FE 535: Introduction to Financial Risk Management

Exam I

Spring 2024

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Due 11:59 PM March 3rd, 2024

Instructions

1. The exam is due Sunday at **11:59 PM March 3rd, 2024**.
2. The exam consists of
   - Multiple choice questions.
   - Three open-ended questions. The first two require data available in the excel sheet distributed with this test.
3. No hand-written answers will be accepted. Submit both a pdf of your answers and a copy of the completed excel sheet. Submit both documents strictly on the cavas portal - no submission via email attachment is accepted. Please show all your work on your submitted pdf document to earn full credit or partial credit.
4. The computations should be conducted using Excel only. No need for coding.

Multiple Choice Questions 60%

1. Consider $X = Z, Y = Z^2$ with $Z = N(0, 1)$. Which of the following statements is correct?

   (a) X and Y are independent and their correlation is zero.

   (b) X and Y are dependent and their correlation is zero.

   c (c) X and Y are dependent and their correlation is non-zero.

   (d) X and Y are independent and their correlation is non-zero.

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2. In the capital asset pricing model, what does the stock beta stand for?

   (a) The volatility of the security

   (b) The joint volatility of any two securities in a portfolio

   c (c) The relative co-movement of a security with respect to the market portfolio

   (d) The volatility of a security divided by the volatility of the market index

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3. Which of the following statements about a stock's beta is true?

   (a) A beta greater than one is less risky than the market

   (b) A beta less than one is risk-free

   (c) A beta greater than one is overvalued

   c (d) A beta greater than one is riskier than the market

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4. Which of the following statements about the Sharpe ratio is **false**?

   (a) The Sharpe ratio is equal to the excess return of a portfolio over the market return divided by the total risk of the portfolio.

   (b) The Sharpe ratio can be used to evaluate absolute performance of undiversified portfolios.

   c (c) The Sharpe ratio considers both the systematic and unsystematic risks of a portfolio.

   (d) The Sharpe ratio is derived from the capital market line.

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5. Assume that the market portfolio is the only systematic risk. Which of the following would be the best measure to use to rank either active or passive index funds based on their risk-return relationship with the market portfolio? Choose the best answer.

   (a) Maximum drawdown (MDD)
   (b) Treynor ratio
   (c) Sharpe ratio
   (d) Information ratio

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6. An equity investment manager is given the task of beating the S&P 500 index. Hence the risk should be measured in terms of

   (a) Loss relative to the bond benchmark
   (b) Loss relative to the S&P 500 index
   (c) Loss relative to the initial investment
   (d) Loss relative to the peak

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7. Consider a portfolio with 80% invested in asset X and 20% invested in asset Y. The volatilities of asset X and Y are 0.3 and 0.2, respectively. The **correlation coefficient** between the two assets is 50%. What is the portfolio volatility?

   (a) 19.70%
   (b) 26.23%
   (c) 43.51%
   (d) 12.99%

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8. Consider the information from Question 7; **however**, suppose that the correlation coefficient increases significantly, while holding all else equal. What happens to the portfolio volatility?

   (a) Stays the same
   (b) Decreases
   (c) Increases
   (d) Cannot determined due to insufficient information

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9. Similar to Question 7, consider a portfolio with w invested in asset X and $1 - w$ invested in asset Y for $w \in (0, 1)$. The volatilities of asset X and Y are 0.3 and 0.2, respectively. Suppose that the correlation coefficient between the two assets is **zero**. Express the portfolio variance as a function of w and find the value w^* that yields the minimum portfolio variance, i.e. $\sigma_p^2(w^*)$. Select the best answer that corresponds to w^*:

   (a) 17%

   (b) 31%

   (c) 83%

   (d) 14%

10. Stock J has a beta of 1, an expected return of 15%. The equity risk premium is 10%, and the risk-free rate is 2.5%. Calculate Jensen's alpha measure for Stock J.

   (a) 2.5%

   (b) -2.5%

   (c) 0%

   (d) -7.5%

11. Currently, shares of MJ Corp. trade at \$100. The probability of the price increasing by \$20 in one day is 30% and the probability of the price decreasing by \$20 in one day is 70%. Let R_2 denote log-return on the asset over two days, i.e.

$$R_2 = \ln\left(\frac{S_2}{S_0}\right) \tag{1}$$

   with S_d denoting the stock price at day d. Given that $S_0 = 100$, what is the volatility of R_2? Choose the best answer.

