

FE535WS: Introduction to Financial Risk Management

Session 1

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Agenda

- Overview

- ▶ The Instructor
- ▶ The Class (syllabus)
- ▶ GARP and FRM

- Introductory Class

- ▶ Building blocks of risk management
- ▶ Modern Portfolio Theory (MPT)
- ▶ Capital Asset Pricing Model

This Class...

- The class materials are available online at <https://sit.instructure.com/courses/72559>
- I will keep the website updated overtime
 - ▶ Syllabus
 - ▶ Slides
 - ▶ Handouts
 - ▶ Projects
 - ▶ Others

Introduction to Financial Risk Management

The Building Blocks of Risk Management

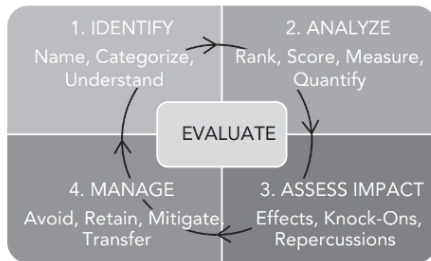
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The Building Blocks of Risk Management

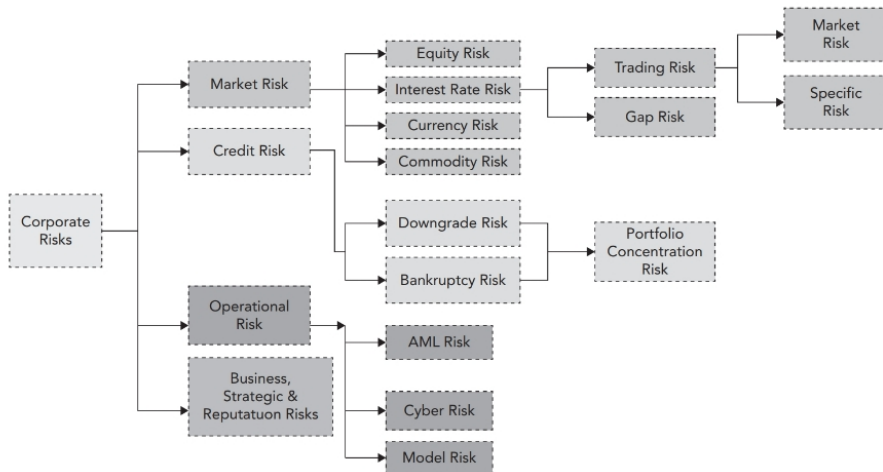
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 - ▶ identify risks
 - ▶ measure them properly
 - ▶ assess their impact - sensitivity
 - ▶ and succeed in managing them

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A Typology of Risks for The Banking Industry



● Source - Chapter 1 from FRM Part 1: Foundations of Risk Management

Market Risk

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- These movements create the potential for loss, as price volatility is the engine of market risk

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 - **Specific market risk** is the risk that an individual asset will fall in value more than the general asset class (idiosyncratic)
 - For risk managers, mismatching between price movements creates what is known **basis risk**
 - ▶ A position intended to hedge market price might do so imperfectly
 - ▶ *covered in advanced linear risk hedging*

Credit Risk

- Credit risk arises from the failure of one party to fulfill its financial obligations to another party
- Examples include
 - ① Failure to pay interest/principle on a loan
 - ② Downgrade risk, which leads to loss in value
- Risk managers use sophisticated credit portfolio models to uncover potential risk factors:
 - ▶ key financial ratios, industry sectors, etc
 - ▶ concentration versus diversification

Liquidity Risk

- Liquidity risk is used to describe two quite separate kinds of risk:
 - ① **Funding Liquidity Risk** limited access to cash to meet obligations or investment
 - ② **Market Liquidity Risk** (Trading Liquidity Risk) is the risk of a loss in asset value when markets temporarily seize up

