CS6690: Pattern Recognition Assignment #2

Group 3: Akshay & Suthirth

September 26, 2014

1 Bayesian Classifiers

According to Bayes Theorem, for a dataset x with classes ω_i,
Probability of a datapoint belonging to class ω_i is defined as:

$$P(\omega_i|x) = \frac{(P(x|\omega_i)P((\omega_i))}{P(x)} \tag{1}$$

- Here, $P(x|\omega_i)$ is known as the class likelihood.
  To estimate this value, we require the distribution of ω_i. Based on the central limit theorem, we can assume that this would be Gaussian distribution for large datasets.
- The value $P(\omega_i)$ is the class prior and is calculated using:

  $$P(\omega_i) = N_i/N \tag{2}$$

  This term becomes irrelevant if the classes have equal probabilities.
- P(x) is termed as 'evidence' and can be calculated as:

  $$P(x) = \sum_i P(x|\omega_i)P(\omega_i) \tag{3}$$

2 Gaussian Likelihood Distribution

For multi-dimensional data, the Gaussian Distribution is:

$$P(x;\mu,\Sigma) = \frac{1}{2\pi^{k/2}|\Sigma|^{1/2}} e^{-(x-\mu)^T\Sigma^{-1}(x-\mu)} \tag{4}$$

where

- μ is the mean
- Σ is the covariance matrix

The above parameters are calculated for the following cases:

2.1 Data

Red - Class 1, Green - Class 2, Blue - Class 3, Cyan - Class 4
Black (solid) - 1st Eigenvector
Black (dashed) - 2nd Eigenvector

