M.Sc. Simulation Sciences, Winter Semester 2016 / 2017

Numerical Methods for PDE

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Homework 8

Due: Friday, March 14, 2017 (by Midnight)

Consider the two-dimensional Poisson problem

$$\begin{aligned} -\nabla^2 u &= f, \qquad x \in \Omega \equiv (0,1)^2 \\ u &= 0, \qquad x \in \partial\Omega, \end{aligned}$$

where the right-hand side is chosen such that the solution is given by $u(x,y) = \sin(\pi x)\sin(\pi y)$. As discussed in class, use the Ritz-Galerkin Method with piecewise linear approximation space and nodal basis, to solve the proper variational form of this equation.

Further guidelines and instructions:

- Use the provided grids to test your implementation.
- Report the L^2-error for each grid.
- It is ok to use a numerical quadrature to compute the integrals for the right-hand side and the error. (Use a three-point quadrature rule, as discussed in the tutorial.)
- In the end, you will have to solve a linear system $A\mathbf{u} = \mathbf{f}$, where A is a matrix of size $n \times n$, and n is the number of *interior* grid nodes. You may store this matrix, and solve the linear system, in any way you see fit. (In particular you may use built-in functions or external libraries.)