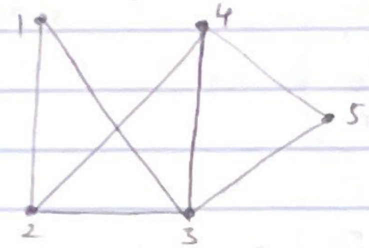
Homework 4 574.

1) The adjacent matrix of a network with k nodes $N_1, \ldots, N_k$ is defined to be that $k \times k$ matrix A, whose i, j^{th} entry equals to 1 if edge between nodes.



$i, j = 2, 4$.

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

let $A^2 = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0+1+1+0+0 & 0+0+1+0+0 & 0+1+0+0+0 & 0+1+1+0+0 & 0+0+1+0+0 \\ 0+0+1+0+0 & 1+0+1+1+0 & 1+0+0+1+0 & 0+0+1+0+0 & 0+0+1+1+0 \\ 0+1+0+0+0 & 1+0+0+1+0 & 1+1+0+1+1 & 0+1+0+0+1 & 0+0+0+1+0 \\ 0+1+1+0+0 & 0+0+1+0+0 & 0+1+0+0+1 & 0+1+1+0+1 & 0+0+1+0+0 \\ 0+0+1+0+0 & 0+0+1+1+0 & 0+0+0+1+0 & 0+0+1+0+0 & 0+0+1+1+0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 3 & 2 & 1 & 2 \\ 1 & 2 & 4 & 2 & 1 \\ 2 & 1 & 2 & 3 & 1 \\ 1 & 2 & 1 & 1 & 2 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 3 & 2 & 1 & 2 \\ 1 & 2 & 4 & 2 & 1 \\ 2 & 1 & 2 & 3 & 1 \\ 1 & 2 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$A^3 = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{bmatrix} 2 & 5 & 6 & 3 & 3 \\ 5 & 4 & 7 & 7 & 3 \\ 6 & 7 & 6 & 7 & 6 \\ 3 & 7 & 7 & 4 & 5 \\ 3 & 3 & 6 & 5 & 2 \end{bmatrix} \end{array}$$

$i, j = 2,4 \Rightarrow 7.$

There are 7 steps from red node to blue node.

2) Given :- $A = \begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 8 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$R_1' = R_1 - R_4; \quad R_3' = R_3 - 6R_4.$

$$A = \begin{array}{ccccccc} \left[\begin{array}{ccccccc} \boxed{1} & 0 & 4 & 1 & 0 & 3 & 0 \\ 0 & \boxed{1} & 5 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \boxed{1} \end{array}\right] & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} \\ \begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{array} & \end{array}$$

let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}$

$\therefore Ax = 0.$

$x_1 + 4x_3 + x_4 + 3x_6 = 0$ — ①

$x_2 + 5x_3 + 2x_4 = 0$ — ②

$x_5 + 8x_6 = 0$ — ③

$x_7 = 0$ — ④.

There are three free variable columns.

they are x_3, x_4, x_6 → solutions should be in terms of free variables.

let $x_3 = a,\ x_4 = b,\ x_6 = c$

$x_1 = -4a - b - 3c$.

$x_2 = -5a - 2b$

$x_5 = -8c$

$x_7 = 0$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -4 \\ -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ -8 \\ 1 \\ 0 \end{bmatrix} x_6.$$

5) Given:- A and B are 10x10 matrices.

det(A) = 4 & det (B) = 5.

R.T.P:- That exchange rows 5 & 7 of matrix A and scale row 9 by 3 to get matrix.

$\det(A) = -\det(B)$

$\det(C) = -3\det(A)$

$= -3 \times 4$

$= -12.$

$A \rightarrow n \times n$ matrix,

if A is an $n \times n$ matrix and A is $\det(A^{-1}) = \frac{1}{\det(A)}$.

$\det(C^{-1}) = -\frac{1}{12}.$

compute the determinant of matrix D.

$\det(D) = 2^{10} \det(B)$

$\Rightarrow 1024 \times 5.$

$= 5120.$

by computing $\det(A^{-1})$

$\det(A^{-1}) = \frac{1}{\det(A)}.$

$\Rightarrow 1/4.$

If A and B are $n \times n$ matrix,

$\det(AB) = \det(A).\det(B).$

$\det(A^{-1}BC^{-1}D) = \det(A^{-1})\det(B)\det(C^{-1})\det(D).$

$\Rightarrow 1/4 \times 5 \times (\frac{-1}{3}) \times 5120$

$\Rightarrow -320/3.$

The value of $\det(A^{-1}BC^{-1}D) = -320/3$//

4) Given :- $A = \begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix}$

Inverse by guass-Jordan elimination.

