IFT-6390 Fundamentals of Machine Learning
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Homework 1 - Theoretical part

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1. **Probability warm-up: conditional probabilities and Bayes rule** [5 points]

   (a) Give the definition of the conditional probability of a discrete random variable X given a discrete random variable Y.
   Answer. The conditional probability of a X is the probability of X given that we know the certain value of a discrete random variable Y.

   $$P(X = x|Y = y) = \frac{P(\{Y = y\} \cap \{X = x\})}{P(Y = y)}$$

   (b) Consider a biased coin with probability 2/3 of landing on heads and 1/3 on tails. This coin is tossed three times. What is the probability that exactly two heads occur (out of the three tosses) given that the first outcome was a head?
   Answer. 3 coin tosses will have 8 cases, but since the first toss is head and we need exactly two heads in three tosses, this leaves us with just two cases i.e. [HHT, HTH]

   $$\begin{aligned} Probability &= P(HHT) + P(HTH) \\ &= \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} \\ &= \frac{4}{27} + \frac{4}{27} \\ &= \frac{8}{27} \end{aligned}$$

   (c) Give two equivalent expressions of $P(X, Y)$:

   (i) as a function of $\mathbb{P}(X)$ and $\mathbb{P}(Y|X)$
   Answer.
   $$\mathbb{P}(X, Y) = \mathbb{P}(X|Y)\mathbb{P}(Y) \tag{1}$$

   (ii) as a function of $\mathbb{P}(Y)$ and $\mathbb{P}(X|Y)$
   Answer.
   $$\mathbb{P}(X, Y) = \mathbb{P}(Y|X)\mathbb{P}(X) \tag{2}$$

(d) Prove Bayes theorem:

$$\mathbb{P}(X|Y) = \frac{\mathbb{P}(Y|X)\mathbb{P}(X)}{\mathbb{P}(Y)}.$$

Answer. Using 1 and 2 in the Bayes equation above.

$$\begin{aligned}\mathbb{P}(X|Y) &= \frac{\mathbb{P}(Y|X)\mathbb{P}(X|Y)\mathbb{P}(X,Y)}{\mathbb{P}(Y|X)\mathbb{P}(X,Y)} \\ &= \mathbb{P}(X|Y)\end{aligned}$$

(e) A survey of certain Montreal students is done, where 55% of the surveyed students are affiliated with UdeM while the others are affiliated with McGill. A student is drawn randomly from this surveyed group.

i. What is the probability that the student is affiliated with McGill?
Answer.

$$\begin{aligned}P(StudentfromMcGill) &= 1 - P(StudentfromUdeM) \\ &= 1 - 0.55 \\ &= 0.45\end{aligned}$$

ii. Now let's say that this student is bilingual, and you know that 80% of UdeM students are bilingual while 50% of McGill students are. Given this information, what is the probability that this student is affiliated with McGill ?

Answer. Using Bayes theorem

$$\begin{aligned}\mathbb{P}(Mcgill|Bilingual) &= \frac{\mathbb{P}(Bilingual|Mcgill)\mathbb{P}(Mcgill)}{\mathbb{P}(Bilingual)} \\ &= \frac{0.50 \times 0.45}{0.50 \times 0.45\, +\, 0.8 \times 0.55} \\ &= \frac{0.225}{0.665} \\ &= 0.338\end{aligned}$$

2. **Bag of words and single topic model** [10 points] We consider a classification problem where we want to predict the topic of a document from a given corpus (collection of documents). The topic of each

document can either be *sports* or *politics*. 2/3 of the documents in the corpus are about *sports* and 1/3 are about *politics*.

We will use a very simple model where we ignore the order of the words appearing in a document and we assume that words in a document are independent from one another given the topic of the document.

In addition, we will use very simple statistics of each document as features: the probabilities that a word chosen randomly in the document is either "goal", "kick", "congress", "vote", or any another word (denoted by *other*). We will call these five categories the vocabulary or dictionary for the documents: $V = \{"goal", "kick", "congress", "vote", other\}$.

Consider the following distributions over words in the vocabulary given a particular topic:

	$\mathbb{P}(\text{word} \mid \text{topic} = sports)$	$\mathbb{P}(\text{word} \mid \text{topic} = politics)$
word = "*goal*"	1/100	7/1000
word = "*kick*"	1/200	3/1000
word = "*congress*"	0	1/50
word = "*vote*"	5/1000	1/100
word = *other*	980/1000	960/1000

Table 1:

This table tells us for example that the probability that a word chosen at random in a document is "vote" is only 5/1000 if the topic of the document is *sport*, but it is 1/100 if the topic is *politics*.

