

*A Project Report*

*submitted by*

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*under the guidance of*  
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## THESIS CERTIFICATE

This is to certify that the thesis entitled , submitted by **D Akshay Rangasai**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work carried out by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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I want to thank myself for this completely pointless endeavour and my parents for paining me constantly about this. This is in the end, quite depressing.

## ABSTRACT

Most materials have two regimes of operation when it comes to the relationship between stress and strain of the material, namely linear and non-linear. While phenomenon and material characterization in the linear regime of operation is pretty well understood, the non-linear regime is not as well understood. This is an intriguing part of the problem as materials that undergo plastic deformation and fatigue loading operate under this non-linear regime, and characterization of these properties help in various manufacturing processes.

This study aims to statistically model and extract relevant parameters to measure non-linearity and its effects on a material by the use of ultrasonic waves, which provide a high strain rate, but very low strain, which is ideal to test the material without changing any of its properties at the current state. We first characterize parameters through harmonics generation and then proceed to non-linear wave mixing, a technique which gives us spatial specificity in our measurements.

The forward model was first built by creating a Finite Difference Time Domain (FDTD) solution to a set of differential equations that represent two dimensional non-linear wave propagation in an isotropic solid medium in a euclidean coordinate system. Wave mixing was simulated using a transverse and a longitudinal wave mixing in collinear path, with a phased array simulated as the transducer. Sensitivity analysis was performed for this solution and this formed the basis of our inverse model that helped predict material parameters.

The inverse model for the forward model was first built using linear regression and the results were compared with a statistical learning technique. We used Gaussian Process modelling to model the predictive model, which we further used to build the inverse model. To evaluate the model, noise was added to the measurements at various Signal to Noise Ratio (SNR) and the error percentage was measured. This model proved to be sufficient for the inverse model. From this, we could effectively estimate model parameters from wave mixing measurements.

# TABLE OF CONTENTS

<b>ACKNOWLEDGEMENTS</b>	<b>i</b>
<b>ABSTRACT</b>	<b>ii</b>
<b>LIST OF TABLES</b>	<b>v</b>
<b>LIST OF FIGURES</b>	<b>vi</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Background . . . . .	1
1.1.1 The stress-strain curve . . . . .	1
1.1.2 Non-Destructive Testing . . . . .	2
1.1.3 Wave Propagation and Mixing . . . . .	2
1.1.4 Statistical Inversion and Learning . . . . .	3
1.2 Outline of the Report . . . . .	3
1.3 Applications of present work . . . . .	3
<b>2 Literature Review</b>	<b>4</b>
2.1 Introduction . . . . .	4
<b>3 Description Of Simulations</b>	<b>5</b>
3.1 2D FDTD Simulation in Cartesian Coordinates . . . . .	5
3.2 Governing Equations . . . . .	5
3.2.1 Discretization of the wave-equation . . . . .	6
3.2.2 Numerical Considerations . . . . .	8
3.3 Simulation . . . . .	9
3.3.1 Sources . . . . .	9
3.3.2 Simulation Parameters . . . . .	11
3.4 Simulation Results . . . . .	11

3.4.1	Snapshots of Simulation . . . . .	11
3.4.2	Validation of Simulation . . . . .	13
<b>4</b>	<b>Sensitivity Analysis</b>	<b>16</b>
<b>5</b>	<b>Inverse Model</b>	<b>17</b>
<b>6</b>	<b>Summary and Future Work</b>	<b>18</b>
<b>A</b>	<b>Appendix</b>	<b>19</b>
A.1	Solver Code . . . . .	19
A.2	Problem Formulation Code . . . . .	28
A.3	Material Data Setting Code and Constants . . . . .	34
A.4	Code for Automating Simulations . . . . .	38
A.5	Code to Analyse Sensitivity . . . . .	40
A.6	Inverse Model Code . . . . .	42
A.7	Defaults Code . . . . .	44
<b>B</b>	<b>Material Properties Used in Simulations</b>	<b>45</b>

