① Matrix $A = \begin{bmatrix} 1 & 3 & 0 & -2 & 7 & 3 \\ 3 & 9 & 1 & -7 & 23 & 8 \\ 1 & 3 & 1 & -3 & 9 & 2 \\ 1 & 3 & -1 & -1 & 5 & 4 \end{bmatrix}$

Ⓐ R.T.P :- $\|A\|$, $\|A\|_\infty$, $\|A\|$.

$\|A\|_F \to$ Frobenius nom is given by the

def :- $\sqrt{\Sigma\{i,j\}|a_{ij}|^2}$

$$= \sqrt{(1)^2 + (3)^2 + (1)^2 + (3)^2 + (1)^2 + (-2)^2 + (-7)^2 + (-3)^2 + (-1)^2 + 7^2 + (23)^2 + 9^2 + 5^2 + 3^2 + 8^2 + 2^2 + 4}$$

$= \sqrt{(1,162)}$

$= 34.072$ app

$\|A\|_\infty \to$ Infenity nom is given by the

def $= \max\{i\}\ \Sigma_j\ |a_{ij}|$

$= \max\{6, 15, 3, 13\}$

$= 15$

$\|A\|_1 \to$ 1. nom is given by the

$= \max_j \Sigma_i\ |a\{i,j\}|$

$= \max\{13, 40, 14, 12\}$

$= 40.$

(B) R. T. P :- Spectral norm $\|A\|_2$

$$A^TA = \begin{bmatrix} 1 & 3 & 0 & -2 & 7 & 3 \\ 3 & 9 & 1 & -7 & 23 & 8 \\ 1 & 3 & 1 & -3 & 9 & 2 \\ 1 & 3 & -1 & -1 & 5 & 4 \end{bmatrix}$$

By computing of A^TA

we get app 828.057, 36.343, 4.028 & 0.573.

$$\|A\|_2 = 828.057$$
$$\approx 28.707$$

(C) R. T. P : The cosine of the angle b/w the 1st and last row of vectors.

$$: \begin{bmatrix} 1 & 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 8 \\ 2 \\ 4 \end{bmatrix} = 29$$

The magnitude $= \| [1 \;\; 3 \;\; 1 \;\; 1] \|$

$$= \left[(1)^2 + (3)^2 + (1)^2 + (1)^2 \right]^{1/2}$$

$$= 3$$

$$\left\| \begin{bmatrix} 3 \\ 8 \\ 2 \\ 4 \end{bmatrix} \right\| = \left[3^2 + 8^2 + 2^2 + 4^2 \right] = 89$$

The magnitude $= 2\sqrt{3}$ and $\sqrt{89}$

$$\cos\theta = \frac{[1 \ 3 \ 1 \ 1] \begin{bmatrix} 3 \\ 8 \\ 2 \\ 4 \end{bmatrix}}{\left\| [1 \ 3 \ 1 \ 1] \right\| \left\| \begin{bmatrix} 3 \\ 8 \\ 2 \\ 4 \end{bmatrix} \right\|} = 0.523$$

$\therefore$ The $(\cos\theta)$ cosine of the angle b/w the 1st and last two row vectors of A.

② vector $x = (1, 2, 3, 4)^T$. the subspace $U \subset \mathbb{R}^4$ where, $U = \text{span}\{(1, -2, 2, 0)^T, (-1, 1, 1, -1)^T\}$

$$x = [1, 2, 3, 4]^T$$

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$U = \{(1, -2, 2, 0)^T, (-1, 1, 1, -1)^T\}$$

$$U = \left\{ \underbrace{\begin{bmatrix} 1 \\ -2 \\ 2 \\ 0 \end{bmatrix}}_{u_1}, \underbrace{\begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}}_{u_2} \right\}$$

$$v_1 = u_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \\ 0 \end{bmatrix}$$

$$v_2 = u_2 - \frac{v_1 \cdot u_2 . v_1}{v_1 \cdot v_1}$$

$$= \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} - \frac{[-1-2+2+0]}{[1+4+4+0]} \begin{bmatrix} 1 \\ -2 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -8/9 \\ 7/9 \\ 11/9 \\ -1 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} -8 \\ 7 \\ 11 \\ -9 \end{bmatrix}$$

$\therefore$ It is equivalent to $v_2 = \begin{bmatrix} 8 \\ -7 \\ -11 \\ 9 \end{bmatrix}$

Now, $U = \left\{ \begin{bmatrix} 1 \\ -2 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 8 \\ -7 \\ -11 \\ 9 \end{bmatrix} \right\} = \{(v_1, v_2)\}$

(A) R.T.P:-

orthogonal Projection of x onto U

$$\text{Proj}_U x = \frac{x \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{x \cdot v_2}{v_2 \cdot v_2} v_2$$

$$= \frac{[1-4+6+0]}{1+4+4+0} \begin{bmatrix} 1 \\ -2 \\ 2 \\ \end{bmatrix} + \frac{8-14-33+36}{64+49+121+81} \begin{bmatrix} 8 \\ -7 \\ -11 \end{bmatrix}$$

$$= \frac{3}{9}\begin{bmatrix} 1 \\ -2 \\ 2 \\ 0 \end{bmatrix} + \frac{-3}{315}\begin{bmatrix} 8 \\ -7 \\ -11 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3 \\ -2/3 \\ 12/3 \\ 0 \end{bmatrix} + \begin{bmatrix} -8/105 \\ 7/105 \\ 11/105 \\ -9/105 \end{bmatrix}$$

$$= \begin{bmatrix} 27/105 \\ -63/105 \\ 81/105 \\ -9/105 \end{bmatrix} \Rightarrow \begin{bmatrix} 9/35 \\ -21/35 \\ 27/35 \\ -3/35 \end{bmatrix}$$

