FE 535: Introduction to Financial Risk Management
Project 2

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Instructions

1. This is a teamwork project
2. Each member should contribute equally.
3. Feel free to use the handouts and the published codes to do your project.
4. You are welcome to use any programming language or statistical software. Also, you are welcome to use any library/package unless stated otherwise.
5. You will need to download data on your own - unless provided otherwise.
6. The final report should be written using a special document editor, e.g. Word, Latex, Markdown, etc. **Any form of document with a handwriting will not be accepted**.
7. Please submit a pdf copy of your final report, please include your code as an appendix or as part of the your Markdown output. Note that Canvas will not accept any other formats than pdf.
8. Not including the code appendix, the maximum report's length **should be no more than 15 pages** - with font 11 point size, 1.5 line space, and 1in margin (just like this document).
9. Please **avoid taking picture snapshots**. You should report your results in an organized table. The same applies to plots and other visualizations - do not paste any low resolution figures.
10. Please use a special equation editor to write any math, in case needed.

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1 Bond Portfolio Management (30 Points)

The file `bonds.csv` contains Treasury bond data, which was hand-collected using different dates over the last few years from https://www.bloomberg.com/markets. Given this data, address the following questions:

1. Specifically, the data was collected from 6 different periods. For each period, the data corresponds to the yield curve for 2, 5, 10, and 30 years maturity. Given historical data on the yield curve, can you identify these six dates? (5 Points)

   Note: To answer this, you need to download data on Treasury yields of different maturities using the FRED database. In particular, you need to download data for the following codes DGS2, DGS5, DGS10, and DGS30. After merging and dropping missing values, the final dataset is daily and should date between Jan 2nd, 2018 and March 31, 2022.

2. Use the pricing equation of a fixed-coupon bond to price each bond from the `bonds.csv` file. I recommend writing a function that takes yield, coupon, face value, and maturity as its main arguments. The resulting prices should correspond to the ones reported in the data. Hence, you should plot the computed prices against the given ones. To confirm, make sure you observe a 45-degree line. (5 Points)

3. Prices should reflect investors' perception of future interest rates. Rather than computing the prices using yields as the case in the previous question, in practice, it is the other way around. We try to deduce yields from market prices. Hence, given a pricing function, you need to find the yield that matches the market price. For each bond, find the implied yield and plot it against the corresponding yield from the `bonds.csv` data. Again, this should result in a 45-degree line. (5 Points)

   Hint: This relies on a numerical solution. Recall that the solution for function f is the x^* that satisfies $f(x^*) = 0$. Since the price of the bond is a function of yield, i.e. $f(y) = P$, design a function $g(y) = f(y) - P_0$, where P_0 is fixed using the values from the given prices. As a result, the implied yield is the solution y^* that satisfies $g(y^*) = 0$, i.e. $f(y^*) = P$. In R, you may refer to the `uniroot` function. In Excel, this can be attained using "goalseek".

4. Compute and report the Macaulay duration for each bond in the `bonds.csv` data file. This should correspond to 24 numbers. As a summary, report the min and max for each duration by maturity. Your summary should correspond to 4×2 table, where each row corresponds to a distinct maturity and columns correspond to statistics (i.e., min and max). How do the results compare within columns? What about within rows? **Provide some rationale**. (5 Points)

5. Using first order Taylor expansion, calculate the change in the Treasury bond prices, if the yield curve in the US shifts **up** by 50 bps. Focus only on the recent bond data to answer this

part, i.e. bonds numbered 21, 22, 23, and 24. To summarize, plot both the original and new prices against maturity. How do you justify this observation? (5 Points)

Note: since you have a pricing function for a fixed coupon bond, you should confirm whether the new price is correct. For instance, if the price P is a function of yield y, then we know that price is $P = f(y)$. To check whether your answer is correct, you should compare your Taylor expansion results with the exact price, which would be $P_1 = f(y + \Delta y)$.