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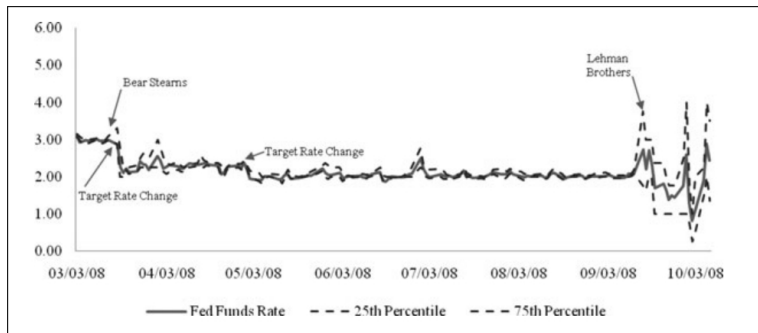


Figure: Source from Afonso et al., 2011

Operational Risk

- Operational risk can be defined as the “risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events.”
 - ▶ It includes legal risk, but excludes business, strategic, and reputational risk
- The definition of operational risk broad
 - ▶ from anti-money laundering risk and cyber risk to risks of terrorist attacks and rogue trading
 - ▶ corporate governance scandals, e.g. Enron 2001
 - ▶ **Model Risk**: the LTCM debacle in 1998
- The outbreaks of rogue trading in the 1990s helped persuade regulators to include operational risk in bank capital calculations

Other Risks

Business and Strategic Risk

- Business risk includes the usual business concerns, such as
 - ▶ consumer demand
 - ▶ pricing decisions
 - ▶ managing product innovation
- Strategic risk involves making large, long-term decisions about the firm's direction
 - ▶ often accompanied by major investments of capital, human resources, and management reputation

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Reputation Risk

- Reputation risk is the danger that a firm will suffer a sudden fall in its market standing/brand with economic consequences
 - ▶ losing customers or counterparties
- A large failure in credit risk management can lead to rumors about a bank's financial soundness

The Risk Management Process

- We take risks in pursuit of reward, whether that reward is food, shelter, or bitcoin
- The key questions are twofold:
 - ① is the risk commensurate with the reward?
 - ② could we lower the risk and still get the reward?

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- The key questions are twofold:
 - ① is the risk commensurate with the reward?
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- Our attempt to address these questions gives rise to our first building block: the classic risk management process

Risk Management Process

- It is the process in which financial risks are
 - ① identified
 - ② measured (quantify?)
 - ③ assessed (sensitivity?)
 - ④ managed (hedging?)
- For the purpose of creating/enhancing economic value

Modern Portfolio Theory

Risk-Reward Trade-off

- In the early half of the 20th century, investors were mainly concerned with return (reward)
- It was later in the 1950s that investors start thinking about reward in terms of risk-taking
 - ▶ Mean-Variance Model (MV) by Markowitz, 1952
- Risks that can be measured are easier to manage
- Investors bear risk to earn reward
- The balance between risk and reward requires risk management

Example 1

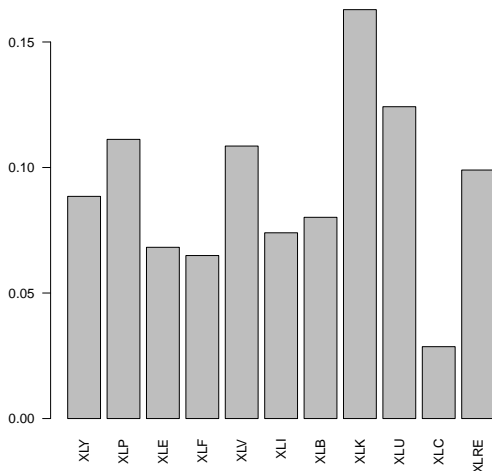
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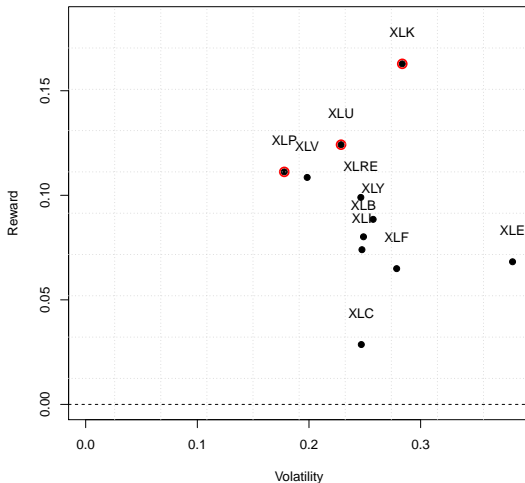
- 1 evaluate the assets
 - ▶ potential gain (reward)
 - ▶ potential loss (risk)
- 2 quantify the above
- 3 finally, construct an optimal strategy

- Consider the annual mean return of each 11 sector ETFs between June 2018 and Aug 2022
 - ▶ For further info about these assets, see this [link](#)



- Which one would you choose?