(B) R.T.P :-

To find the Projection matrix :

$$P = A(A^T A)^{-1} A^T$$

$$A = [v_1 \cdot v_2] = \begin{bmatrix} 1 \\ -2 \\ 2 \\ 0 \end{bmatrix}\begin{bmatrix} 8 \\ -7 \\ -11 \\ 9 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -2 & 2 & 0 \\ 8 & -7 & -11 & 9 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & -2 & 2 & 0 \\ 8 & -7 & -11 & 9 \end{bmatrix}\begin{bmatrix} 1 & 8 \\ -2 & -7 \end{bmatrix}$$

$$\begin{pmatrix} \cdots & 2 & -11 \\ & 0 & 9 \end{pmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 0 & 315 \end{bmatrix}$$

$$(A^T \cdot A)^{-1} = \begin{bmatrix} 9 & 0 \\ 0 & 315 \end{bmatrix}^{-1} = \frac{1}{2835} \begin{bmatrix} 315 & 0 \\ 0 & 9 \end{bmatrix}$$

$$(A^T \cdot A^{-1})^{-1} = \begin{bmatrix} \frac{1}{9} & 0 \\ 0 & 1/315 \end{bmatrix}$$

Now $P = A(A^T A)^{-1} A^T$

$$= \begin{bmatrix} 1 & 8 \\ -2 & -7 \\ 2 & -11 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{315} \end{bmatrix} \begin{bmatrix} 1 & -2 & 2 & 0 \\ 8 & -7 & -11 & 9 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 8 \\ -2 & -7 \\ 2 & -11 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} \frac{1}{9} & \frac{-2}{9} & \frac{2}{9} & 0 \\ \frac{8}{315} & \frac{-7}{315} & \frac{-11}{315} & \frac{9}{315} \end{bmatrix}$$

$$P = \begin{bmatrix} 11/35 & -2/5 & -2/35 & 8/35 \\ -2/5 & 3/5 & -1/5 & -1/5 \\ -2/35 & -1/5 & 29/35 & -11/35 \\ -8/35 & -1/5 & 11/35 & 9/35 \end{bmatrix}$$

(c) R.T.P:-

$d \, \| x - Px \|$

$$Px = P \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 9/35 \\ -3/5 \\ 27/35 \\ -3/35 \end{bmatrix}$$

Then, $x - Px = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 9/35 \\ -3/5 \\ 27/35 \\ -3/35 \end{bmatrix}$

$$\text{distance } (d) = \begin{bmatrix} 26/35 \\ 13/5 \\ 78/35 \\ 137/35 \end{bmatrix}$$

$$d = \frac{1}{35} \left\| \begin{bmatrix} 26 \\ 91 \\ 78 \\ 137 \end{bmatrix} \right\| \Rightarrow \frac{1}{35} \times \sqrt{(26)^2 + (91)^2 + (78)^2 + (137)^2}$$

$$d = \frac{\sqrt{33810}}{35} = \frac{183.87}{35}$$

$$\boxed{d = 5.25} \, //$$

③ Given,
vectors $n = (1, 2, 3)$, $b = (5, 7, 4)$ R^3.

$L \subset R^3$

Ⓐ $\vec{n}^T \vec{n} = [1\ 2\ 3] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 14$

$\cdot (\vec{n}^T \cdot \vec{n})^{-1} = \frac{1}{14}$

Projection matrix $= P = \vec{n} (\vec{n}^T \vec{n})^{-1} \vec{n}^T$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \frac{1}{14} [1, 2, 3]$$

$$P = \frac{1}{14} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

Ⓑ Rank of P is given as

$$P = \frac{1}{14} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

by using the row transformation

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$

$$= \frac{1}{14} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank $= 1$ {Non zero rows}

Ⓒ To find the Projection of the vector b onto L

$$\vec{Pb} = \frac{7}{14}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix}$$

$$\vec{Pb} = \begin{bmatrix} \frac{31}{14} \\ \frac{62}{14} \\ \frac{93}{14} \end{bmatrix}$$

(D) Equation passing through the origin and orthogonal to a vector $\langle a, b, c \rangle$ is given as

$$ax + by + cz = 0$$

so, $x + 2y + 3z = 0$

(E) To find the projection matrix Q that projects elements of R^3 onto V.

$$x + 2y + 3z$$

$$\Rightarrow x = -2y - 3z$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2y - 3z \\ y \\ z \end{bmatrix}$$

$$= y\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ The plane P is spanned by the vectors

$v_1 = (-2, 1, 0)$, $v_2 = (-3, 0, 1)$

$$A = [\vec{v}_1\ \vec{v}_2] = \begin{bmatrix} -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 6 & 10 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{(5)(10)-(6)(6)} \begin{bmatrix} 10 & -6 \\ -6 & 5 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} 10 & -6 \\ -6 & 5 \end{bmatrix}$$

$$Q = A(A^T A)^{-1} A^T$$

$$= \begin{bmatrix} -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{14} \begin{bmatrix} 10 & -6 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} 13 & -2 & -3 \\ -2 & 10 & -6 \\ -3 & -6 & 5 \end{bmatrix}$$

(F) Rank of Q

$$Q = \frac{1}{14} \begin{bmatrix} 13 & -2 & -3 \\ -2 & 10 & -6 \\ -3 & -6 & 5 \end{bmatrix}$$

Rank of Q = dimensions of the Plane V.

= 2

G