6. Assume that the prices in the above table reflect the dollar price of each bond, e.g. the price of bond 9 is \$101.57. As a portfolio manager, you need to allocate \$100,000 between bonds 21 and 22 from the `bonds.csv` data file. If you believe that the Federal Reserve will increase interest rates in the near future, you need to limit your portfolio duration to 4 years. As a result, how many units of each bond you need to purchase to satisfy this? How would your answer change if you target a duration of 8 years instead? Explain why these numbers make sense. (5 Points)

7. **Bonus Question** Consider the details from the previous question. However, in this case, you need to allocate \$100,000 among the four Treasury bonds numbered 21, 22, 23, and 24. If you are targeting a portfolio duration of 8 years, how many units of each bond you need to buy? The position in each individual bond should not be zero. (5 Points)

   Hint: In this case, you need to satisfy two conditions by choosing four unknowns. This results in an under-determined linear system of equations. To solve this, you need to think in terms of a generalized solution. A possible suggestion is to look into a generalized matrix inverse - for instance, see Moore-Penrose pseudoinverse (Wiki page). As a confirmation, check whether the proposed solution satisfies the two requirements.

2 Forward Contracts and No-Arbitrage Pricing (30 Points)

Under no-arbitrage pricing it follows that future price of a stock index corresponds to the following geometric Brownian motion (GBM):

$$S_T = S_t \times \exp\left((r - d - \frac{\sigma^2}{2})\tau + \sigma Z_\tau\right) \tag{1}$$

with r is the risk-free rate, d is the continuous annual dividend yield, and Z_τ is a standard Brownian motion.

To address the following questions, assume that $r = 1\%$ and $\sigma = 0.1$, while $d = 0$, i.e. the underlying stocks of the index pay no dividends. Additionally, suppose that the spot price is \$100. Given this information, address the following questions:

1. Under no-arbitrage pricing, what is the fair value of a k-years forward contract on the above stock index? Report your answer for $k = 1, 2, 3, 4, 5$. As a summary, plot the forward price versus k. What does the graph say? (5 Points)

2. Repeat the previous part but using Monte Carlo simulation. In particular, you will need to simulate the future price of the index for $k = 1, 2, 3, 4, 5$ years. Using a boxplot, plot the distribution of the simulated price for each year and highlight the forward price. How does your answer compare with the previous part? (5 Points)
   Hint: Remember the economic implications of the forward contract.

3. Suppose you are bullish about the stock index, i.e., you believe that the stock market will go up one year from now. At the same time, you have zero capital today. You are planning to materialize a trading strategy using either one-year forward contract, stock index, cash, or a combination of which. Assume you can borrow and lend at the r rate. Given this information, address the following:

   (a) Explain one potential trading strategy that depends on the forward contract. (5 Points)

   (b) Explain one potential trading strategy that depends on the underlying asset. (5 Points)

   (c) Evaluate the profit and loss (P&L) of the each trading strategy using the MC simulation. As a summary, report both the expected and Value-at-Risk of the P&L for each trading strategy. (10 Points)

3 Managing Linear Risk (70 Points)

Case: Today's date is Nov 13, 2023, and a U.S. exporter has been promised a payment of 1,250,000 GBP in Oct 1st, 2024. The spot exchange rate on Nov 13, 2023 is \$1.2273. According to the Chicago Mercantile Exchange (CME Group), the exporter can trade GBP/USD contracts with face amount of 62,500 GBP that will expire in Dec 2024. The CME Group states that futures contracts are terminated "on the second business day immediately preceding the third Wednesday of the contract month."

Calibrating the Exchange Rate Process (10 Points)

Table 1: On Nov 13, 2023, according to investing.com, the spot price of the exchange rate is \$1.2273, while the quotes for the GBP/USD forward contracts are given in the table below. The data set is available via the `FE535_Forward_Prices.csv` file.

