The CESD scale items (C1–C20) from the depression data set in Chapter 3 were used to obtain the factor loadings listed in Table 15.7. The initial factor solution was obtained from the principal components method, and a varimax rotation was performed. Analyze this same data set by using an oblique rotation such as the direct quartimin procedure. Compare the results.

Code:

```
fit_model <- princomp(Data1, cor=TRUE)
summary(fit_model)

varimax1<- principal(Data1,nfactors=4,rotate='varimax')
varimax1

oblique1<- principal(Data1,nfactors=4,rotate='promax')
oblique1
```

Output:

```
> summary(fit_model)
Importance of components:
                          Comp.1     Comp.2     Comp.3     Comp.4     Comp.5     Comp.6     Comp.7     Comp.8     Comp.9    Comp.10
Standard deviation     2.6562036 1.21883931 1.10973409 1.03232021 1.00629648 0.98359581 0.97304489 0.87706188 0.83344885 0.81248191
Proportion of Variance 0.3527709 0.07427846 0.06157549 0.05328425 0.05063163 0.04837304 0.04734082 0.03846188 0.03473185 0.03300634
Cumulative Proportion  0.3527709 0.42704935 0.48862483 0.54190909 0.59254072 0.64091375 0.68825457 0.72671645 0.76144830 0.79445464
                          Comp.11    Comp.12    Comp.13    Comp.14    Comp.15    Comp.16    Comp.17    Comp.18    Comp.19     Comp.20
Standard deviation     0.77950975 0.74117295 0.73255278 0.71324438 0.67149280 0.61252016 0.56673129 0.54273638 0.51804873 0.445396635
Proportion of Variance 0.03038177 0.02746687 0.02683168 0.02543588 0.02254513 0.01875905 0.01605922 0.01472814 0.01341872 0.009918908
Cumulative Proportion  0.82483641 0.85230328 0.87913496 0.90457083 0.92711596 0.94587501 0.96193423 0.97666237 0.99008109 1.000000000
```

PCA output with varimax rotation

```
> varimax1
Principal Components Analysis
Call: principal(r = Data1, nfactors = 4, rotate = "varimax")
Standardized loadings (pattern matrix) based upon correlation matrix
      RC1   RC2   RC3   RC4   h2   u2 com
C1   0.64  0.15  0.27  0.28 0.58 0.42 1.9
C2   0.77  0.30  0.27  0.00 0.76 0.24 1.6
C3   0.73  0.05  0.27  0.05 0.61 0.39 1.3
C4   0.63 -0.06  0.17  0.43 0.61 0.39 2.0
C5   0.80  0.17  0.16  0.02 0.69 0.31 1.2
C6   0.62  0.23 -0.02  0.03 0.44 0.56 1.3
C7   0.59  0.16  0.36  0.34 0.62 0.38 2.5
C8   0.09 -0.05  0.11  0.74 0.57 0.43 1.1
C9   0.24  0.03  0.62  0.11 0.45 0.55 1.4
C10  0.56  0.25  0.38  0.18 0.55 0.45 2.5
C11  0.50  0.15  0.41  0.15 0.46 0.54 2.3
C12  0.45  0.39 -0.05 -0.06 0.36 0.64 2.0
C13  0.07  0.50 -0.17  0.54 0.58 0.42 2.2
C14  0.12  0.70  0.18  0.13 0.55 0.45 1.3
C15  0.49  0.42 -0.12  0.09 0.44 0.56 2.2
C16  0.20  0.67  0.26 -0.07 0.56 0.44 1.5
C17  0.27  0.66  0.19  0.00 0.55 0.45 1.5
C18  0.41  0.21 -0.03  0.22 0.26 0.74 2.1
C19 -0.01  0.24  0.75 -0.09 0.62 0.38 1.2
C20  0.36  0.09  0.51  0.43 0.58 0.42 2.9

                       RC1  RC2  RC3  RC4
SS loadings           4.80 2.38 2.11 1.55
Proportion Var        0.24 0.12 0.11 0.08
Cumulative Var        0.24 0.36 0.46 0.54
Proportion Explained  0.44 0.22 0.19 0.14
Cumulative Proportion 0.44 0.66 0.86 1.00

Mean item complexity =  1.8
Test of the hypothesis that 4 components are sufficient.

The root mean square of the residuals (RMSR) is  0.07
 with the empirical chi square  526.92  with prob <  1.1e-53

Fit based upon off diagonal values = 0.96>
```