## LIST OF TABLES

3.1	General Numerical Properties of Simulation . . . . .	11
3.2	Pulse Properties . . . . .	11

## LIST OF FIGURES

1.1	The Stress Strain Curve for a Ductile Material . . . . .	2
3.1	Schematic of the solid material as an FDTD grid . . . . .	7
3.2	Schematic of the setup the FDTD simulation is mimicking . . . . .	7
3.3	Longitudinal Pulse Excited at Source . . . . .	10
3.4	Transverse Pulse Excited at Source . . . . .	10
3.5	Snapshot of the Longitudinal Wave . . . . .	11
3.6	Snapshot of the Transverse Wave . . . . .	12
3.7	Snapshot of the Transverse Wave . . . . .	12
3.8	A-scan of the longitudinal wave . . . . .	13
3.9	A-scan of the transverse wave . . . . .	14
3.10	Zoomed A-scan of transverse wave . . . . .	14
3.11	Fourier Transform of the New Wave Generated . . . . .	15
3.12	Simulation Results From DE solver[8] . . . . .	15
3.13	Zoomed solution and Fourier Transform of the solution . . . . .	15



# **CHAPTER 1**

## **Introduction**

The subject of the present work deals with the propagation, non-linear mixing effects of high-frequency shear and longitudinal waves which help us characterize material properties. This section is a brief introduction containing the background, outline and applications of the presented work.

### **1.1 Background**

The stress-strain curve for most ductile materials starts in a linear relationship and moves into a non-linear relationship with a few invariants that define the said relationship. Determining these constants is of great use to characterize the material and predict its behaviour under various conditions and also optimize processes with respect to these constants. While the physical relationship between the linear constants and their estimation has been studied to a great extent, the non-linear constants are more difficult to estimate. Our work aims to estimate these non-linear constants through statistical estimation techniques.

#### **1.1.1 The stress-strain curve**

The significant points from fig 1.1.1 are as follows:

1. A - Proportional Limit
2. B - Elastic Limit
3. y - Yield Point
4. u - Ultimate Tensile Strength
5. f - Fracture point

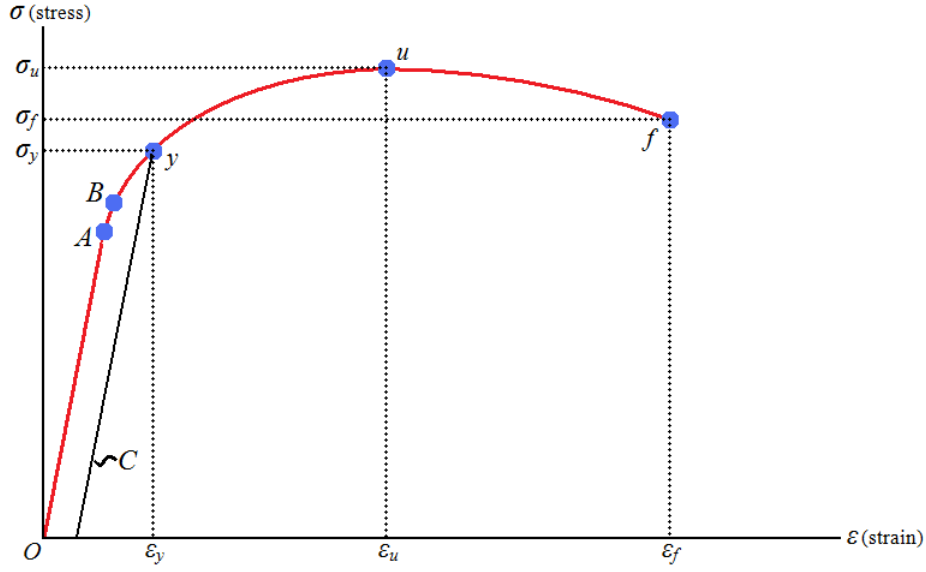


Figure 1.1: The Stress Strain Curve for a Ductile Material

### 1.1.2 Non-Destructive Testing

There are multiple methods of figuring out the elastic constants of a material, most significant of them being destructive methods, where the material is stressed to its Ultimate Tensile Strength and then, a curve is fit to get the second order constants of the material. This results in the material specimen being destroyed and deemed useless. This method is good for laboratory conditions, but not in a real-world scenario.

