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Parametric Timed Formalisms for Specification and Monitoring

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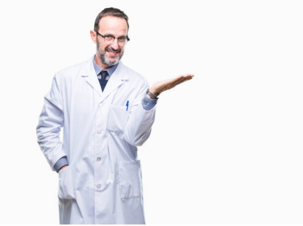
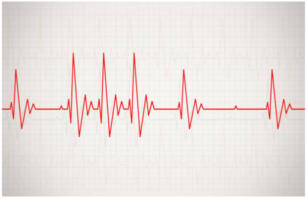
Introduction: First Example

- Electrocardiogram (ECG): Electrocardiogram signals capture information about electrical activity of the heart and can help detect anomalies in its functioning
- Doctors read the signal and detect problems



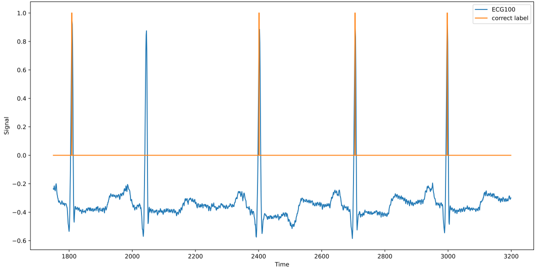


Help doctors by automating

Introduction: First Example

Possible solutions for automation

- Neural networks. Black-box. Not explainable
- Formal specifications. Interpretable and rigorous



Something wrong in the ECG

Introduction: First Example

Possible solutions for automation

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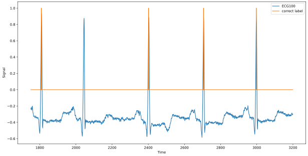
- Formal specifications. Interpretable and rigorous



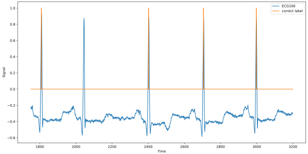
Explaining above ECG formally

- Pulse arrives prematurely (anomalous)
- Normal beat: `pulse` and (`eventually in`$_{[0,316]}$ `pulse`)
- `pulse` : `height` $\geq$ 1.24 on a time `window` of width 44 and `variation` less than 0.49 outside

Introduction: First Example

Possible solutions for automation

- Neural networks. Black-box. Not explainable
- Formal specifications. Interpretable and rigorous



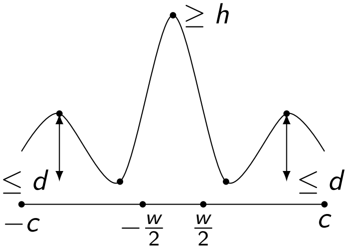
Explaining above ECG formally

- Pulse arrives prematurely (anomalous)
- Normal beat: `pulse` and (`eventually in`$_{[0,316]}$ `pulse`)
- `pulse` : `height` $\geq \cancel{1.24}$ (h) on a time `window` of width $\cancel{44}$ (w) and `variation` less than $\cancel{0.49}$ (d) outside

Introduction: Parameters

Parameter learning

Expert (doctor) gives shape, we learn parameters that fit real (ECG) data.



Depending on the patient and the scaling the parameters differ

Introduction: General Context

- Cyber-Physical Systems (CPS): Computer system + Physical environment
- Monitoring, runtime verification, trace analysis for CPS
  - Given a trace of a system detect special fragments (dangerous manoeuvres, arrhythmia, circadian rhythms, a melody)
  - On-line (train accident prevention) or off-line (plane black box)

In This Thesis

We consider two parametric timed formalisms for specification and monitoring:

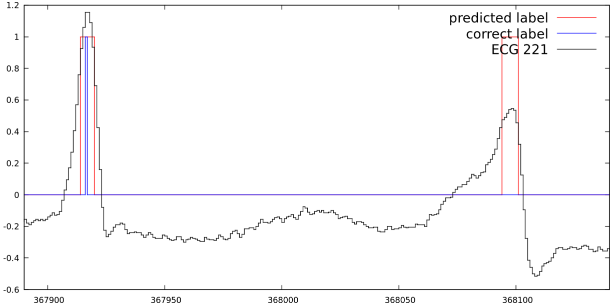
- Parametric Signal Temporal Logic (and extensions)
- Parametric Timed Regular Expressions

Outline

Learning Parameters for STL Specifications

Our Learning Framework

- Patterns in time series: e.g. ECGs
- *Labelling* of patterns: given by experts (doctors)
- Pattern predictor: STL formula which gives *prediction*
- Class of pattern predictors: parametric STL formula
- Learning: Find parameters such that prediction ≈ labelling



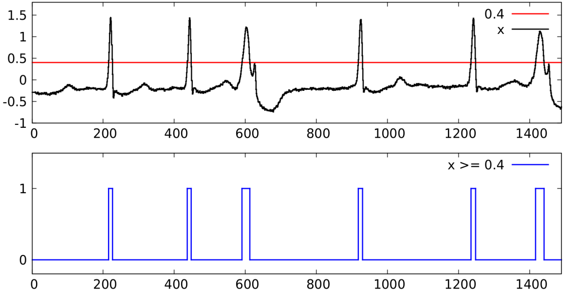
Outline

1. Learning Parameters for STL Specifications
   - Background: Signal Temporal Logic (STL)
   - STL for Accurate Pattern Prediction
   - Monotonic Formulae
   - Algorithm
   - Experiments

Background: Signal Temporal Logic

From [Maler and Ničković, 2004]. Deals with:

- Real-valued signals (x)
- Boolean signals ($x \geq 0.4$)



Background: Signal Temporal Logic

Predicates for signals

$(x \geq c) \mid (x \leq c) \mid p$

Examples

- Speed of a vehicle is below the prescribed limit ($V \leq 40$)
- Indicating sleeping status (`True` or `False`) over time: `sleep`

Background: Signal Temporal Logic

Temporal operators

$F_{[a,b]}\varphi \mid G_{[a,b]}\varphi \mid \varphi_1 \; U_{[a,b]} \; \varphi_2$
Eventually, Always and Until

Examples

- Student finishes exam within one hour ($F_{[0,1]}$ `finish`)
- Student is always awake for two hours during a lecture ($G_{[0,2]}$ `awake`)
- Student does not look at others' sheet until the exam is over (($\neg$`look`)$U_{[0,\infty)}$`over`)

Background: Extended Signal Temporal Logic

Need for min/max operators

- We have a peak in ECG signal ($x \geq 2$). How to arrive at 2?
- ECG signal stabilizes ($-0.5 \leq x \leq 0.2$). What if we do not know the limits?