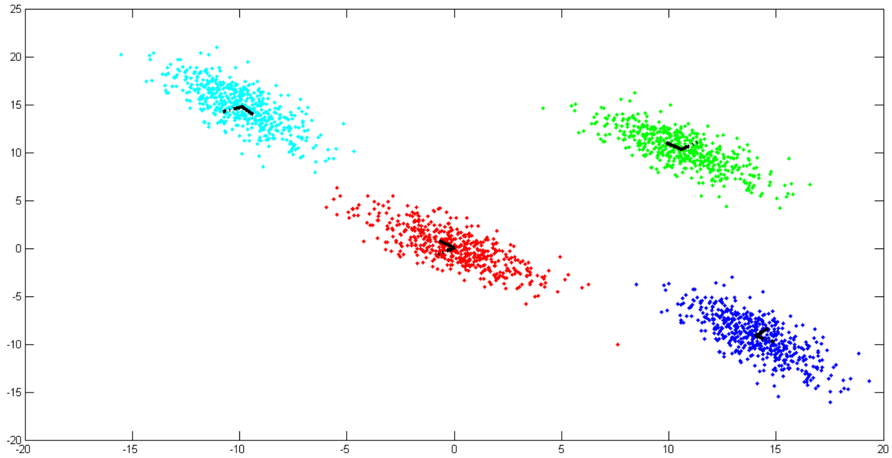


Figure 1: Linearly Separable Data

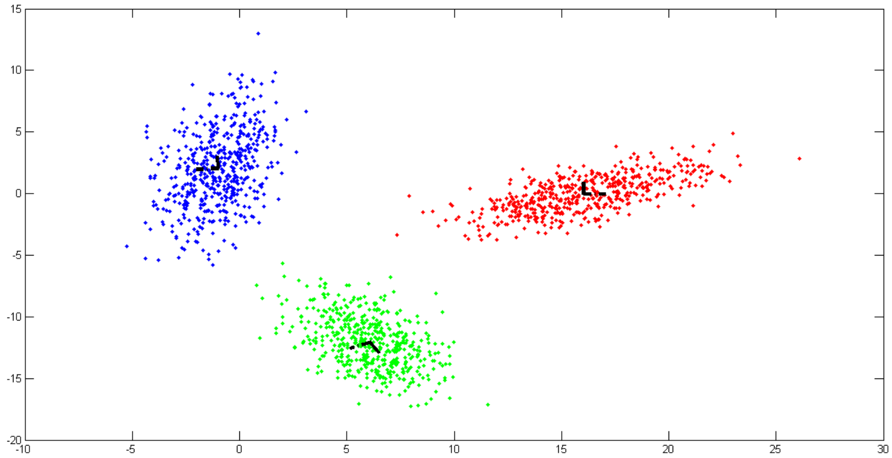


Figure 2: Non-linearly Separable Data

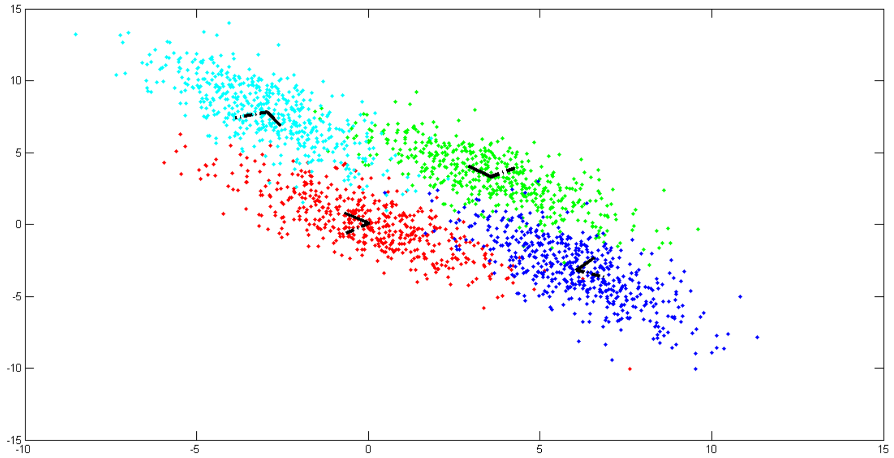


Figure 3: Overlapping Data

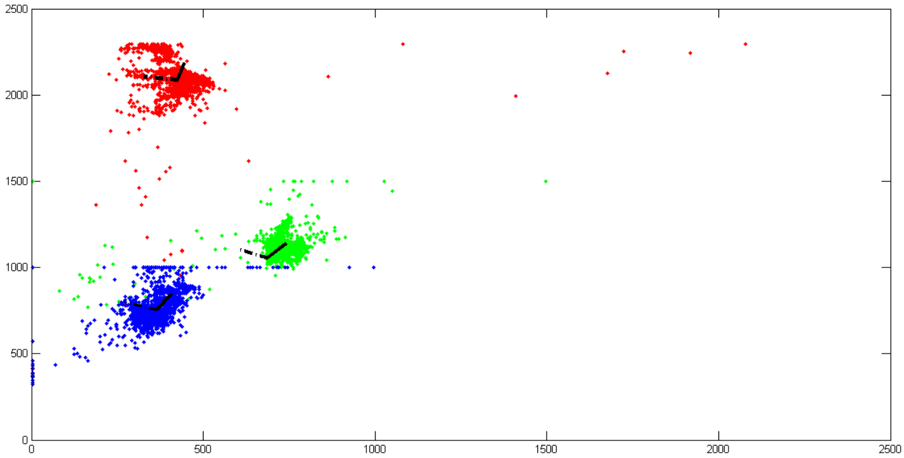
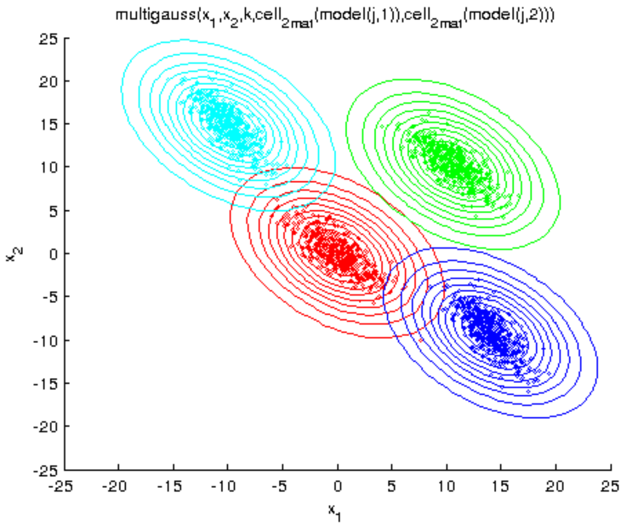


