FE535: Introduction to Financial Risk Management
Session 5

Anthony Diaco

Agenda

- Introduction to Derivatives
  - Derivative Market
- Forward Contracts
  - No-Arbitrage Pricing
  - Evaluating Forward Contracts
- Future Contracts

Derivatives Market

Derivative Market

Derivative Instrument

- It is a financial contract whose value is **derived** from an underlying asset
  - index - stocks, bonds, currency, or commodity
  - reference rate - interest rate
- Derivatives are traded in
  1. Private over-the-counter (OTC) markets
  2. organized exchanges

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- Securities, such as stocks and bonds, are issued to raise capital
- On the other hand, derivatives, is an agreement between two parties
  - ideally they should be a zero-sum game
  - the gain of one party is the loss of the other

Exchange Traded Markets

- Exchanges have been used for trading for many years
  - New York Stock Exchange (NYSE)
  - Chicago Board Options Exchange (CBOE)
  - CME Group - options and futures
- The role of the exchange is to
  - define the contracts and organize trading
  - assure market participants that trade agreement will be honored

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- The role of the exchange is to
  - define the contracts and organize trading
  - assure market participants that trade agreement will be honored
- Traditionally individuals have met at the exchange and agreed on the prices for trades
  - known as the open outcry system
- However, the outcry system have been replaced with *electronic trading* (**link**)

Exchange Traded Markets - Size

Table: Exchange-Traded Instruments: summary by contract type

	March 1995 (amount)	Dec 2009 (amount)	March 1995 (percentage)	Dec 2009 (percentage)
Total	8838	73140	15.81%	10.63%
Interest rate	8380	67057	14.99%	9.75%
Futures	5757	20628	10.30%	3.00%
Options	2623	46429	4.69%	6.75%
Foreign exchange	88	311	0.16%	0.05%
Futures	33	164	0.06%	0.02%
Options	55	147	0.10%	0.02%
Stock index	370	5772	0.66%	0.84%
Futures	128	965	0.23%	0.14%
Options	242	4807	0.43%	0.70%
Both Markets	55910	687814	100.00%	100.00%

- The first two columns report the notional amount in billion $, for year 1995 and 2009, respectively
- The last two columns report the percentage, for year 1995 and 2009, respectively

Over the Counter (OTC) Markets

- The OTC market is a huge network of traders who work for financial institutions, large corporations, or fund managers
- It is used for trading
  - bonds
  - foreign exchange (FX)
  - derivatives
- Banks are very active participants and act as market makers
  - e.g., banks provide bid ask on a range of FX

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- A number of important facts about the OTC market
  - terms of the contract are independent of an exchange
  - flexibility to negotiate any mutually attractive deal
  - trades in the over-the-counter market are much larger than trades in the exchange-traded market

OTC Markets - Size

Table: OTC Instruments: summary by contract type

	March 1995 (amount)	Dec 2009 (amount)	March 1995 (percentage)	Dec 2009 (percentage)
Total	47530	614674	85.01%	89.37%
Interest rate	26645	449793	47.66%	65.39%
Forwards (FRAs)	4597	51749	8.22%	7.52%
Swaps	18283	349236	32.70%	50.77%
Options	3548	48808	6.35%	7.10%
Foreign exchange	13095	49196	23.42%	7.15%
Forwards and forex swaps	8699	23129	15.56%	3.36%
Swaps	1957	16509	3.50%	2.40%
Options	2379	9558	4.26%	1.39%
Equity-linked	579	6591	1.04%	0.96%
Forwards and swaps	52	1830	0.09%	0.27%
Options	527	4762	0.94%	0.69%
Commodity	318	2944	0.57%	0.43%
CDS	0	32693	0.00%	4.75%
Others	6893	73456	12.33%	10.68%
Both Markets	55910	687814	100.00%	100.00%

OTC Markets - Size

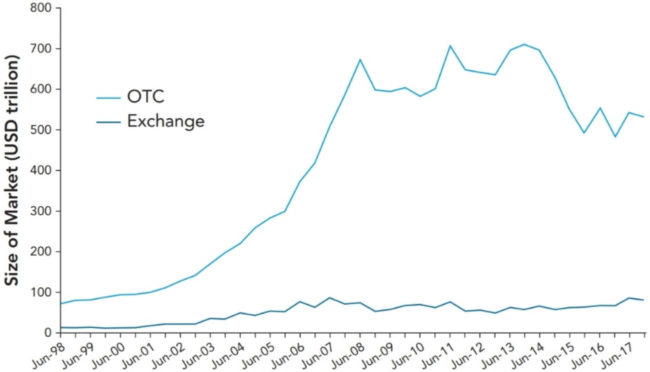

Figure 4.1 **Size of Exchange-traded and OTC market measured in terms of the value of underlying assets between 1998 to 2017.**

