1. A $\frac{d}{dx}\int_{\cos x}^{\sin x} \sqrt{1-t^2}\, dt$

Let Say $F(x) = \int_{\cos x}^{\sin x} \sqrt{1-t^2}\, dx$

using Leibniz rule,

If $F(x) = \int_{g(x)}^{h(x)} f(x,t)\, dt$

$$f'(x) = \int_{g(x)}^{h(x)} \frac{\partial}{\partial x} f(x,t)\, dt + f(x, h(x)) \cdot \frac{d}{dx} h(x)$$

$$- f(x, g(x)) \cdot \frac{d}{dx} g(x)$$

Here, $g(x) = \cos x$ $\quad f(x,t) = \sqrt{1-t^2}$

$h(x) = \sin x$

$\therefore \quad g'(x) = -\sin x$

$h'(x) = \cos x$

$\frac{\partial}{\partial x} f(x,t) = \frac{\partial}{\partial x} \sqrt{1-t^2}$

$\frac{\partial}{\partial x}$ $\frac{d}{dx}$

Now, Substituting these inputs,

we get

$$F'(x) = \int_{\cos x}^{\sin x} -t \frac{\partial t}{\partial x} \sqrt{1-t^2} \cdot \partial t +$$

$$\sqrt{1-\sin^2 x}\,\cos x - \sqrt{1-\cos^2 x}\,(-\sin x)$$

$$= F'(x) = \int_{\cos x}^{\sin x} -t \frac{dt}{dx} \sqrt{1-t^2}\, \partial t + |\cos x| \cos x$$

$$+ |\sin x| \sin x$$

$$= |\cos x| \cos x + |\sin x| \sin x$$

B

$$\int \frac{x^3}{\sqrt{4-x^2}}\, dx$$

Let $x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$$= \int \frac{8\sin^2\theta}{2\cos\theta} \cdot 2\cos\theta \cdot d\theta$$

$$= 8\int (1-\cos^2\theta)\sin\theta \cdot d\theta$$

$$= \cos\theta = t$$

$$= -\sin\theta\, d\theta = dt$$

$$= 8\int (1-t^2)(-dt)$$

$$= -8\int 1-t^2\, dt$$

$$= -8t + 8\frac{t^3}{3}$$

$$= -8\cos\theta + \frac{8\cos^3\theta}{3}$$

$$= -8\sqrt{1-\sin^2\theta} + \frac{8}{3}\sqrt{1-\sin^2\theta}\left(1-\sin^2\theta\right)$$

$$= -8\sqrt{1-\frac{x^2}{4}} + \frac{8}{3}\sqrt{1-\frac{x^2}{4}}\left(1-\frac{x^2}{4}\right)$$

$$= -8\sqrt{1-\frac{x^2}{4}} + \frac{8}{3}\left(1-\frac{x^2}{4}\right)^{3/3} + C$$

C. $\int x^2 \arctan x \, dx = \int x^2 \tan^{-1} x \, dx$

Using By Parts

$$\int u\,dv = uv - \int v\,du$$

$$u = \tan^{-1} x \qquad du = \frac{1}{1+x^2}\,dx$$

$$dv = x^2\,dx \Rightarrow v = \frac{1}{3}x^3$$

$$\int x^2 \tan^{-1} x\,dx = \tan^{-1} x\,\frac{1}{3}x^3 - \int \frac{1}{3}x^3 \cdot \frac{1}{1+x^2}\,dx$$

$$\int \frac{x^3}{1+x^2}\,dx \Rightarrow \text{Take } 1+x^2 = t \quad \text{— (1)}$$