(a) What is the probability that a random word in a document is "goal" given that the topic is *politics*?
Answer.
The conditional probability is given in the question and can be inferred directly from there.

$$\mathbb{P}(w = goal|t = politics) = \frac{7}{1000}$$

(b) In expectation, how many times will the word "goal" appear in a document containing 200 words whose topic is *sports*?

Answer. Since the probability of the word goal in 100 words is 1/100, therefore the expectation of the word goal in 200 words is

$$200 \times 1/100 = 2$$

(c) We draw randomly a document from the corpus. What is the probability that a random word of this document is "goal"?
Answer.

$$\begin{aligned}\mathbb{P}(goal) &= \mathbb{P}(goal|sports)\mathbb{P}(sports) \; + \; \mathbb{P}(goal|politics)\mathbb{P}(politics)\\ &= \frac{1}{100} \times \frac{2}{3} + \frac{7}{1000} \times \frac{1}{3}\\ &= \frac{9}{1000}\end{aligned}$$

(d) Suppose that we draw a random word from a document and this word is "kick". What is the probability that the topic of the document is *sports*?
Answer. Using Bayes theorem

$$\begin{aligned}\mathbb{P}(sports|kick) &= \frac{\mathbb{P}(kick|sports)\mathbb{P}(sports)}{\mathbb{P}(kick)}\\ where, P(kick) &= P(kick|sports)P(sports) + P(kick|politics)P(sports)\\ \mathbb{P}(sports|kick) &= \frac{\frac{1}{200} \times \frac{2}{3}}{\frac{1}{200} \times \frac{2}{3} + \frac{3}{1000} \times \frac{1}{3}}\\ &= \frac{10}{13}\end{aligned}$$

(e) Suppose that we randomly draw two words from a document and the first one is "kick". What is the probability that the second word is "goal"?
Answer.

$$\begin{aligned}\mathbb{P}(w = goal|w_{prev} = kick) = \mathbb{P}(w = goal, t = sports|w_{prev} = kick)\\ + \mathbb{P}(w = goal, t = politics|w_{prev} = kick)\end{aligned} \quad (3)$$

Using P(A,B|C) = P(A|B,C) P(B|C) and P(A|B,C)=P(A|B) because A and C independent on B

$$\begin{aligned}\mathbb{P}(w = goal, t = sports | w_{prev} = kick) &= P(goal|sports, kick)P(sports|kick) \\ &= 1/100 \times 10/13\end{aligned} \tag{4}$$

$$\begin{aligned}\mathbb{P}(w = goal, t = politics | w_{prev} = kick) &= P(goal|politics, kick)P(politics|kick) \\ &= 7/1000 \times 1 - 10/13\end{aligned} \tag{5}$$

Using 4 and 5 in 3

$$\begin{aligned}\mathbb{P}(w = goal | w_{prev} = kick) &= 1/100 \times 10/13 + 7/1000 \times 3/13 \\ &= 121/13000 \\ &= 0.0093\end{aligned}$$

(f) Going back to learning, suppose that you do not know the conditional probabilities given a topic or the probability of each topic (i.e. you don't have access to the information in table 1 or the topic distribution), but you have a dataset of N documents where each document is labeled with one of the topics *sports* and *politics*. How would you estimate the conditional probabilities (e.g., $\mathbb{P}(\text{word} = "\textit{goal}" \mid \text{topic} = \textit{politics})$) and topic probabilities (e.g., $\mathbb{P}(\text{topic} = \textit{politics})$) from this dataset?

Answer.

Since we have the documents labelled with one of the topic, it becomes easy to estimate the topic probabilities.

$$\begin{aligned}\mathbb{P}(topic = politics) &= \frac{No.\ of\ documents\ labelled\ as\ politics}{N} \\ \mathbb{P}(topic = sports) &= \frac{No.\ of\ documents\ labelled\ as\ sports}{N}\end{aligned}$$

The conditional probabilities of the word given a topic can also be calculated by finding the term frequency of the word and dividing it by the total number of words in that document. i.e. fraction of times the word w_i appears among all words in documents of topic t_j.