To account for this, we employ a method of Non-Destructive testing using ultrasonics. Ultrasonic waves have the characteristic of extremely high strain rates, but the magnitude of strain itself is minimal, and the changes to material properties are thus negligible. These high strain rates result in interesting phenomenon occurring, which in turn helps us estimate parameters of the material, without any physical damage to the specimen.

### 1.1.3 Wave Propagation and Mixing

One of the main concepts that are used in this study are that of wave propagation in solid media and mixing of waves in linear and non-linear zones. To solve the differential equations involved, we use FDTD simulations. We employed multiple solvers to evaluate the equations and multiple approaches to solve the problem. This will be discussed in detail in upcoming chapters.

### **1.1.4 Statistical Inversion and Learning**

The forward model is inverted by using purely statistical techniques. We employ various techniques from Support Vector Machines (SVM), Gaussian Mixture Models (GMMs), and Gaussian Processes. The mathematics and results will be discussed in detail in the coming chapters.

## **1.2 Outline of the Report**

This report is organized into 6 chapters.

1. The current chapter gives a basic introduction to the project and explains very briefly what we hope to achieve and techniques we've employed with a little bit of background information.
2. Chapter 2 deals with Literature Review and what we worked on and the subsequent results of the same with reasons as to what method we finally adopted and why with a discussion about the same.
3. Chapter 3 describes the construction of the FDTD model for the forward problem and the collinear wave mixing approach that is taken by us and describes the problem and solution in detail.
4. Chapter 4 explores the sensitivity analysis of our constructed forward model with respect to various parameters of interest and and exploratory analysis of the inverse model.
5. Chapter 5 validates the inverse model and also describes the pitfalls of the model. We make the model more real world friendly and check its performance.
6. Chapter 6 summarizes our work and has a section on how this project can be pursued along with suggestions for experimental validation.

## **1.3 Applications of present work**

The present work has wide range of uses from aircraft industry to the shipping industry. A manufacturing specific application of this current technique will be in the estimation of material parameters in forming process and cold working processes where materials undergo plastic deformation.

This technique will help us understand material deformation better and give us a physical insight into what the constants mean and at the same time help improve existing processes and diagnose issues in current processes.

## **CHAPTER 2**

### **Literature Review**

#### **2.1 Introduction**

Many papers were reviewed

## CHAPTER 3

### Description Of Simulations

This chapter presents the description of the simulations used for the current research. The objective of the simulation is to estimate the amplitude and frequency of the resultant generated wave post mixing. This is primarily due to the non-linear behaviour of the material in the mixing zone. The following are the steps followed to achieve this objective.

#### 3.1 2D FDTD Simulation in Cartesian Coordinates

Finite Difference Time Domain (FDTD) method is widely used to solve wave propagation problems. In the present work the FDTD algorithm is implemented in 2D Orthogonal Cartesian Coordinates. [3]

#### 3.2 Governing Equations

The primary governing equations for the 2D FDTD simulator are the non-linear wave propagation equations in the material. Let  $u(y, t)$ ,  $v(y, t)$  describe the motion of wave propagation for a longitudinal and transverse wave in a solid. The equations can be derived as such from the standard equation of motion in a solid material.

$$\frac{\partial \sigma_{xy}}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2} \quad (3.1)$$

$$\sigma_{xy} = \sigma_{yx} = \frac{\partial u}{\partial y} (\mu + m \frac{\partial v}{\partial y}) \quad (3.2)$$

$$\frac{\partial \sigma_{yy}}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} \quad (3.3)$$