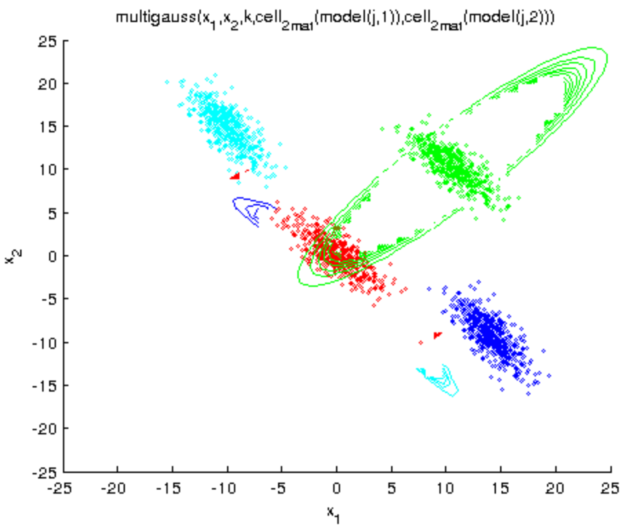
Figure 4: Real World Data: Vowel utterance formant frequencies F1 and F2

2.2 Bayes Classifier with Covariance same for all classes



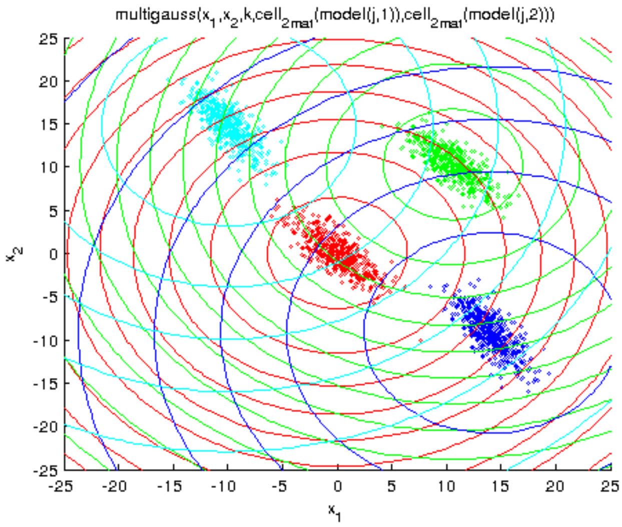
For a bayesian classifier with a common covariance matrix, we combine the matrices by taking the mean of all the class covariance matrices. Geometrically this will result in a linear rotation of the contours, while at the same time, scaling the axes of the contours.

2.3 Bayes Classifier with Covariance different for all classes



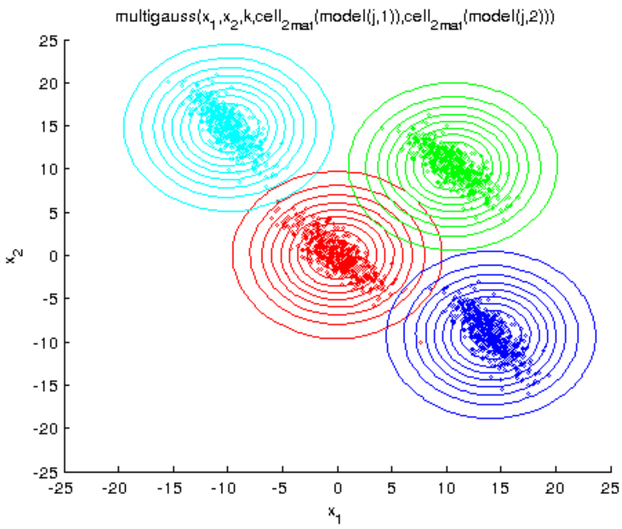
In this case, we take the class matrices and find out standard covariance to get the covariance matrix of each class. This acts as the C in our formula

2.4 Naive Bayes Classifier with C $= \sigma^2 * I$



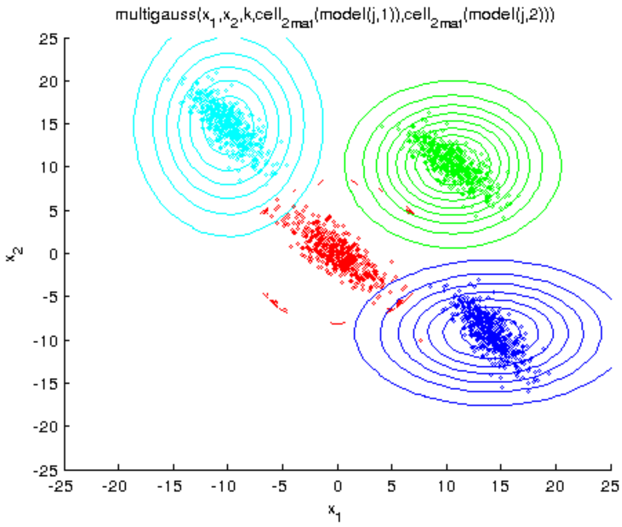
To get one single value of σ, we use a weighted mean for all the features of each class and come up with a common $\sigma_i I$ matrix; We then take a weighted mean of these matrices to come up with one single $\sigma^2 I$ matrix