   (a) 12%

   (b) 14%

   (c) 30%

   (d) 20%

12. If the daily, 90% confidence level value at risk (VaR) of a portfolio is correctly estimated to be USD 10,000, which of the following statements are correct:

    (a) In 1 out of 10 days, the portfolio value will decline by USD 10,000 or more.
    (b) In 1 out of 20 days, the portfolio value will decline by USD 10,000 or more.
    (c) In 19 out of 20 days, the portfolio value will not decline more than USD 10,000.
    (d) In 1 out of 100 days, the portfolio value will decline by USD 10,000 or more.

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13. Assume that portfolio daily returns are independently and identically normally distributed with mean zero. A new quantitative analyst has been asked by the portfolio manager to calculate portfolio VaRs for 10-, 15-, 20-, and 25-day periods. The portfolio manager notices something amiss with the analyst's calculations. Assuming the annualized volatilities of daily returns for the four periods are equal, which of the following VaRs on this portfolio is inconsistent with the others?

    (a) VaR(10-day) = USD 316 million
    (b) VaR(15-day) = USD 426 million
    (c) VaR(20-day) = USD 447 million
    (d) VaR(25-day) = USD 500 million

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14. A risk manager performs an ordinary least squares (OLS) regression to estimate the sensitivity of a stock's return to the return on the S&P 500 Index. This OLS procedure is designed to:

    (a) Minimize the square of the sum of differences between the actual and estimated S&P 500 Index returns.
    (b) Minimize the square of the sum of differences between the actual and estimated stock returns.
    (c) Minimize the sum of squared differences between the actual and estimated stock returns.
    (d) Minimize the sum of differences between the actual and estimated squared S&P 500 Index returns.

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15. A (possibly biased) coin is tossed twice. If the outcome is two heads, \$5 is received. If **any** tail occurs a penalty of -\$1 is imposed. The current price to play this game is \$4. If the game were fair, what is the implied probability of heads?

    (a) 0.81

    (b) 0.61

    (c) 0.71

    (d) 0.91

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16. Which of the following statements best characterizes the relationship between the normal and lognormal distributions?

    (a) The lognormal distribution is the logarithm of the normal distribution.

    (b) If the natural log of the random variable X is lognormally distributed, then X is normally distributed.

    (c) If X is lognormally distributed, then the natural log of X is normally distributed.

    (d) The two distributions have nothing to do with one another.

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17. Consider two exchange rates X/Y and Z/Y. (For example EUR/USD and JPY/USD.) They both follow perfectly correlated geometric Brownian motions with parameters (μ_1, σ_1) and (μ_2, σ_2).

    (a) The cross-exchange rate X/Z (for example EUR/JPY) follows a standard Brownian motion

    (b) The cross-exchange rate X/Z (for example EUR/JPY) follows a general Brownian motion

    (c) The cross-exchange rate X/Z (for example EUR/JPY) follows a geometric Brownian motion

    (d) The cross-exchange rate X/Z (for example EUR/JPY) does not follow a geometric Brownian motion

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18. In the geometric Brownian motion process for a variable S,

    (a) $\Delta ln(S)$ is log-normally distributed.

    (b) $\Delta S/S$ is log-normally distributed.

    (c) S is normally distributed.

    (d) None of the above.

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19. A risk manager would like to simulate the price of a stock using the discretized geometric Brownian motion, where

$$S_{t+\Delta t} = S_t + \mu S_t \Delta t + \sigma\sqrt{\Delta t} S_t \epsilon_t$$

where μ and σ denote, respectively, the stock annual mean return and annual volatility. The data suggest that the weekly mean return on the stock is 0.1% and the weekly volatility is 4%. Assuming a weekly time step of $\Delta t = 1/52$ (in terms of annual units), what is the appropriate estimate of μ?