Source: https://stats.bis.org/statx/toc/DER.html

- Figure was captured from 2020 Financial Risk Management Part I: Financial Markets and Products

OTC Markets - Size

- The magnitude of the notional amount of $688 trillion is several times the world gross domestic product (GDP),
  - which amounted to approximately $61 trillion in 2008
- It is also greater than the total outstanding value of stocks ($34 trillion) and of debt securities ($83 trillion) during that time

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- Notional amounts
  1. give an indication of equivalent positions in cash markets.
  2. however, do not give much information about the risks of the positions.
- The current (positive) market value of OTC derivatives contracts, for instance, is estimated at \$22 trillion.
  - This is only 3% of the notional.
- More generally, the risk of these derivatives is best measured by the potential change in mark-to-market values

OTC Markets - Size

Table 5.1 Statistics Produced for the OTC Market by the Bank for International Settlements in December 2017 (See www.bis.org)

	Value of Underlying Assets (USD Billions)	Value of Transactions (USD Billions)	Ratio of Value of Transactions to Underlying Assets
Foreign Exchange	87,117	2,293	2.63%
Interest Rate	426,649	7,579	1.78%
Equity	6,570	575	8.75%
Commodity	1,862	189	10.15%
Credit Default Swaps	9,577	312	3.26%
Other	137	8	5.84%
Total	531,912	10,956	2.06%

- Figure was captured from 2020 Financial Risk Management Part I: Financial Markets and Products

Clearing Houses

- Exchange-traded instruments are administrated by a clearing house

Clearing Houses

- stand between traders in the exchange-traded market
- require traders to post cash or marketable securities as collateral (*margin*)
- its members contribute to a guarantee fund

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- require traders to post cash or marketable securities as collateral (*margin*)
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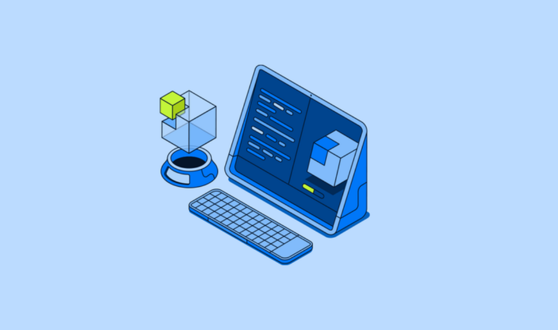
- The margin is set to be sufficiently high that exchange is unlikely to lose money if it has to close out a trader
  - e.g., Value-at-Risk
- This combined with the guaranty fund means that traders are subject to virtually no credit risk
- For instance, see the complete list Clearing Firm Members from CME Group using this **link**

Example of Clearing House

- Suppose that, in a particular exchange-traded market, Trader X agrees to sell one instrument to Trader Y.
- The clearing house (C), e.g., CME, in effect stands between the two traders
  - $X \to C$ and $C \to Y$
- The advantage of this is that X does not need to worry about the creditworthiness of Y, and vice versa
- Both traders deal only with the clearing house.
- If a trader is a clearing house member, such as J.P. Morgan Securities LLC, the trader deals directly with the clearing house, CME.
- Otherwise, the trader deals with the clearing house through a clearing house member.

The Case of Robinhood

Figure: Mechanics behind your Trade. Source: Robinhood **link**



- Due to extraordinary market activity on Jan 28, 2021, Robinhood imposed temporary restrictions on certain securities
  - e.g., users could only sell GME but not buy
- "Market watchers are buzzing about a phone call in the early hours of Thursday morning between Robinhood and its clearinghouse over a $3 billion demand for cash." (**article**)

Central Counterparties (CCPs)

- Some OTC trades have been cleared through clearing houses, known as CCPs
- A CCP plays a similar role to an exchange clearing house
- CCP stands between the two sides,
  - mitigating credit risk concern
- The CCP has members who contribute to a guaranty fund, and provide margin to guarantee their performance

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- After the recent financial crisis, standard derivatives traded between financial institutions must be cleared through CCPs
- CCPs reduce the chance that a financial institution will lose money because of the failure of the other party
- Nonetheless, CCPs also concentrate risk among themselves
  - a failure of one could trigger a systemic risk