- Let risk be measured as the annual standard deviation



- Red dots denote the three assets with highest mean return

Risk-adjusted Performance

- It is common to think about reward (return) with in terms of risk-adjusted terms
- A common measure is the Sharpe-ratio (SR)

$$SR_i = \frac{\mu_i - R_F}{\sigma_i} \quad (1)$$

where μ_i and σ_i denote asset's i **daily** mean return and volatility, respectively, while R_F is the risk-free rate

Annualized SR

It is common to report the SR in annualized terms, such that the SR from (1) is scaled by $\sqrt{252}$, which denotes the average number of trading days in a year.

- **Why?**

How Bad Things Can Get?

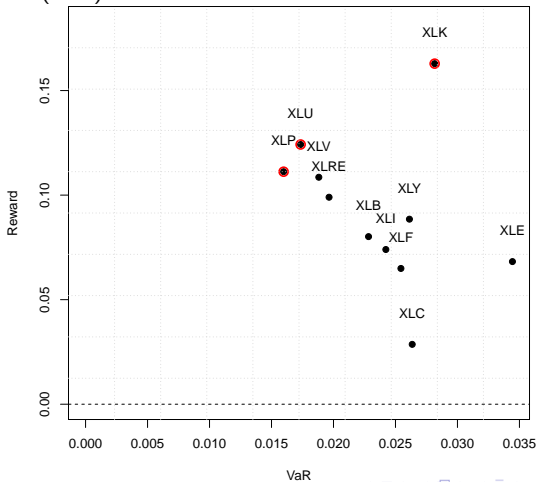
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- For instance, with 95% confidence, what is the expected daily loss on each asset?
- Consider the 5% quantile of the distribution of the daily asset return known as **Value-at-Risk (VaR)**



Portfolio Construction

- The previous illustrations demonstrate how to evaluate potential gains and losses of different assets
- As a risk manager, you face the task of constructing a portfolio by taking into account both risk and reward
 - ▶ How would you do so?

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- The previous illustrations demonstrate how to evaluate potential gains and losses of different assets
- As a risk manager, you face the task of constructing a portfolio by taking into account both risk and reward
 - ▶ How would you do so?
- By setting preference/objective, one can design by construction an optimal portfolio
- Optimal implies taking the best outcome given certain constraints

Optimal Portfolios

- For an optimal portfolio, you would like to either
 - ① maximize your return for a given level of risk
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 - or
 - ② minimize your risk for a given return
- Theoretically, the optimal portfolio weights are given by

$$\min_{\mathbf{w}} \sigma_p^2 = \mathbf{w}' \Sigma \mathbf{w} \quad (2)$$

subject to

$$\mathbf{w}' \mathbf{1} = 1 \quad (3)$$

$$\mathbf{w}' \boldsymbol{\mu} = m \quad (4)$$

where

- ▶ \mathbf{w} denotes a $d \times 1$ vector of portfolio weights
- ▶ Σ is the covariance matrix of the asset returns
- ▶ $\boldsymbol{\mu}$ is the vector of mean returns, and m denotes the mean target

MV Optimal Portfolio

The solution to the optimal portfolio from (2) is given by

$$\mathbf{w} = \mathbf{w}_0 + \frac{1}{A} \mathbf{B} \mu \quad (5)$$

where

$$\mathbf{B} = \Sigma^{-1} [\mathbf{I} - \mathbf{1} \mathbf{w}_0'] \quad (6)$$

and

$$\mathbf{w}_0 = \frac{\Sigma^{-1} \mathbf{1}}{\mathbf{1}' \Sigma^{-1} \mathbf{1}} \quad (7)$$

with

- 1 \mathbf{w}_0 is the global minimum variance (GMV) portfolio, $\mathbf{w}_0' \mathbf{1} = 1$, where $\mathbf{1}$ is a $d \times 1$ column vector and \mathbf{I} is an identity matrix
- 2 A is the risk aversion level of the investor, which can be written as a function of the target mean m , i.e. $A = f(m, \mu, \Sigma)$