Name	Bid	Ask	High	Low
GBPUSD 1M FWD	1.9	2.2	2.08	1.95
GBPUSD 2M FWD	5.48	6.28	5.87	4.82
GBPUSD 3M FWD	8.19	8.49	8.32	7.63
GBPUSD 4M FWD	10.61	11.11	10.92	9.93
GBPUSD 5M FWD	11.68	16.68	14.27	12.86
GBPUSD 6M FWD	15.17	18.17	16.94	16
GBPUSD 7M FWD	17.4	21.8	20.24	18.81
GBPUSD 8M FWD	19.66	24.61	23.2	21.59
GBPUSD 9M FWD	23.71	24.87	26.17	23.6
GBPUSD 10M FWD	26.07	27.45	29.03	26.47
GBPUSD 11M FWD	22.5	35.95	31.93	28
GBPUSD 1Y FWD	28.84	33.84	34.14	28.8

1. Let S_t denote the GBP/USD exchange rate, i.e. the amount of dollars needed to purchase a single GBP at time t. Under no-arbitrage pricing (risk-neutral valuation), S_t follows a Geometric Brownian Motion (GBM), such that the future spot price is given by

$$S_T = S_t \times \exp\left((\theta - \frac{\sigma^2}{2})\tau + \sigma B_\tau\right) \tag{2}$$

where

- $\theta = r - r_f$ denotes the difference between the US risk-free rate and the UK risk-free rate.
- σ is the annual volatility of the exchange rate
- $B_\tau \sim N(0, \tau)$ is a standard Brownian motion

Your first task is to calibrate the GBM:

(a) For θ, you need to refer to the interest rate parity and estimate θ using the forward quotes from Table 1. Note that this a “forward-looking” approach. (5 Points)

(b) For foreign exchange rates, it is common to relate to the interbank lending rate in terms risk-free rate. In particular the data file named `FE535_Libor_USD_GBP.csv` contains LIBOR rates for both USD and GBP. How does your calibrated θ compare with LIBOR rates? (10 Points)

(c) For σ, you need to download data for the daily GBP/USD exchange rate using the “GBPUSD=X” symbol from Yahoo Finance. Your data should be daily and range between 2018-01-01 and 2022-04-03. Given the adjusted prices, you need to calibrate σ using the historical returns. Note that this calibration is backward-looking, which is in line with what you did in Project 1. (5 Points)

VaR for the Unhedged (10 Points)

2. Assume that the exporter does not hedge the exchange rate risk. In this case, the exporter exchanges the GBP on the spot market upon receiving the payment in the future. Let V_T denote the profit/loss (P&L) of the exporter at delivery time. Let $P\&L = S_T - S_t$. What is the 99% VaR of the exporter's P&L in \$? (5 Points)

   Recall that if V_T denotes the P&L at time T, then the $1-\alpha$ VaR is

$$VaR_t(V_T, \alpha) = \mathbb{E}_t[V_T] - Q_t(V_T, \alpha) \tag{3}$$

   Hint: If the pound weakens relative to the dollar, then the exporter gets paid less dollars in the future. Given that the exporter expects 1.25M pounds, the future P&L depends on the future spot rate. You need to address this question using a MC simulation.

Unitary Hedge (30 Points)

3. Consider a unitary hedge, in which the exporter shorts 20 futures contracts today and closes the position when the GBP payment is received. If the risk-free rates are fixed and there is no arbitrage, the price of the futures contract should obey to the interest rate parity. In other words, the futures contract price at time t is given by

$$F_t = S_t \times e^{(r-r_f)\times(t_m-t)} \tag{4}$$

   with t_m denoting the maturity time of the futures contract. Assume that there is no transactions cost, i.e. you are able to buy and sell futures contract with respect to the price implied by the interest rate parity. Using a MC simulation, address the following:

(a) Suppose you use the futures contract expiring in Dec 2024. What is the 99% VaR of the P&L with unitary hedging? (10 Points)

(b) Suppose instead you use the futures contract expiring in Sep 2024 (before delivery). What is the 99% VaR of the P&L now? (10 Points)

(c) How do justify the difference in VaR when comparing your response to Part 2, Part 3 (a), and Part (b). Elaborate in terms of basis risk. (10 Points)

Hedging using ETFs (10 Points)

4. Suppose for some reason the exporter decides to use ETFs (or ETNs) to hedge currency exposure instead of using futures or forward contracts. Your task is to screen 5 different ETFs. For each ETF, provide an economic rationale behind each to serve as a GBP/USD hedge. Justify your reasoning by reporting the hedge effectiveness of each instrument.

   Note this is an open question without a unique answer. However, your reasoning should make sense in terms of economic mechanisms behind the GBP/USD exchange rate movement.