Derivatives Products

Derivatives Products

- Forwards (OTC)
- Futures (exchange listed)
- Swaps (OTC)
- Options (exchange listed and OTC)

Derivative Traders

- Hedgers
- Speculators
- Arbitrageurs

Forward Contracts

Forward Contracts

Definition

A forward contract is an agreement to buy or sell an asset at

- a certain price (**forward price**)
- and a certain future time (**expiration date**)

- Forward contracts trade in the OTC market
- They are popular on currencies and interest rates
- Forward contracts represent contractual obligations

- A position that buys the asset is said to be **long**
- A position that sells the asset is said to be **short**

Forward Contracts - Example

GBP/USD

On Sep 27th, 2018 , Party A signs a forward contract with Party B to sell 1 million British pound (GBP) at 1.30 USD per 1 GBP six month later.

- Today (Sep 27th, 2018), sign a contract without a payment
- March 27th, 2019 (the expiry date), A pays 1 million GBP to B, and receives 1.30 million USD from B in return
- Currently (Sep 27th, 2018), the **spot price** for the pound, i.e. the spot exchange rate, is 1.308. Six month later (March 27th, 2019), the exchange rate is unknown.

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- The question what would happen in six months? Who would win/lose?
  - Scenario 1: the spot price at expiry is $1.35
  - Scenario 2: the spot price at expiry is $1.25

Some Terminology

- **Expiry** date is the time when the contract expires, denoted by T
- **Time to maturity** of the contract is the time length between today and the expiry date, $\tau = T - t$, where t denotes today's date
- The **spot price**/rate, S_t, is the current price of the asset in dollars at time t
- Current **forward price** of the asset at delivery, $F_t(T)$, i.e. delivery price
- **Contract value** at time t, V_t
- Current domestic risk-free rate, r,

- From the previous example, we have
  - $t = 0$ denoting today, while $T = 6/12 = 0.5$ six months maturity
  - The spot price today is $S_0 = 1.308$
  - The forward price is $F_0(0.5) = 1.3$

Two Scenarios

- Suppose that scenario 1 occurs, i.e. $S_{0.5} = 1.35$
  - party A receives \$1.30 per GBP for something which value is \$1.35, netting a loss of \$0.05
  - party B, on the other hand, nets a profit of \$0.05

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- Suppose that scenario 2 occurs, i.e. $S_{0.5} = 1.25$
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- Suppose that scenario 2 occurs, i.e. $S_{0.5} = 1.25$
  - party A receives \$1.30 per GBP for something which value is \$1.25, netting a profit of \$0.05
  - party B, on the other hand, nets a loss of \$0.05
- Clearly, there are multiple (infinite) possible scenarios.
- Nonetheless, if the contract is not a zero-sum game, then one party won't be willing to participate

- In the former example party A is entering a **short** position
  - "betting" on the rate going down
- Whereas, B is entering a **long** position
  - "betting" that exchange rate going up

Put Differently

- For A, the GBP delivery/payment constitutes an asset
- On the other hand, for B it is a liability
- Investors would like to hedge the downside drop in their assets
  - setting a lower bound
- Whereas, traders try to hedge the cost of their liabilities
  - imposing a cap on their cost

- Obviously, party A (respectively B) enjoys a profit if the GBP weakens (respectively strengthen) with respect to the USD
- If we denote the payoff of party A and B as $V^A_{0.5}$ and $V^B_{0.5}$, respectively, then

$$V^A_{0.5} = F_0(0.5) - S_{0.5} \tag{1}$$

and

$$V^B_{0.5} = S_{0.5} - F_0(0.5) = -V^A_{0.5} \tag{2}$$

- Hence, if $V^A_{0.5} > 0$, then $V^B_{0.5} < 0$
  - A wins, B loses
- Whereas, if $V^B_{0.5} > 0$, then $V^A_{0.5} < 0$
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- Obviously, party A (respectively B) enjoys a profit if the GBP weakens (respectively strengthen) with respect to the USD
- If we denote the payoff of party A and B as $V_{0.5}^A$ and $V_{0.5}^B$, respectively, then

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- Hence, if $V_{0.5}^A > 0$, then $V_{0.5}^B < 0$
  - A wins, B loses
- Whereas, if $V_{0.5}^B > 0$, then $V_{0.5}^A < 0$
  - B wins, A loses