PCA output with oblique method, promax rotation

```
> oblique1
Principal Components Analysis
Call: principal(r = Data1, nfactors = 4, rotate = "promax")
Standardized loadings (pattern matrix) based upon correlation matrix
      RC1   RC2   RC3   RC4   h2   u2 com
C1   0.63 -0.01  0.08  0.18 0.58 0.42 1.2
C2   0.86  0.11  0.08 -0.18 0.76 0.24 1.1
C3   0.86 -0.15  0.08 -0.10 0.61 0.39 1.1
C4   0.64 -0.24 -0.05  0.36 0.61 0.39 1.9
C5   0.94 -0.04 -0.05 -0.15 0.69 0.31 1.1
C6   0.71  0.07 -0.19 -0.09 0.44 0.56 1.2
C7   0.53  0.02  0.18  0.25 0.62 0.38 1.7
C8  -0.16 -0.07  0.01  0.82 0.57 0.43 1.1
C9   0.18  0.00  0.57  0.05 0.45 0.55 1.2
C10  0.51  0.14  0.23  0.08 0.55 0.45 1.6
C11  0.49  0.04  0.27  0.05 0.46 0.54 1.6
C12  0.47  0.30 -0.15 -0.17 0.36 0.64 2.2
C13 -0.27  0.55 -0.23  0.61 0.58 0.42 2.7
C14 -0.15  0.76  0.18  0.11 0.55 0.45 1.2
C15  0.47  0.32 -0.26  0.01 0.44 0.56 2.4
C16  0.02  0.71  0.26 -0.13 0.56 0.44 1.3
C17  0.10  0.67  0.16 -0.06 0.55 0.45 1.2
C18  0.37  0.12 -0.16  0.17 0.26 0.74 2.1
C19 -0.15  0.31  0.80 -0.13 0.62 0.38 1.4
C20  0.21  0.03  0.39  0.40 0.58 0.42 2.5

                       RC1  RC2  RC3  RC4
SS loadings           5.26 2.31 1.71 1.56
Proportion Var        0.26 0.12 0.09 0.08
Cumulative Var        0.26 0.38 0.46 0.54
Proportion Explained  0.49 0.21 0.16 0.14
Cumulative Proportion 0.49 0.70 0.86 1.00

 With component correlations of
     RC1  RC2  RC3  RC4
RC1 1.00 0.49 0.35 0.51
RC2 0.49 1.00 0.04 0.19
RC3 0.35 0.04 1.00 0.25
RC4 0.51 0.19 0.25 1.00

Mean item complexity =  1.6
Test of the hypothesis that 4 components are sufficient.

The root mean square of the residuals (RMSR) is  0.07
 with the empirical chi square  526.92  with prob <  1.1e-53

Fit based upon off diagonal values = 0.96
```

=>The 1st component in the oblique rotation is better than varimax. the 2nd and 3rd components are better in the varimax method. 4th component both methods give the same value. The methods don't give outputs that are significantly different.

Repeat the analysis of Problem 15.1 and Table 15.7, but use an iterated principal factor solution instead of the principal components method. Compare the results.

Code:

```
library("GPArotation")

corr_mat<-cor(Data1)
corr_mat_communality<-(1-1/diag(solve(corr_mat)))
diag(corr_mat)<-corr_mat_communality

minimum_error<-0.001

k<-c()

sum1<-sum(diag(corr_mat))

error<-sum1

while (error>minimum_error)
{
  eigen1<-eigen(corr_mat)
  eigen1

  lambda<-as.matrix(eigen1$vectors[,1:2])%*% diag(sqrt(eigen1$values[1:2]))

  corr_mat_Mod <-lambda %*% t(lambda)
  corr_mat_diagonal<-diag(corr_mat_Mod)
  sum2<-sum(corr_mat_diagonal)
  error<-abs(sum1 - sum2)
  sum1<- sum2
  k<- append(k,sum2)
  diag(corr_mat)<-corr_mat_diagonal
}

sum1<-rowSums(lambda^2)
p<-1-sum1
comm1<- rowSums(lambda^2)^2/rowSums(lambda^4)

loadings1<- data.frame(cbind(round(lambda,2),round(sum1,2),round(p,3),round(comm1,2)))

colnames(loadings1)<-c('Factor 1','Factor 2','sum1','p','comm1')
```

```
loadings1

prop_var<-eigen1$values[1:2]/sum(diag(corr_mat))
prop_var

cum_var<-eigen1$values/4

var_factor<-data.frame(rbind(round(prop_var[1:2],2),round(cum_var[1:2],2)))

rownames(var_factor)<- c('Proportion Explained', 'Cumulative Variance')
colnames(var_factor)<-c('Factor_1','Factor_2')
a1<-list(loadings1,var_factor)
a1

varimax2<- fa(cbind(Data1),nfactors = 4,rotate = 'varimax',fm='pa')
varimax2

Quartimin1<- fa(cbind(Data1),nfactors = 4,rotate = 'quartimin',fm='pa')
Quartimin1
```