(a) $\hat{\mu} = 26.4\%$

(b) $\hat{\mu} = 30.16\%$

(c) $\hat{\mu} = 9.36\%$

(d) $\hat{\mu} = 17.94\%$

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20. Suppose that the price of an asset obeys geometric Brownian motion (GBM) with an annual drift μ and an annual volatility of σ. Suppose that these parameters are equal to calibrated values from Question 19. If today's price is \$100, what is the probability that the price two years from now will drop below \$80? **Note** that under GBM, the future price at T, i.e. S_T, given today's spot price, S_t, is

$$S_T = S_t \times \exp\left[(\mu - \frac{\sigma^2}{2}) \times \tau + \sqrt{\tau} \times \sigma \times \epsilon\right]$$

with $\tau = T - t$ and $\epsilon \sim N(0,1)$.

(a) 21%

(b) 35%

(c) 51%

(d) 30%

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21. Which of the following statements most accurately describe an appropriate step in the Monte Carlo (MC) approach for measuring risk of portfolio value?

(a) Simulate ten valuation outcomes for the portfolio and measure the value-at-risk (VaR) based on the simulated values.

(b) Simulate a single price path and compute the mean return of the process.

(c) Simulate thousands of valuation outcomes for the portfolio and measure the value-at-risk (VaR) based on the simulated values.

(d) Simulate a single price path and compute the Sharpe ratio of the process.

22. Which of the following statements is correct regarding the Monte Carlo simulation approach to financial problem-solving?

   (a) In practice, with the use of an initial seed for the start of random draws, it is not possible to replicate results from previous experiments.
   (b) Imprecise results may be present even with a very large number of simulation iterations when the assumptions of model inputs or the data-generating process are realistic.
   (c) If the same assumptions are made in the data-generating process, the results may be very similar.
   (d) The complexity of markets and issues that are examined have also become increasingly complex, leading to more precise simulation analysis.

23. Which of the following statements best reflects the relationship between market efficiency and Geometric Brownian Motion (GBM) in financial modeling?

   (a) Market efficiency implies that stock prices follow a GBM with constant returns.
   (b) GBM assumes market efficiency, as it incorporates all information into current stock prices.
   (c) Market efficiency and GBM are unrelated concepts in financial modeling.
   (d) GBM can accurately predict market inefficiencies and anomalies.

Open-Ended Questions (40%)

The following questions require working with data/excel. The excel file is available in the ***accompanying Excel spreadsheet*** *of the exam - refer to the first page of this exam.*

Question 1. Absolute/Relative Performance

The tab named "Real_Data" from the Excel spreadsheet provides a time series of **monthly prices** for an individual fund (asset) and a given benchmark. Based on this data, fill all the ? cells marked on the Excel. In all cases, assume that the risk-free rate is zero. As a summary, you need to address the following **nine** parts and submit all answers using the Google form:

	Statistics	Value
1.	Annual Jensen's Alpha	??
2.	Market Beta	??
3.	Coefficient of determination	??
4.	Annual Systematic Volatility of Fund	??
5.	Annual Idiosyncratic Volatility of Fund	??
6.	Total Annual Volatility of Fund	??
7.	Annual Sharpe Ratio of Fund	??
8.	Annual Treynor Ratio of Fund	??

9. **Discussion**: Based on the above statistics, briefly describe how the fund performs relative to the benchmark.
Hint: recall the relationship between 4, 5, and 6.

Question 2. Geometric Brownian Motion

In the following, you need to simulate the price path of a risky asset using a geometric Brownian motion (GBM), where $\mu = 0.15$, $\sigma = 0.30$. Consider the time step to be daily, i.e., $\Delta t = 1/252$. Let S_t denote the price of the risky asset at time t, where the initial price is $S_0 = 200$. In order to simulate the future spot price, you are given a sequence of random numbers drawn from a standard normal distribution (daily innovations), denoted by $\epsilon_d \sim N(0,1)$ for $d = 1, ..., 504$. The "Innovations" tab from the excel spreadsheet contains the ϵ_d series.

> Your main task is to simulate the price of the risky asset over the next 504 days. Your analysis should build on the Innovations tab and address the questions below.

1. To make sure your simulation is correct, calibrate σ using the simulated data. To confirm whether the simulated prices makes sense, check whether the calibrated value is consistent with the true value.