From [Bakhirkin and Basset, 2019]. Introduce (local) maximum/minimum operators

$$\mathrm{Max}_{[a,b]}\, x \;\mid\; \mathrm{Min}_{[a,b]}\, x$$

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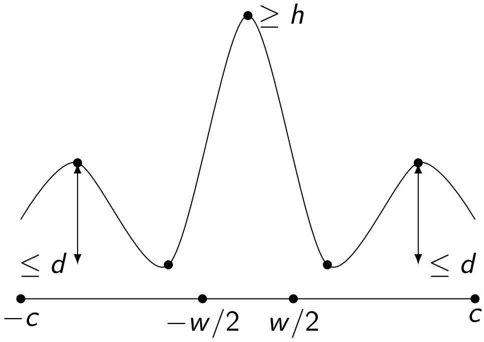
STL for Accurate Pattern Prediction

ECG pulse example:

$$\Psi^{ch}_{(w,d,h)} := ((\mathrm{Max}_{[-c,-w/2]}\, s - \mathrm{Min}_{[-c,-w/2]}\, s) \leq d) \wedge$$
$$((\mathrm{Max}_{[-w/2,w/2]}\, s) \geq h) \wedge ((\mathrm{Max}_{[w/2,c]}\, s - \mathrm{Min}_{[w/2,c]}\, s) \leq d)$$



STL for Accurate Pattern Prediction

ECG pulse example: peak,

$$\Psi^{ch}_{(w,d,h)} := ((\mathrm{Max}_{[-c,-w/2]}\, s - \mathrm{Min}_{[-c,-w/2]}\, s) \leq d) \wedge$$
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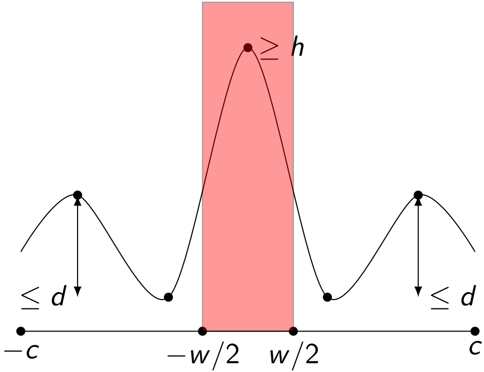
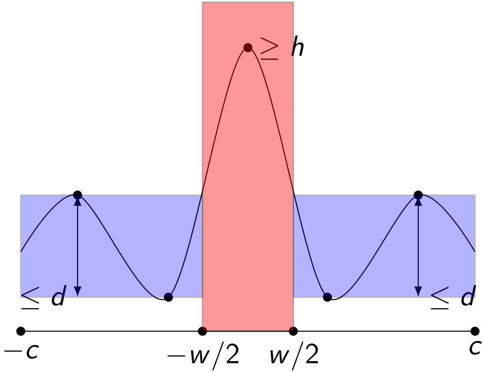


STL for Accurate Pattern Prediction

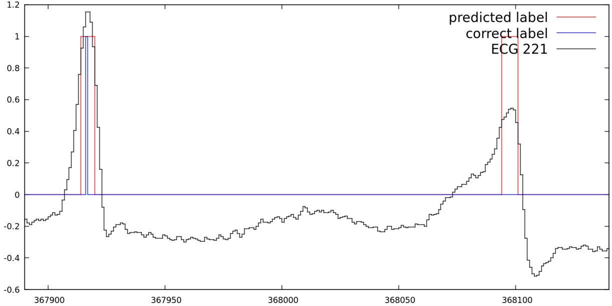
ECG pulse example: peak, stabilization

$$\Psi^{ch}_{(w,d,h)} := ((\mathrm{Max}_{[-c,-w/2]}\, s - \mathrm{Min}_{[-c,-w/2]}\, s) \leq d) \wedge$$
$$((\mathrm{Max}_{[-w/2,w/2]}\, s) \geq h) \wedge ((\mathrm{Max}_{[w/2,c]}\, s - \mathrm{Min}_{[w/2,c]}\, s) \leq d)$$



Learn to Predict Patterns

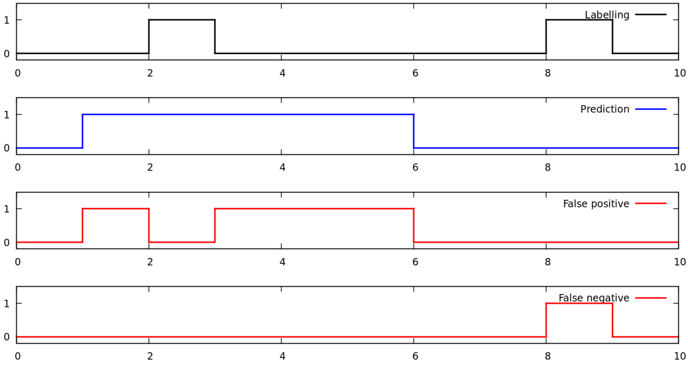
- Labelling and prediction: Both Boolean signals
- Pattern predictor (STL formula): Given observation (real-valued signals) gives predictions (Boolean signals)
- Prediction should match labelling



Defining Mismatch/Error: False Positives and False Negatives

- Class of pattern predictors from which to select: Given by a parametric STL formula
- Learn parameters while bounding mismatches w.r.t labelling
- "How often" does a mismatch occur? How to quantify?
- The <u>false positive signal</u> indicates when the predictor predicts an occurrence when there is none
- The <u>false negative signal</u> indicates when the predictor misses an actual occurrence

Illustration of fn and fp Signals



- How to get a (monotonic) measure from the fp and fn signals?
- Why monotonic?

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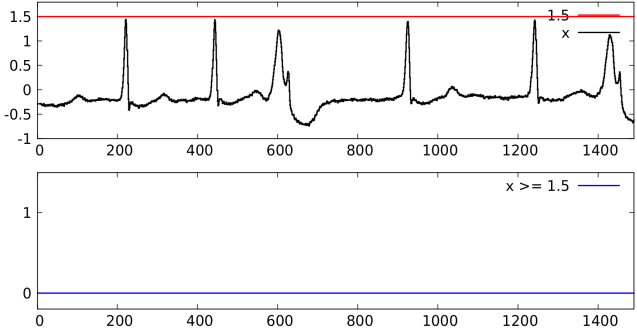
Monotonic Formulae

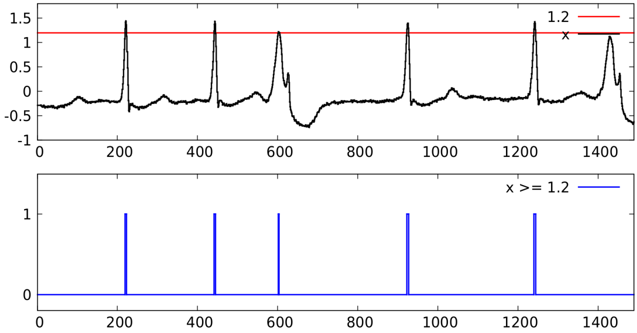
Example monotonic (w.r.t. p) formula: $(x \geq -p)$ $(p = -1.5)$, (No $\texttt{fp}$)