- As a result, A and B would participate in the forward contract, only if they believe that the expected payoff of the contract is zero, i.e.

$$\mathbb{E}(V_{0.5}^A) = \mathbb{E}(V_{0.5}^B) \tag{3}$$

however, since

$$\mathbb{E}(V_{0.5}^B) = -\mathbb{E}(V_{0.5}^A) \tag{4}$$

then it holds true if $\mathbb{E}(V_{0.5}^B) = \mathbb{E}(V_{0.5}^A) = 0$

No-Arbitrage Pricing

Arbitrage

- Arbitrage is a theoretical phrase describing the case in which one engages in trade that yields a profit with 100% probability.
- In more practical terms, it refers to a trading strategy that takes advantage of mispricing of securities to make a profit.

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- In more practical terms, it refers to a trading strategy that takes advantage of mispricing of securities to make a profit.

- If there is mispricing in the delivery price of the forward, then one party may exploit the mispricing to make a profit
  - creating an arbitrage opportunity
- In particular, under no-arbitrage, the price of the forward today, $F_t(T)$, is determined

$$V_t = S_t - e^{-r\tau} F_t(T) = 0 \rightarrow F_t(T) = S_t e^{r\tau} \tag{5}$$

Economic Interpretation

- Consider the following two strategies
  1. Buy share of the SPY today at the spot price of S_0 and hold for one year, $T = 1$
  2. Enter a forward contract to buy the SPY in one year from now for a price of $F_0(1) = K$. To pay \$$K$ in one year from now, invest the present value of K in an continuously interest bearing account with rate r

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  2. Enter a forward contract to buy the SPY in one year from now for a price of $F_0(1) = K$. To pay \$$K$ in one year from now, invest the present value of K in an continuously interest bearing account with rate r
- Both options are economically equivalent
- In the first case, you will pay S_0, whereas in the second case you will pay $e^{-r}K$
- Therefore, the value of K should reflect the following relationship

$$S_0 = e^{-r}K \rightarrow K = e^{r}S_0 \tag{6}$$

- Alternatively, it should also reflect that

$$\mathbb{E}[S_1] = K, \text{such that } \mathbb{E}[S_1] = e^{r}S_0 \tag{7}$$

Arbitrage Opportunity

- Any deviation from (6) would imply an arbitrage opportunity.
- Consider the following case, $S_0 = 100$, $\tau = 1$, and $r = 0.05$, it follows that the forward's value today is

$$F_0(1) = e^{0.05}100 = 105.13 \tag{8}$$

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- For instance, what would happen if $F_0(1) = 110$?

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Case 1: High Price

- For instance, what would happen if $F_0(1) = 110$?
- Borrow \$100 to buy at spot price and enter a short forward contract with $F_0(1) = \$110$, then at maturity
  - you owe \$105.13, principle plus interest
  - deliver the asset and receive \$110
  - netting profit of \$4.87

Short-Sale

- Short selling involves selling securities you do not own
- Today, you receive a total amount of the spot price
- In the future, you will need to deliver/buy the asset

Case 2: Low Price

- For example, how would you utilize an arbitrage opportunity if $F_0(1) =$ \$102?

Short-Sale

- Short selling involves selling securities you do not own
- Today, you receive a total amount of the spot price
- In the future, you will need to deliver/buy the asset

Case 2: Low Price

- For example, how would you utilize an arbitrage opportunity if $F_0(1) =$ \$102?
- Short sale the asset today
  - Receive an amount of \$100
  - Invest in an interest paying account
- Purchase the asset at maturity for a price of \$102
- At maturity,
  - Receive \$105.13 from the interest account
  - Pay \$102 to deliver the asset you shorted
  - Net a profit of \$3.13

Exam Question - FRM Exam 2002 Question 56

Consider a forward contract on a stock market index. Identify the **false** statement:

1. The forward price depends directly on the level of the stock market index
2. The forward price will decrease if the underlying stock increase the level of the dividend payments over the life of the contract
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- Recall that the forward price is a function of the spot price
- Dividend levels denote future possible payments
- Higher maturity denotes more time value of money
- When interest rates go up, the forward price should go up as well

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- Dividend levels denote future possible payments
- Higher maturity denotes more time value of money
- When interest rates go up, the forward price should go up as well
  - Otherwise, an arbitrage is possible, short sell the asset today and enjoy higher interest, while purchasing the asset later for a relatively low price