We can create a mega document by concatenating all the docs from a particular topic j and then finding the frequency of the word w. This word w can come from a self-made vocabulary or it can be done on all the words we see in the document.

$$\mathbb{P}(w_i|t_j) = \frac{count(w_i, t_j)}{\Sigma\, count(w, c_j)}$$

Sometimes, we do see that a word does not appear in one of the topic, therefore the probability for that will become 0 which is not the correct representation, so we can use Laplace Smoothing by adding 1 in both numerator and denominator to correct it.

3. **Maximum likelihood estimation** [5 points]

   Let $x \in \mathbb{R}$ be uniformly distributed in the interval $[0, \theta]$ where θ is a parameter. That is, the pdf of x is given by

$$f_\theta(x) = \begin{cases} 1/\theta & \text{if } 0 \leq \mathbf{x} \leq \theta \\ 0 & \text{otherwise} \end{cases}$$

   Suppose that n samples $D = \{x_1, \ldots, x_n\}$ are drawn independently according to $f_\theta(x)$.

   (a) Let $f_\theta(x_1, x_2, \ldots, x_n)$ denote the joint pdf of n independent and identically distributed (i.i.d.) samples drawn according to $f_\theta(x)$. Express $f_\theta(x_1, x_2, \ldots, x_n)$ as a function of $f_\theta(x_1), f_\theta(x_2), \ldots, f_\theta(x_n)$
   Answer.
   Since the n observations are independent and identically distributed (i.i.d), we can give the joint distribution as..

$$f_\theta(x_1, x_2, \ldots, x_n) = f(x_1, x_2, ..., x_n|\theta)$$

$$= f(x_1|\theta)f(x_2|\theta), ..., f(x_n|\theta)$$

$$= f_\theta(x_1)f_\theta(x_2), ..., f_\theta(x_n)$$

   (b) We define the maximum likelihood estimate by the value of θ which maximizes the likelihood of having generated the dataset D from the distribution $f_\theta(x)$. Formally,

$$\theta_{MLE} = \arg\max_{\theta \in \mathbb{R}} f_\theta(x_1, x_2, \ldots, x_n),$$

Show that the maximum likelihood estimate of θ is $max(x_1, \ldots, x_n)$

Answer.

$$f_\theta(x) = \begin{cases} 1/\theta & \text{if } 0 \leq \mathbf{x} \leq \theta \\ 0 & \text{otherwise} \end{cases}$$

$$\theta_{MLE} = \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^{n} 1/\theta$$

To maximize the likelihood function we must minimize the value of θ as they have an inverse relation.
Since $f_\theta(x) = 0 \, for \, \theta \leq max \, x_n$, the θ has to be at least max x_n

Therefore the maximum likelihood estimate of θ is max$(x_1, ..., x_n)$

4. **Maximum likelihood estimation 2** [10 points]

Consider the following probability density function:

$$f_\theta(x) = 2\theta x e^{-\theta x^2}$$

where θ is a parameter and x is positive real number.

Using the same notation as in exercise 3, compute the maximum likelihood estimate of θ.
(hint: you may simplify computations by proving that the maximizer of $f_\theta(x_1, x_2, \ldots, x_n)$ is also the maximizer of $log[f_\theta(x_1, x_2, \ldots, x_n)]$)

Answer.
Assuming the observations are independent and identically distributed and taking log likelihood for simplification.

Taking first derivative of $LL(\theta;x)$ function w.r.t θ and equate it to 0

$$f_\theta(x) = 2\theta x e^{-\theta x^2}$$

$$L(\theta;x) = f_\theta(x_1, x_2, x_3..x_n|\theta)$$

$$= f_\theta(x_1|\theta) f_\theta(x_2|\theta) f_\theta(x_3|\theta)...f_\theta(x_n|\theta)$$

$$LL(\theta;x) = log(f_\theta(x_1))log(f_\theta(x_2))log(f_\theta(x_3))...log(f_\theta(x_n))$$

$$= log(2\theta x_1 e^{-\theta x_1^2})log(2\theta x_2 e^{-\theta x_2^2})log(2\theta x_3 e^{-\theta x_3^2})...log(2\theta x_n e^{-\theta x_n^2})$$

$$= log(2\theta x_1) + log(e^{-\theta x_1^2}) + log(2\theta x_2) + log(e^{-\theta x_n^2})...log(2\theta x_1) + log(e^{-\theta x_n^2})$$

$$\frac{\partial LL(\theta;x)}{\partial \theta} = \frac{2x_1}{2\theta x_1} - x_1^2 + \frac{2x_2}{2\theta x_2} - x_2^2 + ... \frac{2x_n}{2\theta x_n} - x_n^2 = 0$$

$$= \frac{n}{\theta} - (x_1^2 + x_2^2 + ... + x_n^2) = 0$$

$$\theta = \frac{n}{x_1^2 + x_2^2 + ... + x_n^2}$$

5. ***k*-nearest neighbors** [10 points]