In terms of the Lamé constants  $\lambda$  and  $\mu$ , and the third order elastic constants  $l$ ,  $m$ , and  $n$  (the Mur-naghan coefficients), the stress components can be related to the displacement gradients through + [4]

$$\sigma_{yy} = (\lambda + 2\mu) \frac{\partial v}{\partial y} + (l + 2m) \frac{\partial v^2}{\partial y} + \frac{m}{2} \frac{\partial u^2}{\partial y} \quad (3.4)$$

Substituting the previous two relations to the standard wave equation, we get

$$\frac{\partial^2 u}{\partial t^2} - c_t^2 \frac{\partial^2 u}{\partial y^2} = \beta_t c_t^2 \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} \right) \quad (3.5)$$

$$\frac{\partial^2 v}{\partial t^2} - c_l^2 \frac{\partial^2 v}{\partial y^2} = \beta_l c_l^2 \frac{\partial v}{\partial y} \frac{\partial^2 v}{\partial y^2} + \beta_t c_t^2 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \quad (3.6)$$

Where,  $c_L = \sqrt{(\lambda + 2\mu)\rho}$ ,  $c_T = \sqrt{\mu\rho}$

$$\beta_L = 3 + \frac{2(l + m)}{\lambda + 2\mu} \quad (3.7)$$

$$\beta_T = \frac{\lambda + 2\mu}{\mu} + \frac{m}{\mu} \quad (3.8)$$

### 3.2.1 Discretization of the wave-equation

This wave equation, can be discretized as a FDTD grid, which using the staggered method of FDTD solver along with the central differencing technique results in a equation similar to this:

$$\begin{aligned} \frac{u_k^{n+1} - 2 * u_k^n + u_k^{n-1}}{\Delta t^2} = & c_T^2 \frac{u_{k+1}^n - 2u_k^n + u_{k-1}^n}{\Delta y^2} \\ & + \beta_T c_T^2 \left( \frac{u_{k+1}^n - 2u_k^n + u_{k-1}^n}{\Delta y^2} \frac{v^n k + 1 - v_{k-1}^n}{2\Delta y} \right. \\ & \left. + \frac{v_{k+1}^n - 2v_k^n + v_{k-1}^n}{\Delta y^2} \frac{u^n k + 1 - u_{k-1}^n}{2\Delta y} \right) \end{aligned} \quad (3.9)$$

$$\begin{aligned} u_k^{n+1} = & 2 * u_k^n - u_k^{n-1} + \Delta t^2 c_T^2 \left( \frac{u_{k+1}^n - 2u_k^n + u_{k-1}^n}{\Delta y^2} \right. \\ & \left. + \beta_T \left( \frac{u^n k + 1 - u_{k-1}^n}{2\Delta y} \frac{u_{k+1}^n - 2u_k^n + u_{k-1}^n}{\Delta y^2} + \frac{v_{k+1}^n - 2v_k^n + v_{k-1}^n}{\Delta y^2} \frac{u^n k + 1 - u_{k-1}^n}{2\Delta y} \right) \right) \end{aligned} \quad (3.10)$$