Number of true intervals: 0
Cumulative length of true intervals: 0

Monotonic Formulae

Example monotonic (w.r.t. p) formula: $(x \geq -p) \quad (p = -1.2)$

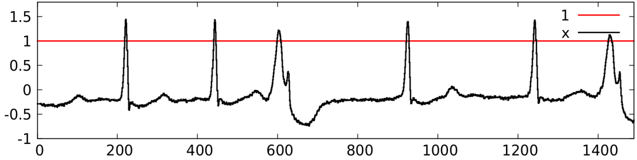


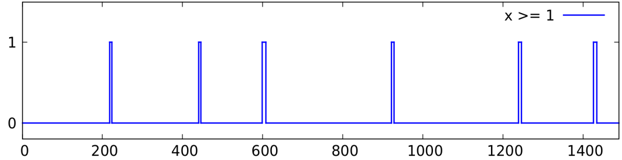
Number of true intervals: 5

Cumulative length of true intervals: 19

Monotonic Formulae

Example monotonic (w.r.t. p) formula: $(x \geq -p) \quad (p = -1.0)$



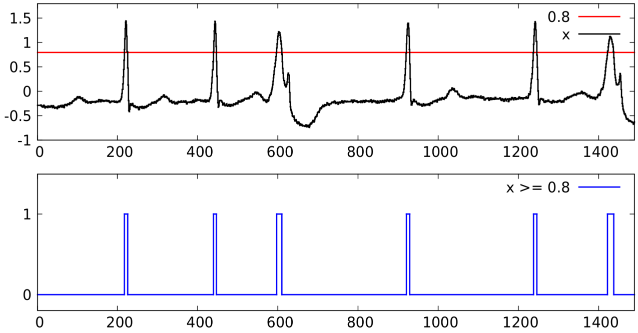


Number of true intervals: 6

Cumulative length of true intervals: 43

Monotonic Formulae

Example monotonic (w.r.t. p) formula: $(x \geq -p)$ $(p = -0.8)$

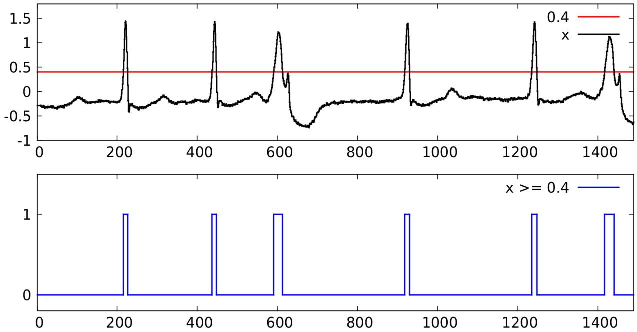


Number of true intervals: 6

Cumulative length of true intervals: 60

Monotonic Formulae

Example monotonic (w.r.t. p) formula: $(x \geq -p) \qquad (p = -0.4)$

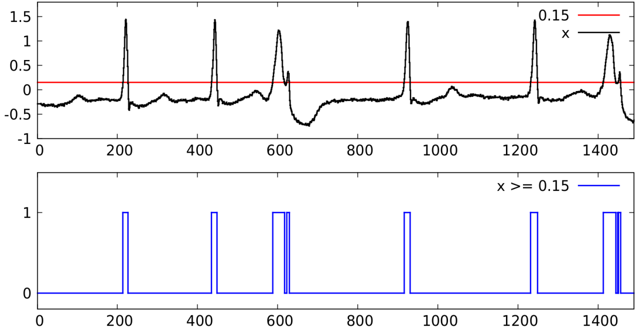


Number of true intervals: 6

Cumulative length of true intervals: 93

Monotonic Formulae

Example monotonic (w.r.t. p) formula: $(x \geq -p) \qquad (p = -0.15)$

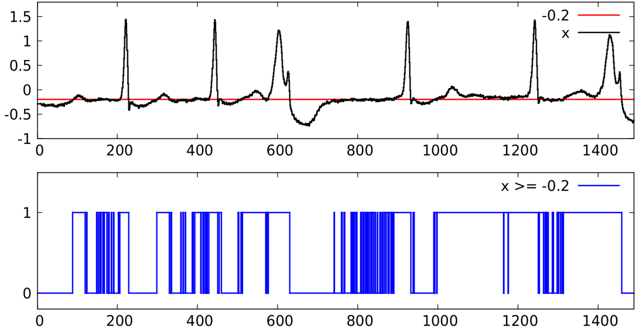


Number of true intervals: 9

Cumulative length of true intervals: 133

Monotonic Formulae

Example monotonic (w.r.t. p) formula: $(x \geq -p)$ $\qquad (p = 0.2)$

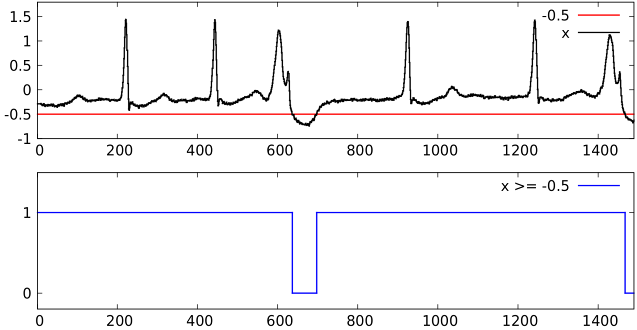


Number of true intervals: 66

Cumulative length of true intervals: 817

Monotonic Formulae

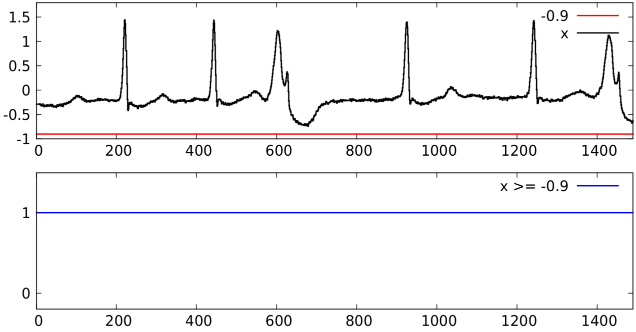
Example monotonic (w.r.t. p) formula: $(x \geq -p)$ $(p = 0.5)$



Number of true intervals: 2
Cumulative length of true intervals: 1407

Monotonic Formulae

Example monotonic (w.r.t. p) formula: $(x \geq -p)$ $(p = 0.9)$(No `fn`)



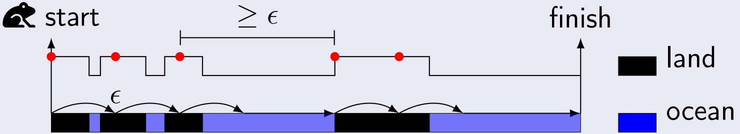
- $p \nearrow$: `fn` $\searrow$: Therefore, given bound on `fn` easy to satisfy
- $p \nearrow$: `fp` $\nearrow$: Therefore, given bound on `fp` difficult to satisfy
- Formula monotonic then `fn` and `fp` also monotonic (w.r.t. p)
- Number of true intervals in Boolean signals is not monotonic (w.r.t. p)

Introducing ϵ-count

- For a Boolean signal w, the maximum number of ϵ-separated points that can be contained in true zone is ϵ-count (monotonic)
- Close to the notion of ϵ-capacity by [Kolmogorov and Tikhomirov, 1959]

Frog analogy for our greedy algorithm

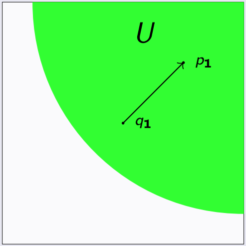
- A primordial one legged frog. It can jump by exactly ϵ when on land
- It can swim any distance in water
- The number of footsteps it leaves is ϵ-count ($= 5$ below)



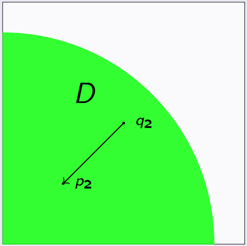
Upset, Downset and Intersection

Subsets of parameter space

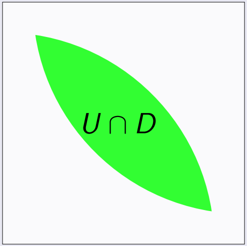
- $\{p \mid$ ϵ-count of false negatives $\leq \texttt{f}_-$ $\}$ is an Upset (upward closed)
- $\{p \mid$ ϵ-count of false positives $\leq \texttt{f}_+$ $\}$ is a Downset (downward closed)
- Intersection: Both the above conditions