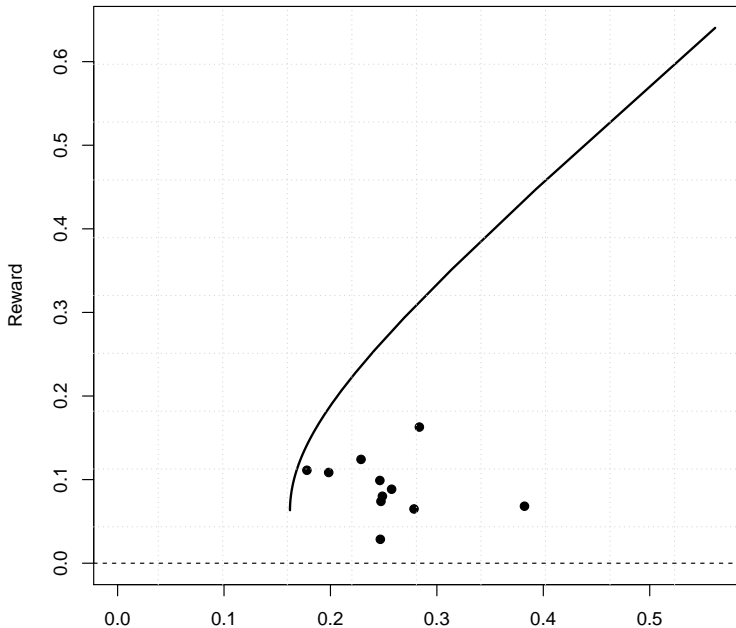
Mean-Variance Efficient Frontier

- If the investor knows μ and Σ , then for each m , there is a unique portfolio \mathbf{w} that returns the minimum variance for the mean target, m .
- The set of all optimal portfolios is known as the Mean-Variance Efficient Frontier (MVEF)
- To construct the MVEF, one can do the following steps
 - ▶ estimate μ and Σ using historical returns
 - ▶ for a given $A = 1, \dots, 100$, compute $\mathbf{w}(A)$
 - ▶ for each $\mathbf{w}(A)$, find the portfolio mean and variance, i.e.

$$\mu_p(A) = \mathbf{w}(A)' \mu \quad (8)$$

$$\sigma_p^2(A) = \mathbf{w}(A)' \Sigma \mathbf{w}(A) \quad (9)$$

- ▶ finally, plot $\mu_p(A)$ against $\sigma_p(A)$



Volatility

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Special Portfolios

Under certain conditions, the solution to (2) can be approximated as

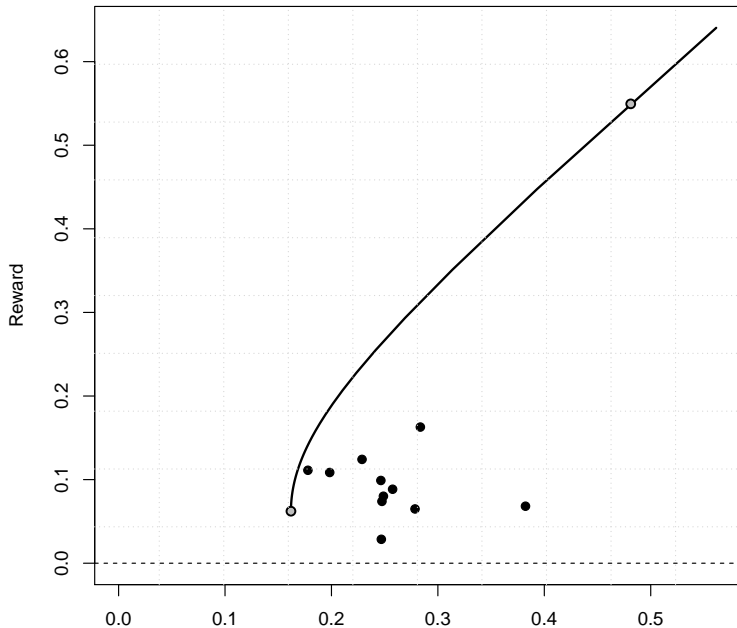
- Global Minimum Variance (GMV) Portfolio: weight is reciprocal to risk

$$w_i^{\sigma} = \frac{\frac{1}{\sigma_i^2}}{\sum_{j=1}^d \frac{1}{\sigma_j^2}} \quad (10)$$

