MA540 Homework 2

Date 25/9/23 No. 11

Q36. Two cards are chosen at random from a deck of 52 cards. What is the probability that they
(a) are both aces.
(b) have same value.

→ (a) Lets find the total no of outcomes.

$$\binom{52}{2} = \frac{\cancel{52}^{26} \times 51 \times \cancel{50!}}{\cancel{50!} \times \cancel{2!}}$$

$$= 26 \times 51 \Rightarrow 1326.$$

Ace Ace

let the four suits be A, B, C, D respectively.

∴ we need to chose 2 Ace from 4.

$$\left(4_{C_2}\right) = \frac{\cancel{4}^{2} \times 3 \times \cancel{2!}}{\cancel{2!} \times \cancel{2!}} \Rightarrow 6$$

AB, BC, CD.
AC, BD
AD

$\underbrace{\qquad\qquad}_{\text{6 outcomes.}}$

$$\therefore \frac{4_{C_2}}{52_{C_2}} \Rightarrow \frac{6}{1326}$$

$$\boxed{P = \frac{1}{221}} \approx 0.0045$$

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(B) lets find total num of outcomes.

$$\binom{52}{C_2} = \frac{\overset{26}{\cancel{52}} \times 51 \times \cancel{50!}}{\cancel{50!} \times \cancel{2!}} = 1326 \text{ possible outcomes.}$$

we need to select 2 cards having same values.

consider Ace as as example.

num of ways to ~~select~~ chose 2 Aces from 52 cards are

$$\frac{4C_2}{52C_2} \Rightarrow \frac{6}{1326} \Rightarrow \frac{1}{221}.$$

we have 13 different cards in a suit.

$$\therefore \quad 13 \times \frac{1}{221}.$$

$$= \frac{13}{221} \Rightarrow \approx 5.88\%$$

$$= 0.058$$

Q41. If a dice is rolled 4 times, what is the probability that 6 comes atleast once?

→ To find 6 atleast once in 4 rolls we can ~~to~~ substract no six in 4 rolls by 1.

$p(\text{six} \leq 1) = 1 - p(\text{no six})$

- There are total 6 outcomes for one roll.
  ∴ there are 6^4 outcomes in 4 rolls.

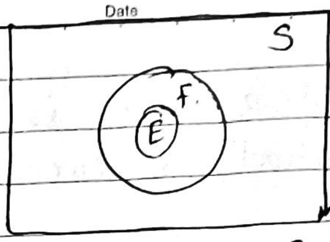
There are 5 ~~out~~ outcomes without six in one roll
∴ there are 5^4 outcomes without six in 4 rolls.

$$\therefore p(\text{six} \leq 1) = 1 - \frac{5^4}{6^4}$$

$$= 1 - \left(\frac{5}{6}\right)^4$$

- $p(\text{six} \leq 1) = \underline{0.523}$

Thereotical exercise.



Q2. If $E \subset F$, then $F^c \subset E^c$.

$E \subset F$ states that every element of E is in F

for all x, if $x \in E$, then $x \in F$

Assume y is the element of F^c. that means. y is not in F

$\therefore$ from $E \subset F$ & $y \notin F$ we can say that $y \notin E$ ($\because$ E is a subset of F).

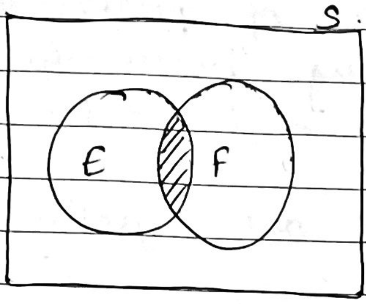
$\therefore$ $y \notin E = y \in E^c$

Hence, we can conclude that if y is in F^c, it is also in E^c.

Thus $F^c \subset E^c$.

Hence proved

Q13. Prove that $P(EF^c) = P(E) - P(EF)$



let E & F be two events.

first lets express EF as the intersection of E & F.

$EF = E \cap F$ & $P(Ef) = P(E \cap f)$ ↳ Property of probability

$\because P(Ef^c) = 1 - P(Ef)$

we can substitute $P(Ef)$ as $P(E \cap f)$

$P(Ef^c) = 1 - P(E \cap f)$

To find $P(E)$ we can do.

$P(E) = P(E \cup F) - P(F)$ as E can be expressed as union of EF & EF^c

$\therefore P(E) = P(EF) + P(Ef^c)$

Hence

$P(EF^c) = P(E) - P(Ef)$

Hence proved.

Date No.

Q20 Consider an experiment whose sample space consist of a countably infinite number of points. Show that not all points can be equally likely. Can all points have a positive probability of occuring.

→ To show that not all points are equally likely in infinite number of points.

Lets take an example of rolling dice.

In this case there are 6 outcomes with $1/6$ probability each. ie equal probability.
However not all points are equally likely when considering specific events.

Eg. probability of rolling an even number.

$P(E) = 1/2$

OR

$P(\geq 3) = 2/3$

In specific events rolling ~~as even~~ As shown above. have different possibilities compared to individual outcomes.

for given space, the sum of probability must be equal to one. Hence all the events cannot have sum of 1 in infinite point sample space.

If all points have positive probability of occuring is infinite sample space, the total probability would be infinity.

for eg. $S = \{1, 2, 3, 4, \ldots \infty\}$.

$p(1) + p(2) + p(3) + \ldots p(\infty) = 1$

according to probability axioms.

But since, all ~~nt~~ probability has to be +ve, even if we take smallest number, the final result would be ∞ (infinity).

$p(1) + p(2) + \ldots p(\infty) = \underline{\underline{\infty}}$. given that all probability are +ve which is not true.

Thus, all points cannot have positive probability