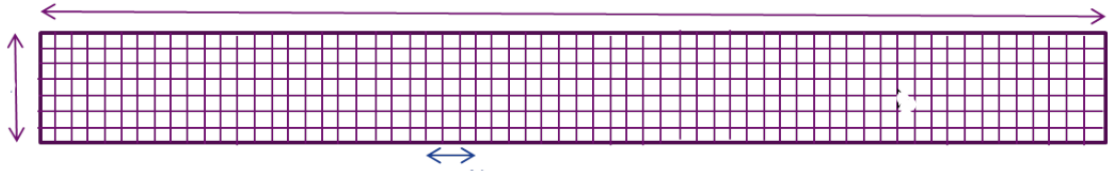


Figure 3.1: Schematic of the solid material as an FDTD grid

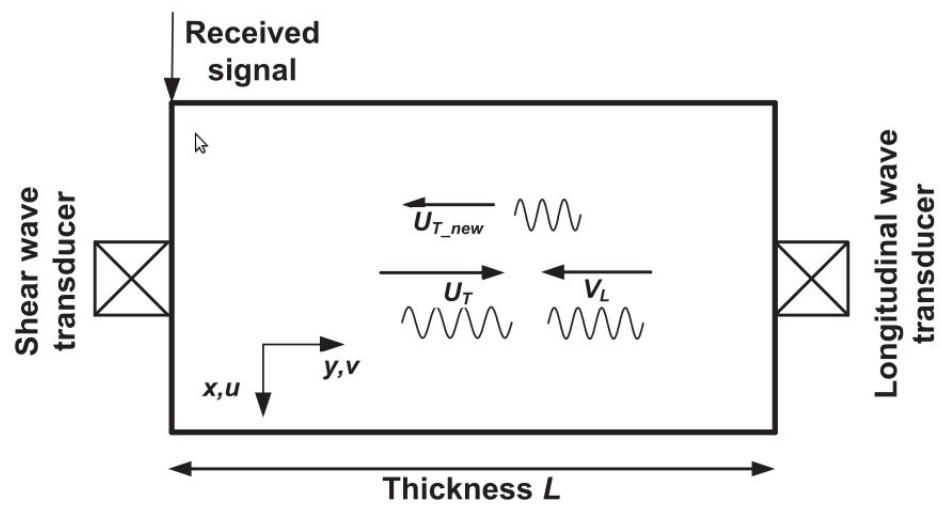


Figure 3.2: Schematic of the setup the FDTD simulation is mimicking

### 3.2.2 Numerical Considerations

#### Stability Criteria

A finite difference scheme is stable if the errors made at one time step of the calculation do not cause the errors to increase as the computations are continued. A neutrally stable scheme is one in which errors remain constant as the computations are carried forward. If the errors decay and eventually damp out, the numerical scheme is said to be stable. If, on the contrary, the errors grow with time the numerical scheme is said to be unstable. The stability of numerical schemes can be investigated by performing von Neumann stability analysis. For time-dependent problems, stability guarantees that the numerical method produces a bounded solution whenever the solution of the exact differential equation is bounded. Stability, in general, can be difficult to investigate, especially when the equation under consideration is non-linear.

In the FDTD for sound waves, the main stability criteria is the **Courant** condition. This constraint ensures that the errors in the numerical simulation get damped out to give a fairly accurate estimate of the actual solution.

$$\Delta t \leq \frac{1}{C} \frac{1}{\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}}} \quad (3.11)$$

$$\Delta x = \Delta y, \Delta t \leq \frac{1}{C} \frac{1}{\sqrt{2}} \quad (3.12)$$

#### Boundary Conditions

The simulation needs a few boundary conditions to be set up, such that there is a solution to the equation that is solved. In this case, we assume the propagation of the wave as equal to the propagation of a wave through any standard wave guide. Since, we are assuming a metal-to-air interface, we have modeled this as a completely reflecting wave guide with no absorption. For other interfaces or absorption in the wave-guide, we could look at a Perfectly Matched Layers (PML) at the boundaries for complete absorption.

The wave guide is modeled as a free waveguide on all ends and the movement is not restricted anywhere. For such a scenario, in the case of plane wave propagation in solids, it is the particle displacement at the boundary which is free, and nothing else. In case the ends are not free to move, the particle displacements at the boundary are trivial. For both these cases, we see reflections, but there is a phase inversion that happens in the latter case. This is not of great importance in this context, but helps



validate simulations.

The conditions for the same implemented in the code is as follows.

$$u_{-1}^n = u_{-2}^n \quad (3.13)$$

$$v_{-1}^n = v_{-2}^n \quad (3.14)$$

Where  $-1, -2$  denote the grid points from the boundaries, with  $-1$  being the last grid point.

### Initial condition

Another important component of the boundary layer is basically the initial conditions of the simulation itself. This determines the initial state of the system and the excitement that is given to the system and determines the outcome of the simulation.

