MA574 Pre Class Assignment 15 31/10/2023

Q1. $b = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ $a_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $a_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}$$

$$P = A(A^TA)^{-1}A^T.$$

$$A^TA = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 5 & 5 \end{pmatrix}$$

$$(A^TA)^{-1} = \frac{1}{6}\begin{pmatrix} 5 & -3 \\ -3 & 5 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}\frac{1}{6}\begin{pmatrix} 5 & -3 \\ -3 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$= \frac{1}{6}\begin{pmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{pmatrix}$$

Q2. $S = \{(0,6), (1,0), (2,0)\}$

$$A^T A = \begin{pmatrix} m & \Sigma t_i \\ \Sigma t_i & \Sigma t_i^2 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 3 & 5 \end{pmatrix}$$

$$m = 3, \quad \Sigma t_i = 3, \quad \Sigma t_i^2 = 5$$

$$A^T \cdot b = \begin{pmatrix} \Sigma b_i \\ \Sigma t_i b_i \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} A^T A \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} \Sigma b_i \\ \Sigma t_i b_i \end{pmatrix}$$

$$\begin{pmatrix} 3 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} c \\ D \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

$$D = 3$$

$$c = 5.$$

$$y = \underline{5 - 3t}.$$

Q3. $S = \{(-2,0), (-1,0), (0,1), (1,0), (2,0)\}$.

let $y = Ax^2 + Bx + c$ be eq^n.

$0 = 4A - 2B + c = 0$.

$0 = A - B + c = 0$.

$c = 1$

$A + B + c = 0$.

$4A + 2B + c = 0$.

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 4 & 1 & 0 & 1 & 4 \\ 2 & -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 34 & 16 & 10 \\ 16 & 10 & 4 \\ 10 & 4 & 5 \end{bmatrix}$$

$$A^T A \vec{x} = A^T \vec{B}$$

$$\begin{bmatrix} 34 & 16 & 10 \\ 16 & 10 & 4 \\ 10 & 4 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 16/34 & 10/34 & 0 \\ 16 & 10 & 4 & 0 \\ 10 & 4 & 5 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 10/34 & 10/34 & 0 \\ 6 & 1 & -1/2 & 0 \\ 0 & -24/34 & 20/34 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 16/34 & 10/34 & 0 \\ 0 & 1 & -1/2 & 0 \\ 6 & 0 & 1 & 1 \end{array}\right]$$

$C = 1$

$B = 1/2.$

$A = -9/17.$