2. What is the asset price one year from now?

3. What is the asset price two years from now?

4. Suppose that the profit and loss (PnL) of a strategy is captured by the daily log returns of the risky asset. Based on the simulated PnL, what is the 1-day 99% value-at-risk (VaR)?

    - To confirm your answer, check how many times your daily loss exceeds the above-calculated VaR.

5. What is the 6-days 99% VaR?

6. **Discussion**: One can answer the previous question (Part 5) given a certain assumption. Discuss this assumption and its **limitation** for computing VaR briefly.

7. **Discussion**: Consider the following game in which you pay a premium of $\$p$ to participate at the beginning of each day. The game pays a dollar if the daily log return is less than -1%. Suppose that $p = 0.5$. Based on the simulated returns, do you consider this game fair? Elaborate.

Question 3. Investigating Model Risk

You and your co-analyst are attempting to find ideal weights for a portfolio of three assets. You decide that finding weights that maximize the Sharpe Ratio of the resulting portfolio is a good plan. In Lab 1, you did this numerically by evaluating risk-aversion parameters. A more direct way is to apply the closed-form solution for the weights that maximize the Sharpe ratio given by

$$w^* = \frac{\Sigma^{-1}\mu}{\mathbf{1}^\top \Sigma^{-1}\mu}. \tag{2}$$

Based on the above, address the following questions. In all cases, assume that the risk-free rate is zero.

1. Using historical data, you estimate the covariance matrix and mean return vector to be the following:

$$\Sigma = \begin{bmatrix} 0.0540 & 0.0053 & 0.0303 \\ 0.0053 & 0.0909 & -0.0332 \\ 0.0303 & -0.0332 & 0.0315 \end{bmatrix} \qquad \mu = \begin{bmatrix} 0.1246 \\ 0.1782 \\ 0.0035 \end{bmatrix}$$

   Given these estimates, find the weights that maximize the Sharpe ratio with respect to Equation (2), as well as the portfolio mean return and volatility. As a summary, report the Sharpe ratio of this portfolio.

2. To check your work, your co-analyst performs the same optimization. However, he rounds the estimations to 2 decimal points instead of 4. Given this new piece of information, repeat your work from the previous part using this alternate set of estimates.

3. How would you compare the two sets of weights? For instance, do they seem to have the same risk profile? Explain briefly.

4. What does this exercise say about the sensitivity of these portfolios to parameter estimation? Discuss how a situation like this might be encountered in practice and potential mechanisms to mitigate this issue.

Question 1. Absolute/Relative Performance

Discussion: Based on the above statistics, briefly describe how the fund performs relative to the benchmark.

The fund has a negative alpha, indicating underperformance relative to the benchmark.

The high beta implies that the fund is more volatile than the market, potentially leading to higher returns in bullish markets and larger losses in bearish markets.

The strong R-squared suggests a high correlation with the benchmark, indicating that most of the fund's movements can be explained by market trends.

The Sharpe ratio is positive, indicating that the fund provides a positive risk-adjusted return compared to a risk-free rate.

The Treynor ratio suggests a positive risk-adjusted return relative to market risk (beta).

Overall, while the fund may exhibit higher volatility and underperform the benchmark on average (as indicated by the negative alpha), it also provides positive risk-adjusted returns, especially when considering market risk and the risk-free rate. Investors should carefully weigh the fund's performance metrics against their risk tolerance and investment objectives.

Question 2. Geometric Brownian Motion

Discussion: Consider the following game in which you pay a premium of $p to participate at the beginning of each day. The game pays a dollar if the daily log return is less than -1%. Suppose that p = 0.5. Based on the simulated returns, do you consider this game fair? Elaborate.

In order to determine whether the game is fair, we need to assess the expected payoff and compare it to the premium paid to participate. Here's how we can analyze the fairness of the game based on the given information:

- Participants pay a daily premium of $0.5 to play
- The game pays out $1 if the daily log return is less than -1%.

$$E[X] = P(\text{log return} < -1\%) \times \$1 + P(\text{log return} \geq 1) \times (-0.5\$)$$

$$= -0.1200$$

negative expected returns suggests that, over time, participands are likely to experience loss.