2.5 Naive Bayes Classifier with C same for all classes



We follow the same principle as the non-naive case, but with features being independent of each other, thus setting off diagonal elements to 0

2.6 Naive Bayes Classifier with C different for all classes



We follow the same principle as the non-naive case, but with features being independent of each other, thus setting off diagonal elements to 0

3 Bayes Classification

If $P(\omega_1|x) > P(\omega_2|x)$ then x belongs to class ω_1
If $P(\omega_1|x) < P(\omega_2|x)$ then x belongs to class ω_2

Using equation (1), this can be written as:

$$P(x|\omega_1)P(\omega_1) \gtrless P(x|\omega_2)P(\omega_2) \tag{5}$$

The value x_0 at which the RHS and LHS is called the threshold value.

This classification rules minimizes number of misclassifications.

Based on this classification, the following quantities can be defined:

$TruePositiveRate = \frac{\Sigma TruePositive}{\Sigma ConditionPositive}$

$FalsePositiveRate = \frac{\Sigma FalsePositive}{\Sigma ConditionNegative}$

A plot of these two quantities is known as a DET Curve.

4 Experiments

4.1 Decision Boundaries

Following plots describe the decision boundaries for various datasets with different Bayesian classifiers.

For every figure,
Row 1: Bayes with: (L) Same covariance for all classes, (R) Different covariance for all classes
Row 2: Naive Bayes with: (L) $C = \Sigma^2 * I$, (R) Same C for all classes
Row 3: Naive Bayes with different C for all classes

Legends:
Red - Class 1
Green - Class 2
Blue - Class 3
Cyan - Class 4
White - $\mu_1 - \mu_2, \mu_3, \mu_4$
Yellow - Decision Boundary b/w Class 1 and 2
Magenta - Decision Boundary b/w Class 1 and 3
Cyan (line) - Decision Boundary b/2 Class 1 and 4

