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Homework - 5

1) Given:

A. $S = \{(x,y,z) \in \mathbb{R}^3 \mid x = y\}$

S contains a zero vector for $x=y=z=0$ and it also is closed under vector addition and scalar multiplication.

So, Yes, S is a vector subspace of $\mathbb{R}^3$.

B. $T = \{(x,y,z) \in \mathbb{R}^3 \mid y=1\}$

$y=1$, so no zero vector and it is not a vector subspace in $\mathbb{R}^3$.

C. $U = \{(x,y,z) \in \mathbb{R}^3 \mid xyz = 0\}$

Yes, it has zero vector $x=y=z=0$. and it is. Closed under vector addition and scalar multiplication.

So, S is a vector subspace in $\mathbb{R}^3$.

2) Given α and β does the matrix $\begin{pmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & \alpha & 2 & 2 \\ 0 & 0 & 0 & \beta & 2 \end{pmatrix}$

lets check for what α and β it has rank 2.

We have to reduce the matrix to euchelon form,

$$\begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & \alpha & 2 & 2 \\ 0 & 0 & 0 & \beta & 2 \end{bmatrix} \xrightarrow[R_2 = R_2/2]{R_3 = R_3 - R_2} \begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & \alpha/2 & 1 & 1 \\ 0 & 0 & -\alpha & \beta-2 & 0 \end{bmatrix}$$

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For the rank of matrix = 2,
3rd row = 0
x_1 and x_4, ~~Ch~~ x_1 and x_4 be 'I'.

$-\alpha = 0, \ \beta - 2 = 0$

$\alpha/2 = 0$

$\boxed{\alpha = 0, \beta = 2}$

3) Given 3 different matrices of 2×2 shape, transform the plane $\mathbb{R}^2$.

a) ① ⟶ Matrix A rotates the plane clockwise by 30°

The matrix B projects the elements of the plane onto the line $x+y=0$.

Matrix C shears the plane horizontally by a factor of 0.5.

$$\left.\begin{array}{l} \hat{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \hat{y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{array}\right\} \text{Basic Vectors.}$$

From ①

$x' = \hat{x}\cos\theta + \hat{y}\sin\theta$ $\qquad \theta = -30°$

$y' = -\hat{x}\sin\theta + \hat{y}\cos\theta$

$x' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}(\sqrt{3}/2) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}(-1/2)$, $y' = -\begin{bmatrix} 1 \\ 0 \end{bmatrix}(-1/2) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}(\sqrt{3}/2)$

$\rightarrow x' = \begin{bmatrix} \sqrt{3}/2 \\ -1/2 \end{bmatrix} \quad y' = \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix}$

$$A = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$$

rank = 2

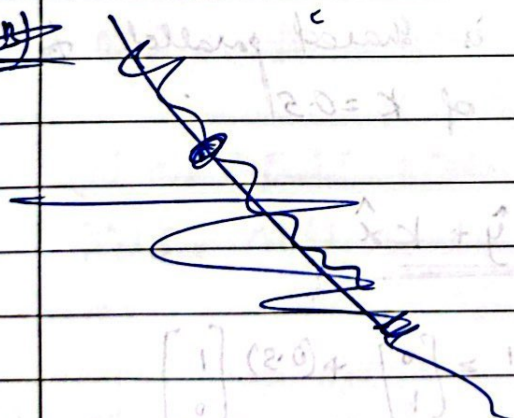
Dimensions for the fundamental subspaces.

(i) for null space of (A), $\dim N(A) = n - r = 2 - 2 = 0$

(ii) for null space of (A^T), $\dim N(A^T) = m - r = 2 - 2 = 0$

(iii) for column space and row space.

$\dim(A) = \dim(A^T) = r = 2$.



(b) $x + y = 0$

lets say $|\hat{x}| = 1/2$

$|\hat{y}| = 1/2$.

