip)) \tag{15}$$

$$((Y(i, ip) + Yp(i, ip) - 1) + (A(i, ip) + A(ip, i) - 1)) - 1 \leq U * adj(i, ip) \tag{16}$$

$$1 \leq ((Y(i, ip) + Yp(i, ip) - 1) + (A(i, ip) + A(ip, i) - 1)) - 1 + U * (1 - adj(i, ip)) \tag{17}$$

$$\sum_{i \in I} Xf(i) = 1 \tag{18}$$

$$\sum_{i \in I} Xs(i) = 1 \tag{19}$$

$$lf \geq f(i) \tag{20}$$

$$es \leq s(i) \tag{21}$$

$$lf \leq U * (1 - Xf(i)) + f(i) \tag{22}$$

$$es + U * (1 - Xs(i)) \geq s_i \tag{23}$$

Table 1: Variable Table

Variable	Type	Significance
m(i,j)	Binary	1 if order i is processed on machine j
A(i, ip)	Binary	1 iff order i occurs on a machine with number less than or equal to that of ip
B(i, ip)	Binary	1 iff order i finishes before or at start time of ip with setup time required if i is processed before ip in between
Z(i , ip)	Binary	1 iff order i and ip on same machine and i finishes at or before ip
Y(i, ip)	Binary	1 iff no of orders after i on m(i) - no of orders after ip on m(ip) $\geq$ 1
Yp(i, ip)	Binary	1 iff no of orders after i on m(i) - no of orders after ip on m(ip) $\leq$ 1
adj(i, ip)	Binary	1 iff order ip is the order next after i on the same machine
Xf(i)	Binary	1 if i is the Latest Order
Xs(i)	Binary	1 if i is the Earliest Order
s(i)	Integer	start time of order i
f(i)	Integer	finish time of order i
C(i)	Integer	number of orders scheduled after order i on the same machine
lf	Integer	largest finish time
es	Integer	earliest start time
U	Scalar	Very Large Constant