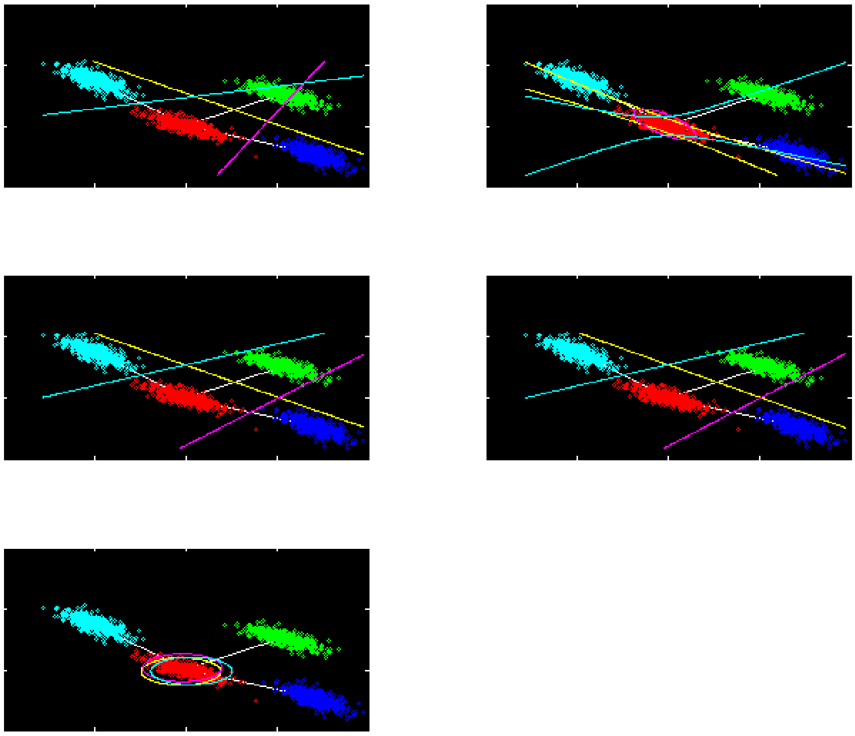


Figure 5: Decision Boundary for Dataset 1

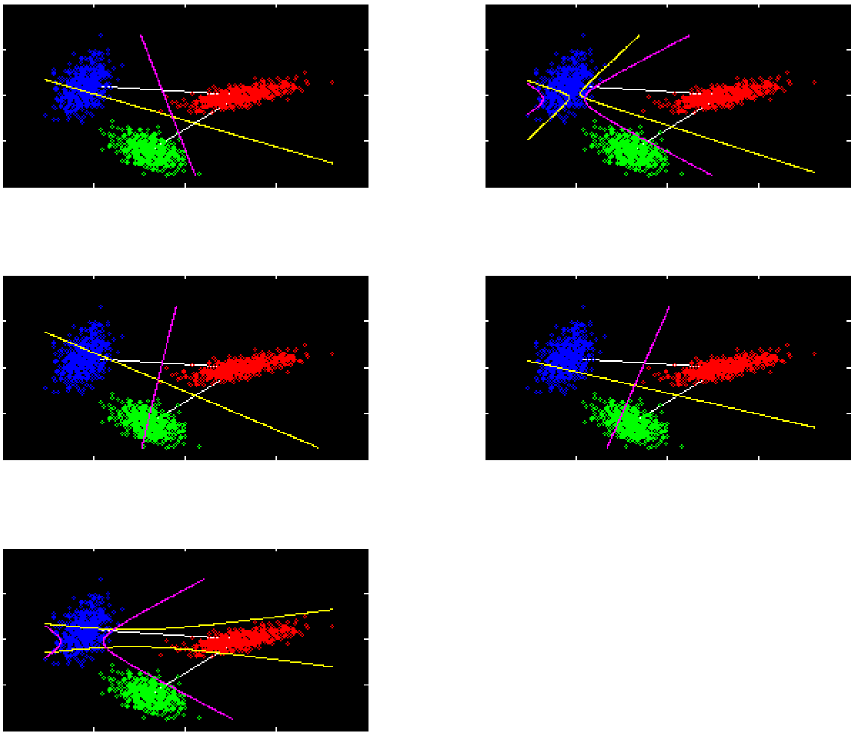


Figure 6: Decision Boundary for Dataset 2

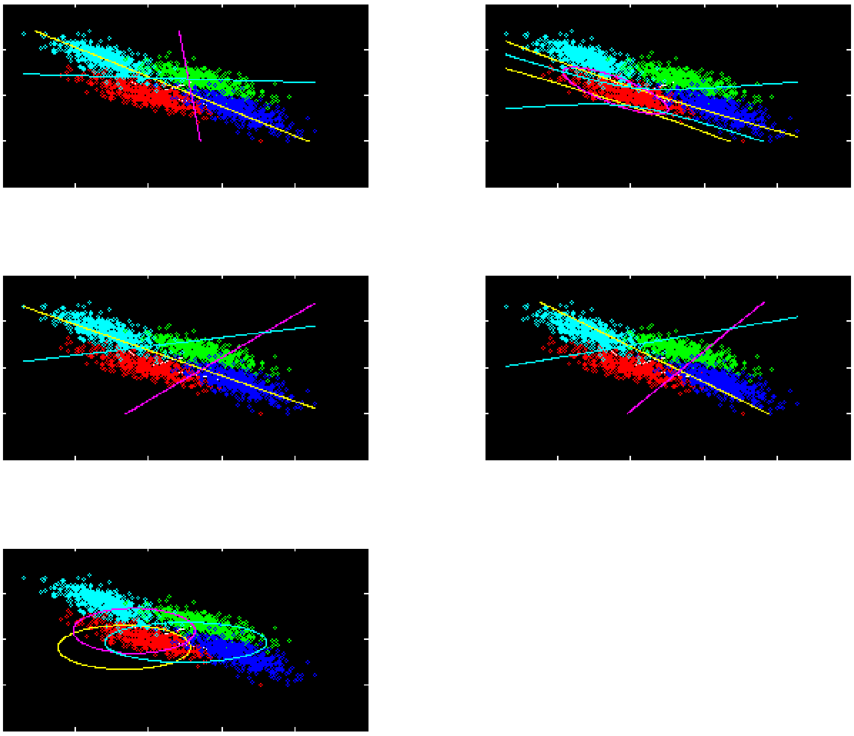


Figure 7: Decision Boundary for Dataset 3

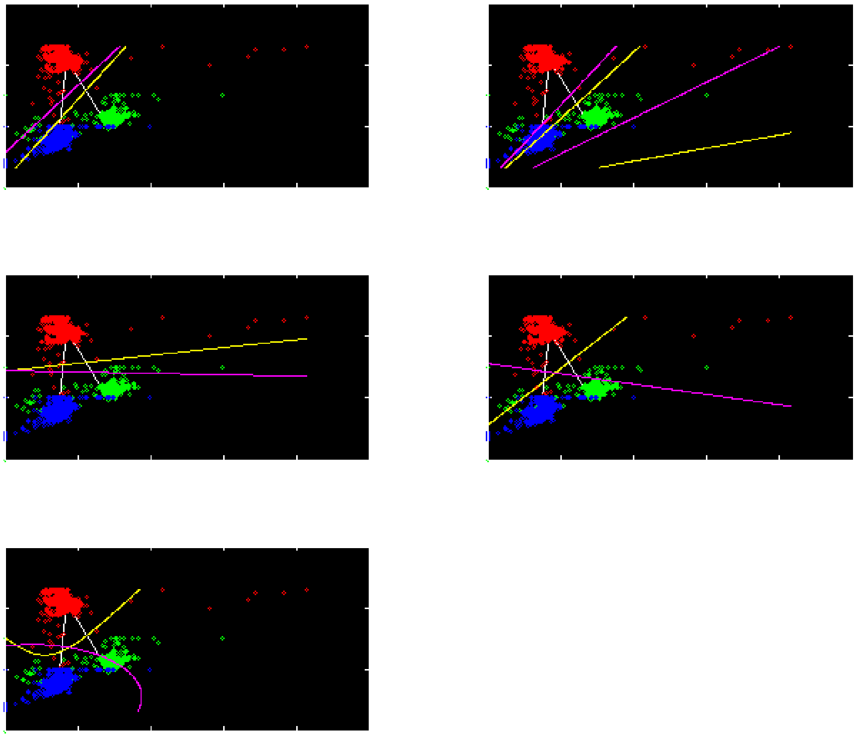


Figure 8: Decision Boundary for Dataset 4

4.2 DET Curves

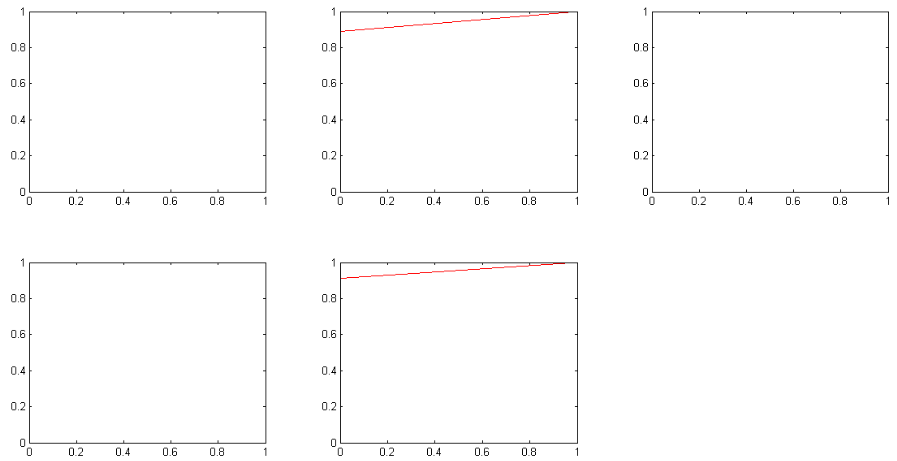


Figure 9: DET Curves for Linearly Separable Data

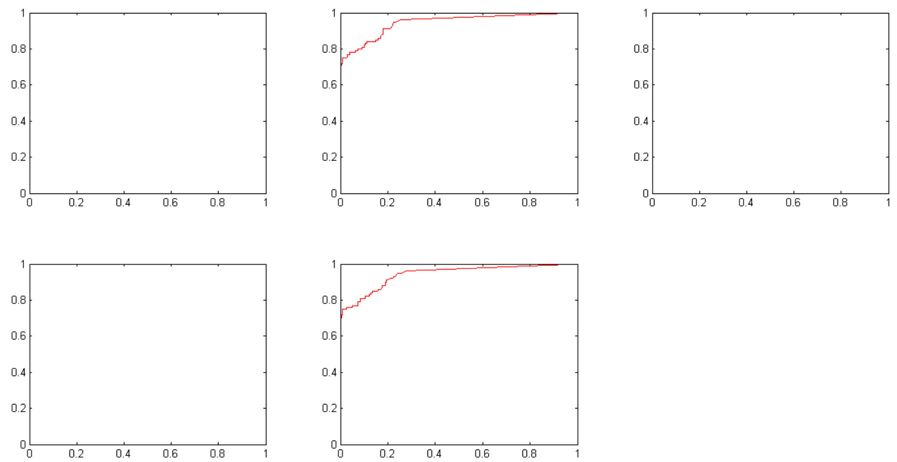


Figure 10: DET Curves for Non-linearly Separable Data

