Lesson 6 Board-work

Plan for Lesson 6[(1)]

- Understanding multivariate functions and their domain.
- Visualizing functions of two variables.
- Understanding and finding partial derivatives.
- Using chain rules for multivariate functions

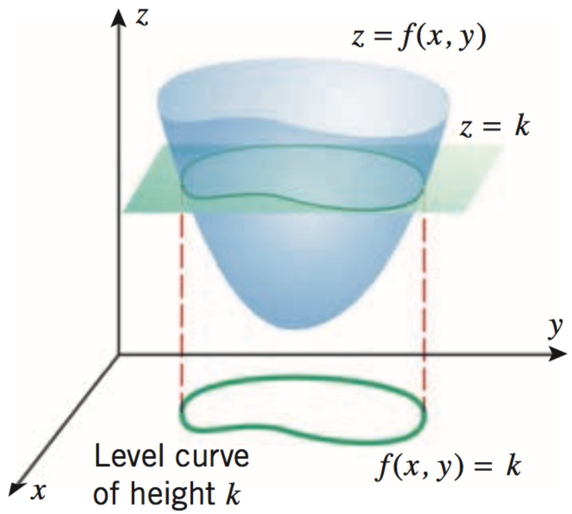
(1)Image source: Calculus by Anton, Bivens and Davis

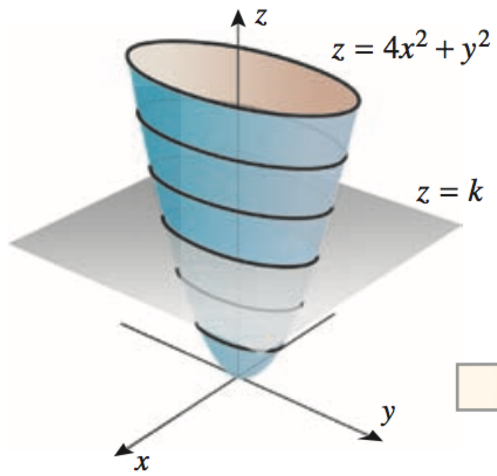
Q 1. Find and sketch the domains of the following functions of two variables.

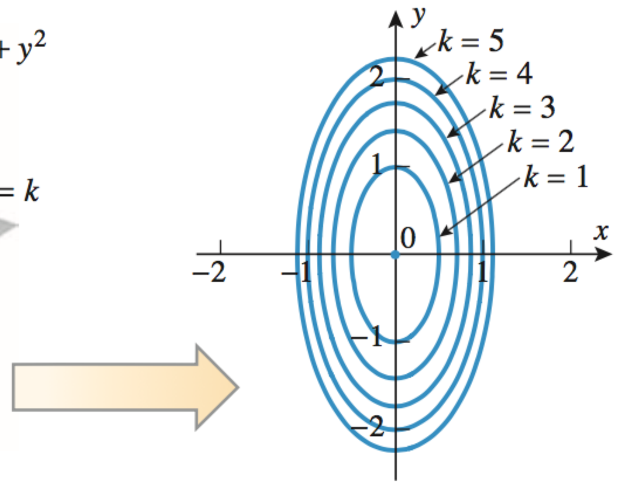
A. $f(x,y) = \ln(y - x^2)$

B. $g(x,y) = \sqrt{x} + \sqrt{y} + \sqrt{1 - x^2 - y^2}$

Cross-sections, Level Curves, and Contour Plots







Q 2. Sketch a contour diagram (i.e. contour map) for each of the following functions by sketching several level curves. Then sketch a rough graph of the function.