(a) Upset: $q_1 \in U \wedge q_1 \leq p_1 \rightarrow p_1 \in U$



(b) Downset: $q_2 \in D \wedge p_2 \leq q_2 \rightarrow p_2 \in D$



(c) Intersection

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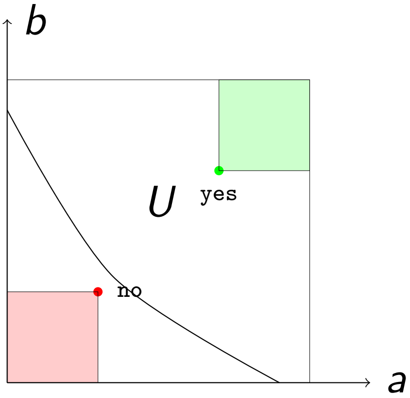
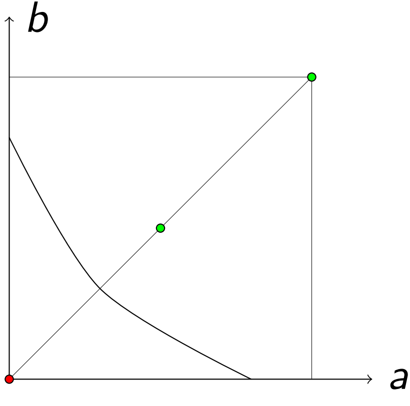
Learning an Upset Via Membership Queries [ParetoLib: Basset, Bakhirkin, Maler, Requeno 2019]

Infer from queries



Learning an Upset Via Membership Queries [ParetoLib: Basset, Bakhirkin, Maler, Requeno 2019]

- Binary search on the diagonal using queries



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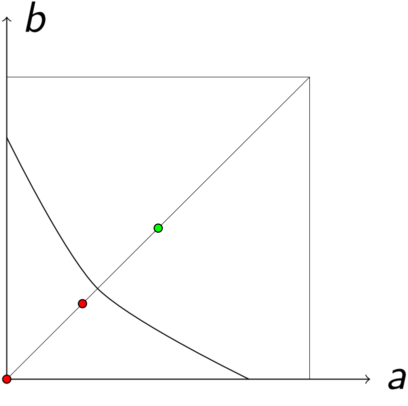

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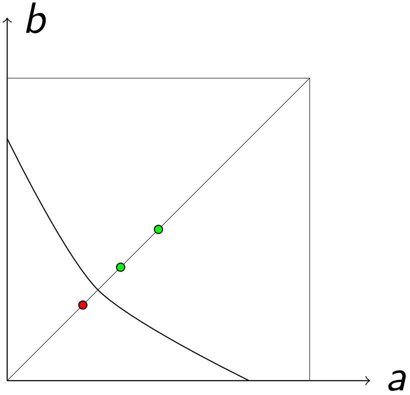
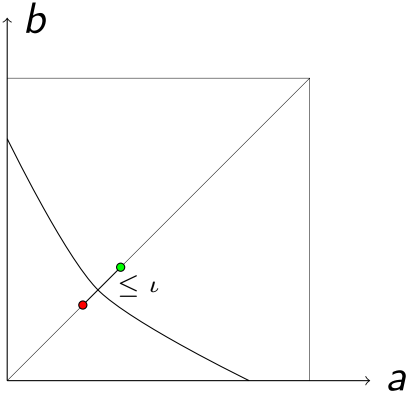


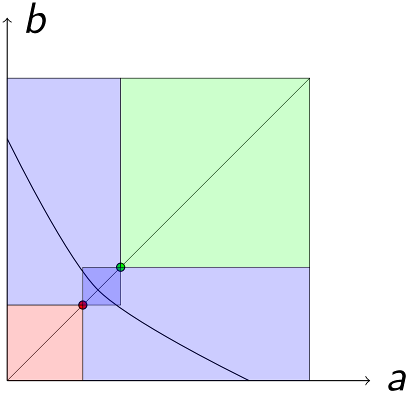
Learning an Upset Via Membership Queries [ParetoLib: Basset, Bakhirkin, Maler, Requeno 2019]

- Binary search on the diagonal using queries
- Find interval (length $\leq \iota$) containing the boundary



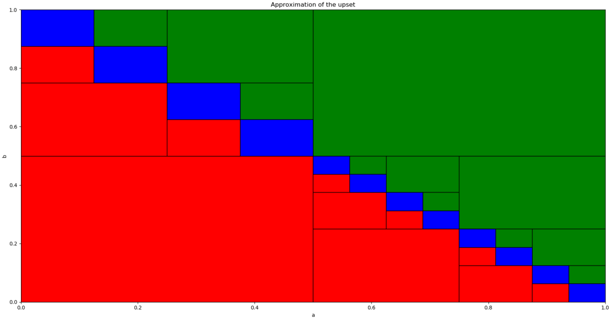
Learning an Upset Via Membership Queries [ParetoLib: Basset, Bakhirkin, Maler, Requeno 2019]

- Binary search on the diagonal using queries
- Find interval (length $\leq \iota$) containing the boundary
- Divide and recurse (on a blue box)



Learning an Upset Via Membership Queries [ParetoLib: Basset, Bakhirkin, Maler, Requeno 2019]

- Approximating upset $(a + b \geq 1)$ after a few iterations
- $\iota = 10^{-5}$ negligible



Learning Intersection of an Upset and a Downset Via Membership Queries
In ParetoLib 2.0

Algorithm Sketch to Process a Box

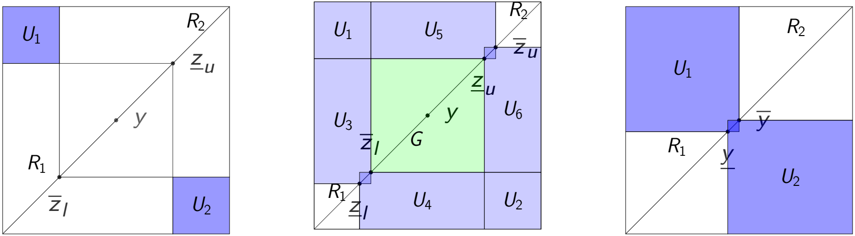
- Binary search on the diagonal to find a point p_c in the intersection
  - $p_c \in U \cap D$ (positive intersection) or
  - $p_c \in U^c \cap D^c$ (negative intersection)
- Binary search two times to find boundary points p_+ and p_-

(a) Negative intersection.

(b) Positive intersection.

Decomposing the Box

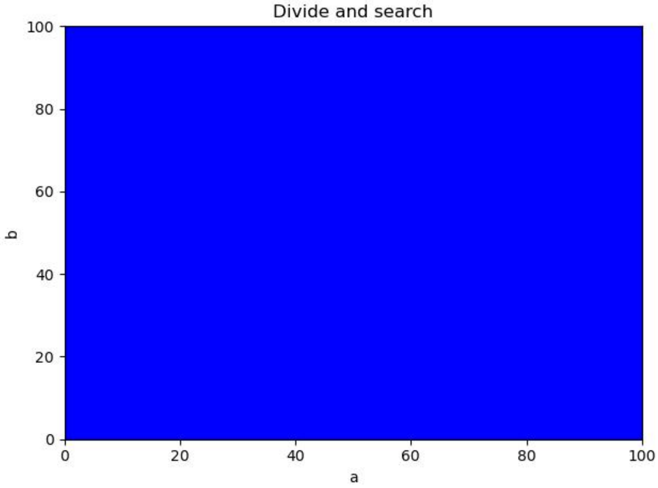
- Deduce the green (positive intersection), white (no solution) and blue (undecided) regions by drawing hyper-rectangles (boxes)
- The number of boxes generated is at most quadratic in dimension of the parameter space
- Add the blue boxes to the processing queue and repeat