4 Option Pricing using Monte Carlo (80 Points)

In this question, you will be pricing a European call option using Monte Carlo simulation. In all cases, assume that the annual mean return and volatility are given by $\mu = 0.10$ and $\sigma = 0.15$, respectively. Further, assume that the initial stock price is $S_0 = \$120$, the risk-free rate is a constant $r = 0.03$, the strike price for the option is $K = \$125$, and the option expires in 1 year ($T = 1$). Additionally, assume that the stock pays no dividends.

4.1 Stock Price Simulation - 15 Points

Recall that risk-neutral valuation is consistent with no-arbitrage pricing, which assures that the option is fairly priced. Hence, your first step is to simulate a thousand terminal prices, i.e., $S_T(n)$ for $n = 1, ..., 1000$. Under risk-neutral valuation, note that

$$S_T(n) = S_0 \times \exp\left((r - \frac{\sigma^2}{2})T + \sigma\sqrt{T}Z(n)\right) \tag{5}$$

where $Z(n) \overset{IID}{\sim} N(0,1)$ for $n = 1, ..., 1000$.

1. Your first task is simulate $\{S_T(n)\}_{n=1}^{N}$ for $N = 1000$. To do so, you need simulate $\{Z(n)\}_{n=1}^{N}$ and use Equation (5) to map $Z(n)$ into $S_T(n)$. Make sure to save the $\{Z(n)\}_{n=1}^{N}$ sequence, as you will be working with it later on. Plot the density of the simulated stock price using the sequence $\{S_T(n)\}_{n=1}^{N}$. (5 Points)

2. As a summary, estimate $\mathbb{E}^{\mathbb{Q}}[S_1 \mid S_0]$ and $\mathbb{V}^{\mathbb{Q}}[S_1 \mid S_0]$ using the above simulation, where the superscript $\mathbb{Q}$ implies that these expectations are computed under risk-neutral valuation. (10 Points)
   Hint: We did cover the closed-form solution of both in Lab 2. Therefore, **you should not move forward to the next part until you get a close approximation of the true answer.** You are not going to get exact answers for small N, but the answers should not deviate significantly.

4.2 Option Pricing using Baseline Approach - 15 Points

1. Based on the simulated prices, what is the estimated price of a **European Call** option? (5 Points)

2. As a validation, compute the price of the European Call option using the Black-Scholes model (BSM). (5 Points)

3. How does the price of the BSM compare with the simulated one? Explain briefly. (5 Points)

4.3 Bias-Variance - 20 Points

In this part of the question, we will take a look at the bias-variance of the estimated option price from Section 4.2. As you may remember from statistics courses, it is often insufficient to simply provide an 'answer.' We also may like to assess how accurate our answer is. For instance, how much does the answer deviate from the actual value? Is the estimate biased? What about its variance? In the following task, we will be computing the mean-squared error (MSE) of the estimated option price to assess both its bias and variance. In this regard, the Monte Carlo simulation provides a unique way to do this.

1. Now, you are going to repeat the above experiment a hundred times ($M = 100$). Specifically, you need simulate the sequence $\{Z(n)\}_{n=1}^{N}$ and, thus, $\{S_T(n)\}_{n=1}^{N}$ hundred times for $N = 1000$. For each $m = 1, .., M$, the above experiment results in a single estimate of the European Call option, denoted by $\hat{c}(m)$ for $m = 1, ..., M$. As a summary report the average option price across the hundred experiments (10 Points), i.e.,

$$\frac{1}{M}\sum_{m=1}^{M}\hat{c}(m)$$

2. Next, you need to compute the MSE of the estimated option price, which is proxied by

$$MSE(\hat{c}) = \frac{1}{M}\sum_{m=1}^{M}\left(\hat{c}(m) - c_{BSM}\right)^2 \tag{6}$$

   where c_{BSM} is the **fixed** price according to the BSM. (10 Points)

   Note that the above MSE measure can be decomposed into two components: bias and variance. The bias is given by

$$b^2(\hat{c}) = \left[\frac{1}{M}\sum_{m=1}^{M}\hat{c}(m) - c_{BSM}\right]^2,$$

   whereas the variance part can be computed as

$$\frac{1}{M}\sum_{m=1}^{M}\left[\hat{c}(m) - \frac{1}{M}\sum_{m=1}^{M}\hat{c}(m)\right]^2$$

   Hint: The computation of either metric can be easily computed by applying default `average/mean` or `var` functions from standard statistical software on the sequence $\{\hat{c}(m)\}_{m=1}^{M}$.