A. $f(x,y) = x + y$

B. $g(x,y) = x^2 + y^2$

C. $h(x,y) = x^2 - y^2$

Definition: Partial derivative

Given a function $f(x, y)$, the **partial derivative with respect to** x is defined by

$$f_x(x, y) = \frac{d}{dx} f(x, y) = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h},$$

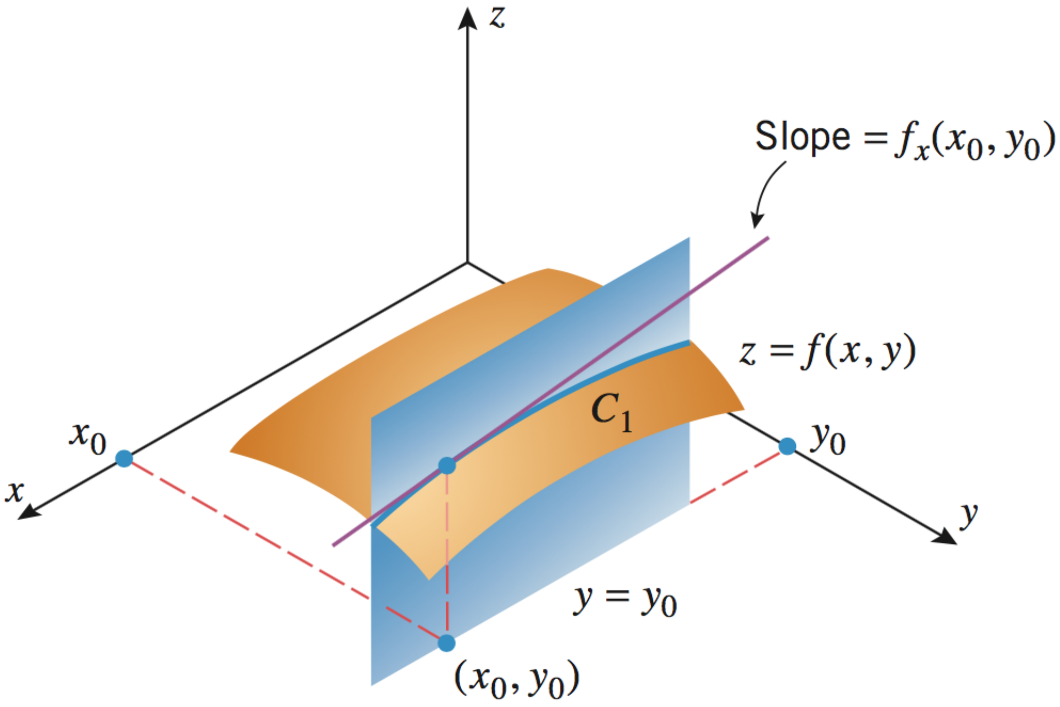
provided the limit exists.

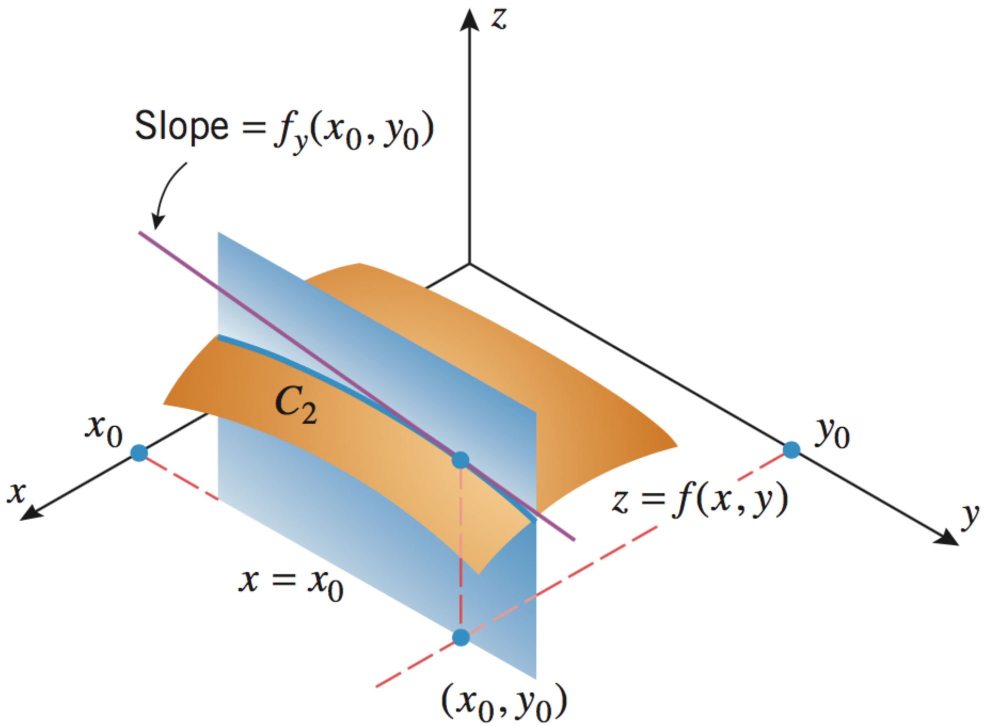
Given a function $f(x, y)$, the **partial derivative with respect to** y is defined by