(a) Negative intersection (b) Positive intersection (c) No intersection found

Algorithm Execution (a and b are Parameters)

Explore: $a + b \geq 100 \wedge a + b \leq 130$. Target unexplored volume (V_δ): 10%



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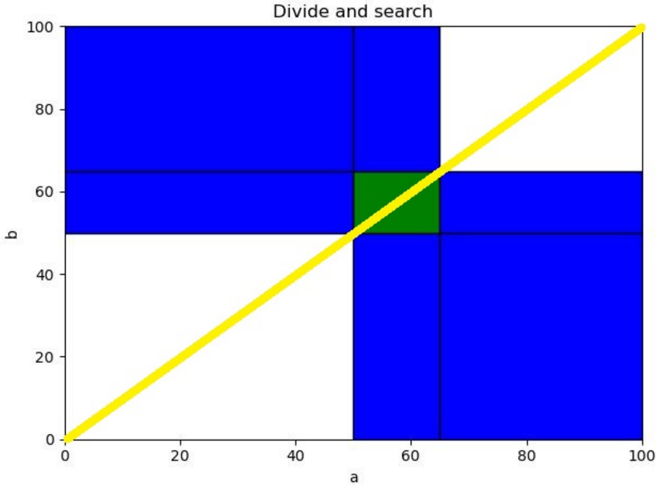

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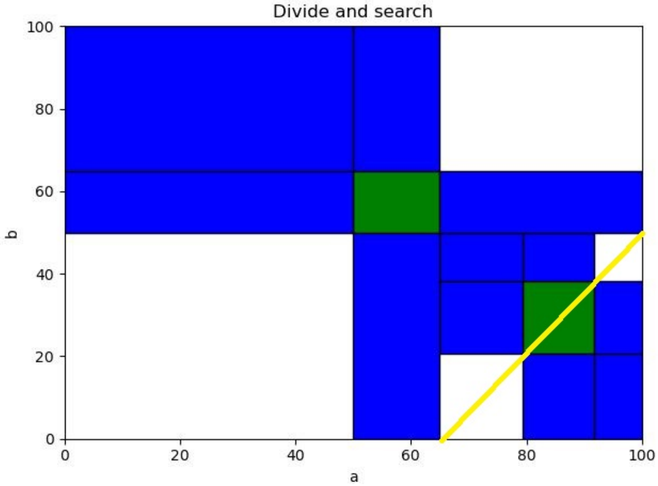

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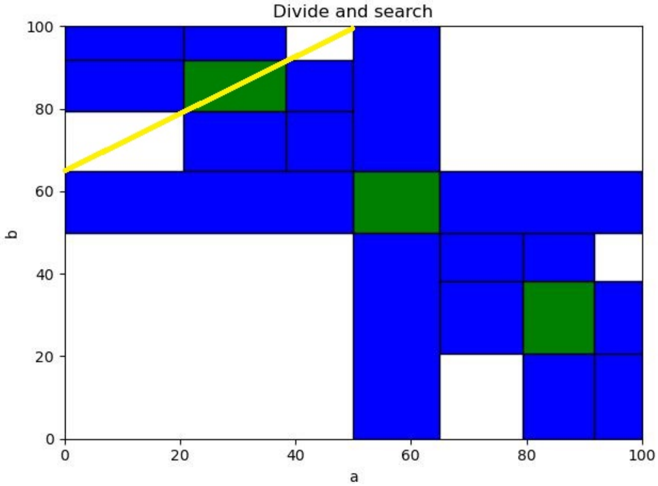

Algorithm Execution (a and b are Parameters)

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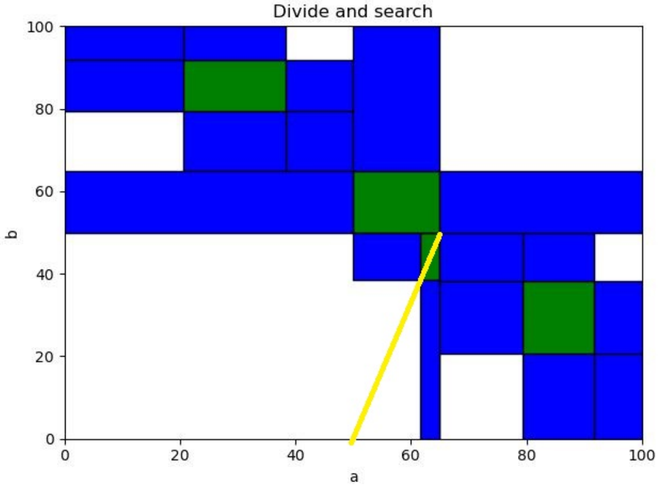

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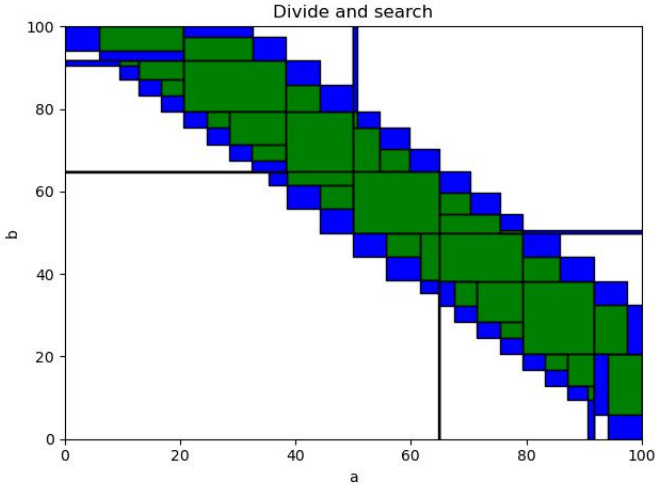

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   - Experiments

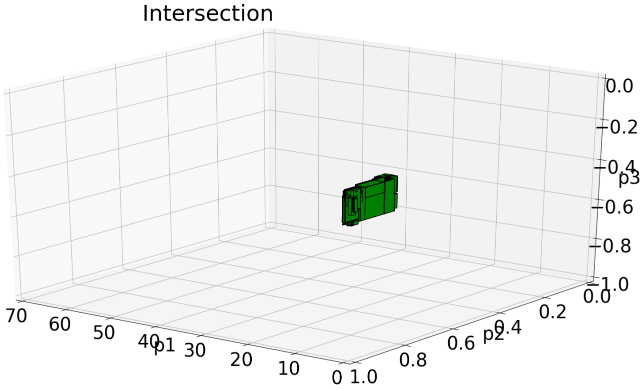
Experiments

- ECG signals taken from MIT-BIH Arrhythmia Database of Physionet [Goldberger et al., 2000, Moody and Mark, 2001]
- Give solution set of parameters with the best possible trade-off between false positives and false negatives