Output:

```
> loadings1
   Factor 1 Factor 2 sum1     p comm1
1      0.72    -0.15 0.54 0.462  1.09
2      0.83     0.04 0.69 0.312  1.00
3      0.69    -0.16 0.50 0.498  1.11
4      0.63    -0.35 0.52 0.483  1.56
5      0.75    -0.06 0.56 0.441  1.01
6      0.55     0.04 0.30 0.695  1.01
7      0.74    -0.17 0.57 0.426  1.10
8      0.26    -0.20 0.11 0.893  1.86
9      0.43    -0.09 0.19 0.806  1.08
10     0.71    -0.02 0.51 0.494  1.00
11     0.61    -0.08 0.38 0.617  1.03
12     0.44     0.20 0.23 0.767  1.38
13     0.30     0.14 0.11 0.889  1.38
14     0.45     0.38 0.35 0.654  1.94
15     0.50     0.16 0.27 0.727  1.20
16     0.49     0.42 0.41 0.585  1.96
17     0.53     0.40 0.45 0.552  1.86
18     0.42     0.01 0.18 0.824  1.00
19     0.32     0.07 0.11 0.894  1.09
20     0.60    -0.20 0.39 0.606  1.22

> prop_var
[1] 0.8823207 0.1176793
```

```
> a1<-list(loadings1,var_factor)
> a1
[[1]]
   Factor 1 Factor 2 sum1     p comm1
1      0.72    -0.15 0.54 0.462  1.09
2      0.83     0.04 0.69 0.312  1.00
3      0.69    -0.16 0.50 0.498  1.11
4      0.63    -0.35 0.52 0.483  1.56
5      0.75    -0.06 0.56 0.441  1.01
6      0.55     0.04 0.30 0.695  1.01
7      0.74    -0.17 0.57 0.426  1.10
8      0.26    -0.20 0.11 0.893  1.86
9      0.43    -0.09 0.19 0.806  1.08
10     0.71    -0.02 0.51 0.494  1.00
11     0.61    -0.08 0.38 0.617  1.03
12     0.44     0.20 0.23 0.767  1.38
13     0.30     0.14 0.11 0.889  1.38
14     0.45     0.38 0.35 0.654  1.94
15     0.50     0.16 0.27 0.727  1.20
16     0.49     0.42 0.41 0.585  1.96
17     0.53     0.40 0.45 0.552  1.86
18     0.42     0.01 0.18 0.824  1.00
19     0.32     0.07 0.11 0.894  1.09
20     0.60    -0.20 0.39 0.606  1.22

[[2]]
                     Factor_1 Factor_2
Proportion Explained     0.88     0.12
Cumulative Variance      1.63     0.22
```

```
> varimax2
Factor Analysis using method =  pa
Call: fa(r = cbind(Data1), nfactors = 4, rotate = "varimax", fm = "pa")
Standardized loadings (pattern matrix) based upon correlation matrix
      PA1  PA2  PA4   PA3   h2   u2 com
c1   0.53 0.19 0.38  0.27 0.53 0.47 2.7
c2   0.71 0.33 0.20  0.30 0.75 0.25 2.0
c3   0.56 0.17 0.37  0.17 0.51 0.49 2.2
c4   0.50 0.02 0.58  0.06 0.58 0.42 2.0
c5   0.72 0.20 0.23  0.19 0.65 0.35 1.5
c6   0.49 0.28 0.24 -0.04 0.38 0.62 2.1
c7   0.44 0.25 0.51  0.25 0.58 0.42 2.9
c8   0.11 0.01 0.34  0.07 0.13 0.87 1.3
c9   0.14 0.09 0.17  0.69 0.53 0.47 1.2
c10  0.48 0.25 0.21  0.51 0.60 0.40 2.8
c11  0.38 0.21 0.28  0.39 0.41 0.59 3.4
c12  0.35 0.33 0.05  0.08 0.24 0.76 2.1
c13  0.20 0.27 0.11 -0.02 0.12 0.88 2.2
c14  0.16 0.58 0.10  0.09 0.38 0.62 1.3
c15  0.45 0.31 0.06  0.06 0.31 0.69 1.9
c16  0.20 0.60 0.04  0.17 0.43 0.57 1.4
c17  0.26 0.61 0.09  0.10 0.46 0.54 1.5
c18  0.33 0.18 0.15  0.12 0.18 0.82 2.3
c19 -0.03 0.31 0.27  0.24 0.22 0.78 2.9
c20  0.17 0.24 0.70  0.20 0.62 0.38 1.5

                       PA1  PA2  PA4  PA3
SS loadings           3.36 2.00 1.92 1.35
Proportion Var        0.17 0.10 0.10 0.07
Cumulative Var        0.17 0.27 0.36 0.43
Proportion Explained  0.39 0.23 0.22 0.16
Cumulative Proportion 0.39 0.62 0.84 1.00


Mean item complexity =  2.1
Test of the hypothesis that 4 factors are sufficient.

The degrees of freedom for the null model are  190  and the objective function was  7.96 with Chi Square of  2272.32
The degrees of freedom for the model are 116  and the objective function was  1

The root mean square of the residuals (RMSR) is  0.04
The df corrected root mean square of the residuals is  0.05

