Due: March 5, 2024

Assignment 2: MLP and Word Vectors

Homework assignments will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited. Electronic submission on Canvas is mandatory.

Instructions: You shall submit a zip file named A2_LastName_FirstName.zip including:

- -A jupyter notebook which includes all your code (and your written part). We provided some code templates. It is up to you to use the templates or not.
- -(optional) a png (or jpg) file contains the word vector plot (vector.png).
- -(optional) a pdf file contains all your solutions for the Written part.
- -Your code may run for several hours depending on your implementation and your device. Please start working on it as early as possible!

1. Multi-Layer Perceptron (MLP) (30 pts)

- (a) (5 pts) Preprocess the data: tokenization, feature extraction. You can reuse the code from Assignment 1.
- (b) (20 pts) Implement the MLP class.
 - (5 pts) Model implementation with numpy
 - (5 pts) AdaGrad implementation
 - (5 pts) Minibatch gradient GD for MLP using AdaGrad
 - (5 pts) Model implementation with Tensorflow
- (c) Run all the code to make sure your implementation works.
- (d) (5 pts) Conclusion.

2. Word2vec - Written (25 pts)

(a) (5 pts) Derive the gradients of the sigmoid function and show that it can be rewritten as a function of the function value (i.e., in some expressions where only $\sigma(x)$, but not x, is present). Assume that the input x is a scalar for this question. Recall, the sigmoid function is:

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

(b) (5 pts) Assume you are given a predicted word vector \mathbf{v}_c corresponding to the center word c for skip-gram, and the word prediction is made with the softmax function:

$$\hat{y}_o = p(o|c) = \frac{\exp(\mathbf{u}_o^{\top} \mathbf{v}_c)}{\sum_{w=1}^{W} \exp(\mathbf{u}_w^{\top} \mathbf{v}_c)}$$

where o is the expected word, w denotes the w-th word and \mathbf{u}_w (w = 1, ..., W) are the "context" (output) word vectors for all words in the vocabulary. The cross entropy function is defined as:

$$J_{\text{CE}}(o, \mathbf{v}_c, U) = CE(\mathbf{y}, \hat{\mathbf{y}}) = -\sum_i y_i \log(\hat{y}_i)$$

where the gold vector \mathbf{y} is a one-hot vector, the softmax prediction vector $\hat{\mathbf{y}}$ is a probability distribution over the output space, and $U = [u_1, u_2, ..., u_W]$ is the matrix of all the context vectors. Assume cross entropy is applied to this prediction, derive the gradients with respect to \mathbf{v}_c .

- (c) (5 pts) Derive gradients for the "context" word vector \mathbf{u}_w (including \mathbf{u}_o) in (b).
- (d) (5 pts) Repeat (b) and (c) assuming we are using the negative sampling loss for the predicted vector \mathbf{v}_c . Assume that K negative samples (words) are drawn and they are 1, ..., K respectively. For simplicity of notation, assume $(o \notin \{1, ..., K\})$. Again for a given word o, use \mathbf{u}_o to denote its context vector. The negative sampling loss function in this case is:

$$J_{\text{neg-sample}}(o, \mathbf{v}_c, U) = -\log(\sigma(\mathbf{u}_o^{\top} \mathbf{v}_c)) - \sum_{k=1}^{K} \log(\sigma(-\mathbf{u}_k^{\top} \mathbf{v}_c))$$

(e) (5 pts) Derive gradients for all of the word vectors for skip-gram given the previous parts and given a set of context words [word_{c-m}, ..., word_{c+m}] where m is the context size. Denote the "center" and "context" word vectors for word k as \mathbf{v}_k and \mathbf{u}_k respectively.

Hint: feel free to use $F(o, \mathbf{v}_c)$ (where o is the expected word) as a placeholder for the $J_{\text{CE}}(o, \mathbf{v}_c...)$ or $J_{\text{neg-sample}}(o, \mathbf{v}_c...)$ cost functions in this part – you'll see that this is a useful abstraction for the coding part. That is, your solution may contain terms of the form $\frac{\partial F(o, \mathbf{v}_c)}{\partial ...}$ Recall that for skip-gram, the cost for a context centered around c is:

$$\sum_{-m \le j \le m, j \ne 0} F(w_{c+j}, \mathbf{v}_c)$$

- 3. Word2vec Coding (45 points)
 - (a) (5pts) Data processing including tokenization.
 - (b) (10 pts) Training data generation.
 - (5 pts) Positive samples
 - (5 pts) Negative samples
 - (c) (20 pts) Skip-gram model implementation using Tensorflow.
 - (10 pts) Loss function.
 - (10 pts) Model.
 - (d) (5 pts) Implement the k-nearest neighbors algorithm, which will be used for visualization and analysis. The algorithm receives a vector, a matrix and an integer k, and returns k indices of the matrix's rows that are closest to the vector. Use the cosine similarity as a distance metric (https://en.wikipedia.org/wiki/Cosine_similarity).
 - (e) Run the jupyter notebook code to make sure your implementation works
 - (f) (5 pts) Conclusion.