MA 574. Pre Class Assignment 22. 30/11/23

Q1. $f(x,y) = x^{2/3} y^{1/3}$ $\qquad x+y = 3780.$

first we will find gradient of the function

$\because$ we know $\nabla f = d \nabla g.$

$g(x,y) = x+y = 3780$

$$\nabla f = \begin{bmatrix} \frac{\partial}{\partial x}\left(x^{2/3} y^{1/3}\right) \\ \frac{\partial}{\partial y}\left(x^{2/3} y^{1/3}\right) \end{bmatrix} = \begin{bmatrix} \frac{2}{3} x^{-\frac{1}{3}} . y^{1/3} \\ \frac{1}{3} x^{2/3} . y^{-2/3} \end{bmatrix}$$

$$\nabla g = \begin{bmatrix} \frac{\partial}{\partial x}\left(x+y=3.78\right) \\ \frac{\partial}{\partial y}\left(x+y=3.78\right) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3} x^{-\frac{1}{3}} . y^{1/3} \\ \frac{1}{3} x^{2/3} . y^{-2/3} \end{bmatrix} = d \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{2}{3} x^{-1/3} . y^{1/3} = d \qquad \text{—(1)}$$

$$\frac{1}{3} x^{2/3} . y^{-2/3} = d. \qquad \text{—(2)}$$

$$x + y = 3780 \qquad \text{—(3)}.$$

$$x = 3780 - y.$$

$$\frac{1}{3}\left(\frac{3.78-y}{y}\right)^{2/3} = d.$$

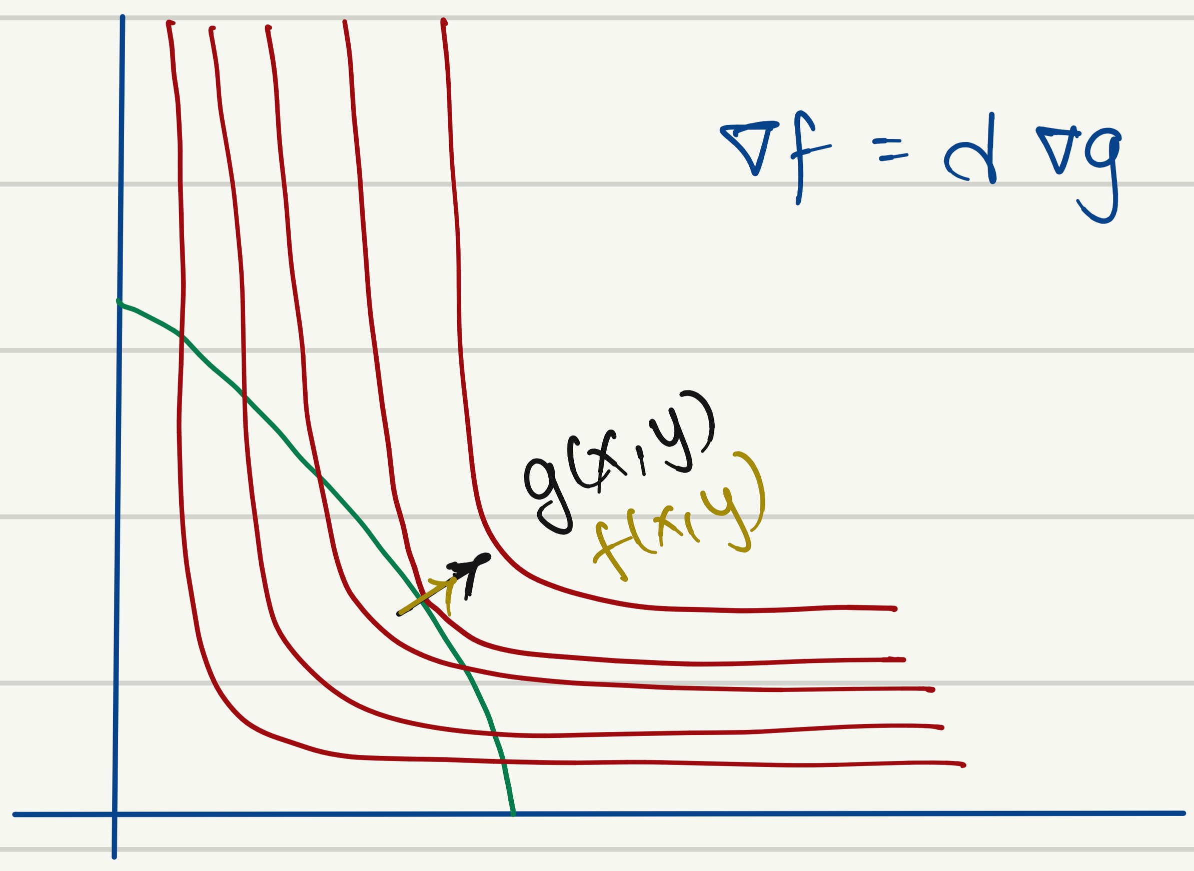
$$(3d)^{2/3} = \frac{3.78-y}{y}$$

$$y.\left(3d^{2/3}+1\right) = 3.78.$$

$$y = \frac{3.78}{(3d)^{2/3}+1}$$

Substitute $y = \frac{3.78}{(3d)^{2/3}+1}$ & $x = 3.780 - y$

in eqⁿ (1).



$\nabla f = d \nabla g$

$$\frac{2}{3}\left(\frac{y}{x}\right)^{1/3} = d.$$

$$\left(\frac{y}{x}\right)^{1/3} = \frac{3d}{2}.$$

$$\left(\frac{y}{3.78-y}\right)^{1/3} = \frac{3d}{2}.$$

$$\left(\frac{\frac{3.78}{(3d)^{2/3}+1}}{3.78 - \left(\frac{3.78}{(3d)^{2/3}+1}\right)}\right)^{1/3} = \frac{3d}{2}.$$

$$\left(\frac{\frac{3.78}{\cancel{(3d)^{2/3}+1}}}{\frac{3.78\left[(3d)^{2/3}+\cancel{1}-\cancel{1}\right]}{\cancel{(3d)^{2/3}+1}}}\right)^{1/3} = \frac{3d}{2}.$$