The harmonic number of observations is  294 with the empirical chi square  179.34  with prob <  0.00015
The total number of observations was  294  with Likelihood Chi Square =  282.36  with prob <  6.9e-16

Tucker Lewis Index of factoring reliability =  0.868
RMSEA index =  0.072  and the 90 % confidence intervals are  0.06 0.08
BIC =  -376.94
Fit based upon off diagonal values = 0.99
Measures of factor score adequacy
                                                  PA1  PA2  PA4  PA3
Correlation of scores with factors               0.86 0.81 0.82 0.78
Multiple R square of scores with factors         0.75 0.65 0.67 0.60
Minimum correlation of possible factor scores    0.49 0.31 0.35 0.21
```

```
> Quartimin1
Factor Analysis using method =  pa
Call: fa(r = cbind(Data1), nfactors = 4, rotate = "quartimin", fm = "pa")
Standardized loadings (pattern matrix) based upon correlation matrix
      PA1   PA2   PA4   PA3   h2   u2 com
C1   0.50  0.03  0.21  0.15 0.53 0.47 1.6
C2   0.72  0.16 -0.04  0.15 0.75 0.25 1.2
C3   0.56  0.00  0.21  0.04 0.51 0.49 1.3
C4   0.49 -0.16  0.47 -0.07 0.58 0.42 2.2
C5   0.78  0.02 -0.01  0.04 0.65 0.35 1.0
C6   0.49  0.19  0.12 -0.19 0.38 0.62 1.8
C7   0.35  0.10  0.40  0.12 0.58 0.42 2.3
C8   0.06 -0.06  0.34  0.02 0.13 0.87 1.1
C9   0.02 -0.03  0.04  0.71 0.53 0.47 1.0
C10  0.42  0.10  0.01  0.43 0.60 0.40 2.1
C11  0.31  0.08  0.14  0.31 0.41 0.59 2.5
C12  0.33  0.28 -0.06 -0.03 0.24 0.76 2.0
C13  0.15  0.25  0.07 -0.10 0.12 0.88 2.2
C14  0.00  0.60  0.08 -0.02 0.38 0.62 1.0
C15  0.46  0.24 -0.08 -0.05 0.31 0.69 1.6
C16  0.05  0.61 -0.01  0.06 0.43 0.57 1.0
C17  0.12  0.62  0.03 -0.04 0.46 0.54 1.1
C18  0.32  0.10  0.05  0.04 0.18 0.82 1.3
C19 -0.22  0.30  0.30  0.20 0.22 0.78 3.6
C20 -0.03  0.13  0.72  0.09 0.62 0.38 1.1

                       PA1  PA2  PA4  PA3
SS loadings           3.80 1.91 1.69 1.22
Proportion Var        0.19 0.10 0.08 0.06
Cumulative Var        0.19 0.29 0.37 0.43
Proportion Explained  0.44 0.22 0.20 0.14
Cumulative Proportion 0.44 0.66 0.86 1.00

 With factor correlations of
     PA1  PA2  PA4  PA3
PA1 1.00 0.49 0.53 0.41
PA2 0.49 1.00 0.26 0.35
PA4 0.53 0.26 1.00 0.34
PA3 0.41 0.35 0.34 1.00
```

```
Mean item complexity =  1.7
Test of the hypothesis that 4 factors are sufficient.

The degrees of freedom for the null model are  190  and the objective function was  7.96 with Chi Square of  2272.32
The degrees of freedom for the model are 116  and the objective function was  1

The root mean square of the residuals (RMSR) is  0.04
The df corrected root mean square of the residuals is  0.05

The harmonic number of observations is  294 with the empirical chi square  179.34  with prob <  0.00015
The total number of observations was  294  with Likelihood Chi Square =  282.36  with prob <  6.9e-16

Tucker Lewis Index of factoring reliability =  0.868
RMSEA index =  0.072  and the 90 % confidence intervals are  0.06 0.08
BIC =  -376.94
Fit based upon off diagonal values = 0.99
Measures of factor score adequacy
                                                 PA1  PA2  PA4  PA3
Correlation of scores with factors             0.94 0.87 0.87 0.83
Multiple R square of scores with factors       0.89 0.75 0.76 0.69
Minimum correlation of possible factor scores  0.77 0.50 0.52 0.39
```

=>Factor 1 Oblique is better than factor 1 from varimax. But, Factor 2,3 and 4 from varimax are better than oblique method. The 2 methods give outputs that are not that significantly different from each other.