4.78)

Given that urn contains

white balls = 4 = P(W)

Black balls = 4 = P(B)

Total no. of ways $= \frac{8!}{4!\,4!} = 70$

$P(1-W) + P(1-B)$

$= 2 = 1-P = 1 - \frac{36}{70} = \frac{34}{70} = \frac{17}{35}$

The probability that we can make Exactly n selections

$P(X=n) = (P)(2)^{n-1}$

$= \left(\frac{17}{35}\right)^{n-1} \times \frac{18}{35}$

Theoretical problem:-

4.11) Let us consider n independent Sequential Events

Probability of success = P

" " failure = 1-P

P(k success & n-k failures) = P(S,F,F,S----F)

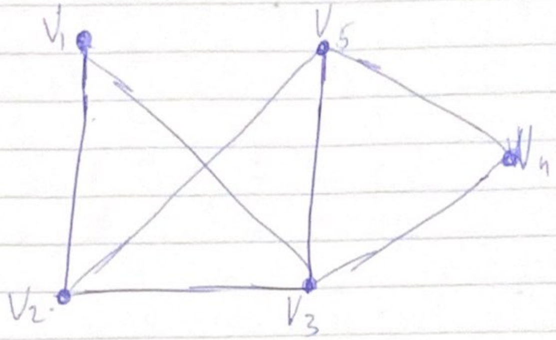
$p^k (1-p)^{n-k}$

We have n! ways to arrange n successes & n-k failures in a sequence

$= \frac{n!}{n!\,(n-k)!}$ Arrangements in Equal likely.

HW-4

1)



∴ The adjacent Matrix for the following V1, V2, V3, V4, V5 is

$$A(G) = \begin{array}{c} \begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

For number of three paths from red node to the blue node 1st we need find the (V_2, V_5)

~~we can tak~~ By Simplifying the Existing Matrix $A = A^2$

$$A^2 = \begin{bmatrix} 0&1&1&0&0 \\ 1&0&1&0&1 \\ 1&1&0&1&1 \\ 0&0&1&0&1 \\ 0&1&1&1&0 \end{bmatrix} \begin{bmatrix} 0&1&1&0&0 \\ 1&0&1&0&1 \\ 1&1&0&1&1 \\ 0&0&1&0&1 \\ 0&1&1&1&0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2&1&1&1&2 \\ 1&3&2&2&1 \\ 1&2&4&1&2 \\ 1&2&1&2&1 \\ 2&1&2&1&3 \end{bmatrix} \Rightarrow A^3 = A^2 - A$$

$$\begin{bmatrix} 2 & 1 & 1 & 1 & 2 \\ 1 & 3 & 2 & 2 & 1 \\ 1 & 2 & 4 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & 6 & 3 & 3 \\ 5 & 4 & 7 & 3 & 7 \\ 6 & 7 & 6 & 6 & 7 \\ 3 & 3 & 6 & 2 & 5 \\ 3 & 7 & 7 & 5 & 4 \end{bmatrix}$$

Therefore V_2, V_5 of $A^3 = 7$
so we have total no. of 7 steps from red node to blue node

Q. 2) Given that

$$A = \begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 8 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

By elimination rows $R_1 = R_1 - 4R_4$
$R_3 = R_3 - 6R_4$

$$A = \begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \Rightarrow Ax = 0$$

from the both A and x we have

$x_1 + 4x_3 + x_4 + 3x_6 = 0$

$x_2 + 5x_3 + 2x_4 = 0$

$x_5 + 8x_6 = 0$

$x_7 = 0$

$x_1 = -4a - b - 3c$

$x_2 = 5a - 2b$

$x_5 = -8c \quad x_7 = 0$

Solutions $= a(-4, -5, 1, 0, 0, 0, 0) + b(-1, -2, 0, 1, 0, 0, 0)$
$+ c(-3, 0, 0, 0, -8, 1, 0)$

3) find general solution $Ax = b$ where $b = (2, 3, -1, 4)^T$

from the above problem we have the solutions to $Ax = 0$; Hence Ax is

$$A = \begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 8 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 4 \end{bmatrix}$$

