Akshay Parate.

MA 541-A – Spring 2024

HOMEWORK 11

(Due date: Sunday, 04/14/2024, at 11:00 pm)

(Write the pledge on top of your work and sign under it.)

I pledge my honor that I have abided by the Stevens honor system

Akshay.

Problem 1:

Using basic statistical properties of the variance, as well as single-variable calculus, derive the formula for α in page 19 of the Week 12-Lecture. In other words, prove that $\alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}$ does indeed minimize Var[αX + (1 − α)Y].

let $z = ax + (1-a)y$.

$\therefore var(z) = a^2 var(x) + (1-a)^2 var(y) + 2a(1-a) cov(x,y)$

we want to minimize var(z) w.r.t a.

$\therefore$ taking derivative w.r.t a.

$\frac{d}{da} var(z) = 2a\, Var(x) - 2(1-a) var(y) + 2(1-2a) cov(x,y)$

let $\frac{d}{da} var(z) = 0$

$\therefore 2a\big(var(x) - var(y)\big) + 2\big(cov(x,y) - 2a\, cov(x,y)\big) = 0.$

$a\,(var(x) - var(y) - 2\,cov(x,y)) = -\big(cov(x,y) - 2a\, cov(x,y)\big)$

$a = \frac{cov(x,y) - 2a\, cov(x,y)}{var(x) - var(y) - 2cov(x,y)}$

$a\, var(x) - a\, var(y) = cov(x,y)$

Rearranging terms:

$a\, var(x) = cov(x,y) + a\, var(y)$

$a = \frac{cov(x,y)}{var(x) - var(y)}$

This proves that a minimizes $var(ax + (1-a)y)$