M.Sc. Simulation Sciences, Winter Semester 2016 / 2017

Numerical Methods for PDE

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Homework 1

Due: Tuesday, Nov 15, 2016 (by midnight)

Problem Definition

Consider a smooth function $x \mapsto u(x)$, and a difference approximation of the form

$$u^{(m)}(x_i) = \sum_{k=-p}^{q} d_k u(x_{i+k}), \qquad p, q \geq 0. \tag{1}$$

where $u^{(m)}(x_i) \equiv u_i^{(m)}$ is the m^{th} derivative, and points x_j are defined on a *grid*,

$$\mathcal{G}_h := \{jh : j \in \mathbb{Z}, h > 0\}. \tag{2}$$

Write a computer program that generates a Taylor table, and determines the coefficients of a difference approximation. The program should be written in such a way that derivatives of arbitrary order (i.e. m in eq. (1)) can be handled, and also different stencil sizes (i.e. p, q) can be selected.

Test the implementation by generating the coefficients (a, b, c, d, e) corresponding to the best possible (i.e. highest order consistent) difference approximation

$$\frac{au_{i-2} + bu_{i-1} + cu_i + du_{i+1} + eu_{i+2}}{h^2} \;=\; u_i'' + \mathcal{O}(h^r). \tag{3}$$

Report the following:

- The coefficients a through e in (3);
- the accuracy of the approximation, i.e. the value r in (3).

Hints and Comments

- The main parameters are the order of the derivative and the stencil size (here given by p and q). These should be input parameters.
- It is obvious that one cannot approximate a derivative of given order if one does not have enough points in the stencil. Your code should include a basic check that the input parameters p, q correspond to sufficiently many points to support approximation of a derivative of desired order.
- Carefully read the handout regarding homework rules.

- In particular, recall that you should submit all code that you wrote to solve the problem, and *additionally* submit a short write-up in which you report the values that were asked for above.
- For this problem it is ok to solve the arising linear system with an intrinsic function (e.g. using $A\backslash b$ in `MATLAB`)