$$\frac{\cancel{3.78}}{\cancel{3.78}\,(3d)^{2/3}} = \frac{3d}{2}.$$

$$(3d)^{2/3} \times 3d = 2.$$

$$(3d)^{5/3} = 2.$$

$$3d = (2)^{5/3}.$$

$$3d = (2)^{5/3}.$$

$$d = 1.05.$$

$$\boxed{d \approx 1}$$

$$\boxed{y = 1.26}$$

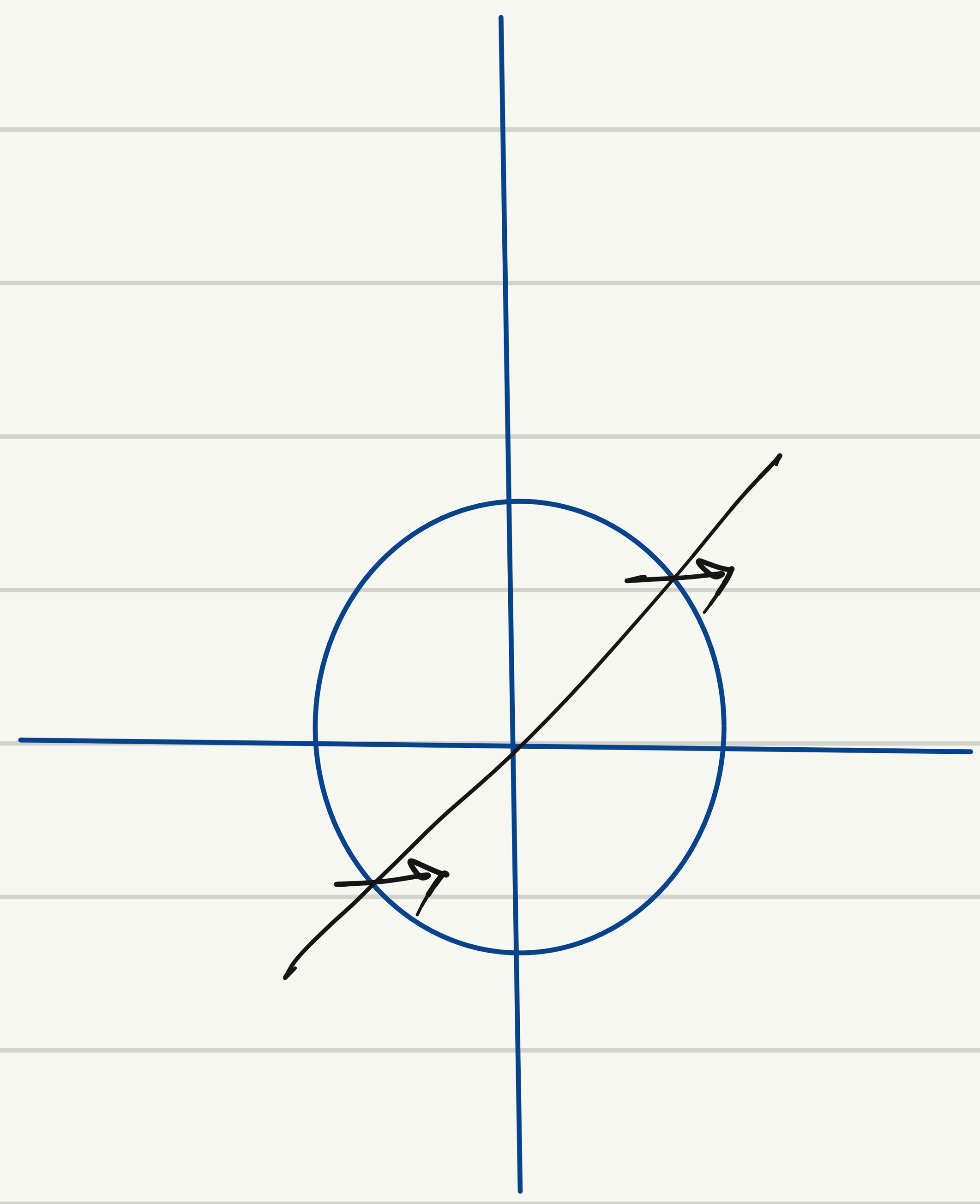
$$\boxed{x = 2.52}$$

Thus max level of productⁿ occurs at $x = 2.52$ & $y = 1.26$.

Q2. $f(x,y) = x+y$ on circle $x^2+y^2=4$.

we know that.

$$\nabla f = \lambda \nabla g. \qquad g = x^2+y^2.$$



$$\nabla f = \begin{bmatrix} \frac{\partial}{\partial x}(x+y) \\ \frac{\partial}{\partial y}(x+y) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\nabla g = \begin{bmatrix} \frac{\partial}{\partial x}(x^2+y^2) \\ \frac{\partial}{\partial y}(x^2+y^2) \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$$2x\lambda = 1 \quad — (1)$$
$$2y\lambda = 1 \quad — (2).$$
$$x^2+y^2=1 \quad — (3)$$

$x = \frac{1}{2\lambda}$, $y = \frac{1}{2\lambda}$. in eq^n (3).

