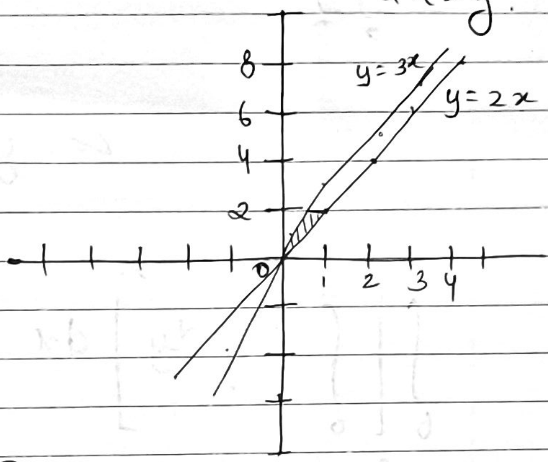
Q1. (A) $\displaystyle\int_0^1\int_{2x}^{3x} (x+y^2)\, dy\, dx.$

$3x = y$

$2x = y$



$$= \int_0^1 \left[\int_{2x}^{3x} (x+y^2)\, dy\right] dx$$

$$= \int_0^1 \left[xy + \frac{y^3}{3}\right]_{2x}^{3x} dx.$$

$$= \int_0^1 \left[3x^2 + \frac{(3x)^3}{3} - 2x^2 - \frac{(2x)^3}{3}\right] dx$$

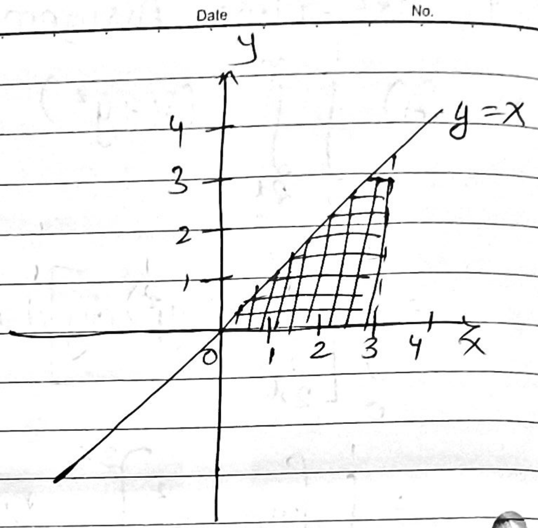
$$= \int_0^1 \left[x^2 + \frac{27x^3}{3} - \frac{8x^3}{3}\right] dx = \int_0^1 \left[x^2 + \frac{19x^3}{3}\right] dx.$$

$$= \frac{1}{3}\left[\frac{\cancel{3}x^3}{\cancel{3}} + \frac{19x^4}{4}\right]_0^1 \Rightarrow \frac{1}{3}\left[(1)\cancel{x}^3 + \frac{19(1)^4}{4} - 0 - 0\right]$$

$$= \frac{1}{3}\left[1 + \frac{19}{4}\right] \Rightarrow \frac{1}{3}\left[\frac{23}{4}\right]$$

$$= \boxed{\frac{23}{12}.}$$

(B) $\int_0^3 \int_0^x e^{x^2} \, dy \, dx$



$y = x$ & $y = 0$.

$\int_0^3 \left[\int_0^x e^{x^2} \, dy \right] dx$

$= \int_0^3 \left[y \cdot e^{x^2} \right]_{y=0}^{y=x} dx \Rightarrow \int_0^3 e^{x^2} [x] \, dx.$

using integration by substitution

let $u = x^2$.