Question 3. Investigating Model Risk

How would you compare the two sets of weights? For instance, do they seem to have the same risk profile? Explain briefly.

Comparison of Results:

Original Estimates:

Weight Vector: [0.6137,0.3078,0.0785][0.6137,0.3078,0.0785]

Portfolio Mean Return: 0.10950.1095

Portfolio Volatility: 0.22560.2256

Sharpe Ratio: 0.4850.485

Rounded Estimates:

Weight Vector: [0.58,0.31,0.11][0.58,0.31,0.11]

Portfolio Mean Return: 0.110.11

Portfolio Volatility: 0.230.23

Sharpe Ratio: 0.4780.478

Comparison Analysis:

The two sets of weights are quite close, with only minor differences in the third decimal place.

Both portfolios have similar risk profiles, as indicated by their similar Sharpe ratios.

The rounded estimates lead to a slightly higher portfolio volatility and a slightly lower Sharpe ratio, but the differences are relatively small.

What does this exercise say about the sensitivity of these portfolios to parameter estimation?
Discuss how a situation like this might be encountered in practice and potential mechanisms to mitigate this issue.

Sensitivity to Parameter Estimation:

The sensitivity of portfolios to parameter estimation is evident in the small variations in weights, mean returns, and volatilities when using rounded estimates.

In practice, parameter estimates such as expected returns and covariance matrices are often based on historical data, which can be subject to noise and uncertainties.

Small changes in these estimates can lead to differences in optimal portfolio weights and performance metrics.

Sensitivity to parameter estimation highlights the importance of robust methods for estimating key parameters and the need for caution in relying heavily on precise estimates.

Mitigating the Sensitivity Issue:

Use more sophisticated methods for parameter estimation that account for uncertainties and incorporate additional information.

Implement robust optimization techniques that are less sensitive to estimation errors.

Regularly update parameter estimates to reflect changing market conditions.

Diversify the portfolio across different asset classes to reduce sensitivity to specific parameter estimates.

In summary, while sensitivity to parameter estimation is a challenge, careful modeling, robust optimization methods, and ongoing monitoring can help mitigate its impact on portfolio decisions.

Q3.

1) $w^* = \Sigma^{-1}\mu / (1^T \Sigma^{-1}\mu)$

$$\Sigma = \begin{bmatrix} 0.0540 & 0.0053 & 0.0303 \\ 0.0053 & 0.0909 & -0.0332 \\ 0.0303 & -0.0332 & 0.0315 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 0.1246 \\ 0.1782 \\ 0.0035 \end{bmatrix}$$

1. Calculate the Inverse of Covariance Matrix.

$$\Sigma^{-1} = \begin{bmatrix} \sigma^{-1}_{11} & \sigma^{-1}_{12} & \sigma^{-1}_{13} \\ \sigma^{-1}_{21} & \sigma^{-1}_{22} & \sigma^{-1}_{23} \\ \sigma^{-1}_{31} & \sigma^{-1}_{32} & \sigma^{-1}_{33} \end{bmatrix}$$

1. wt vector

$$w^* = \Sigma^{-1}\mu / (1^T \Sigma^{-1}\mu)$$

calculate Portfolio Mean Return

$$R_P = w^T \mu$$

Voltility,

$$\sigma p = \sqrt{w^T \Sigma w}$$

Inverse of Covariance Matrix

$$\Sigma^{-1} = \begin{bmatrix} 21.68 & -2.52 & -25.47 \\ -2.52 & 13.41 & 6.77 \\ -25.47 & 6.77 & 38.79 \end{bmatrix}$$

wt. vector

$$w^* = \begin{bmatrix} 0.6137 \\ 0.3078 \\ 0.0785 \end{bmatrix}$$

$$\text{Sharpe Ratio} = \frac{0.1095}{0.2256}$$

$$= 0.485$$

Q3.

2) Rounded wt vector,

$$w^* = \begin{bmatrix} 0.58 \\ 0.31 \\ 0.11 \end{bmatrix}$$

$$R_P = w^T \mu$$

$$R_P = 0.11$$

Sharpe Ratio, $= \dfrac{0.11}{0.23}$

$= 0.478$