$$\left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 = 1.$$

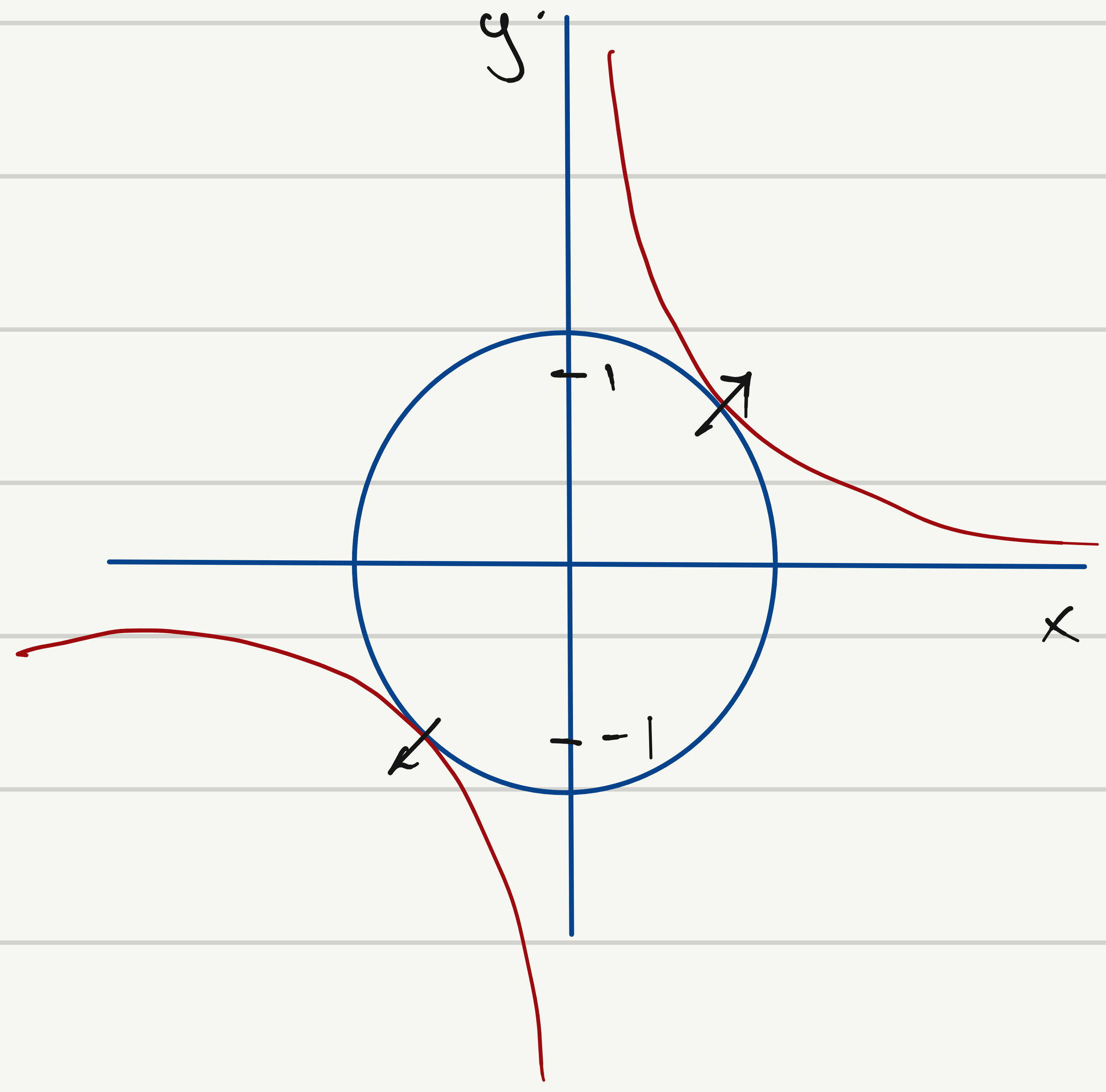
$$\frac{4\lambda^2 + 4\lambda^2}{16\lambda^4} = 1. \Rightarrow \frac{\cancel{\lambda^2}(8)}{\cancel{16}\lambda^{\cancel{4}}} = 1$$

$$\frac{1}{\lambda^2} = 1.$$
$$\lambda^2 = 1 \qquad \boxed{\lambda = \pm 1}$$

$$\boxed{\therefore x = \pm\frac{1}{2}} \qquad \boxed{y = \pm\frac{1}{2}}$$

max & min values at which both the curves & contour plot coincide is at $\boxed{x = \pm\frac{1}{2} \ \& \ y = \pm\frac{1}{2}}$