$[A/I] \Leftrightarrow [I/A^{-1}]$.

$$A/I = \left[\begin{array}{ccc|ccc} -2 & 2 & -1 & 1 & 0 & 0 \\ 3 & -5 & 4 & 0 & 1 & 0 \\ 5 & -6 & 4 & 0 & 0 & 1 \end{array}\right]$$

using Row operations.

$$\left[\begin{array}{ccc|ccc} -2 & 2 & -1 & 1 & 0 & 0 \\ 3 & -5 & 4 & 0 & 1 & 0 \\ 5 & -6 & 4 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \to 2R_3 - 5R_1$

$$\left[\begin{array}{ccc|ccc} -2 & 2 & -1 & 1 & 0 & 0 \\ 0 & 4 & -5 & -3 & -2 & 0 \\ 0 & 2 & -3 & -5 & 0 & -2 \end{array}\right] \quad R_2 \to -2R_2 - 3R_1$$

$R_3 \rightarrow 2R_3 - R_2.$

$$\left[\begin{array}{ccc|ccc} -2 & 2 & -1 & 1 & 0 & 0 \\ 0 & 4 & -5 & -3 & -2 & 0 \\ 0 & 0 & -1 & -7 & 2 & 4 \end{array}\right]$$

$R_1 \rightarrow 2R_1 - R_2.$

$$= \left[\begin{array}{ccc|ccc} -4 & 0 & 3 & 5 & 2 & 0 \\ 0 & 4 & -5 & -3 & -2 & 0 \\ 0 & 0 & 1 & 7 & -2 & 4 \end{array}\right]$$

$R_2 \rightarrow R_2 + 5R_3$

$$\left[\begin{array}{ccc|ccc} -4 & 0 & 0 & -16 & 8 & -12 \\ 0 & 4 & 0 & 32 & -12 & 20 \\ 0 & 1 & 1 & 7 & -2 & 4 \end{array}\right] \quad \begin{array}{l} R_2 \rightarrow R_2/4. \\ R_1 \rightarrow R_1/4. \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -2 & 3 \\ 0 & 1 & 0 & 8 & -3 & 5 \\ 0 & 0 & 1 & 7 & -2 & 4 \end{array}\right]$$

I / A^{-1}

$$A^{-1} = \begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix}$$

3) $Ax = b$

$b = (2, 2, -1, 4)^T = \begin{pmatrix} 2 \\ 3 \\ -1 \\ 4 \end{pmatrix}$

$$A = \begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 8 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \qquad A/B =$$

~~Ax=b~~ $A/b = \left[\begin{array}{ccccccc|c} 1 & 0 & 4 & 1 & 0 & 3 & 1 & 2 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 8 & 6 & -1 \\ 0 & 0 & 0 & 0 & 1 & 6 & 1 & 4 \end{array}\right]$

$\to$ ~~x,44~~

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 5R_2$.

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 4 & 1 & 0 & 3 & 1 & 2 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 8 & 6 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 4 & 1 & 0 & 3 & 1 & 2 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 & 3 \\ 0 & 0 & -25 & -10 & 1 & 8 & 6 & -16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right] \quad R_3 \to \frac{-1}{25} R_3$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 4 & 1 & 0 & 3 & 1 & 2 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2/25 & -1/25 & -8/25 & -6/25 & -16/25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - 4R_3 \\ R_2 \to R_2 - 5R_3 \end{array}$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 4 & 1 & -3/5 & 32/25 & 192/25 & 2 - 64/25 \\ 0 & 1 & 5 & 2 & 1 & 8 & 6 & 15 - 16 \\ 0 & 0 & 1 & 2/5 & -1/25 & -8/25 & -6/25 & 16/25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$$A|b = \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & -3/5 & 32/25 & 192/25 & 48/25 & 2 - 64/25 \\ 0 & 5 & 1 & 0 & \phi & 8 & 6 & 15 - 16 \\ 0 & 0 & 0 & 2/5 & -1/25 & -8/25 & -6/25 & 16/25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$x_1 + 4x_3 + x_4 + 3x_6 = 2.$

$x_2 + 5x_3 + 2x_4 = 3$

$x_5 + 8x_6 = -4$

$x_7 = 0.$

let say $x_3 = a$; $x_4 = b$, $x_6 = c$.

$x_1 = 2 - 4a - b - 3c.$

$x_2 = 3 - 5a - 2b.$

$x_3 = a, \ x_4 = b$

$x_5 = -1 - 8c, \ x_6 = c.$

set of solution of non homogenous.

$(2 - 4a - b - 3c, 3 - 5a - 2b, a, b, -1 - 8c, c).$