MA574. Pre class Assignment 21. 21/11/2023

Q1. Given that,

$f(x) = \sin(x)$ centered at $x = \frac{\pi}{2}$.

$$\begin{aligned} f(x) &= \sin(x) & f(\pi/2) &= 1 \\ f'(x) &= \cos(x) & f'(\pi/2) &= 0 \\ f''(x) &= -\sin(x) & f''(\pi/2) &= -1 \\ f^3(x) &= -\cos(x) & f^3(\pi/2) &= 0 \\ f^4(x) &= \sin(x) & f^4(\pi/2) &= 1 \end{aligned}$$

using taylor polynomial formula.

$$P_n(x) = f(c) + \frac{f'(c).(x-c)}{1!} + \frac{f''(c).(x-c)^2}{2!} + \dots \frac{f^n(c)(x-c)^n}{n!}$$

$$P_0(x) = 1$$

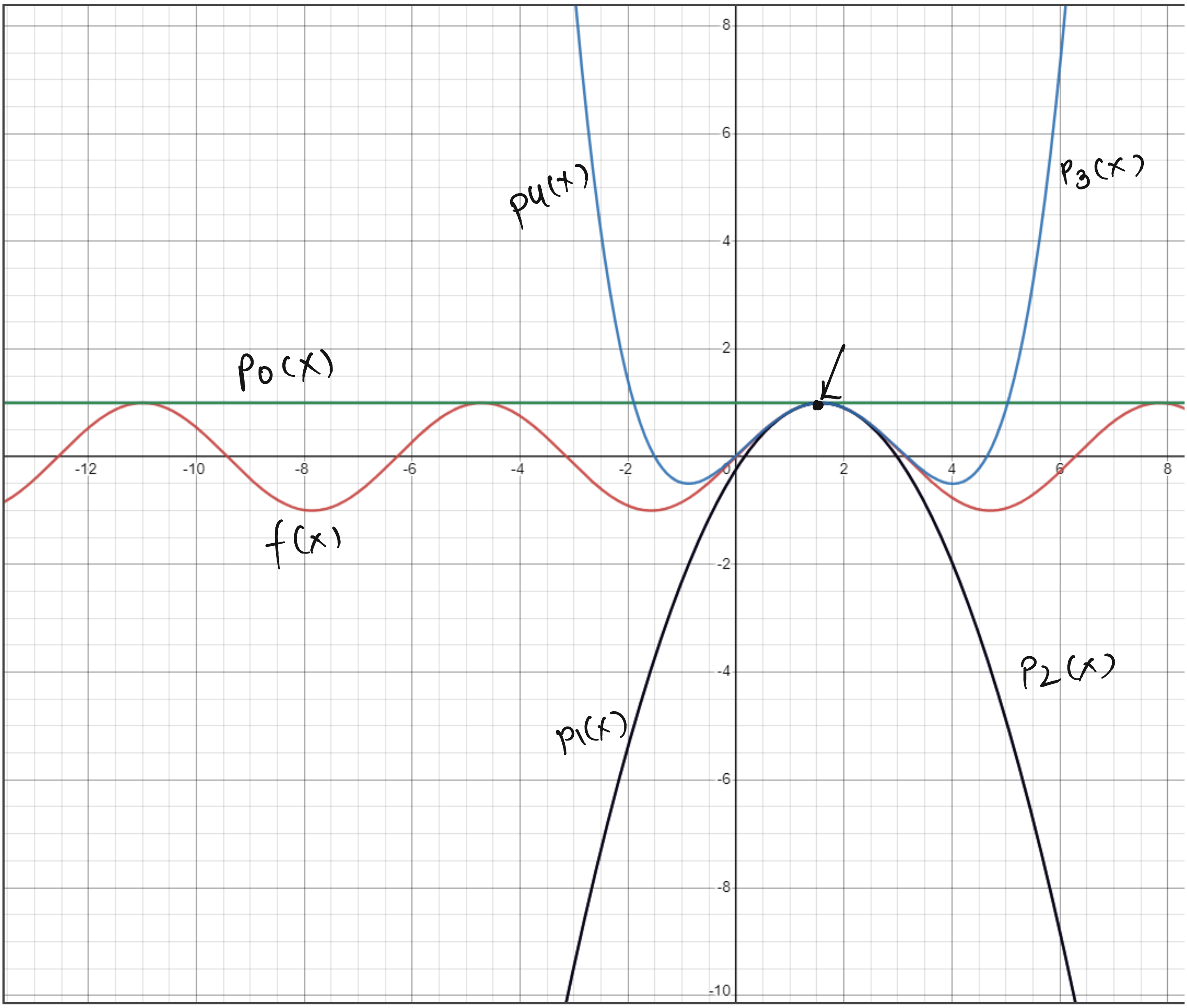
$$P_1(x) = 1$$

$$P_2(x) = 1 + 0 - \frac{1\left(x - \frac{\pi}{2}\right)^2}{2!}$$

$$= 1 - \frac{(x - \pi/2)^2}{2!}$$

$$P_3(x) = 1 - \frac{(x-\pi/2)^2}{2!}$$

$$P_4(x) = 1 - \frac{(x-\pi/2)^2}{2!} + \frac{1\left(x-\pi/2\right)^4}{4!}$$



6
$$y = 1 - \frac{\left(x - \frac{\pi}{2}\right)^2}{2} + \frac{\left(x - \frac{\pi}{2}\right)^4}{24}$$

7
$$y = \sin x$$

8
$$y = 1$$

3
$$y = 1$$

4
$$y = 1 - \frac{\left(x - \frac{\pi}{2}\right)^2}{2}$$

5
$$y = 1 - \frac{\left(x - \frac{\pi}{2}\right)^2}{2}$$

we notice that,
all the polynomials are intersecting at single point $\sin\left(\frac{\pi}{2}\right)$. ie $(1.8, 1)$.

ie $x = 1.8$.