$\boxed{Ax = b}$

Here

$x_7 = 4$

$x_5 + 8x_6 + 6x_7 = -1$

$x_3 + 5x_6 = -25$

Let $x_6 = a$

$x_5 = -25 - 8a$

$x_2 + 5x_3 + 2x_4 = 3$

~~Let~~ Therefore $x_4 = b$; $x_3 = c$

$x_2 = 3 - 5c - 25$

$x_1 + 4x_3 + x_4 + 3x_6 + x_7 = 2$

$\Rightarrow x = -2 - 3c - 5 - 3a$

Here the above general solution depends on

values of a, b, c

4) Given ~~Matr~~ Matrix

$$A = \begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A / I = \left[\begin{array}{ccc|ccc} -2 & 2 & -1 & 1 & 0 & 0 \\ 3 & -5 & 4 & 0 & 1 & 0 \\ 5 & -6 & 4 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \Rightarrow R_1 / -2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0.5 & -0.5 & 0 & 0 \\ 3 & -5 & 4 & 1.5 & 1 & 0 \\ 5 & -6 & 4 & 2.5 & 0 & 1 \end{array}\right]$$

$R_2 \Rightarrow R_2 - 3R_1$, $R_3 \Rightarrow R_2 - 5R_1$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0.5 & -0.5 & 0 & 0 \\ 0 & -2 & 2.5 & 1.5 & 1 & 0 \\ 0 & -1 & 1.5 & 2.5 & 0 & 1 \end{array}\right]$$

$R_2 / -2 \rightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0.5 & -0.5 & 0 & 0 \\ 0 & 1 & -1.25 & -0.75 & -0.5 & 0 \\ 0 & -1 & 1.5 & 2.5 & 0 & 1 \end{array}\right]$$

~~$R_2 \Rightarrow R_2 - 3R_1$ R_3 $R_3 - 5R_1$~~

$$\left[\begin{array}{ccc|ccc} \cancel{1} & \cancel{-1} & \cancel{0.5} & \cancel{-0.5} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{-2} & \cancel{2.5} & \cancel{1.5} & \cancel{1} & \cancel{0} \\ 0 & -1 & 1.5 & 2.5 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 + R_2,\ R_3 + R_2 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -0.75 & -1.25 & -0.5 & 0 \\ 0 & 1 & -1.25 & 0.75 & -0.5 & 0 \\ 0 & 0 & 0.25 & 1.75 & -0.5 & 1 \end{array}\right]$$

$R_3 / 0.25 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -0.75 & -1.25 & -0.5 & 0 \\ 0 & 1 & -1.25 & -0.75 & -0.5 & 0 \\ 0 & 0 & 1 & 7 & -2 & 4 \end{array}\right]$$

$R_1 + 0.75 R_3 \to R_1 :\ R_2 + 1.25\, R_3 \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -2 & 3 \\ 0 & 1 & 0 & 8 & -3 & 5 \\ 0 & 0 & 1 & 7 & -2 & 4 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix}$$

$\therefore$ The Inverse of Matrix $= \begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix}$

5) ① Given that A & B are 10×10 Matrices, such that

$\det(A) = 4$ $\det(B) = 5$

② Matrix C is obtained by Exchanging rows 5 and 7 of A Then Scaling row 9 by 3.

Matrix D obtained by exchanging columns 1 & 3

① -> Det of $A^{-1}BC^{-1}D = ?$

$$\operatorname{Det}(C) = -3 \times \det(A)$$
$$= -3 \times 4$$
$$= -12$$

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(A^{-1}) = \frac{1}{-12}$$

② ->

$$\det(d) = 2^{10} \times \det(B)$$
$$= 1024 \times 5$$
$$= 5120$$

$$\therefore \det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{4}$$

Here $\det(A^{-1}BC^{-1}D)$

$$\det(A^{-1}BC^{-1}D) = \det(A^{-1}) \det(B) \det(C^{-1}) \det(D)$$

$$= \frac{1}{4} \times 5 \times \frac{-1}{12} \times 1024 \times 5$$

$$= \frac{-5120 \times 5}{48} = \frac{-25600}{48} = 533.3$$