   Let $D = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ be a set of n independent labelled samples drawn using the following sampling process:

   - the label of each $\mathbf{x}_i$ is drawn randomly with 50% probability for each of the two classes
   - x_i is drawn uniformly in S^+ if its label is positive, and uniformly in S^- otherwise

   Where S^+ and S^- are two **unit** hyperspheres whose centers are 10 units apart.

(a) Show that if k is odd the average probability of error of the k-NN classifier is given by

$$P_n(e) = \frac{1}{2^n} \sum_{j=0}^{(k-1)/2} \binom{n}{j}.$$

Answer.
There is no overlap in the positive and negative labels as the two units hyperspheres are 10 units apart. To predict a class label for a new test case, we need more than (k+1)/2 classes belonging from one of the label.
Also, P(positive) = P(negative) = $\frac{1}{2}$

$$P_n(e) = P(\text{true label is } S^+ \text{ when more frequent is } S^-)$$
$$+ P(\text{true label is } S^- \text{ when more frequent is } S^+)$$

$$= 2P(\text{true label is } S^+\text{, less labelled are } S^+ \text{ and more frequent is } S^-)$$

$$= 2P(\text{S}^+)\ \text{P}(S^+ \text{ labelled less than k-1/2 and } S^- \text{ labelled more than half of k})$$

$$= (2)(1/2)(\text{choosing } 0,..,\text{j } S^+ \text{ values})(\text{prob of } S^+ \text{ values for } 0,..,\text{j})$$
$$(\text{prob of } S^- \text{ values for j+1},..,\text{n})$$

$$= (1)(\binom{n}{0}\binom{n}{1}..\binom{n}{k-1/2})(\frac{1}{2}\frac{1}{2}\frac{1}{2}..\text{j times})(\frac{1}{2}\frac{1}{2}\frac{1}{2}..\text{n-j times})$$

$$= \sum_{j=0}^{(k-1)/2} (\binom{n}{j})(\frac{1}{2})^j(\frac{1}{2})^{n-j}$$

$$= \frac{1}{2^n} \sum_{j=0}^{(k-1)/2} \binom{n}{j}$$

(b) Show that in this case the single-nearest neighbor classifier ($k = 1$) has a lower error rate than the k-NN classifier for $k > 1$.

Answer.

$$
\begin{aligned}
for\, k = 1, P(e) &= \frac{1}{2^n} \sum_{j=0}^{(0)} \binom{n}{0} \\
&= \frac{1}{2^n} \\
for\, k > 1, P(e) &= \frac{1}{2^n} \sum_{j=0}^{(k-1)/2} \binom{n}{j} \\
&= \frac{1}{2^n} \binom{n}{0} \binom{n}{1} \binom{n}{2} \dots \binom{n}{(k-1)/2}
\end{aligned}
$$

Since for k $>$ 1, the positive values will be added to $\frac{1}{2^n}$

Therefore, P(e) for k = 1 $<$ P(e) for k $>$1

(c) If k is allowed to increase with n but is restricted by $k \leq a\sqrt{n}$ (for some constant a), show that $P_n(e) \to 0$ as $n \to \infty$.