- or the Sharpe-ratio Portfolio, where weight is proportional to SR

$$w_i^{SR} = \frac{SR_i}{\sum_{j=1}^d SR_j} \quad (11)$$

where d is the total assets in the portfolio and $\sum_{i=1}^d w_i = 1$



Volatility

Portfolio Performance

- Let's consider the performance of each portfolio over the sample period
- To construct either portfolio from (10) or (11), we need to estimate μ_i and σ_i

- Black line refers to Portfolio 1, i.e., the minimum variance portfolio from (10)
- Red line refers to Portfolio 2, i.e., the Sharpe-ratio portfolio from (11)



Absolute Performance

- In annualized terms, the absolute performance can be summarized

	Minimum Variance Portfolio	SR Portfolio
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Relative Performance

- How is the performance of Portfolio 2?
- To answer this, one needs to take into consideration two things:
 - ① **Benchmarking**: one may compare the above with a market index or a passive index fund, e.g., SPY
 - ② **Model Validation**: we need to **rethink** about the **assumptions** of the model

Capital Asset Pricing Model (CAPM)

CAPM

- Consider the case in which we have a risk-free asset with return R_F
- If we reformulate the MV optimization problem in the presence of risk-free asset, the vector of weights allocated to the risky assets becomes

$$\mathbf{w} = \frac{1}{A} \Sigma^{-1} [\mu - R_F \mathbf{1}] \quad (12)$$

with

$$w_F = 1 - \mathbf{1}^\top \mathbf{w} \quad (13)$$

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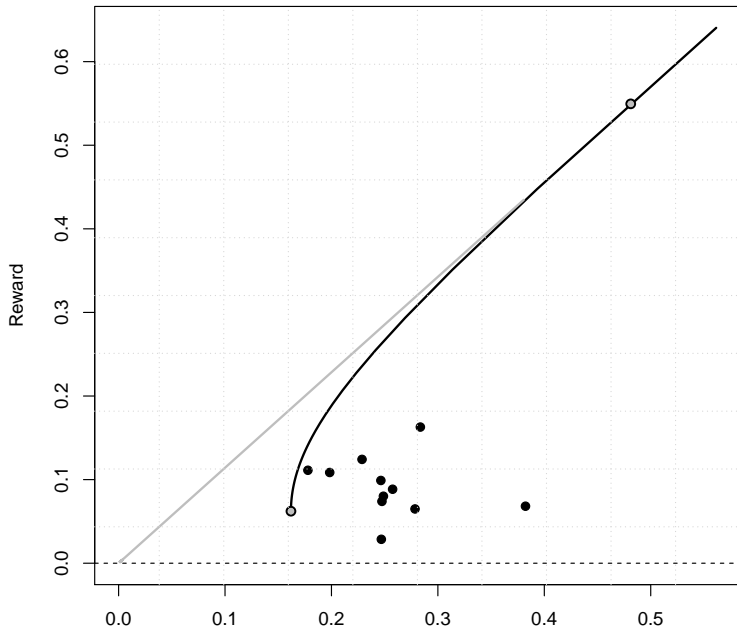
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- Similar to the previous case, the MV efficient frontier is attained by computing the mean return and variance of the portfolio for different values of A
- Using a closed-form solution, the MV efficient frontier can be represented as

$$\mu_p = R_F + \sigma_p \times \theta \quad (14)$$

for some constant θ that depends on μ , Σ , and R_F

- Equation (14) which states for any level of risk, σ_p , there is a linear relationship between mean and return



Volatility

- Since the Sharpe portfolio yields the highest SR, investors wind up holding it in equilibrium, which denotes the market portfolio
- With access to the risk-free asset, the CAPM states that in equilibrium, investors allocate their wealth between the risk-free asset and the market portfolio
- The weight allocated to the risk-free asset versus the market portfolio depends on the investor's risk tolerance
 - ▶ More risk-averse (tolerant), invest more (less) the riskless asset