For this FDTD simulation, we need to excite the system with one transverse and one longitudinal wave on either end. As the system we have currently taken is collinear, the only criteria is the excitation, and nothing else. The initial wave forms must also be smooth[7] to prevent numerical dispersion and dissipation errors in the simulations [5] [6]

**Mesh and Sampling Sampling** To further get a more accurate solution, we have to choose a suitable size of mesh for which the solution is acceptable and at the same time not too time consuming. The solution must be approximately right, without too much noise in the data. The solution is also sampled at a rate which is extremely high to prevent aliasing of the data. We sample at a rate that is proportional to the meshing, a criteria we get from the courant condition.

## 3.3 Simulation

### 3.3.1 Sources

There are two main sources in the simulations, one is a longitudinal wave with a specific frequency and another is a transverse wave coming in from the opposite direction. These are limited in time, and thus are pulses of waves. The pulses are raised cosine pulses with a pulse width of 10.

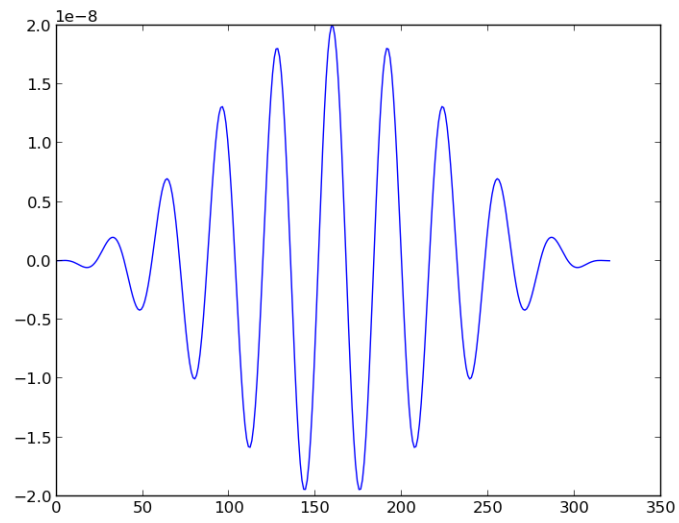


Figure 3.3: Longitudinal Pulse Excited at Source

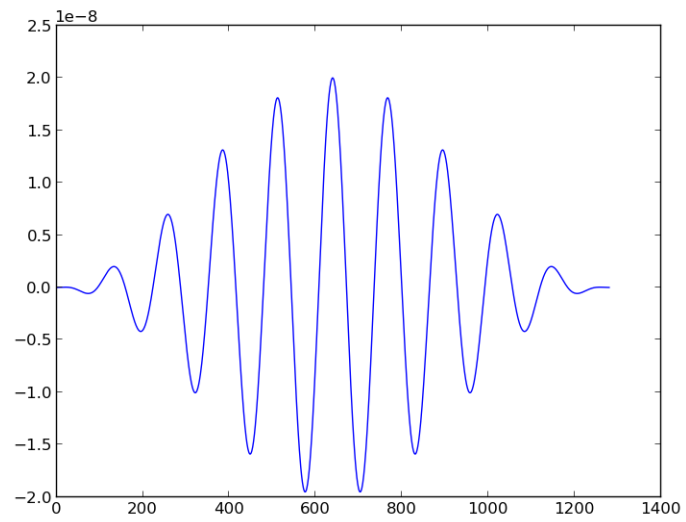


Figure 3.4: Transverse Pulse Excited at Source

### 3.3.2 Simulation Parameters

Property	Value
$\Delta t$	$3.125 \times 10^{-09}\text{s}$
$\Delta x$	$3.94 \times 10^{-05}\text{m}$
Sampling Frequency	$3.2 \times 10^8\text{Hz}$

Table 3.1: General Numerical Properties of Simulation

Property	Longitudinal	Transverse
Number of pulses	10	10
Pulse Frequency	10MHz	2.5MHz
Pulse Amplitude	$2 \times 10^{-8}\text{m}$	$2 \times 10^{-8}\text{m}$
Pulse Duration	$10^{-6}\text{s}$	$4 \times 10^{-6}\text{s}$
Pulse Velocity	$6299.5\text{ms}^{-1}$	$3100\text{ms}^{-1}$

