MA 574
Fall 2023
Exam 1
10/24/2023
Time Limit: 85 Minutes

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- This exam contains 7 pages (including this cover page) and 4 questions. Make sure you have every page!
- Make sure to show as much work as you can. There is no such thing as explaining too much! You also increase your chances for getting partial credit by including more explanation.
- There will be no calculators allowed during the exam.
- There will be no collaboration permitted during the exam. Any questions must be presented to me directly.

Question	Points	Score
1	23	
2	15	
3	20	
4	20	
Total:	67	

1. (12 points) Determine whether each of the following statements are True or False. **You must include a short explanation for each answer.**

   (a) (3 points) True or False: If the **coefficient matrix** of a linear system has at least one row of zeros, then the system must either have no solution or infinitely many solutions.

False - It is true that a system with a zero row will have a solution, but that solution may have endless num of solutions. However it is also possible for a system to have a unique solution.

   (b) (3 points) True or False: If $f : \mathbb{R} \to \mathbb{R}$ is invertible and differentiable, and its inverse $f^{-1}(x)$ is **NOT** differentiable at $x = x_0$, then the tangent line to the curve $y = f(x)$ at the point $x = f^{-1}(x_0)$ is given by $y = c$ for some constant c.

True

   (c) (3 points) True or False: The line $\mathbf{r}(t) = \langle 1 - t, 3t, 2 + 2t \rangle$ intersects the plane $2x - 2y + 4z = 10$ at a single point.

False

$x = 1 - t, \ y = 3t, \ z = 2 + 2t$

$2(1-t) - 2(3t) + 4(2+2t) = 10$

$2 - 2t - 6t + 8 + 8t = 10.$

$\cancel{-8t} + 10 + \cancel{8t} = 10$

$10 = 10.$

since $10 = 10$ will always be true for any value of t

   (d) (3 points) True or False: If $f(x, y)$ is a strictly positive function, where each level curve $f(x, y) = c$ is a circle of radius $r = -\ln(c)$ centered at a fixed point (x_0, y_0), then the partial derivatives of f are always negative.

False, The statement does not necessarily hold. The information provided about the level curves does not imply anything about the signs of partial derivatives of function.

The above information does not specify the behaviour of the function in terms of +ve or -ve value along particular direct

2. (15 points) Select all correct answer(s) in each of the following problems.

   (a) (3 points) The trajectory of the vector-valued function $\mathbf{r}(t) = \langle \sin(t^2), \cos(t^2) \rangle$ most closely resembles what?

   - ✓ A circle
   - A parabola
   - A graph of the tangent function
   - A line

   (b) (3 points) Let A be an invertible 3×3 matrix, and assume that $A^{-1} = A^T$. Select all (if any) of the following statements that are also true.

   - $A = A^{-1}$
   - ✓ Any row echelon form of A has a pivot in every column.
   - ✓ The determinant of A is equal to either 1 or -1
   - ✓ The unique solution to $A\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ is the first row of A

   (c) (3 points) Let R denote the bounded region between the curves $y = \sqrt{x}$ and $y = \frac{x}{3}$. If $f(x, y)$ is some continuous function, which of the following integrals compute $\iint_R f(x, y)dA$?

   - $\int_0^9 \int_{\frac{x}{3}}^{\sqrt{x}} f \ dy \ dx$
   - ✓ $\int_0^9 \int_{\sqrt{x}}^{\frac{x}{3}} f \ dy \ dx$
   - $\int_0^3 \int_{3y}^{y^2} f \ dx \ dy$
   - ✓ $\int_0^3 \int_{y^2}^{3y} f \ dx \ dy$

$y = \sqrt{x}$, $y = x/3$

$\sqrt{x} = \frac{x}{3}$

$x = \frac{x^2}{9}$

$9\cancel{x} = x^2$

$\underline{x = 9}$.

$y = \sqrt{9}$, $y = 9/3$.

$y = 3$

(d) (3 points) Let $A = \begin{pmatrix} 1 & 1 & 0 \\ -2 & 0 & -1 \\ 1 & 2 & 0 \end{pmatrix}$. Which of the following correctly compute $\det(A)$?

- $\begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} + \begin{vmatrix} -2 & -1 \\ 1 & 0 \end{vmatrix}$
- $2\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$
- $\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$
- $\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} - 2\begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix}$

$\begin{matrix} 1 & 1 & 0 & 1 & 1 \\ -2 & 0 & -1 & -2 & 0 \\ 1 & 2 & 0 & 1 & 2 \end{matrix} = 1$

$\begin{pmatrix} 2 & 0 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

$\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} \Rightarrow 6 - 5 = 1$

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ -4 & -2 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \Rightarrow 2 - 1 = 1$

$\begin{pmatrix} -1 & 0 \\ 4 & 1 \end{pmatrix}$ $-1 - 0 = -1$

(e) Which (if any) among the following expressions is a representation of the line in $\mathbb{R}^3$ that has direction vector $\mathbf{v} = \langle -2, 1, 5 \rangle$ and passes through the point $(0, 3, -2)$?