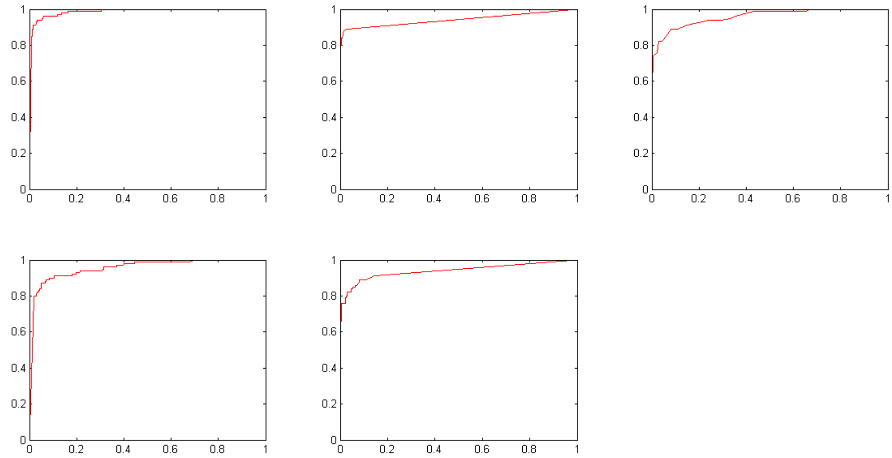


Figure 11: DET Curves for Overlapping Data

5 Confusion Matrix

5.1 Case 1

$$LS = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$$

$$OD = \begin{bmatrix} 91 & 0 & 1 & 2 \\ 0 & 92 & 13 & 1 \\ 3 & 2 & 86 & 0 \\ 6 & 6 & 0 & 97 \end{bmatrix}$$

5.2 Case 2

$$LS = \begin{bmatrix} 71 & 0 & 0 & 0 \\ 10 & 100 & 0 & 0 \\ 8 & 0 & 100 & 0 \\ 11 & 0 & 0 & 100 \end{bmatrix}$$

$$OD = \begin{bmatrix} 64 & 0 & 0 & 0 \\ 12 & 95 & 13 & 4 \\ 14 & 2 & 87 & 0 \\ 10 & 3 & 0 & 96 \end{bmatrix}$$

5.3 Case 3

$$LS = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$$

$$OD = \begin{bmatrix} 86 & 0 & 1 & 2 \\ 0 & 86 & 14 & 4 \\ 6 & 7 & 85 & 0 \\ 8 & 7 & 0 & 94 \end{bmatrix}$$

5.4 Case 4

$$LS = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$$

$$OD = \begin{bmatrix} 86 & 0 & 1 & 4 \\ 0 & 87 & 14 & 2 \\ 6 & 7 & 85 & 0 \\ 8 & 6 & 0 & 94 \end{bmatrix}$$

5.5 case 5

$$LS = \begin{bmatrix} 84 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 6 & 0 & 100 & 0 \\ 10 & 0 & 0 & 100 \end{bmatrix}$$

$$OD = \begin{bmatrix} 66 & 0 & 0 & 0 \\ 2 & 89 & 14 & 4 \\ 14 & 5 & 86 & 0 \\ 18 & 6 & 0 & 96 \end{bmatrix}$$