Table 3.2: Pulse Properties

## 3.4 Simulation Results

### 3.4.1 Snapshots of Simulation

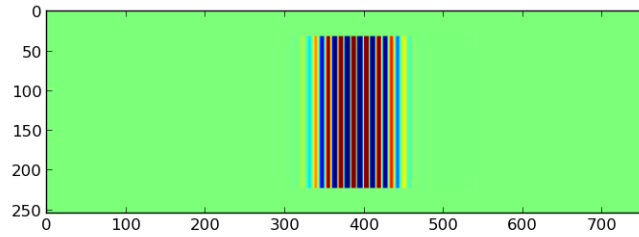


Figure 3.5: Snapshot of the Longitudinal Wave

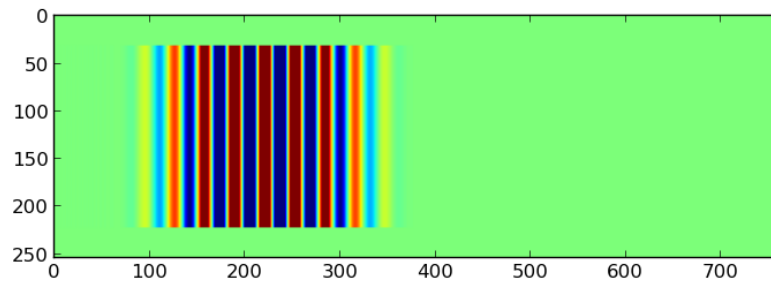


Figure 3.6: Snapshot of the Transverse Wave

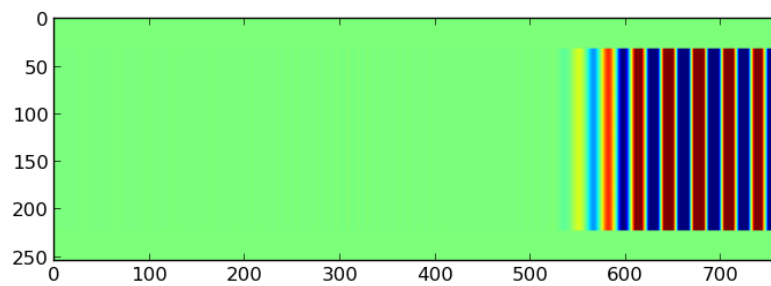


Figure 3.7: Snapshot of the Transverse Wave

### 3.4.2 Validation of Simulation

The solution from the simulation was obtained from an Amplitude Scan at both the sources. The wave forms and the fourier transforms of the waves after the A-scans were consistent with what is expected from an FDTD simulation. There was minimal dispersion or dissipation error and thus the simulation parameters were satisfactory. To check if the FDTD simulation is accurate enough to be used for more complex cases with the same solver and engine, we compared the results of our FDTD simulation with the one by Liu Et. al [8] which solved this case by the use of an ODE solver.

From the comparisons of the two solutions, we compared the scale of amplitude as well as the frequency of the generated wave. Due to our input being markedly different from that of the reference paper, the deviations in wave shape are acceptable. All the other criteria match with that of the reference paper. The comparisons and Validation plots are given below.