- $\mathbf{s}(t) = \langle -2, 1 + 3t, 5 - 2t \rangle$
- $\mathbf{r}(t) = \langle -2t, 3 + t, -2 + 5t \rangle$
- $x(t) = -2\sqrt[3]{t}, 3 + \sqrt[3]{t}, -2 + 5\sqrt[3]{t}$
- $-2x = 3 + y = -2 + 5z$

$r(t) = r_0 + tv$

$L = \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

3. (20 points) Throughout this problem, let $f(x, y) = \ln(x + \ln(y))$.

(a) (5 points) What is the domain of f?

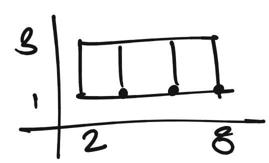
$x + \ln(y) > 0$
$\ln(y) > 0$
$\because \ln$ is increasing
$y > 0$

$\because y > 0$
$\therefore x > 0.$
$f(x,y) = \ln(x + \ln(y))$
$x > 0$ & $y > 0$.

(b) (5 points) Compute $\dfrac{\partial f}{\partial y}$

(c) (5 points) **Using implicit differentiation**, compute the slope of the tangent line to the level curve $f(x, y) = 1$ at the point $(e, 1)$.

(d) (5 points) Approximate the volume under the surface defined by f, but above the rectangle $[2, 8] \times [1, 3]$, using Riemann sums. You should break the region into 3 smaller squares and use the lower right corners of each square. Your answer may be left as a sum.



$\Delta x = \frac{8-2}{3} \Rightarrow 2$

$\Delta y = \frac{3-1}{1} \Rightarrow 2$

volume of small rectangular prism

$v = f(x,y) \cdot \Delta x \, \Delta y.$

$V = 4\,f(4,1) + 4f(6,1) + 4\,f(8,1)$

$V = 4(\ln 4 + \ln 6 + \ln 8)$

$V = 21 \cdot 03.$

$(4,1), (6,1), (8,1)$

$\ln(4 + \ln 1) = \ln 4$

$\ln(6 + \ln 1) = \ln 6$

$\ln(8 + \ln 1) = \ln 8.$

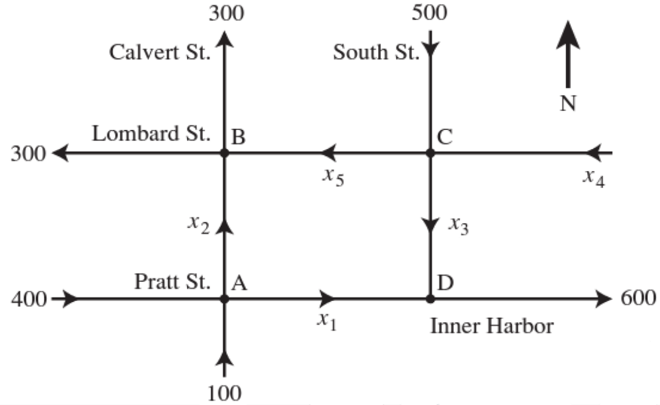
4. (20 points) Use Gaussian elimination to find **all** solutions to the given system of equations.

   (a) (5 points)

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 3 \\ 2x_1 - x_2 - x_2 &= 0 \\ -2x_1 + 2x_2 + x_3 &= 1 \end{aligned}$$

   (b) (5 points) $A\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, where $A = \begin{pmatrix} 1 & -2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

(c) (10 points) Pictured below is a network that displays the traffic flow of every street in a city (measured in cars/hour). Notice that every street pictured only has traffic flowing in one direction. Also notice that a number of the streets have undetermined traffic flows indicated by variables x_1, x_2, x_3, x_4, and x_5. Being that this is a "real world" network, you know that at every intersection the amount of traffic flow into the intersection is equal to the amount of traffic flow out. Moreover, you know that the total amount of traffic flowing into the network must equal the amount flowing out. Putting all of this together, write down a system of equations that models this situtation, and find all possible values of x_1, x_2, x_3, x_4 and x_5 giving solutions to the system.



Node A $\rightarrow$ $x1 + x2 = 500$

node B $\rightarrow$ $x2 + x5 = 600$

node c $\rightarrow$ $x3 + x5 - x4 = 500$

node d $\rightarrow$ $x1 + x3 = 600$.

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 500 \\ 0 & 1 & 0 & 0 & 1 & 600 \\ 0 & 0 & 1 & -1 & 1 & 500 \\ 1 & 0 & 1 & 0 & 0 & 600 \end{array}\right] R4 - R1.$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 500 \\ 0 & 1 & 0 & 0 & 1 & 600 \\ 0 & 0 & 1 & -1 & 1 & 500 \\ 0 & -1 & 1 & 0 & 0 & 100 \end{array}\right] R4 + R2$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 500 \\ 0 & 1 & 0 & 0 & 1 & 600 \\ 0 & 0 & 1 & -1 & 1 & 500 \\ 0 & 0 & 1 & 0 & 1 & 700 \end{array}\right] R4 - R3.$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 500 \\ 0 & 1 & 0 & 0 & 1 & 600 \\ 0 & 0 & 1 & -1 & 1 & 500 \\ 0 & 0 & 0 & 1 & 0 & 200 \end{array}\right] R2 - R4.$$

$$\boxed{\begin{array}{l} x4 = 200 \\ x3 + x5 = 700 \\ x2 + x5 = 600 \\ x1 + x2 = 500. \end{array}}$$