$\frac{du}{dx} = 2x.$

$dx = \frac{1}{2x} \, du.$

$\int_0^3 e^u \cdot [\cancel{x}] \times \frac{1}{2\cancel{x}} \, du \Rightarrow \int_0^3 e^u \, du.$

$= \left[e^u \right]_0^3$

resubstitute

$$= \left[e^{x^2}\right]_0^3$$

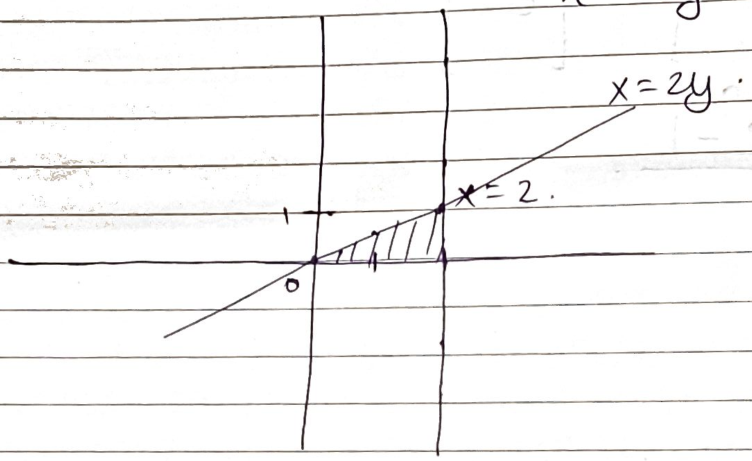
$$= \left[e^9\right] - \left[e^0\right]$$

$$= \underline{e^9 - 1}.$$

Q2. (A) $\displaystyle\int_0^1\int_{2y}^{2} f(x,y)\,dx\,dy.$

$x = 2.$

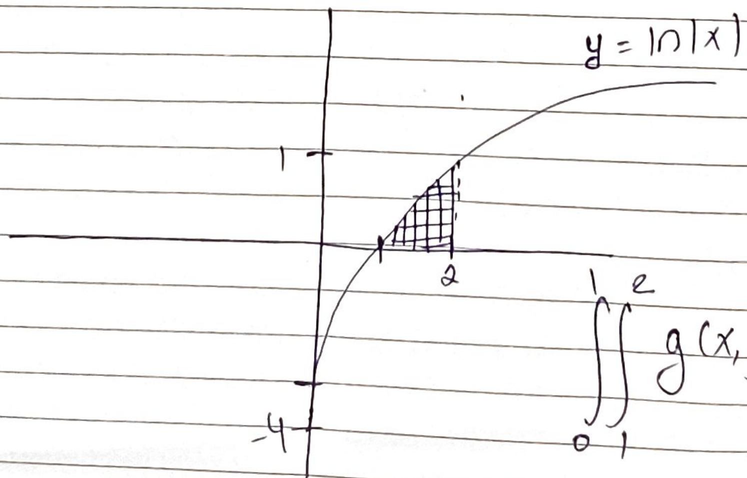
$x = 2y$ $\qquad y = \frac{x}{2}.$



reversing the order of integration.

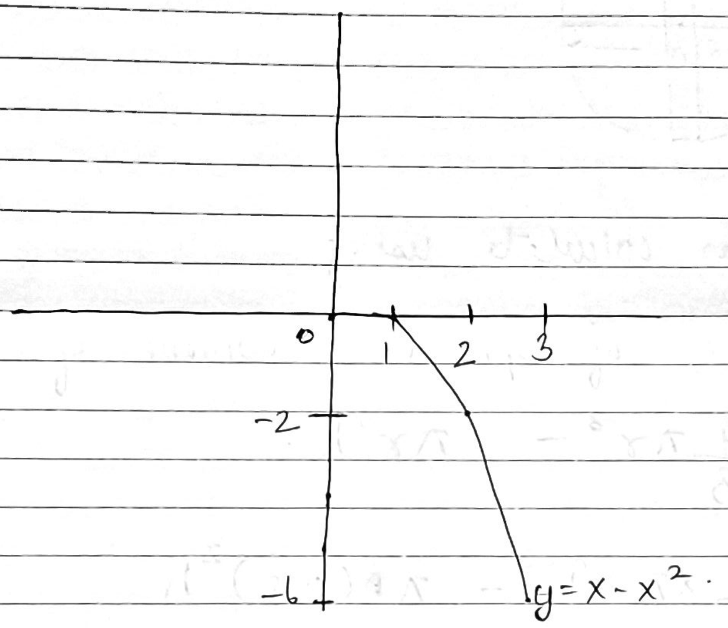
$$\int_0^2\int_0^1 f(x,y)\,dy\,dx.$$

(B) $\displaystyle\int_1^2\int_0^{\ln|x)} g(x,y)\,dy\,dx.$



$$\int_0^1\int_1^e g(x,y)\,dx\,dy.$$

Q3. $y = 0$ & $y = x - x^2$.