- The end result of the CAPM states that the mean return on each risky asset i , μ_i , satisfies the following

$$\mu_i - R_F = \beta_i(\mu_p - R_F) \quad (15)$$

with

$$\beta_i = \frac{\sigma_{ip}}{\sigma_p^2} \quad (16)$$

σ_{ip} is the covariance between asset i and the market portfolio

CAPM and Relative Performance I

- The main takeaway of the CAPM is that the risk of individual asset can be decomposed into
 - ▶ specific risk - can be diversified away
 - ▶ systematic risk - cannot be eliminated
- The model allows us to “price” the return on asset/portfolio i

$$R_{i,t} - R_F = \alpha_i + \beta_i(R_{M,t} - R_F) + \epsilon_{i,t} \quad (17)$$

- Clearly, by taking the expectation on both sides of (17), we end up with the same result as (15)

CAPM and Relative Performance II

Relative Performance Measures

- 1 **Jensen's α** estimated from (17)
- 2 **Treynor ratio (TR)**, computed as

$$TR_i = \frac{\mu_i - R_F}{\beta_i} \quad (18)$$

- 3 **Information Ratio (IR)**

$$IR_i = \frac{\mu_i - \mu_M}{\omega} \quad (19)$$

where μ_M is the market expected return and ω is the tracking error measured as

$$\omega = \sqrt{\text{Var}(R_{i,t} - R_{M,t})} \quad (20)$$

with $\text{Var}(X)$ denotes the variance of a random variable X

- Going back to our two portfolio strategies, let's take a look at the relative performance of each with respect to SPY

	Portfolio 1 to SPY	Portfolio 2 to SPY
Beta	0.54	0.97
Alpha	0.00	0.00
Treynor Ratio	0.10	0.64
Tracking Error	0.15	0.52
Information Ratio	-0.20	1.04

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Question

Would you invest in Portfolio 2 versus SPY or Portfolio 1?
Why?

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 - ▶ For example, the price of gold was fixed to \$35 per ounce from 1934-1967
- How do we evaluate the quality of a risk measurement process?
- To help answer this question, it is useful to classify risks into various categories:
 - 1 known knowns
 - 2 known unknowns
 - 3 unknown unknowns

Evaluation of the Risk Measurement Process Cont.

Known Knowns

- Risks that are properly identified and measured
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Known Unknowns

- Includes model weaknesses that are known to exist but are not properly measured by risk managers.
- For example, the risk manager could have ignored important known risk factors.
- The distribution of risk factors, including volatilities and correlations, could be measured inaccurately.
- The mapping process, which consists of replacing positions with exposures on the risk factors, could be incorrect.^a

^aRead more on the mapping process [here](#)

Evaluation of the Risk Measurement Process Cont.

Unknown Unknowns

- These risks tend to be the most difficult and represent events totally outside the scope of most scenarios.
 - ▶ regulatory risks, e.g. restrictions on shore-sales
- Similarly, it is difficult to account fully for counterparty risk
 - ▶ It is not enough to know your counterparty; you need to know your counterparty's counterparties, too.
 - ▶ There are network externalities, e.g. understanding the full consequences of Lehman's failure

Challenges in the Previous Example

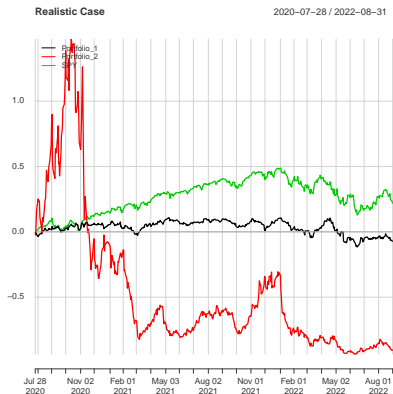
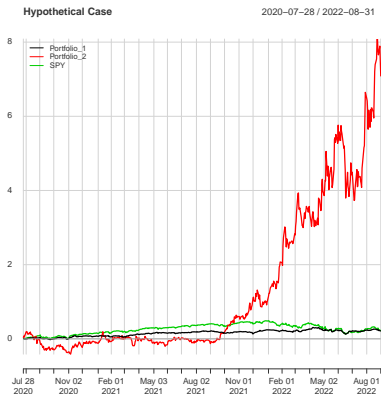
- The construction of Portfolio 1 and 2 depends on μ_i and σ_i
- In practice, portfolio decision are made ex-ante (before event)
- To construct an optimal portfolio, you need to evaluate risks and rewards beforehand
- This is typically called **Model Risk**
 - ▶ known unknown
- Such risk can be evaluated using **back-testing** and **stress testing**