Back to GBM

- Suppose that the spot price at time t obeys to a geometric Brownian motion (GBM)
- We know that

$$S_T = S_t \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\tau + \sigma Z_\tau\right) \tag{9}$$

with $Z_\tau \; N(0, \tau)$ denoting a BM

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- Since S_T follows a log-normal distribution, then it holds true that

$$\mathbb{E}_t[S_T] = S_t e^{\mu\tau} \tag{10}$$

- The forward price of the asset should be equal to the expected price, i.e.

$$F_t(T) = \mathbb{E}_t[S_T] = S_t e^{\mu\tau} \tag{11}$$

- Hence, for no aribtrage opportunity, it should hold true that

$$S_t e^{\mu\tau} = S_t e^{r\tau} \Rightarrow \mu = r \tag{12}$$

Risk-Neutral Valuation

- The condition that $\mu = r$ reflects that investors do not require an extra expected return for bearing risks
  - hence, the idea of risk-neutral
- Risk-neutral evaluation and is a key assumption in pricing derivatives
  - and consistent with no-arbitrage pricing
- Indeed, for (12) to hold true, then it should be reflected that the expected present value of the asset is equal to S_t, which is today's spot price, i.e.

$$e^{-r\tau}\mathbb{E}_t[S_T] = S_t \tag{13}$$

  which holds true if $\mu = r$

Forward Contracts with Income Payment

- When an asset provides payment prior to maturity, we can invest less today to get a \$1 unit of the asset later on
- Let's consider again the previous example for a stock index forward contract with spot price $S_0 = 100$ and $r = 0.05$
- Under no arbitrage, the forward price $F_0(1) = 105.13$
- Now assume, that the underlying stock index pays continuously compounded dividend with yield $q = 0.01$
- If the forward price does not reflect the dividend payments, then there is an arbitrage opportunity

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- If the forward price does not reflect the dividend payments, then there is an arbitrage opportunity
  - Borrow \$100 and buy the asset
  - Hold the asset until maturity and receive divided payments of $\approx$ \$1
  - At maturity, pay back the interest with principle, i.e. \$105.13
  - Deliver the asset for a price of $F_0(1) = \$105.13$
  - Overall, you earned a risk-free profit of \$1

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  - At maturity, pay back the interest with principle, i.e. \$105.13
  - Deliver the asset for a price of $F_0(1) = \$105.13$
  - Overall, you earned a risk-free profit of \$1
- Therefore, under no-arbitrage pricing, the future price should be lower to take into account possible future payments

$$F_t(T) = S_t e^{r\tau} \times e^{-q\tau} = S_t e^{(r-q)\tau} \tag{14}$$

Forward Contracts with Income Payment - Relation to GBM

- In fact, under no-arbitrage pricing or risk-neutral valuation, the future spot price is

$$S_T = S_t \exp\left(\left((r-q) - \frac{\sigma^2}{2}\right)\tau + \sigma Z_\tau\right) \tag{15}$$

- Clearly, if $q = 0$, then we get the same motion as in (9)
- Taking the conditional expectation of S_T, we can see that the expected spot price is

$$\mathbb{E}_t[S_T] = S_t \times e^{(r-q)\tau} \tag{16}$$

- Which is equal to the forward contract price at time t
  - $F_t(T)$ from Equation (14)

Exam Question FRM Exam 2007 Question 119

A three-month forward contract on a stock index is trading at \$1000. The current index level is \$990. Assuming a continuous payment dividend rate of 2% with an continuously compounded interest rate of 4%. The potential arbitrage profit per contract is

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2. \$7.5
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- According to (14), the fair price of the forward contract is

$$F_0(0.25) = \$990 \times e^{(0.04-0.02)\frac{3}{12}} \approx \$995 \tag{17}$$

- Hence, there is a \$5 price discrepancy
- Since the forward price is higher, one can engage in the following arbitrage opportunity
  - Borrow \$990, purchase the underlying index
  - Enjoy dividends payment and sell the index for \$1000
  - After paying the principle and interest, you netted a profit of \$5

Interest Rate Parity

- A foreign currency can be regarded as an investment asset that provides a yield equal to the foreign interest rate r^*
- Let's consider the previous GBP/USD example, where you are party B, who needs to deliver 1 million GBP six months from now
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- By not accounting for a foreign interest rate, one can engage in the following arbitrage
  - Borrow \$1.308 and buy 1 million GBP
  - Invest the 1 million GBP in a saving account with $r^* = 0.01$ interest
  - At maturity, deliver the 1 million GBP for \$1.321 and pay interest plus principle on domestic loan of $\$1.308e^{0.02\times 0.5}$, which has a zero net
  - Finally, you earned interest on the foreign saving account

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The spot rate CAD/USD is \$0.7539 and the market price for the three months forward contract is \$0.7553. Given this data, what is the implied foreign interest rate differential (FIRD)?[a]

[a] This example was taken from earlier this year, before the Fed cut the interest rates twice in Jul and Sept 2019.