$$f_y(x, y) = \frac{d}{dy} f(x, y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h},$$

provided the limit exists.

Geometric Interpretation: $f_x(x_0,y_0)$ gives the slope of the surface in the direction of x whereas $f_y(x_0,y_0)$ gives the slope in the direction of y.





Q 3. Compute the following partial derivatives.

A. $\dfrac{\partial}{\partial v}\left(\dfrac{2\pi r}{v}\right)$

B. f_x, where $f(x,y) = \sin(5x^3y - 3xy^2)$

C. g_y, where $g(x,y,z) = xe^{1-z^2}$

Theorem: Tangent Plane

The equation of the tangent plane to the surface $z = f(x,y)$ at the point (x_0, y_0, z_0) is given by

$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

Q 4. Find the equation of the tangent plane to the paraboloid $z = 2x^2 + 3y^2$ at point $(1,1,5)$.

Q 5. Consider $f(x,\, y,\, z) = \sin{(xy)} + e^{yz}$.

- Find $\frac{\partial^2 f}{\partial x \partial y}$.

- Find f_{yxz}.

Definition: The Chain Rule

Let $f(x_1,\ldots,x_n)$ be a function in n variables, and let $g_i(t_1,\ldots,t_m)$ be a function in m variables for each i. Then the composition $f(g_1,g_2,\ldots,g_n)$ is a function in the variables $t_1,\ldots,t_m$. In this case the partial derivatives are given by

$$f_{t_i} = \frac{\partial f}{\partial x_1}(g_1,\ldots,g_n)\cdot\frac{\partial g_1}{\partial t_i} + \ldots + \frac{\partial f}{\partial x_n}(g_1,\ldots,g_n)\cdot\frac{\partial g_n}{\partial t_i}$$

Q 6. Find $\dfrac{\partial w}{\partial u}$ at $(u,\, v) = (1,\, 1)$ for $w = e^{x^2yz}$ where $x = 3u + 2v$, $y = 3u - 2v$ and $z = uv$.

Q 7. The radius of a right circular cone is increasing at a rate of 1.8 in/s while its height is decreasing at a rate of 2.5 in/s. At what rate is the volume of the cone changing when the radius is 120 in and the height is 140 in?

Theorem: Implicit Differentiation

If x and y are independent variables, and z is the dependent variable given by the implicit function $F(x, y, z) = C$, for some constant C, then

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \quad \text{and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}.$$

Q 8. Find $\dfrac{\partial z}{\partial x}$ by using implicit differentiation: $x^2 z + \sin xyz = 0$.

Survey 6 (Sept. 21): Name in Capitals

Q 9. The level curves of the function $f(x,y) = e^{x^2+y^2}$ are:

(a). Parallel Lines

(b). Concentric circles

(c). Exponential curves

(d). None of the above

Q 10. True or False: If the level curves of a function $f(x,y)$ are all straight parallel lines, then $f(x,y)$ must be a plane.

Indicate your level of understanding of the following by putting a number from 1 to 5.

- Computing the partial derivatives and tanget planes of a function
  [ ]

- Using the chain rule to compute the derivatives of a composition of multi-variate functions.
  [ ]