Characterization of ECG Pulses

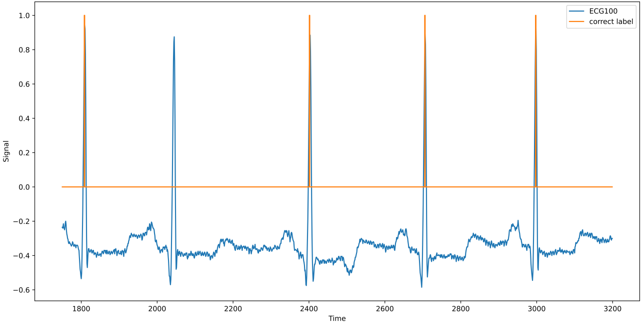
- $\Psi^{ch}_{(p_1,p_2,p_3)} := ((\mathrm{Max}_{[-\mathrm{c},-\mathrm{p}_1]}\, s - \mathrm{Min}_{[-\mathrm{c},-\mathrm{p}_1]}\, s) \leq p_2) \wedge ((\mathrm{Max}_{[-\mathrm{p}_1,\mathrm{p}_1]}\, s) \geq -p_3) \wedge ((\mathrm{Max}_{[\mathrm{p}_1,\mathrm{c}]}\, s - \mathrm{Min}_{[\mathrm{p}_1,\mathrm{c}]}\, s) \leq p_2)$
- For ECG-221, no false negatives (fn) and a single false positive (fp) out of 2462 annotations
- For ECG-100, neither the number of fp nor fn can go below 30 out of 2274 annotations
- Reason: Expressed only the shape of the heart pulses not their rhythm

3D Intersection/Solution Set (ECG 221) (zero false negatives and one false positive)



ECG 100: Atrial Premature Beat

- $\Psi^{ch_b}_{(p_1,p_2,p_3,p_4)} := \Psi^{ch}_{(p_1,p_2,p_3)} \wedge F_{[0,p_4]} \Psi^{ch}_{(p_1,p_2,p_3)}$
- Above peak detector has 3 false positives and 1 false negative



Outline

2. Parametric Timed Regular Expressions
   - Pattern Matching
   - Parametric Signal Regular Expressions (PSRE)
   - Parametric Timed Regular Expressions (Event-Based)
   - Parametric Identification of PSRE

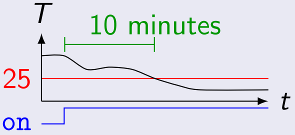
Matching Words and Timed Pattern Matching

Classical pattern matching

Search for "cow" in "The cow gives milk."

Timed pattern matching [Ulus et al., 2014]

Find time intervals (t, t'): $\uparrow \texttt{on} \cdot \langle T \geq 25 \rangle_{[10,\infty)}$ (air-conditioning malfunction)



Parametric Timed Pattern Matching (PTPM)

- Done for event-based Parametric Timed Automata (PTA) by [Waga and André, 2019]
- In this thesis, for three kinds of parametric TRE

Three Types of Parametric TRE

Parametric Signal Regular Expressions (PSRE)

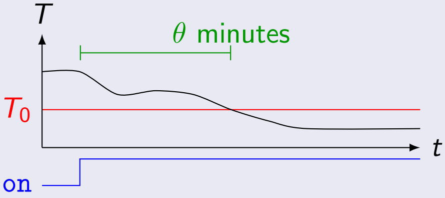
Parametric Timed Regular Expressions (PTRE) (event-based)

Parametric Event-Bounded Timed Regular Expressions (PE-TRE)

Three Types of Parametric TRE

Parametric Signal Regular Expressions (PSRE) extends SRE [Bakhirkin et al., 2017]

- $\langle T \geq T_0 \rangle_{[\theta,\infty)}$



Parametric Timed Regular Expressions (PTRE) (event-based)

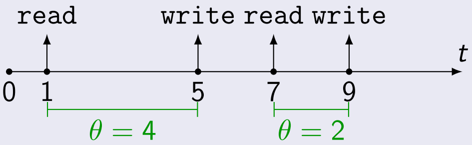
Parametric Event-Bounded Timed Regular Expressions (PE-TRE)

Three Types of Parametric TRE

Parametric Signal Regular Expressions (PSRE)

Parametric Timed Regular Expressions (PTRE) (event-based) extends TRE [Asarin et al., 2002]

- $\underline{\texttt{read}} \cdot \langle \underline{\texttt{write}} \rangle_{[\theta,\theta]}$



Parametric Event-Bounded Timed Regular Expressions (PE-TRE)

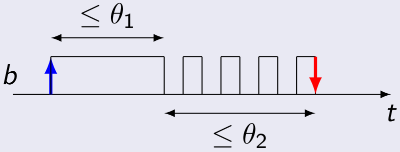
Three Types of Parametric TRE

Parametric Signal Regular Expressions (PSRE)

Parametric Timed Regular Expressions (PTRE) (event-based)

Parametric Event-Bounded Timed Regular Expressions (PE-TRE) extends E-TRE [Ferrère et al., 2015]

- Rising and falling edges as events
- $\uparrow b \cdot \langle b \rangle_{[0,\theta_1]} \cdot \langle \neg b \cdot b \rangle^{+}_{[0,\theta_2]} \cdot \downarrow b$



Outline

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   - Pattern Matching
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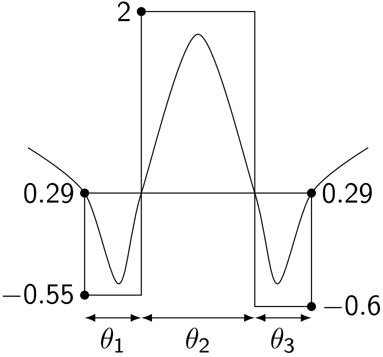
   - Parametric Timed Regular Expressions (Event-Based)
   - Parametric Identification of PSRE

Specification with PSRE: ECG Example

PSRE (with simple concatenation):

$$\langle -0.55 \leq x \leq 0.29 \rangle_{[\theta_1,\theta_1]} \cdot \langle 0.29 \leq x \leq 2.0 \rangle_{[\theta_2,\theta_2]} \cdot \langle -0.6 \leq x \leq 0.29 \rangle_{[\theta_3,\theta_3]}$$



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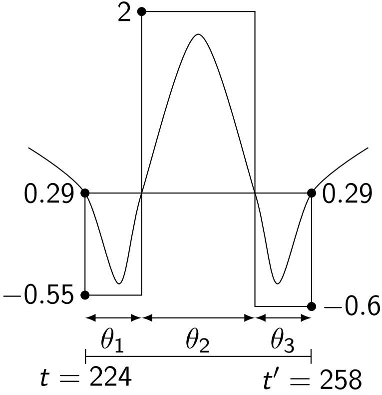
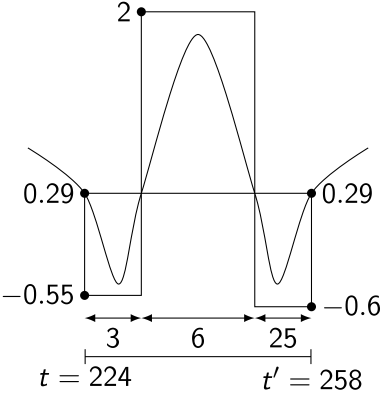


timed match

Specification with PSRE: ECG Example

PSRE (with simple concatenation):

$$\langle -0.55 \leq x \leq 0.29 \rangle_{[\theta_1,\theta_1]} \cdot \langle 0.29 \leq x \leq 2.0 \rangle_{[\theta_2,\theta_2]} \cdot \langle -0.6 \leq x \leq 0.29 \rangle_{[\theta_3,\theta_3]}$$



parametric timed match

Parametric Signal Regular Expressions (PSRE)

PSRE (syntax):

$$x \geq \lambda \mid x \leq \lambda \mid x \geq c \mid x \leq c \mid \langle \phi \rangle_{[a(\overline{\theta}), b(\overline{\theta})]} \mid \phi \cdot \psi \mid \phi^* \mid \phi \wedge \psi \mid \phi \vee \psi$$

- x: signal variable
- $\overline{\theta}$: parameters
- $a(\overline{\theta})$, $b(\overline{\theta})$: affine expressions over $\overline{\theta}$
- $\lambda \in \overline{\theta}$: a parameter
- $c \in \mathbb{R}$: a constant