Q3. $f(x,y) = x^2 + y^2$ $\qquad$ $xy = 1$



we know that:

$$\nabla f = \lambda \nabla g.$$

$$\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}, \quad \nabla g = \begin{bmatrix} y \\ x \end{bmatrix}$$

$$2x = \lambda y. \quad \text{—} (1)$$
$$2y = \lambda x \quad \text{—} (2)$$
$$xy = 1 \quad \text{—} (3)$$
$$x = \frac{1}{y}$$

$$2 \times \frac{1}{\cancel{y}} = \lambda \cancel{y}.$$

$$\boxed{\lambda = 2}$$

$$2y = 2\frac{1}{y}$$

$$y^2 = 1$$
$$\boxed{y = \pm 1}$$

$$\boxed{x = \pm 1}$$

Hence max value of x & y where the tangent is getting created is +1

and min value of x & y where the tangent is getting created is -1

Q4. $f(x,y,z) = xy^2z^3$ $\qquad x^2+y^2+z^2 \leq 1$

we know that,

$\nabla f = \lambda \nabla g.$ $\qquad g(x,y,z) = x^2+y^2+z^2$

$$\nabla f = \begin{bmatrix} y^2z^3 \\ 2xyz^3 \\ 3xy^2z^2 \end{bmatrix}, \quad \nabla g = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix}$$

$y^2z^3 = 2x\lambda$ —— ①

$2xyz^3 = 2y\lambda$ —— ②

$3xy^2z^2 = 2z\lambda.$ —— ③

$x^2+y^2+z^2=1$ —— ④

$$x = \frac{y^2z^3}{2\lambda}$$

$$\left(\frac{y^2z^3}{2\lambda}\right)z^3 = \lambda. \Rightarrow \frac{y^2z^6}{2\lambda} = \lambda.$$

$$3\left(\frac{y^2z^3}{2\lambda}\right)y^2 \cdot z^{\cancel{2}} = 2\cancel{z}\lambda$$

$$\frac{3}{4}\left[y^4 \cdot z^4\right] = \lambda^2.$$

$$\lambda = \sqrt{\frac{3}{4}(y^4z^4)}$$

$$\left(\frac{y^2z^3}{2\left(\cancel{\sqrt{\frac{3}{4}(y^4z^4)}}\right)}\right)z^3 = \cancel{\sqrt{\frac{3}{4}(y^4z^4)}}$$

$$y^2z^6 = 2.$$

$$y = \sqrt{\frac{2}{z^6}}$$

$$\lambda = \sqrt{\frac{3}{4}\left(\frac{\sqrt{2}}{\cancel{z^3}} \times \cancel{z^4}\right)} \Rightarrow \lambda = \sqrt{\frac{3}{4}z}.$$

$$x = \frac{y^2z^3}{\cancel{2} \times \left(\frac{\sqrt{3}}{\cancel{2}} \times \sqrt{z}\right)} \quad \therefore x = \frac{y^2z^3}{\sqrt{3z}}$$

$$x^2+y^2+z^2=1$$

$$\frac{y^4z^6}{3z} + \frac{2}{z^6} + z^2 = 1.$$

$$\frac{\frac{2}{\cancel{z^6}} \times \cancel{z^6}}{3z} + \frac{2}{z^6} + z^2 = 1$$

$$\frac{2}{3z} + \frac{2}{z^6} + z^2 = 1.$$

$$\frac{2z^6 + 6z}{3z^7} + z^2 = 1.$$

$$\frac{2z^6 + 6z + 3z^9}{3z^7} = 1.$$

$$2z^6 + 6z + 3z^9 = 3z^7.$$

$$3z^9 - 3z^7 + 2z^6 + 6z = 0.$$

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Q5) For this question,

we can solve the eq^n & find the values of $x_1, x_2 \ldots x_n$ which has max value being inside the constraint.