Pre-Class Assignment 3

New Attempt

Due Sep 12 by 5pm **Points** 5 **Submitting** a file upload **File Types** pdf

Along with the concept of a derivative, the concept of an integral is the other big idea in calculus. Whereas derivatives calculate instantaneous rates of change, integrals calculate the *accumulation* of change. Although the operations of differentiation and integration seem, at first glance, to do different things and solve different kinds of problems, they are intimately connected with one another. The aptly named Fundamental Theorem of Calculus provides the connection between them.

Reading. Review **Section 5.1** (https://openstax.org/books/calculus-volume-1/pages/5-1-approximating-areas), **Section 5.2** (https://openstax.org/books/calculus-volume-1/pages/5-2-the-definite-integral), and **Section 5.3** (https://openstax.org/books/calculus-volume-1/pages/5-3-the-fundamental-theorem-of-calculus) of the *Calculus, Volume I* OpenStax textbook. Then answer the following reading questions.

1. What is a definite integral, and how may it be defined as a limit of Riemann sums?

2. Choose at least three of the properties of definite integrals listed in Section 5.2 (see equations 5.9-5.14) and explain why they are true.

   Hint: Use the definition of a definite integral in terms of Riemann sums.

3. According to the Fundamental Theorem of Calculus (Part 1), $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$ for any continuous function f.
   A. Explain the different roles of the variables x and t in this formula.
   B. Explain what's going on with the lower bound of integration a. Can a be any number?

4. Discuss the Fundamental Theorem of Calculus (Part 2) in your own words. How does it unite the theories of differentiation and integration?

5. Go to the Module 1 Discussion Board and do at least one of the following:
   - Ask a question about the Fundamental Theorem of Calculus that you think is interesting, or that's got you confused.
   - Answer someone else's question. Partial answers and conjectures are acceptable!

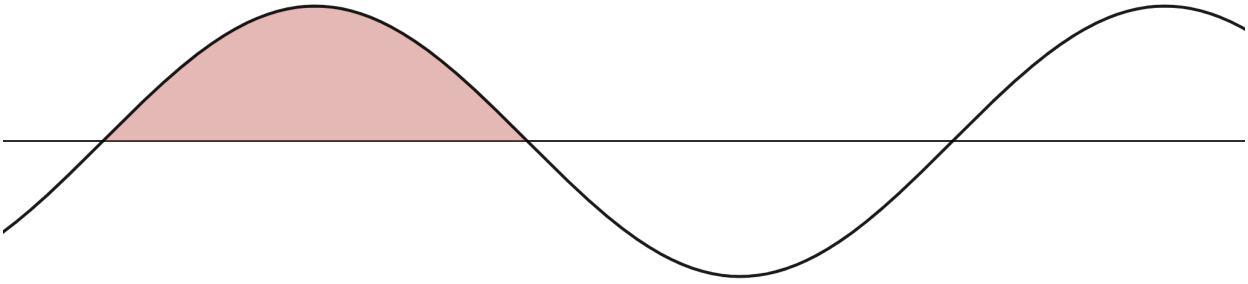
Exercises. Work on the exercises given below and submit your work along with the answers to the reading questions. If you're not sure how to solve an exercise, that's OK, but do not leave it blank.

Instead, indicate (a) What you tried, (b) Where you got stuck, and (c) At least one idea for how you might make progress.

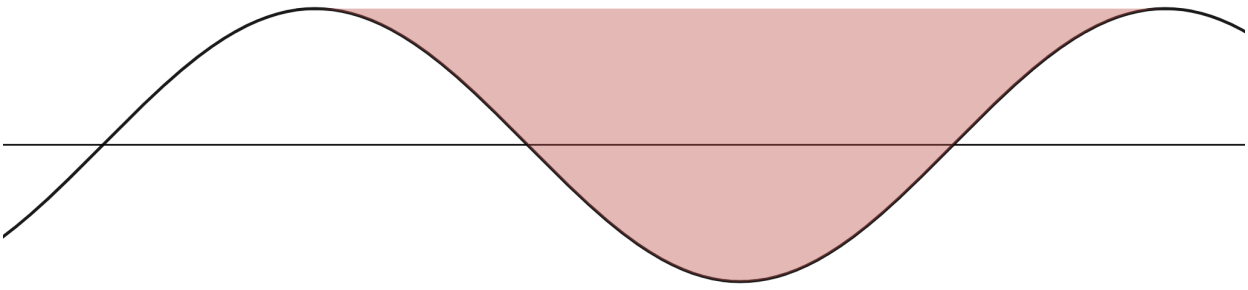
1. Suppose $f : [a, b] \to \mathbb{R}$ is increasing, and let $A = \int_a^b f(x)\,dx$.
   A. Will a left Riemann sum for $f(x)$ underestimate or overestimate A?
   B. How about a right Riemann sum?
   C. What about the *average* of the left and right Riemann sums? Explain.