Parametric match set:

$$\mathcal{M}(w, \phi(\overline{\theta})) = \{(t, t', \overline{\theta}) \mid (w, t, t') \models \phi(\overline{\theta})\}$$

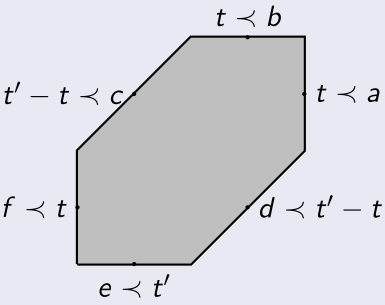
Main aspects

- Describe sequences naturally
- Duration and magnitudes
- Parameters for more expressiveness

Main theorem of timed pattern matching [Ulus et al., 2014]

The match-set $\mathcal{M}(\varphi, w)$ is a finite union of zones

Zone



$$\begin{pmatrix} t \prec a \\ t' \prec b \\ t' - t \prec c \\ d \prec t' - t \\ e \prec t' \\ f \prec t \end{pmatrix}$$

Method for proof and algorithm

Structural induction over φ

Main theorem of parametric timed pattern matching

The **parametric** match-set $\mathcal{M}(\varphi, w)$ is a finite union of **parametric** zones

Parametric zone (z)

$$\begin{pmatrix} t \prec \min(a_1(\overline{\theta}), \ldots, a_{n_1}(\overline{\theta})) \\ t' \prec \min(b_1(\overline{\theta}), \ldots, b_{n_2}(\overline{\theta})) \\ t' - t \prec \min(c_1(\overline{\theta}), \ldots, c_{n_3}(\overline{\theta})) \\ \max(d_1(\overline{\theta}), \ldots, d_{n_4}(\overline{\theta})) \prec t' - t \\ \max(e_1(\overline{\theta}), \ldots, e_{n_5}(\overline{\theta})) \prec t' \\ \max(f_1(\overline{\theta}), \ldots, f_{n_6}(\overline{\theta})) \prec t \end{pmatrix} \wedge \mathrm{C_z}(\overline{\theta}) \text{ where } \mathrm{C_z}(\overline{\theta}) \text{ is a polytope}$$

over $\overline{\theta}$

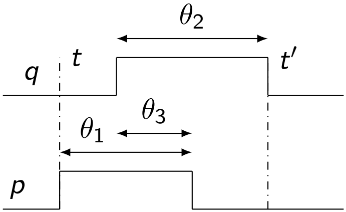
Method for proof and algorithm

Structural induction over φ

Using PSRE Intersection: Example

PSRE:

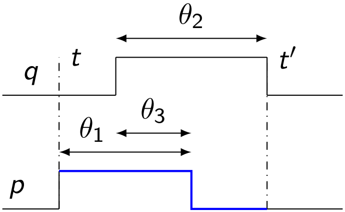
$$(\langle p\rangle_{[\theta_1,\theta_1]}\cdot\neg p)\wedge(\neg q\cdot\langle q\rangle_{[\theta_2,\theta_2]})\wedge(\mathtt{true}\cdot\langle p\wedge q\rangle_{[\theta_3,\theta_3]}\cdot\mathtt{true})$$



Using PSRE Intersection: Example

PSRE:

$$(\langle p \rangle_{[\theta_1,\theta_1]} \cdot \neg p) \wedge (\neg q \cdot \langle q \rangle_{[\theta_2,\theta_2]}) \wedge (\mathtt{true} \cdot \langle p \wedge q \rangle_{[\theta_3,\theta_3]} \cdot \mathtt{true})$$



Using PSRE Intersection: Example

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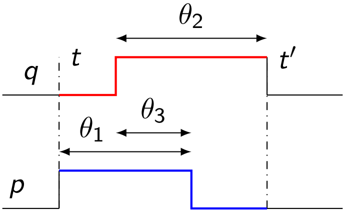

Using PSRE Intersection: Example

PSRE:

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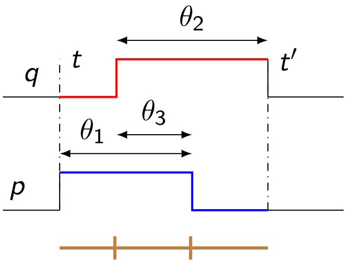
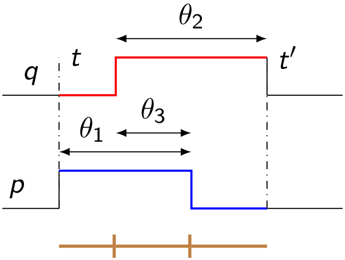


Using PSRE Intersection: Example

PSRE:

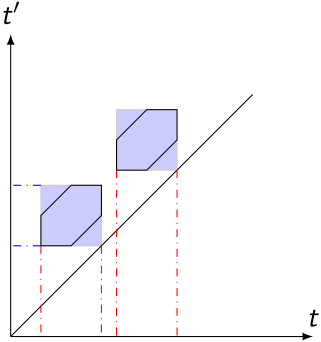
$$(\langle p\rangle_{[\theta_1,\theta_1]}\cdot\neg p)\wedge(\neg q\cdot\langle q\rangle_{[\theta_2,\theta_2]})\wedge(\texttt{true}\cdot\langle p\wedge q\rangle_{[\theta_3,\theta_3]}\cdot\texttt{true})$$

A parametric zone in parametric match-set: $(t'-\theta_2=3)\wedge(t+\theta_1=7)\wedge(\theta_3\leq 4)\wedge(t'\leq 10)\wedge(t\leq 3)\wedge(\theta_3\geq 1)\wedge(t\geq 0)\wedge(t'\geq 7)$



Algorithms for Intersection and Concatenation: Plane-Sweep

- Intersect and concatenate formulae: intersect and sequential composition of zones respectively
- All possible combinations: quadratic
- Handle parametric zones by ordering them along the time dimensions



- Project on time dimensions
- Infer (~~compute~~) empty intersection and sequential composition
- Linear number of polytope operations for usual cases

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Parametric Timed Regular Expressions (PTRE) (Event-Based) Syntax

Syntax:

$$\underline{a} \mid \langle \varphi \rangle_I \mid \varphi_1 \cdot \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \wedge \varphi_2 \mid \varphi^*$$

where $I = [\alpha, \beta]$ and each of α, β is either a non-negative constant or a timing parameter s_i

Σ is the event alphabet

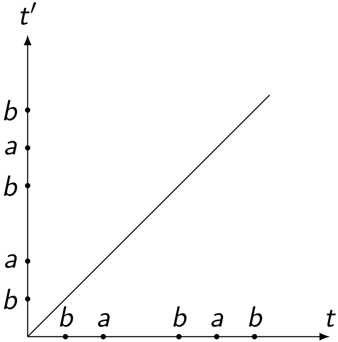
For $a \in \Sigma$, $\underline{a}$ represents an arbitrary passage of time followed by event a

Express

- Sequences of events
- Distribution of events in time and their relations

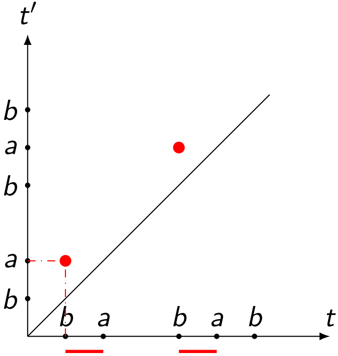
PTRE (Event-Based) Matching Illustration

