MA574 Pre Class Assignment - 18. 09/11/23

Q1. To solve the problem, we need to find the U & V column vectors such that their outter product results is given matrix.

Let Blue be 1 & White be 0

matrix :
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\vec{U} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \vec{V} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad V^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\vec{U} \cdot \vec{V}^T = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_{7\times 1} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}_{1\times 9}$$

$$\vec{U} \cdot \vec{V}^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Q2. $A_0 = \begin{pmatrix} 5 & 4 & 3 & 2 & 1 \\ -1 & 1 & 0 & 1 & -1 \end{pmatrix}$

i) row centred matrix.

mean of row1 $= \frac{5+4+3+2+1}{5} \Rightarrow 3.$

mean of row 2 $= \frac{-1+1+0+1-1}{5} \Rightarrow 0.$

$$A = \begin{pmatrix} 5-3 & 4-3 & 3-3 & 2-3 & 1-3 \\ -1-0 & 1-0 & 0-0 & 1-0 & -1-0 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 1 & 0 & -1 & -2 \\ -1 & 1 & 0 & 1 & -1 \end{pmatrix}.$$

Sample covariance matrix is given by.

$$S = \frac{AA^T}{n-1}$$

$$AA^T = \begin{pmatrix} 2 & 1 & 0 & -1 & -2 \\ -1 & 1 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 1 & 1 \\ 0 & 0 \\ -1 & 1 \\ -2 & -1 \end{pmatrix}$$

$$= \begin{bmatrix} 10 & 0 \\ 0 & 4 \end{bmatrix}$$

$$S = \frac{\begin{bmatrix} 10 & 0 \\ 0 & 4 \end{bmatrix}}{4} \Rightarrow \begin{bmatrix} 5/2 & 0 \\ 0 & 1 \end{bmatrix}$$

Eigen values of S are.

$$\det(S - \lambda I) = 0.$$

$$\det\left(\begin{pmatrix} 5/2 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right) = 0.$$

$$\det\left(\begin{pmatrix} 5/2 - \lambda & 0 \\ 0 & 1-\lambda \end{pmatrix}\right) = 0.$$

$$\left(\frac{5}{2} - \lambda\right)(1-\lambda) - 0 = 0.$$

$$\frac{5}{2} - \frac{5}{2}\lambda - \lambda + \lambda^2 = 0.$$

10
52.

$$\lambda^2 - \frac{7}{2}\lambda + \frac{5}{2} = 0.$$

$$2\lambda^2 - 7\lambda + 5 = 0.$$

$$2\lambda^2 - 5\lambda - 2\lambda + 5 = 0.$$

$$\lambda(2\lambda - 5) - 1(2\lambda - 5) = 0.$$

$$(\lambda - 1)(2\lambda - 5) = 0.$$

$\lambda = 1$ & $\lambda = 5/2$. (Eigen values)

<u>d</u> $y = x$ line is the closest to these sample points in column A.

Q3. $A_0 = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 2 & 2 \end{pmatrix}$

mean value of row1 $= \frac{6}{3} \Rightarrow 2.$

mean value of row2 $= \frac{9}{3} \Rightarrow 3.$

$A = \begin{pmatrix} -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix}$

$\therefore S = \frac{AA^T}{n-1}$ $\qquad AA^T = \begin{pmatrix} -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix}_{2\times 3} \begin{pmatrix} -1 & 2 \\ 0 & -1 \\ 1 & -1 \end{pmatrix}_{3\times 2}$

$AA^T = \begin{bmatrix} 2 & -3 \\ -3 & 6 \end{bmatrix}.$

$S = \frac{AA^T}{2} \Rightarrow \begin{bmatrix} 1 & -3/2 \\ -3/2 & 3 \end{bmatrix}.$

Q4. To find the centered matrix, we would substract the elements in each row with the mean value of the individual rows.

The sample covariance can be found using $S = \frac{AA^T}{n-1}$.

where, A = centered matrix.
A^T = A transpose.
n = num of elements in a row.

leading eigenvector of S represents that eigenvector of covariance matrix is always equal to the principal components, as the eigenvector with the largest eigenvalue is the direction along which that data set has the maximum variance.