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- First note that the forward is trading at premium, i.e. market expects USD to strengthen w.r.t CAD
- Second, if the forward is trading at premium, then it should imply that $r^* < r$
- From (19), we have

$$FIRD = r - r^* = \frac{1}{\tau}\log\left(\frac{F_0(\tau)}{S_0}\right) = \frac{1}{0.25}\log\left(\frac{0.7553}{0.7539}\right) = 0.74\% = 74\text{bps} \tag{20}$$

Interest Rate Parity - Relation to GBM

- Let S_t denote the spot exchange rate between foreign and domestic currencies
- If the spot rate behaves according to geometric Brownian motion, then we have

$$S_T = S_t \times \exp\left((\theta - \frac{\sigma^2}{2})\tau + \sigma Z_\tau\right) \tag{21}$$

with

1. σ is the volatility of the exchange rate
2. θ is the drift component, i.e. expected return (growth)
3. $\tau = T - t$ time to maturity
4. $Z_\tau \; N(0, \tau)$ is a standard Brownian motion

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- Under no-arbitrage pricing (risk-neutral), it holds true that

$$\theta = r - r^* \tag{22}$$

- In other words, the expected future spot rate is

$$\mathbb{E}_t[S_T] = S_t \times e^{(r-r^*)\tau} \tag{23}$$

Futures Contracts

Futures Contract

- Similar to forward contract, it is an agreement to buy or sell an asset for a certain price at a certain time
- Forward contracts allow counter-parties to take high leverage with no upfront payment
  - inducing credit risk
- Unlike forwards, futures contract are
  - traded on organized exchanges
  - standardized
  - have clearing houses
  - marked to market
  - involve posting a collateral **margin**

Margin for Future Contracts

- The margin ensures that they will honor their commitments under the contract
- Consider a futures contract on 1000 units of an asset worth \$100
- A long position is equivalent of holding \$100,000 position in the asset
- Suppose that the trader has to post only 5%, \$5,000, margin to maintain such position

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- If the next day the asset drops by \$3, i.e. the position drops by \$3,000
- The trader incurs the loss and his margin now is below 5%, i.e. \$2,000
- For him to maintain his position, he needs to raise capital of \$3,000
- If he fails to meet the margin call, the broker closes his position

Margin for Future Contracts II

- The the goal of margins to provide buffer against future losses
- An *initial margin* must be posted when initiating the position.
- If the equity in the account falls below the *maintenance margin*, the customer is required to provide additional funds to cover the initial margin.
- The level of which needed to be restored depends on the instrument and the type of position
  - when trading futures, it is restored to the initial level
  - when trading equities, it suffices to restore to maintenance level

Valuing Futures Contract

- Valuation of futures contract is very similar to forward contract
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Exam Question - FRM Exam 2004 Question 38

An investor enters a **short** position in a gold future contracts at \$294.2. Each contract controls 100 troy ounces. The initial margin is \$3,200 and the maintenance margin is \$2,900. At the end of the first day, the price of the gold drops to \$286.6. What is the minimum amount the investor needs to maintain his position?

FRM Exam 2004 Question 38 - Solution

Short Position

- This is a tricky question.
- Note that the investors is entering a short position
  - only incurring losses when the price goes up
- As a result, there is no need to raise additional capital

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Long Position Scenario

- However, if it were a long position, then, losses will be incurred
- Since the futures contract controls 100 troy ounces, then the loss

$$(294.2 - 286.6) \times 100 = 760 \tag{24}$$

- The current equity in the account becomes $3200 - 760 = 2440$, which is below the maintenance margin
- As a result, trader needs to raise capital to bring back to initial margin, which is \$760

Summary

- This lecture provides an introduction to derivatives market
  - We focused mainly on forward and futures contract
- The implication of no-arbitrage pricing are crucial to pricing other derivatives
  - option pricing
- Moving forward, we will focus pricing other derivatives and risk management applications
  - Managing linear risk - hedging using futures
  - Non-linear risk models - option hedging