2. Evaluate the following definite integrals *without* using the Fundamental Theorem of Calculus.
   A. $\int_3^{10} 2\,dx$
   B. $\int_0^5 (1 + 3x)\,dx$
   C. $\int_0^3 \sqrt{9 - x^2}\,dx$
   D. $\int_{-2}^2 (1 - |x|)\,dx$

3. How much area is underneath one "hump" of the sine function $f(t) = \sin t$?



How much area is contained in between two successive peaks of $f(t) = \sin t$?



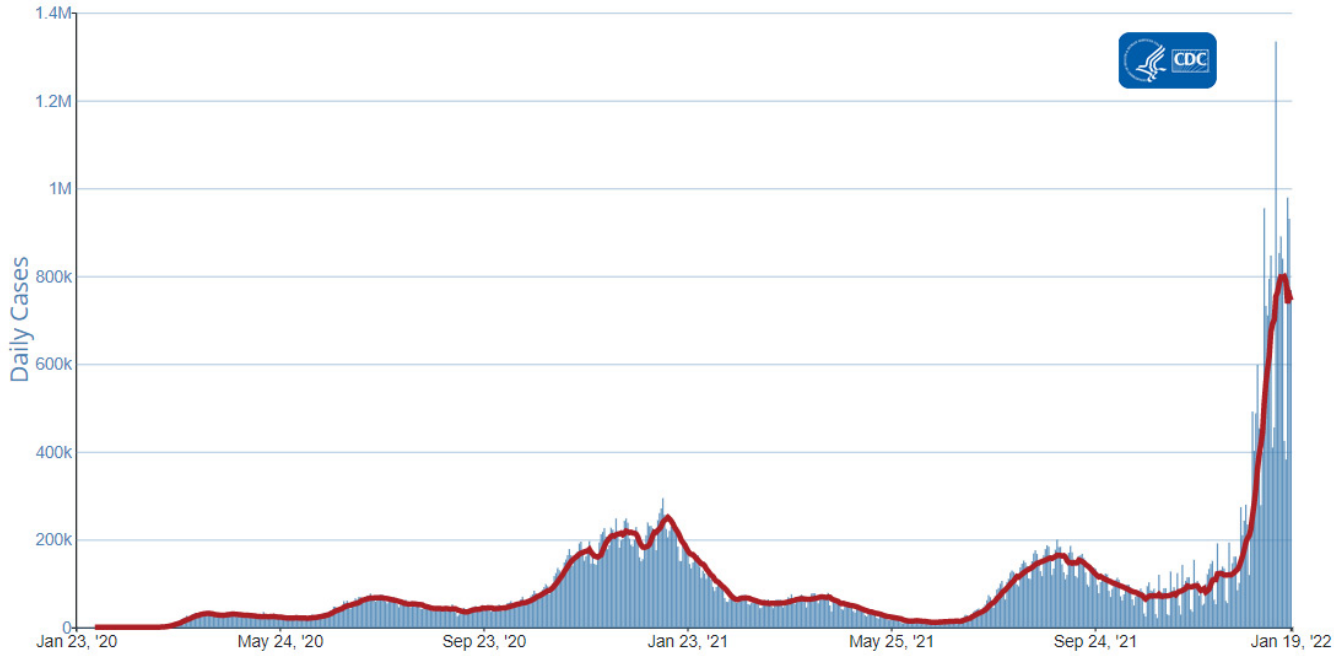
4. Compute the derivatives of the following functions. (By the way, each of these functions involves an important function used in statistics and data analysis. Can you identify them?)
   A. $F(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$

B. $G(x) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_{100}^{x} e^{-t^2}\, dt$

C. $H(x) = \displaystyle\int_{x}^{\pi} \frac{1}{1+e^{-t}}\, dt$

D. $I(x) = \displaystyle\int_{0.1}^{x^2} \ln\left(\frac{t}{1-t}\right) dt$

5. Consider the following image, obtained from the **CDC** (https://www.cdc.gov/coronavirus/2019-ncov/covid-data/covidview/index.html) , which illustrates daily COVID-19 cases in the United States from January 23, 2020 to January 19, 2022. How can you use this data to determine the total number of COVID-19 cases that were recorded during some period of time, and what's the connection with the Fundamental Theorem of Calculus?



6. **Working with Data.** Suppose you collect sample data from a function $\psi : [a, b] \to \mathbb{R}$, resulting in a large data set $S = \{(t_1, \psi_1), (t_2, \psi_2), \ldots, (t_N, \psi_N)\}$, where $a = t_1 < t_2 < \ldots < t_N = b$. Write down an expression that approximates $\displaystyle\int_a^b \psi(t)\, dt$.