Answer. Finding an upper bound on the binomial distribution to prove that P(e) → 0 as n → ∞

As per tail bounds inequality for a binomial distribution for a function F

$$
F(k, n, p) = \binom{n}{k} p^k (1-p)^{n-k}, F(k, n, p) \leq \text{exp(-2 } \tfrac{(np-k)^2}{n} \tag{6}
$$

We can use 6 in our case

$$
\frac{1}{2^n} \sum_{j=0}^{(k-1)/2} \binom{n}{j} \leq exp - 2\frac{(np-k)^2}{n}
$$

For the function $e^{-x} \to 0$, x $\to \infty$

$$\lim_{n\to\infty} 2\frac{(n(1/2)-(k-1)/2)^2}{n}$$

$$\lim_{n\to\infty} 2\frac{(\frac{n-(k-1)}{2})^2}{n}$$

$$\lim_{n\to\infty} \frac{n^2+k^2+1-2k-2nk+2n}{2n}$$

Since k $\leq$ a$\sqrt{n}$

$$\lim_{n\to\infty} n/2+1/2n-a/\sqrt{n}+\mathit{constant}$$

Clearly this is going to $\infty,\ when,\ n\to\infty$

$Hence,\ P(e)\to 0,\ when\ n\to\infty$

6. **Gaussian Mixture** [10 points]

Let $\mu_1,\mu_2 \in \mathbb{R}^2$, and let Σ_1,Σ_2 be two 2x2 positive definite matrices (i.e. symmetric with positive eigenvalues).
We now introduce the two following pdf over $\mathbb{R}^2$:

$$f_{\mu_1,\Sigma_1}(\mathbf{x}) = \frac{1}{2\pi\sqrt{det(\Sigma_1)}}e^{-\frac{1}{2}(\mathbf{x}-\mu_1)^T\Sigma_1^{-1}(\mathbf{x}-\mu_1)}$$

$$f_{\mu_2,\Sigma_2}(\mathbf{x}) = \frac{1}{2\pi\sqrt{det(\Sigma_2)}}e^{-\frac{1}{2}(\mathbf{x}-\mu_2)^T\Sigma_2^{-1}(\mathbf{x}-\mu_2)}$$

These pdf correspond to the multivariate Gaussian distribution of mean μ_1 and covariance Σ_1, denoted $\mathcal{N}_2(\mu_1,\Sigma_1)$, and the multivariate Gaussian distribution of mean μ_2 and covariance Σ_2, denoted $\mathcal{N}_2(\mu_2,\Sigma_2)$.

We now toss a balanced coin Y, and draw a random variable X in $\mathbb{R}^2$, following this process : if the coin lands on tails $(Y=0)$ we draw X from $\mathcal{N}_2(\mu_1,\Sigma_1)$, and if the coin lands on heads $(Y=1)$ we draw X from $\mathcal{N}_2(\mu_2,\Sigma_2)$.

Calculate $\mathbb{P}(Y = 0|X = \mathbf{x})$, the probability that the coin landed on tails given $X = \mathbf{x} \in \mathbb{R}^2$, as a function of μ_1, μ_2, Σ_1, Σ_2, and $\mathbf{x}$. Show all the steps of the derivation.

Answer. Using Bayes Theorem

$$\mathbb{P}(Y = 0|X = x) = \frac{\mathbb{P}(X = x|Y = 0)\mathbb{P}(Y = 0)}{\mathbb{P}(X)}$$

$$= \frac{\mathbb{P}(X = x|Y = 0)\mathbb{P}(Y = 0)}{\mathbb{P}(X = x|Y = 0)\mathbb{P}(Y = 0) + \mathbb{P}(X = x|Y = 1)\mathbb{P}(Y = 1)}$$

$$= \frac{\frac{1}{2\pi\sqrt{det(\Sigma_1)}}e^{-\frac{1}{2}(\mathbf{x}-\mu_1)^T\Sigma_1^{-1}(\mathbf{x}-\mu_1)}}{\frac{1}{2\pi\sqrt{det(\Sigma_1)}}e^{-\frac{1}{2}(\mathbf{x}-\mu_1)^T\Sigma_1^{-1}(\mathbf{x}-\mu_1)} + \frac{1}{2\pi\sqrt{det(\Sigma_2)}}e^{-\frac{1}{2}(\mathbf{x}-\mu_2)^T\Sigma_2^{-1}(\mathbf{x}-\mu_2)}}$$

$$= \frac{1}{1 + \frac{\sqrt{det(\Sigma_1)}}{\sqrt{det(\Sigma_2)}}e^{-\frac{1}{2}[(\mathbf{x}-\mu_2)^T\Sigma_2^{-1}(\mathbf{x}-\mu_2)-(\mathbf{x}-\mu_1)^T\Sigma_1^{-1}(\mathbf{x}-\mu_1)]}}$$

Homework 1 - Practical part - REPORT

Graduates 10 pts

Question. 5. Implement two functions **ErrorRate.hard_parzen** and **ErrorRate.soft parzen** that compute the error rate (i.e. the proportion of missclassifications) of the HardParzen and SoftRBFParzen algorithms. The expected behavior is as follows :

test_error = ErrorRate(x_train, y_train, x_val, y_val) initiates the class and stores the training and validation sets, where x_train and x_val are matrices with 4 feature columns, and y_train and y_val are arrays containing the labels.

test_error.hard_parzen(h) takes as input the window parameter h and returns as a float the error rate on x_val and y_val of the HardParzen algorithm that has been trained on x_train and y_train.

test_error.soft_parzen(σ) works just like with Hard Parzen, but with the SoftRBFParzen algorithm.

