

Algebraic Geometry Exercises

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1.1 Affine varieties

Exercise 1. (a) Let Y be the plane curve $y = x^2$ (i.e., Y is the zero set of the polynomial $f = y - x^2$). Show that $A(Y)$ is isomorphic to a polynomial ring in one variable over k .

(b) Let Z be the plane curve $xy = 1$. Show that $A(Z)$ is not isomorphic to a polynomial ring in one variable over k .

Solution. (i) $y - x^2$ is irreducible, so $I(Y) = (y - x^2)$. Therefore $A(Y) = k[x, y]/(y - x^2) = k[x]$.

(ii) Similarly, $A(Z) = k[x, y]/(xy - 1) = k[x, x^{-1}]$, which is clearly not isomorphic to a polynomial ring in one variable over k .

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