MA574 Pre class Assignment 19. 14/11/23.

Q1. $f(x) = ax^2 + bx + c$ where $a > 0$

→ critical points:

$$f'(x) = 2ax + b = 0.$$

$$x = \frac{-b}{2a}.$$

$\because$ $a > 0$, x will always be < 0.

$f''(x) = 2a$

$\because$ $f''(x) > 0$, $x = \frac{-b}{2a}$ is local minimum & global minimum

$f''(x) = 2a > 0$. (concave up).

Q2. (A) $f(x) = x^2 + 3$ on $[-1, 4]$

critical points:

$$f'(x) = 2x = 0.$$
$$x = 0.$$

$f'(x)_{x=-1} = -2$ ← global minima.

$f'(x)_{x=0} = 0$

$f'(x)_{x=4} = 8$ ← global maxima

(B) $g(x) = (x - x^2)^2$ on $[-1, 1]$.

$$g(x) = x^2 - 2x^3 + x^4$$

$$g'(x) = 2x - 6x^2 + 4x^3 = 0.$$
$$x - 3x^2 + 2x^3 = 0.$$
$$2x^3 - 3x^2 + x = 0$$
$$x(2x^2 - 3x + 1) = 0.$$
$$x(2x^2 - 2x - x + 1) = 0.$$
$$x(2x(x-1) - 1(x-1)) = 0.$$
$$x(2x-1)(x-1) = 0.$$
$$x = 0, \quad x = \frac{1}{2}, \quad x = 1.$$

$g'(x)_{x=-1} = -6$ ← global minima

$g'(x)_{x=0} = 0$

$g'(x)_{x=1/2} = 0$ ← global maxima.

$g'(x)_{x=1} = 0$

(c) $h(x) = 4\sin(x) - 3\cos(x)$ on $[0, 2\pi]$

Critical points :

$$h'(x) = 4\cos x + 3\sin(x) = 0.$$

$$4\cos x = -3\sin(x).$$

$$\cos x = \frac{-3}{4}\sin(x)$$

$$x = 2\pi$$

Q3. at $t=0$, $h'(t)$ is increasing.
at $t=1$, $h'(t)$ is 0

There is no maxima or minima since,
$h'(t)$ is always non negative .

Q4. $f(x,y,z) = x^2z + y^3z^2 - xyz$

$\frac{\partial}{\partial x}(x^2z + y^3z^2 - xyz)$.

$= 2xz - yz$

$\frac{\partial}{\partial y}(x^2z + y^3z^2 - xyz)$

$= 3y^2z^2 - xz$.

$\frac{\partial}{\partial z}(x^2z + y^3z^2 - xyz)$

$= x^2 + 2y^3z - xy$.

$\langle 2xz - yz,\ 3y^2z^2 - xz,\ x^2 + 2y^3z - xy \rangle$

at point $(1,1,1)$

$\langle 1, 2, 2 \rangle$

(A) max rate of charge is equal to magnitude of the vector

$$\nabla F(1,1,1) = \sqrt{(1)^2+(2)^2+(2)^2}.$$

$$= \sqrt{9}.$$

$$= 3.$$

(b) direction in which maxima occurs:

$$\frac{\nabla F(1,1,1)}{|\nabla F(1,1,1)|} = \frac{\langle 1, 2, 2\rangle}{3}.$$

$$= \langle 1/3, 2/3, 2/3 \rangle$$

(c) direction towards. (1,2,3)

$$\sqrt{(1)^2+(2)^2+(3)^2} \Rightarrow \sqrt{14}$$

$$\text{direction} = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$$

Q6. $f(x,y) = x^3 - 3xy - y^3.$

$\rightarrow$ $\frac{\partial}{\partial x}\left(f(x,y)\right) = 3x^2 - 3y$

$\frac{\partial}{\partial y}\left(f(x,y)\right) = -3x - 3y^2$

$3x^2 - 3y = 0.$

$-3x - 3y^2 = 0. \Rightarrow x = -y^2$

$3(-y^2)^2 - 3y = 0.$

$-3y^4 - 3y = 0.$

$-y^4 - y = 0.$

$y^4 + y = 0.$

$y(y^3 + 1) = 0.$

$y = 0, \quad y = -1$

$x = 0 \quad x = -1$

critical points are $(0,-1)$ & $(0,-1)$