Then, include in your report a single plot with two lines:

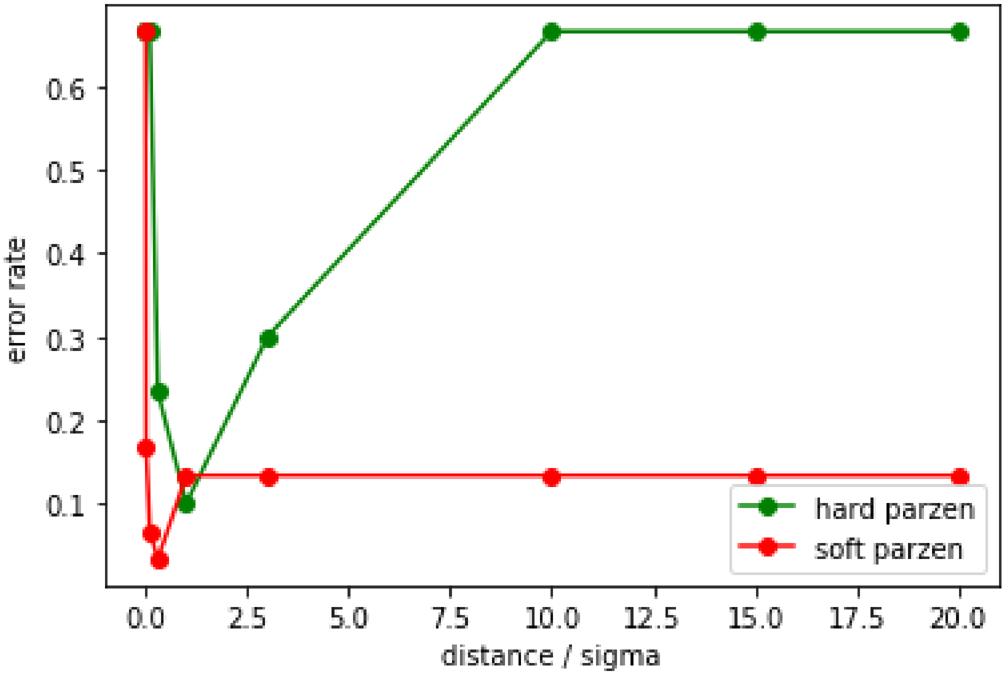
1. Hard Parzen window's classification error on the validation set of iris, when trained on the training set (see question 4) for the following values of h:

$$h \in \{0.001, 0.01, 0.1, 0.3, 1.0, 3.0, 10.0, 15.0, 20.0\}$$

2. RBF Parzen's classification error on the validation set of iris, when trained on the training set (see question 4) for the following values of σ:

$$\sigma \in \{0.001, 0.01, 0.1, 0.3, 1.0, 3.0, 10.0, 15.0, 20.0\}$$

The common x-axis will represent either h or σ. Always label your axes and lines in the plot! Give a detailed discussion of your observations.



Answer. As evident, soft parzen has a lower error rate when compared to hard parzen because it takes a weighted vote for all the training points.
Soft Parzen: The bandwidth or the sigma exhibits a strong influence on the resulting estimate. A large width will over smooth the density and mask the structure in the data whereas a small bandwidth will yield a density estimate that is spiky and very hard to interpret. For σ=0.3 the error is minimized between the estimated density and the true density giving an optimal solution. But once we increase the sigma, the error increases and then becomes constant because the weight of all training points will not change with the further increase in bandwidth/width.

Hard Parzen: Similarily, the radius h plays an important role in evaluating the performance of our method. Our classifier becomes blind to the overall data when the h is too small whereas high value of h will smoothen the decision boundaries and give a more erratic prediction as the classifier tends to get biased towards the dominant class with more training points being added for a vote. On further increase of the radius, the error rate seems to become constant because by then we had already covered all the points in the training data and there are no more points left to change the error rate.