$x^2+y^2 = 1/2$

$2x^2 = 1/4$ (or) $\boxed{x = 1/2}$

$$x' = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$

$$y' = \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix} \quad \underline{\text{rank} = 2}$$

(i) for column space and Row space
$\dim(B) = \dim(B^T) = r = 2$

(ii) for null space, $\dim N(B) = n - r = 2 - 2 = 0$

(iii) for null space of B^T, $\dim N(B^T) = m - r = 2 - 2 = 0$

(4)

$$\hat{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \hat{y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The plane is shared parallel to y-axis by a factor of $K = 0.5$.

$$\underline{x' = \hat{x}, \quad y' = \hat{y} + k\hat{x}}$$

$$x' = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow y' = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + (0.5) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y' = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}_{rank = 2}$$

(i) for column space and row space.

$\dim(C) = \dim(C^T) = r = 2$.

(ii) for null space ; $\dim(N(C)) = n - r = 2 - 2 = 0$

(iii) for null space of C^T, $\dim N(C^T) = m - r = 2 - 2 = 0$

(D) the transformation are applied to the order A, B and C. We need to find the dimensions C.B.A

$$CBA = \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$CBA = \begin{bmatrix} 1/4 & -1/4 \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$M = CBA = \begin{bmatrix} \frac{\sqrt{3}+1}{8} & \frac{1-\sqrt{3}}{8} \\ \frac{-1-\sqrt{3}}{4} & \frac{-1+\sqrt{3}}{4} \end{bmatrix}_{rank = 2}$$

(i) for column space and row space

$\dim(M) = \dim(M^T) = r = 2$

(ii) for null space, $\dim N(M) = n - r = 2 - 2 = 0$

(iii) for null space of M^T, $\dim(N(M^T)) = m - r = 2 - 2 = 0$.

4) $$H = \text{Span}\left\{ \begin{bmatrix} 5 \\ 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 14 \\ 3 \\ 2 \\ 8 \end{bmatrix}, \begin{bmatrix} 38 \\ 8 \\ 8 \\ 24 \end{bmatrix}, \begin{bmatrix} 47 \\ 10 \\ 7 \\ 28 \end{bmatrix}, \begin{bmatrix} 10 \\ 2 \\ 3 \\ 12 \end{bmatrix} \right\}$$

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lets write it as a matrix.

$$\begin{bmatrix} 5 & 14 & 38 & 47 & 10 \\ 1 & 3 & 8 & 10 & 2 \\ 1 & 2 & 6 & 7 & 3 \\ 4 & 8 & 24 & 28 & 12 \end{bmatrix}$$

Now reduce it row echelon form.

$R_4 = R_4 - 4R_3$

$R_1 = R_1 - 4R_2$

$$\begin{bmatrix} 1 & 2 & 6 & 7 & 2 \\ 1 & 3 & 8 & 10 & 2 \\ 1 & 2 & 6 & 7 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$R_3 = R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 6 & 7 & 2 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4^{th} row is all zeros.

x_1, x_2 and x_5 are the leading variables

rank = 3

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(i) For column space and row space.

$\dim(H) = \dim(H^T) = r = 3.$

ii) for null space, $\dim N(H) = n - r = 5-3=2.$

iii) for null space of H^T, $\dim N(H^T) = m-r = 4-3=1$

x_1, x_2 and x_5

$$\begin{bmatrix} 5 \\ 1 \\ 1 \\ 4 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 8 \end{bmatrix} \begin{bmatrix} 10 \\ 2 \\ 3 \\ 12 \end{bmatrix}$$

5) Given $A = \begin{bmatrix} 1 & 3 & 0 & -2 & 7 & 3 \\ 3 & 9 & 1 & -7 & 23 & 8 \\ 1 & 3 & 1 & -3 & 9 & 2 \\ 1 & 3 & -1 & -1 & 5 & 4 \end{bmatrix}$

reduce into echelon form.

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - R$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 7 & 3 \\ 0 & 0 & 1 & -1 & 2 & -1 \\ 0 & 0 & 1 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - R_2 \\ R_4 \to R_4 + R_2 \end{matrix}$$

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$$\begin{pmatrix} 1 & 3 & 0 & -2 & 7 & 3 \\ 0 & 0 & 1 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

3rd and 4th row are zeros

x_1 and x_3 are the leading variable.

$r = 2$

i) $\dim C(A)$ $\dim C(A)^T = r = 2$.

ii) ~~de~~ null space, $\dim(N(A)) = n - r = 6 - 2 = 4$

iii) ~~null space~~, $\dim N(A^T) = m - r = 4 - 2 = 2$.