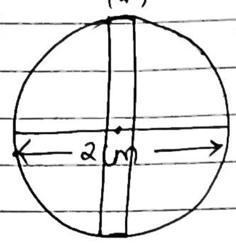


$$y = \frac{1}{A}\iint_R y \, dA \longrightarrow \text{average distance formula } \cancel{b/w}$$

$$y = \frac{1}{A}\iint_R \cancel{x - x^2}\, y \; dA$$

$$A = \iint 1 \cdot dA$$

Q5.



we can calculate using

Volume of sphere − Volume of cylinder.

$$\frac{4}{3}\pi r^3 - \pi r^2 h.$$

$$\frac{4}{3} \times \pi \times (1)^3 - \pi (0.5)^2 h.$$

$$\frac{4}{3} \times \pi - \frac{1}{4} \times \pi \times 2.$$

$$\frac{4}{3}\pi - \frac{\pi}{2}.$$

$$\frac{8\pi - 3\pi}{6} \Rightarrow \frac{5\pi}{6}.$$

$\frac{5\pi}{6}$ Sphere is remaining

Q4.

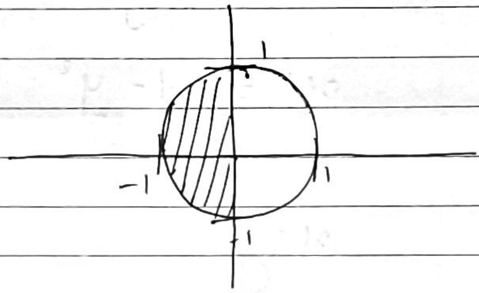
Q4. (A) $\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x \cdot dy\, dx$.

→ converting the eq^n in polar form.

$y = \sqrt{1-x^2}$.

$y^2 = 1 - x^2$.

$\underline{x^2 + y^2 = 1}$.



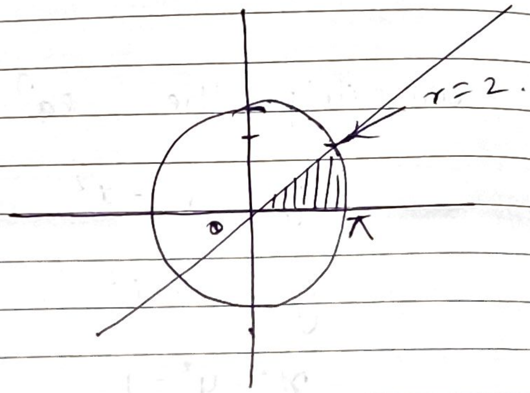
$\cancel{\int_{0}^{\pi}\int_{1}^{-1} x \, dy \, dx}$.

$\int_{0}^{\pi}\int_{-1}^{1} x \; \overset{r \cdot dr \cdot d\theta}{\cancel{dy \, dx}}$. $\cancel{=} \; \cancel{\int_{0}^{\pi} [xy]_{-1}^{1}}$

$\cancel{\int_{0}^{\pi} \frac{r^2}{2}}$.

(B) $\int_{0}^{\sqrt{2}} \int_{y}^{\sqrt{4-y^2}} xy \, dx \, dy$.

Q6.



$x = \sqrt{4-y^2}$

$x^2 = 4 - y^2$.

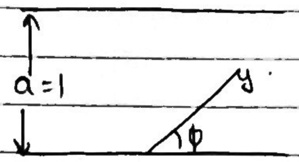
$x^2 + y^2 = 2^2$.

$x = y$

$\int_{0}^{\sqrt{2}} \int_{0}^{\pi} xy \cdot r \cdot dr \, d\theta$.

Stuck in converting xy to polar form.

Q6.



→ Buffon's needle problem

The probability that it will intersect the lines is
$p = 2\,\frac{1}{a\pi}$

where, a → distance between ∥ lines.

$$p = 2 \times \frac{1}{1 \times \pi}$$

$$= \underline{0.64}$$