Graduates 5 pts

Question. 7. Include in your report a discussion on the running time complexity of these two methods. How does it vary for each method when the hyperparameter h or σ changes? Why ?

Answer.

Both the algorithms have a run time complexity of $O(\text{n})$ where n is the number of training data. Since the distance has to be calculated on all the training points at the time of prediction, the value of radius h or bandwidth σ will have no affect on the time complexity.

Although both the methods have the same complexity, but in real time the hard parzen runs quite faster than soft parzen because of less computations.

Graduates 10 pts

Question. 9. Similar to Question 5, compute the validation errors of Hard Parzen classifiers trained on 500 random projections of the training set, for

$$h \in \{0.001, 0.01, 0.1, 0.3, 1.0, 3.0, 10.0, 15.0, 20.0\}$$

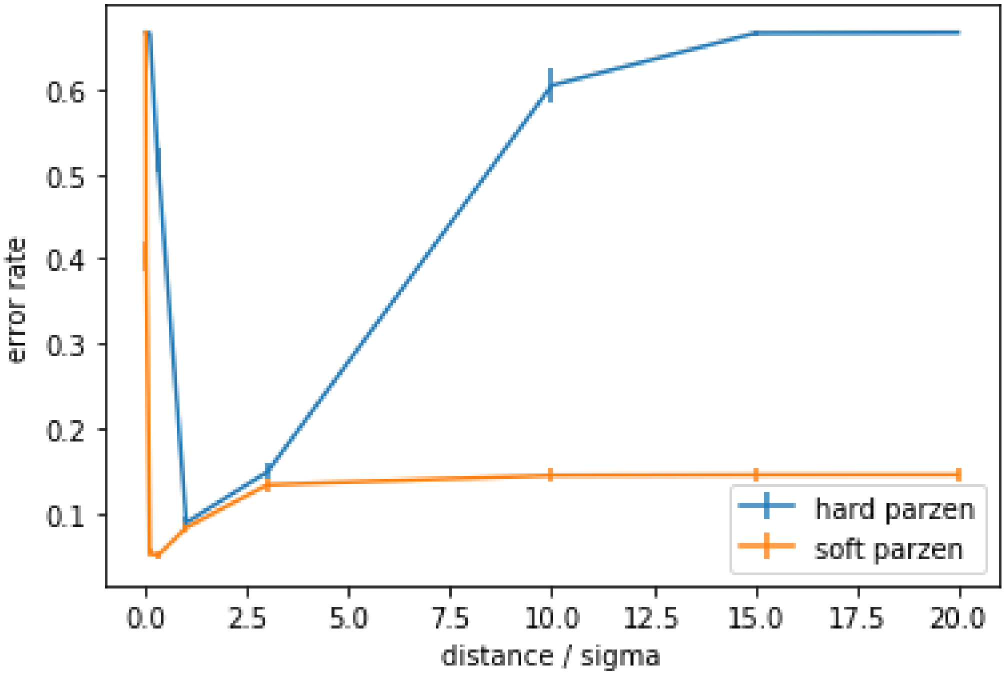
The validation errors should be computed on the projected validation set, using the same matrix A. To obtain random projections, you may draw A as 8 independent variables drawn uniformly from a gaussian distribution of mean 0 and variance 1.

You can for example store these validation errors in a 500×9 matrix, with a row for each random projection and a column for each value of h.

Do the same thing for RBF Parzen classifiers, for

$$\sigma \in \{0.001, 0.01, 0.1, 0.3, 1.0, 3.0, 10.0, 15.0, 20.0\}$$

Plot and include in your report in the same graph the average values of the validation errors (over all random projections) for each value of h and σ, along with error bars of length equal to $0.2\times$ the standard deviations.How do your results compare to the previous ones?



Answer.

The random projection from a gaussian distribution doesnt seem to have any affect on the error rate plot as the results in this graph seems to corroborate the one we had earlier. The error for soft parzen is lower than that of hard parzen. As usual, the hard parzen error rises with increase in radius because it takes too many training data points in consideration for prediction and then becomes constant when all the points are considered. Whereas in the soft parzen, the constant error rate is achieved much in advance because all the training samples are in the same region even if we increase the bandwidth anymore.