MAS714. Pre Class Assignment -20. 16/11/23

Q1. $f\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \end{bmatrix} = \begin{bmatrix} x^2y \\ 5x+\sin y \end{bmatrix}$

Then we have

$f_1(x,y) = x^2y \quad , f_2(x,y) = 5x+\sin y.$

$$Jf(x,y) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \begin{bmatrix} 2xy & x^2 \\ 5 & \cos y \end{bmatrix}$$

$$\det(Jf(x,y)) = 2xy\cos y - 5x^2.$$

Q2. A) we can express the mapping f^n as:

$f(x,y) = (r, \theta)$

$r = \sqrt{x^2+y^2}$

$\theta = \arctan\left(\frac{y}{x}\right)$

(B) $f(r,s,t) = (r\cos\theta,\ r\sin\theta,\ \theta)$

$f(r,s,t) = (x,y,z)$

$$\frac{\partial(x,y,z)}{\partial(r,s,t)} = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial s} & \frac{\partial z}{\partial t} \end{bmatrix}$$

$$= \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= r\cos^2\theta + r\sin^2\theta.$$

$$= \underline{\underline{r.}}$$

Q 3. $f(x) = \sin(x_1)\cos(x_2)$ where $x \in R^2$.

$$J = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} \end{bmatrix}$$

$$\frac{\partial f}{\partial x_1} = \cos(x_1) \cdot \cos(x_2)$$

$$\frac{\partial f}{\partial x_2} = -\sin(x_1)\ \sin(x_2)$$

$$J = \cos(x_1) \cdot \cos(x_2) - \sin(x_1)\sin(x_2)$$

(B) $g(x,y) = x^T y$ where $x, y \in R$.

$$J = \begin{bmatrix} \frac{\partial g}{\partial x_1} & \frac{\partial g}{\partial x_2} \\ \frac{\partial g}{\partial y_1} & \frac{\partial g}{\partial y_2} \end{bmatrix}$$

$$\frac{\partial g}{\partial x_1} = y_1 \qquad \frac{\partial g}{\partial x_2} = y_2$$

$$\frac{\partial g}{\partial y_1} = x_1 \qquad \frac{\partial g}{\partial y_2} = x_2$$

$$J = \begin{bmatrix} y_1 & y_2 \\ x_1 & x_2 \end{bmatrix}$$

(c) $h(x) = x x^T$ where $x \in R$.

$$J = \frac{\partial h}{\partial x}.$$

$$h(x) = x x^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}$$

$$\frac{\partial h(x)}{\partial x_1} = \begin{bmatrix} 2x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \frac{\partial h(x)}{\partial x_2} = \begin{bmatrix} x_1 \\ 2x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\frac{\partial h(x)}{\partial x_n} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ 2x_n \end{bmatrix}.$$

$$J = \begin{bmatrix} 2x_1 & x_2 \dots & x_n \\ x_1 & 2x_2 \dots & x_n. \\ \vdots & & \\ x_1 & x_2 \dots & 2x_n \end{bmatrix}$$

Q4. The matrix of 2^{nd} order derivative often denoted by hessian matrix, is derived by taking the partial derivative of first order PD of a multivariate function.

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \vdots & & & \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n} \end{bmatrix}$$