Q. Prove $\sqrt[n]{\frac{1}{\prod_{i=1}^{N} P(w_i)}} = e^{\left(-\frac{1}{N}\sum_{i=1}^{N}\log(P(w_i))\right)}$

$\rightarrow$ Perplexity $= \sqrt[n]{\frac{1}{\prod_{i=1}^{N} P(w_i)}}$

Taking log on both the sides.

$$\log(\text{Perplexity}) = \log\left(\frac{1}{\prod_{i=1}^{N} P(w_i)}\right)^{\frac{1}{N}}$$

using log property : $\log a^b = b\log(a)$

$$\therefore \log(\text{Perplexity}) = \frac{1}{N}\log\left(\frac{1}{\prod_{i=1}^{N} P(w_i)}\right)$$

using log property : $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$

$$\therefore \log(\text{Perplexity}) = \frac{1}{N}\left[\log(1) - \log\left(\prod_{i=1}^{N} P(w_i)\right)\right]$$

$\because \log(1) = 0$ & $\because \log(a.b) = \log(a+b)$

$$\therefore \log(\text{Perplexity}) = -\frac{1}{N}\sum_{i=1}^{N}\log P(w_i)$$

apply exponential term on both sides.

$$\cancel{e}^{\log(\text{Perplexity})} = e^{-\frac{1}{N}\sum_{i=1}^{N}\log P(w_i)}$$

$$\text{Perplexity} = e^{-\frac{1}{N}\sum_{i=1}^{N}\log P(w_i)}$$

$$= \text{RHS}$$

Hence proved.