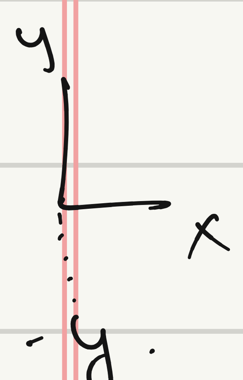
MA574 Pre Class Assignment 13. 19/10/23

Q1. A) The basis vectors are $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



for reflection across x axis.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

$$\therefore \quad 1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$0\begin{pmatrix} 1 \\ 0 \end{pmatrix} + (-1)\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$

$$\text{matrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

B) $$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ y \end{pmatrix}$$

$$= 2\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$= 0\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

$$\text{matrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

C) $$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$ collapsed to single point.

$$\therefore \text{matrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

d) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}(x-y) \\ \frac{1}{\sqrt{2}}(x+y) \end{bmatrix}$

matrix = $\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

e) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ y \end{pmatrix}$

matrix = $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

f) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x+y \end{pmatrix}$

matrix = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

Q2. $A = \begin{pmatrix} 1 & 3 & 5 & 0 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{3 \times 5}$

$m \times n$.

Rank = 2

$\because$ num of independent rows are 2

dimension of column space.

$\dim C(A) = r$.

$\therefore \dim C(A) = 2$.

dimension of row space

$\dim C(A^T) = r$.

$\therefore \dim C(A^T) = 2$.

dimension of null space.

$$\begin{aligned} \dim N(A) &= n - r. \\ &= 5 - 2 \\ &= 3. \end{aligned}$$

dimension of null space^T.

$$\begin{aligned} \dim N(A^T) &= m - r \\ &= 3 - 2 \\ &= 1 \end{aligned}$$

Q3 a) consider a vector (u, v, w, x) in R^4
given $u = v = w = x$
this vector can be written as $x(1,1,1,1)$
Hence $(1,1,1,1)$ is the basis

b) consider a vector (u, v, w, x) in R^4
given $u + v + w + x = 0.$
$u = -v - w - x.$
this vector can be written as
$v(-1,1,0,0) + w(-1,0,1,0) + x(-1,0,0,1)$
Hence $(-1,1,0,0)$, $(-1,0,1,0)$, $(-1,0,0,1)$

c) consider a vector (u, v, w, x) in R^4
given $u + v = 0$
$u + w + x = 0.$
$v = -u$
$x = -u - w.$
this vector can be written as
$u(1,-1,0,-1) + w(0,0,1,-1)$
Hence basis is $(1,-1,0,-1)$ $(0,0,1,-1)$

d) doubt.