- The timed word: $(1, b), (2, a), (4, b), (5, a), (6, b)$
- Whole expression: $\langle \underline{a} \cdot \underline{b} \rangle_{[\theta_1, \theta_2]}$
-



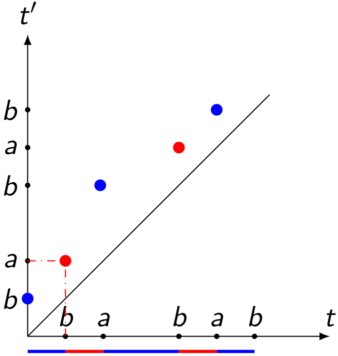
PTRE (Event-Based) Matching Illustration

- The timed word: $(1, b), (2, a), (4, b), (5, a), (6, b)$
- Whole expression: $\langle \underline{a} \cdot \underline{b} \rangle_{[\theta_1, \theta_2]}$
- $\underline{a}$



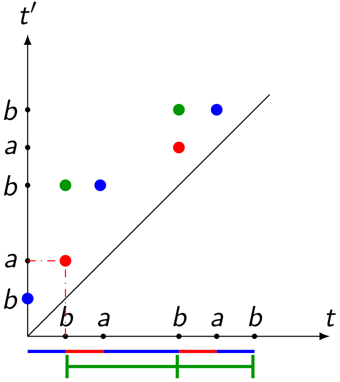
PTRE (Event-Based) Matching Illustration

- The timed word: $(1, b), (2, a), (4, b), (5, a), (6, b)$
- Whole expression: $\langle \underline{a} \cdot \underline{b} \rangle_{[\theta_1, \theta_2]}$
- $\underline{a}$ $\underline{b}$



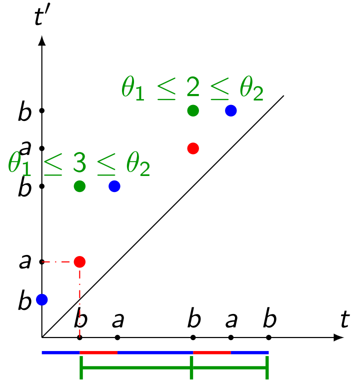
PTRE (Event-Based) Matching Illustration

- The timed word: $(1, b), (2, a), (4, b), (5, a), (6, b)$
- Whole expression: $\langle \underline{a} \cdot \underline{b} \rangle_{[\theta_1, \theta_2]}$
- $\underline{a} \cdot \underline{b}$



PTRE (Event-Based) Matching Illustration

- The timed word: $(1, b), (2, a), (4, b), (5, a), (6, b)$
- Whole expression: $\langle \underline{a} \cdot \underline{b} \rangle_{[\theta_1,\theta_2]}$
- $\langle\ \underline{a}\ \cdot\ \underline{b}\ \rangle_{[\theta_1,\theta_2]}$



Match-Set and Parametric Intervals

Main theorem for PTPM of PTRE (event-based)

For a given timed word the parametric match-set of a PTRE is a finite union of parametric intervals

Parametric interval (y)

$$(t = d_1 \wedge t' = d_2 \wedge \boldsymbol{C_y})$$

where d_1, d_2 are real constants and $\boldsymbol{C_y}$ is a polytope over the parameter space

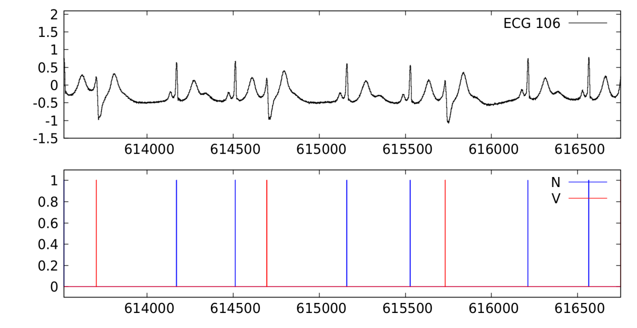
Algorithms for intersection and concatenation

Sort according to the start and end of intervals. Then use binary search

Example: Ventricular Trigeminy

- Repetition of one premature **V**entricular contraction (V) followed by two **N**ormal pulses (N) (previously in [Bartocci et al., 2014])
- $\langle\langle \underline{V} \cdot \underline{N} \cdot \underline{N} \rangle_{[\theta_1,\theta_1]} \cdot (\langle \underline{V} \cdot \underline{N} \cdot \underline{N} \rangle_{[\theta_2,\theta_3]})^{+} \cdot \underline{V} \rangle_{[\theta_4,\theta_4]}$

An example of match (parametric interval)



$$(t = 613516) \wedge (t' = 616750) \wedge (\theta_1 = 995) \wedge (0 \leq \theta_2 \leq 1016) \wedge$$
$$(1037 \leq \theta_3) \wedge (\theta_4 = 3234)$$

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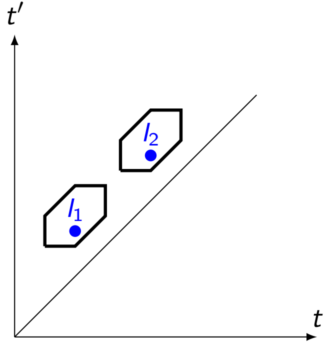
Parametric Identification of PSRE

- Given a PSRE ψ (defined over parameter vectors u and v)
- Signal w
- A list $\mathcal{I}$ of n intervals $I_1 = [a_1, b_1], \ldots, I_n = [a_n, b_n]$
- Compute the set of parameters that produce a match at each of these n intervals



Parametric Identification of ECG

PSRE for characterizing and identifying ECG:

$$\phi_7 := \langle -\lambda_0 \leq x \leq \lambda_1 \rangle_{[0,\theta_0]} \cdot \langle \lambda_1 \leq x \leq \lambda_0 \rangle_{[0,\theta_1]} \cdot \langle -\lambda_0 \leq x \leq \lambda_1 \rangle_{[0,\theta_0]}$$

- We label the first ten pulses of ECG using ten intervals of 80 time units each
- Parametric identification takes 478s. Inefficient for large data

How to side-step this inefficiency?

- Bottleneck is computing match-set when magnitude parameters (λ) for expressions like $x \leq \lambda$
- Avoid magnitude parameters. Pre-process signals using Booleanizers
- Then detect patterns using expressions with only timing parameters

Conclusions: PSTL

- Theoretical contributions
  - Use of STL formulae as pattern predictors
  - Inaccuracy of prediction represented using false positive and false negative signals
  - ϵ-count to quantify the above signals
  - Intersection algorithm for parametric identification of monotonic predictors using queries
- Implementation of intersection algorithm in ParetoLib
- Application to analysis of ECGs
  - Parametric identification of normal pulse/peak detection in ECGs
  - Classification of ECGs via feature learning

Conclusions: parametric TRE

- Theoretical contributions
  - Three types of parametric TRE suitable for expressing different types of behaviours (in signals and timed words)
  - Algorithms for parametric timed pattern matching of the above
  - Parametric identification of PSRE
- Implemented `parameTRE` tool
- Applications:
  - Analysis of ECG signals
  - Detection of marine traffic rule violation

Conclusions

Not presented today:

- Counting operators for (parametric) STL
- Extension of (parametric) TRE with Min/Max operators
- Concept of “decisive region” in (parametric) timed pattern matching

Future Work

- Monotonic PSRE (timing parameters only?) defined over PSTL Booleanizers (magnitude parameters only?) in ParetoLib
- Diagnostics of TRE and parametric TRE. Use match-set graph

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