Back-Testing

- Let's split the sample into two: in-sample (IN) and out-of-sample (OUT)
- Given IN, we estimate both μ_i and σ_i for each i , and find the weights of Portfolio 1 and 2 using (10) and (11)

Back-Testing

- Let's split the sample into two: in-sample (IN) and out-of-sample (OUT)
- Given IN, we estimate both μ_i and σ_i for each i , and find the weights of Portfolio 1 and 2 using (10) and (11)
- Finally, we test each portfolio performance OUT



- Hypothetically, if we knew the weights OUT, we would have

	Portfolio 1	Portfolio 2	SPY
Mean	0.10	1.30	0.11
Std	0.13	0.79	0.18
SR	0.77	1.65	0.61
Beta	0.50	0.75	1.00
Annualized Alpha	0.04	2.37	0.00
Treynor Ratio	0.19	2.27	0.10
Tracking Error	0.13	0.78	0.00
Information Ratio	-0.03	2.07	

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- However, what we get in reality is the following

	Portfolio 1	Portfolio 2	SPY
Mean	-0.02	-0.32	0.11
Std	0.16	1.25	0.18
SR	-0.15	-0.26	0.61
Beta	0.50	2.59	1.00
Annualized Alpha	-0.08	-0.46	0.00
Treynor Ratio	-0.07	-0.26	0.10
Tracking Error	0.16	1.20	0.00
Information Ratio	-0.83	-0.65	

Stress Testing Example

- In fact, Portfolio 2 has a big short position in the energy sector
- Suppose that energy companies recover in the OUT period
 - ▶ this recovery does not spillover to other assets
- In that case, the “stressed” asset return in the OUT is given by

$$\tilde{R}_{XLE,t} = R_{XLE,t} + U_t \times \sigma_{XLE}$$

with U_t is an iid uniform random variable between 0 and 1



Risk Management Failures

- Generally, the role of risk management involves several tasks:
 - ① Identifying all risks faced by the firm
 - ② Assessing and monitoring those risks
 - ③ Managing those risks if given the authority to do so
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- In the end, the objective of risk management is not to prevent losses
 - ▶ take a smart risk
- Risk management can fail if
 - ▶ risks could go unrecognized
 - ▶ mismeasurement of risk can occur due to model risk, liquidity risk, or distributions are not adequately measured.
 - ▶ ineffective communication of risks

Summary

We talked about

- Risk-reward trade-off
- Optimal decision making
- Absolute/relative risk-reward performance
- Failure of risk models
 - ▶ Model Risk (known unknowns)
 - ▶ Maximizing return or error? (Michaud, 1989)
- Example of stress testing

In the next class, we will

- Focus on scenario evaluation and analysis
- Talk about Monte Carlo methods in risk management

Appendix

CAPM Multivariate

- Let R_t denote the return on the d risky assets at time t
- According to the CAPM, it can be represented as

$$R_t - \mathbf{1} \times R_F = \alpha + \beta \times (R_{M,t} - R_F) + U_t \quad (21)$$

where

- 1 U_t is the vector of the residuals, $U_t \sim D(\mathbf{0}, \Sigma_\epsilon)$
 - 2 Σ_ϵ is a diagonal covariance matrix, i.e. $\text{Cov}(\epsilon_i, \epsilon_j) = 0$, for all $i \neq j$
- It follows that

$$\text{Var}(R_t) = \beta \beta' \sigma_M^2 + \Sigma_\epsilon \quad (22)$$

- Note that in practice it is very hard of impossible to estimate the idiosyncratic volatility of each asset.
- Hence, to estimate the covariance matrix of the returns, one needs to estimate d betas and the market volatility (total of $d + 1$ parameters)
- Without an asset pricing model, one needs to estimate $d \times (d - 1)/2$ parameters

References I

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