$$2x\,dx = dt$$

$$x\,dx = \frac{dt}{2}$$

$$\int \frac{(t-1)}{t} \cdot dt/2$$

$$= \frac{1}{2}\int 1-\frac{1}{t} \cdot dt$$

$$= \frac{1}{2}t - \frac{1}{2}\ln t$$

$$= \frac{1}{2}(1+x^2) - \frac{1}{2}\ln(1+x^2)$$

$\therefore$ The final answer from 1 is

$$\frac{1}{3}x^2\tan^{-1}x - \frac{1}{6}(1+x^2) + \frac{1}{6}\ln(1+x^2) + C$$

2) given

$$\vec{r}_1 = (6+t+0.5t^2,\ t^2+2t,\ 5t+2t^2)$$

$$\vec{r}_2 = (7t-0.5t^2,\ 1+0.5t^2-t,\ t^2-9t)$$

$$V_1' = \frac{d\vec{r_1}}{dt} = (1+t,\ 2t+2,\ 5-4t)$$

$$\vec{V_2} = \frac{d\vec{r^2}}{dt} = (7-t,\ t-1,\ 2t-9)$$

Lets say V_2 at $U = V_1$ at t

$$\Rightarrow (1+t,\ 2t+2,\ 5-4t,) = (7-4,\ U-1,\ 2U-9)$$

$$\therefore \left.\begin{aligned} 1+t &= 7-U \\ 2t+2 &= U-1 \\ 5-4t &= 2U-9 \end{aligned}\right\} \begin{aligned} U &= 5 \\ t &= 1 \end{aligned}$$

$=$ $\vec{V_2}$ at 5 units $= \vec{V_1}$ at 1 unit

$$\vec{V_2} = (7-5,\ 5-1,\ 2(5)-9) = (2,4,1)$$

$$\vec{V_1} = (1+1,\ 2(1)+2,\ 5-4(1)) = (2,4,1$$

$$V_1 = \sqrt{2^2+4^2+1} = \sqrt{21} \text{ units}$$

3. Two Vectors of Tetrahedron are $(-1,0,0)$ and $(1,0,0)$

Lets assume other two are A & B

Mid-Point of this line segment

$$= \left(\frac{-1+1}{2}, \frac{0+0}{2}, \frac{0+0}{2}\right) = (0,0,0)$$

Eq^n of the Plane is $z=0$

Let $A(x,y,z)$

Distance b/w A & $(-1,0,0)$ be s

$$\therefore s^2 = (x+1)^2 + y^2 + z^2 = (x+1)^2 + y^2$$

Distance b/w A & $(1,0,0)$ is also s

$\therefore s^2 = (x+1)^2 + y^2 + z^2 = (x+1)^2 + y^2$

Distance b/w A and $(1,0,0)$ is also s

$\therefore s^2 = (x-1)^2 + y^2 + z^2 = (x-1)^2 + y^2$

$\therefore (x+1)^2 + y^2 = (x-1)^2 + y^2 \Rightarrow x = 0$

$\Rightarrow y^2 = \dfrac{s^2 - 1}{4}$

$\therefore A = \left(0, \dfrac{\sqrt{s^2-1}}{2}, 0\right)$

$B = \left(0, \dfrac{\sqrt{s^2-1}}{2}, 0\right)$

B) Centroid of the Tetrahedron

$= \dfrac{\text{Sum of all co-ordinates}}{4}$

'

$= (0,0,0)$

C. We know $O(0,0,0)$ is the Centoid

$\therefore OA = (-1,0,0)$

$OB = (1,0,0)$

$OC = \left(0, \frac{\sqrt{s^2-1}}{2}, 0\right)$

$OD = \left(0, -\frac{\sqrt{s^2-1}}{2}, 0\right)$

$OA \cdot OB = -1 = |OA||OB|\cos\theta = -1$

Using this dot Product, we can find that all angles on the centroid By the Edges are 108°

4. $D_f = (-\infty, 1)$ $D_g = [0, \pi]$ $D_h = (-1, 1)$

$$F(x,y) = f(x) - g(x)\,h(y)$$

$$D_F = (D_f \cap D_g) \times D_h$$

$$= \{(-\infty, 1) \cap [0, \pi)\} \times (-1, 1)$$

$$= [0, 1) \times (-1, 1)$$

5. A. there, it is Positive at C as contour value increases as we move along the x-axis keeping Y - constant

At A & B the contour value Decrease as we move along x-axis keeping y-constant.

At A & B the contour value decreases as one move along x-axis.

So it is negative at A & B so it is highest at C.

B. Here it is positive at B, as one move along +ve y axis the contour value remains const. So it is zero at B. At C the contour value decreases as one move along +ve y axis. So it is highest at B.

C. The red curve can't be a level curve as it corresponds to two different values at two different points. But a level has a single value.