Q2. $N(x) = e^{-x^2}$ centered at $x = 0$.
$f(x) = e^{x}$ centered at $x = 0$.

Hint: $N(x) = f(-x^2)$

→ $f(x) = e^{x}$ $\quad f(0) = 1$
$f'(x) = e^{x}$ $\quad f'(0) = 1$
$f''(x) = e^{x}$ $\quad f''(0) = 1$
$f'''(x) = e^{x}$ $\quad f'''(0) = 1$
$f^{4}(x) = e^{x}$ $\quad f^{4}(0) = 1$

$$P_4(x) = 1 + \frac{1x}{1} + \frac{1x^2}{2} + \frac{1x^3}{6} + \frac{1x^4}{24}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$N(x) = e^{-x^2}$ $\quad N(0) = 1$
$N'(x) = -2xe^{-x^2}$ $\quad N'(0) = 0$

$$N''(x) = -\left(2x \cdot (-2xe^{-x^2}) + e^{-x^2} \cdot (2)\right)$$
$$= 4x^2 \cdot e^{-x^2} + 2e^{-x^2}$$
$N''(0) = 2.$

$$N'''(x) = -8x^3 e^{-x^2} + e^{-x^2} \cdot 8x + -4xe^{-x^2}$$
$N^3(0) = 0.$
$$N^4(x) = \left(\cancel{8x^3 \cdot (-2x) \cdot e^{-x^2}} + \cancel{e^{-x^2} \cdot 24x^2}\right) + e^{-x^2} \cdot 8 + \cancel{8x \cdot (-2x) e^{-x^2}} + -\left(\cancel{4x \cdot (-2x) e^{-x^2}} + e^{-x^2}(4)\right)$$
$N^4(0) = 8 - 4$
$= 4$

$$N(x) = 1 + 0 + \frac{2x^2}{2!} + 0 + \frac{4x^4}{4!}$$

$$e^{-x^2} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}$$

$\ni \quad ? \quad ! \quad \circ$

Q3. $f(x) = 2x^7 - 3x^4 + x^2 - 5x + 1$ at $x = \sqrt{2}$.

