Lesson 1 Board-work

Plan for Lesson 1

- Refreshing the idea of derivatives.
- Using various rules for derivatives.
- Implicit differentiation.
- Numerical Differentiation (Python File).

Derivatives

Definition: The Derivative Function

The derivative of a function of $f(x)$ is the function $f'(x)$ defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The domain of f' is the set of values of x for which the above limit exists.

NOTE: The function is said to be **differentiable** at a points $x = a$ if the above limit exists (or $f'(a)$ exists). The process of finding derivatives is called **differentiation**.

Q 1.

Pickup **the correct setups** for the derivative of the function $f(x) = \sqrt{3x+1}$ **at x=1** by definition.

- $f'(1) = \lim_{h\to 0} \frac{\sqrt{4+3h}-2}{h}$ **A**
- $f'(1) = \lim_{h\to 0} \frac{\sqrt{4-3h}-2}{h}$ **B**
- $f'(1) = \lim_{h\to 0} \frac{\sqrt{4+3h}+2}{h}$ **C**
- $f'(1) = \lim_{h\to 0} \frac{\sqrt{3+4h}-2}{h}$ **D**

Q 2. Use the limit definition of the derivative to find the tangent line to the curve $x^2 + 1$ at the point $a = 1$.

Question: When does the derivative not exist at a point in the domain of the function? Let's understand this pictorially.

Vital Derivative Rules

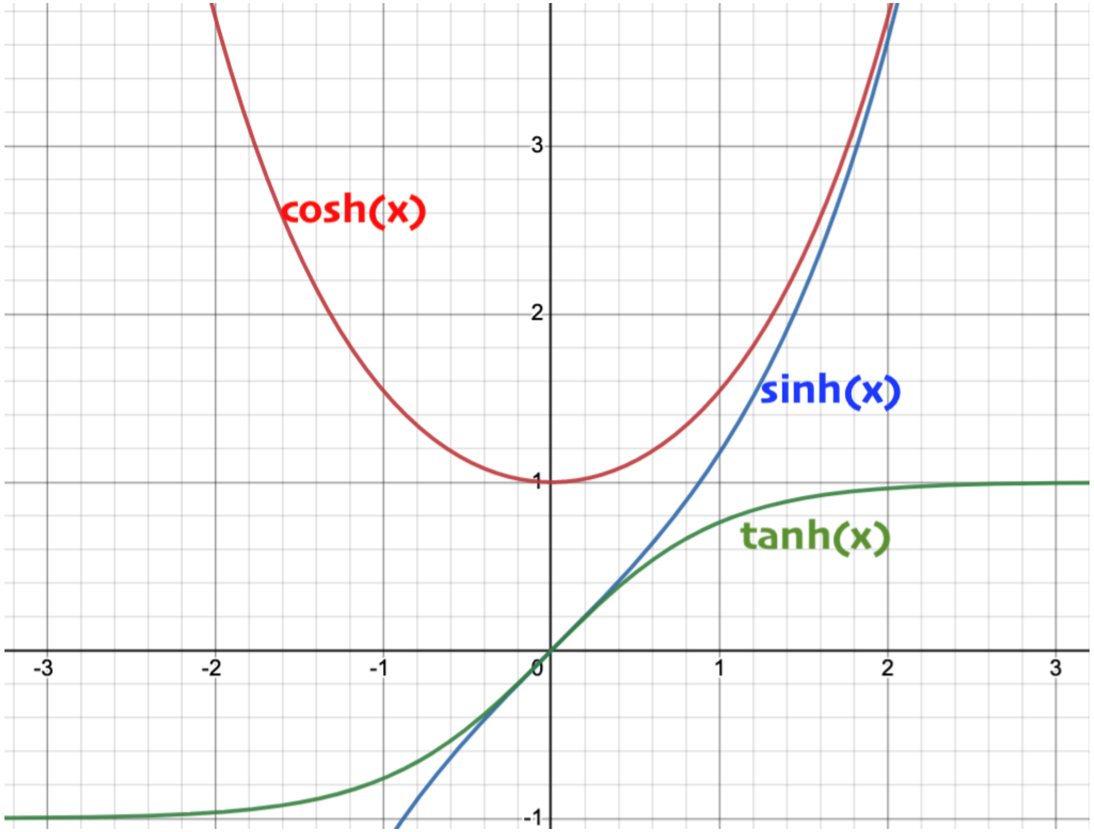
- **Product Rule** $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
- **Linearity Rule** $(f(x) + g(x))' = f'(x) + g'(x)$
- **Quotient Rule** $(\frac{f(x)}{g(x)})' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$
- **Chain Rule** $f(g(x))' = f'(g(x))g'(x)$

Very Important Derivatives to memorize

- **Power Rule** $(x^n)' = nx^{n-1}$
- **Exponential** $(e^x)' = e^x$
- **Logarithm** $(\ln(x))' = \frac{1}{x}$
- **Trig functions** $\sin(x)' = \cos(x)$, $\cos(x)' = -\sin(x)$.