4.4 Variance Reduction Approach - 30 Points

Computational costs are often a significant consideration when applying Monte Carlo simulation. More accurate answers can always be obtained by performing more simulations at the cost of more computational expense. However, many techniques have been developed to minimize the deviation of estimates without increasing the number of simulations. In this question, you will examine one of the simplest approaches called **Antithetic Variates**.

1. The premise behind the above approach can be explained using the theoretical sum of two dependent random variables. For the variance of their sum to be less than the sum of their variances, the covariance between the two must be negative. Based on this insight, let us go back to Section 4.1 and perform the following. First, select the first half of the simulated prices ($N/2$ in total). Second, rather than simulating another half of $N/2$ prices, let us utilize the first half by multiplying it by -1, such

$$\{Z(n)\}_{n=N+1}^{N} = (-1) \times \{Z(n)\}_{n=1}^{\frac{N}{2}}.$$

   Note that the new sequence is based only on one half of the original simulation, whereas the second half takes the opposite sign.

   Given this "new simulation," utilize Equation (5) to simulate 1000 stock prices and estimate $\mathbb{E}^{\mathbb{Q}}[S_1 \mid S_0]$ and $\mathbb{V}^{\mathbb{Q}}[S_1 \mid S_0]$. (5 Points)

2. Given the new 1000 prices, repeat the analysis from Section 4.2 to estimate the price of the European Call option. (5 Points)

3. The final part of this question is the replication of Section 4.3. In particular, you need to measure the MSE of the option price from the previous part. As a summary, report the MSE. (10 Points)

4. Finally, draw conclusions about the use of antithetic variates. Specifically, make sure to address the following:

   (a) Did the method reduce the MSE of the option price? (5 Points)

   (b) If yes, where does the effect stem from, i.e., variance or bias? Explain briefly. (5 Points)

5 Bonus - Stochastic Interest Rate Models (20 Points)

To qualify for this bonus question, the team is expected to present the solution to the instructor via Zoom by the end of the semester.

This is a bonus question that requires working with stochastic interest rate models. In particular, you will be exposed to the famous Vasicek (1977) model, which is considered the archetype of bond pricing models. You will need to refer to the discrete version of the model with respect to Section 4.1 from Backus et al. (1998). Following the formulation of Backus et al. (1998), your main task is the following:

1. First, you need to replicate Figure 1 of Backus et al. (1998). Note that there is no need to download the original data. You can reconstruct the figure using the the summary statistics provided in Table 1 of the paper.

2. Second, the table below provides a sequence of 4 innovations for ε_{t+s} for $s = 1, 2, 3, 4$, which come from $N(0, 1)$ distribution. Given these innovations, your final task is to simulate the yield curve over the next four periods. This should result in four yield curves. In line with Figure 1 from Backus et al. (1998), your summary should illustrate the four yield curves on the same plot. Make sure to use a legend to denote which line is which.

ε_{t+1}	0.55
ε_{t+2}	-0.28
ε_{t+3}	1.78
ε_{t+4}	0.19

Note: Backus et al. (1998) discertize the model into monthly increments. Hence, the ultimate goal behind the above exercise to simulate the yield curve over 4 months. The major challenge behind this formulation is to construct the yield curve under no-arbitrage pricing. For a given short-term rate, i.e. 1-month Treasury yield, the no-arbitrage pricing dictates how the short-term rate propagate through the yield curve. Section 4.1 Backus et al. (1998) provides the closed-form solution for this no-arbitrage yield curve.

References

Backus, D., Foresi, S., Telmer, C., 1998. Discrete-time models of bond pricing. Tech. rep., National bureau of economic research.

Vasicek, O., 1977. An equilibrium characterization of the term structure. Journal of financial economics 5, 177–188.