$\rightarrow$ $f'(x) = 14x^6 - 12x^3 + 2x - 5$

$f'(\sqrt{2}) = 14(\sqrt{2})^6 - 12(\sqrt{2})^3 + 2(\sqrt{2}) - 5$

$f''(x) = 84x^5 - 36x^2 + 2$

$f''(\sqrt{2}) = 84(\sqrt{2})^5 - 36(\sqrt{2})^2 + 2.$

$f'''(x) = 420x^4 - 72x$

$= 420(\sqrt{2})^4 - 72(\sqrt{2})$

$f^4(x) = 1680x^3 - 72$

$= 1680(\sqrt{2})^3 - 72$

$$P_4(x) = \left(2(\sqrt{2})^7 - 3(\sqrt{2})^4 + (\sqrt{2})^2 - 5(\sqrt{2}) + 1\right) + \left(14(\sqrt{2})^6 - 12(\sqrt{2})^3 + 2(\sqrt{2}) - 5\right)\frac{(x-\sqrt{2})}{1!}$$
$$+ \left(84(\sqrt{2})^5 - 36(\sqrt{2})^2 + 2\right)\frac{(x-\sqrt{2})^2}{2!} + \left(420(\sqrt{2})^4 - 72(\sqrt{2})\right)\frac{(x-\sqrt{2})^3}{3!} + \left(1680(\sqrt{2})^3 - 72\right)\frac{(x-\sqrt{2})^4}{4!}$$

$$\sum_{n=0}^{\infty} 2(n-1).(\sqrt{2})^{7-n} \qquad \frac{(x-\sqrt{2})^n}{n!}$$

doubt.

Q4 Given that,

$f(x) = e^{x}$ centered at $\alpha = 0$. on $[-0.5, 0.5]$.

→ $$R_n(x) = \frac{f^{n+1}(z)\,(x-c)^{n+1}}{(n+1)!}$$

$f(x) = e^{x}$ $f(0) = 1$

$f'(x) = e^{x}$ $f'(0) = 1$

$f''(x) = e^{x}$ $f''(0) = 1$

$f'''(x) = e^{x}$ $f'''(0) = 1$

$f^{4}(x) = e^{x}$ $f^{4}(0) = 1$

$$P_4(x) = 1 + \frac{1x}{1} + \frac{1x^2}{2} + \frac{1x^3}{6} + \frac{1x^4}{24}$$

$n = 4$

$c = 0$.

$z \to (-0.5 \text{ to } 0.5)$

$$R_n(x) = \frac{f^{5}(z)\, x}{5!}$$

$f^{5}(x) = e^{x}$

$f^{5}(z) = e^{z}$.

$\because$ $f^{5}(z)$ will be max at 0.5

$\therefore$

$f^{5}(0.5) = e^{(0.5)} = 1.649$.

$$R_n(x) = \frac{1.649\, x}{5!}$$

Q5. find 2^{nd} degree polynomial

$$f(x,y) = \ln(1+x-2y) \quad f(0,0) = 0.$$

$$f_x(x,y) = \ln(1+x-2y) \cdot 1 \quad f_x(0,0) = 0$$

$$f_y(x,y) = -\ln(1+x-2y)2 \quad f_y(0,0) = 0$$

$$f_{xx}(x,y) = \ln(1+x-2y) \quad f_{xx}(0,0) = 0$$

$$f_{yy}(x,y) = \ln(1+x-2y)4 \quad f_{yy}(0,0) = 0$$

$$f_{xy}(x,y) = -2\ln(1+x-2y) \quad f_{xy}(0,0) = 0.$$

1^{st} degree:

$$L(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$$

$$= 0 + 0 + 0.$$

$$= 0.$$

2^{nd} degree.

$$Q(x,y) = L(x,y) + f_{xx}\frac{(x,y)(x-0)^2}{2} + f_{yy}\frac{(x,y)(y-0)^2}{2}$$

$$+ f_{xy}(x,y)(x-0)(y-0)$$

$$= 0 + 0 + 0 + 0.$$

$$= 0.$$