Q 3. Hyperbolic tangent functions are used as activation functions in deep learning. Find the derivative of this function:

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



Q 4.

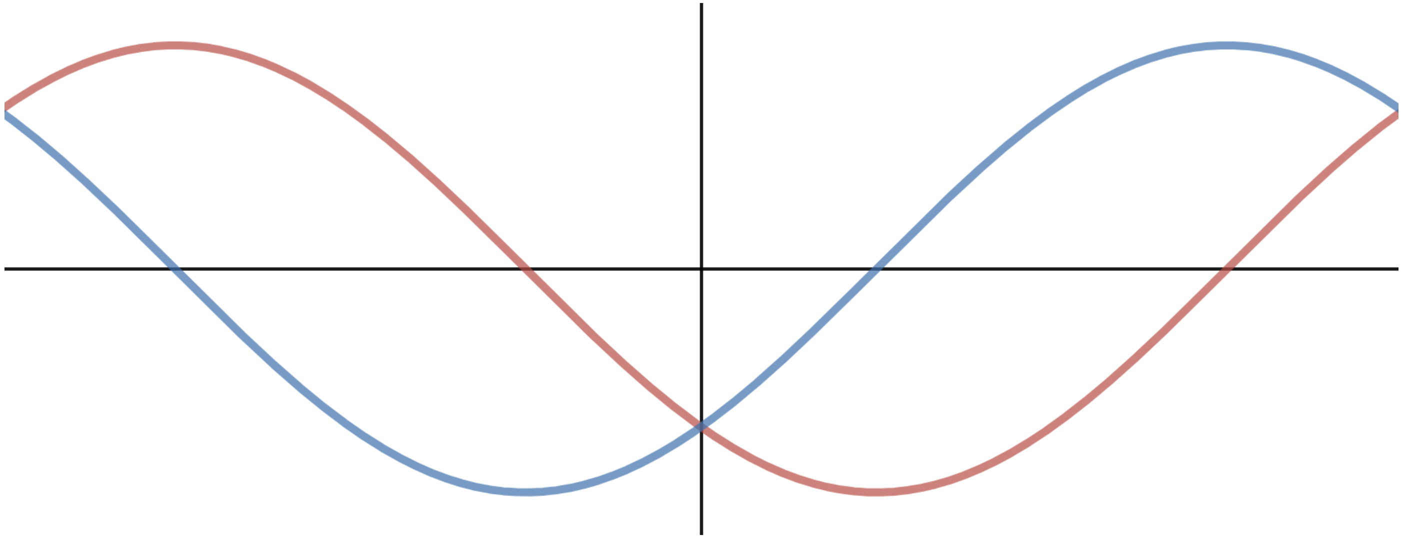
Pick the **three correct options** from below.

- ☐ $\frac{d}{dx}\left[\pi^5\right] \neq 5\pi^4.$ **A**
- ☐ $\frac{d}{dx}\left[4x^{2.5} - \frac{2}{x}\right] = 10x^{1.5} + \frac{2}{x^2}$ **B**
- ☐ $\frac{d}{dx}\left[(x^5 + x^2)(9x^3 - 7x)\right] = (5x^4 + 2x)(27x^2 - 7)$ **C**
- ☐ $\frac{d}{dx}\left[\frac{x^5 - 4x^4}{2x^3}\right] = x - 2$ **D**
- ☐ $\frac{d}{dx}\left[\frac{x^5 - 4x^4}{2x^3}\right] = \frac{5x^4 - 16x^3}{6x^2}$ **E**

Increasing/decreasing behavior: A function defined on an interval (a, b) is ..

- **increasing ($\nearrow$)** on the interval if $f'(x) > 0$ at every point on the interval.
- **decreasing ($\searrow$)** on the interval if $f'(x) < 0$ at every point on the interval.

Q 5. Is the blue curve the derivative of the red curve? Is the red curve the derivative of the blue curve? Explain.



Q 6. Compute $y'(0)$ provided that

$$3x^2 - 2xy = 5$$

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Q 7. Application to Data. Suppose you are presented with a large data set $S = \{(x_1,y_1),(x_2,y_2),\ldots,(x_N,y_N)\}$, where $0 < x_{n+1} - x_n < \Delta x$ for some (small) increment $\Delta x > 0$ when $1 \leq n < N$.

A. Given an index n such that $1 \leq n < N$, explain how the data set S can be used to estimate the value of a derivative at x_n.

B. Explain another way in which the data set can be used to estimate the value of a derivative at x_n. Can you think of a third or even fourth way?

C. Explain when and how the data set S can be used to estimate the value of a second derivative at x_n.

Python Resources available on Canvas for numerical differentiation.

Survey 1 (Sept 5th):

Q 8. Find the equation of the tangent line the the curve $y = |3x - 2|$ at $x = 0$.

Indicate your level of understanding of the following by putting a number from 1 to 5.

- How refreshed you feel around the derivatives and their meaning.
  [ ]

- Can you comfortably apply basic derivative rules.
  [ ]