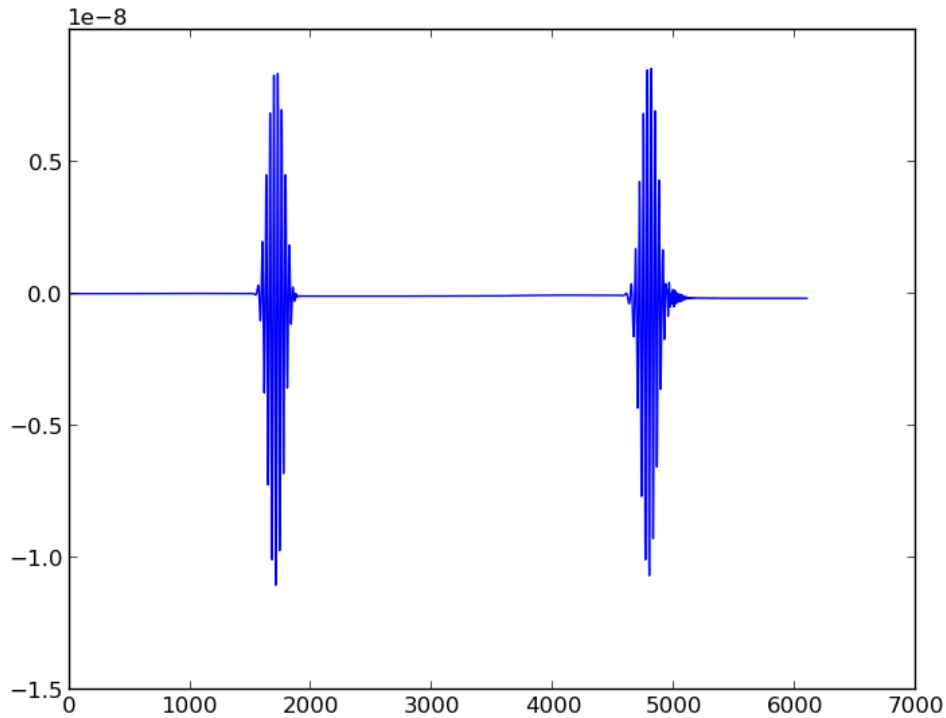


Figure 3.8: A-scan of the longitudinal wave

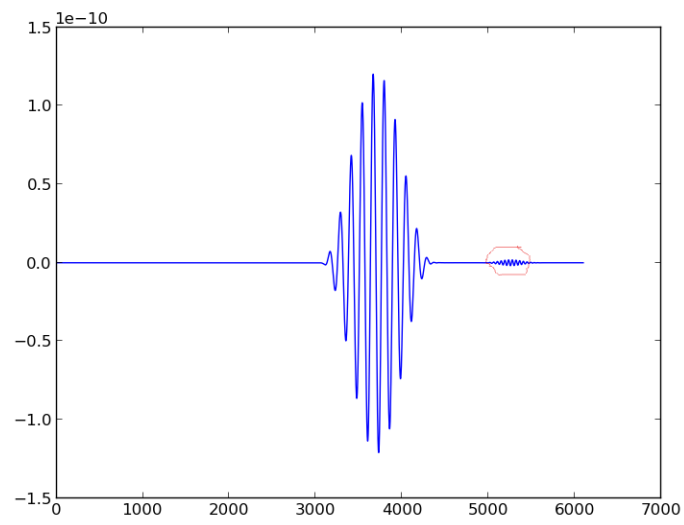


Figure 3.9: A-scan of the transverse wave

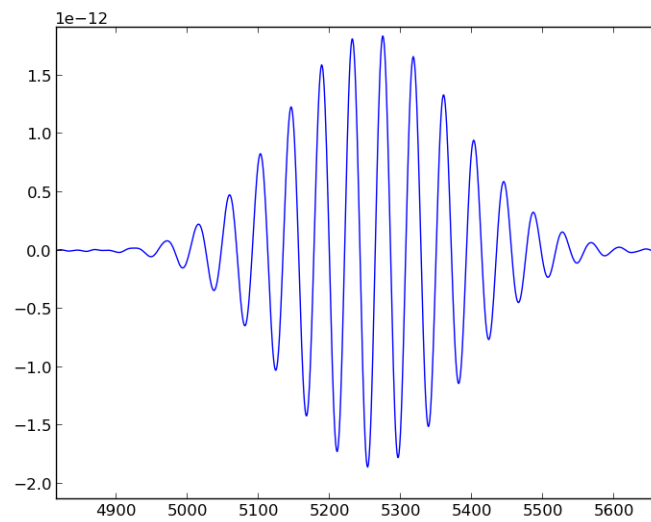


Figure 3.10: Zoomed A-scan of transverse wave

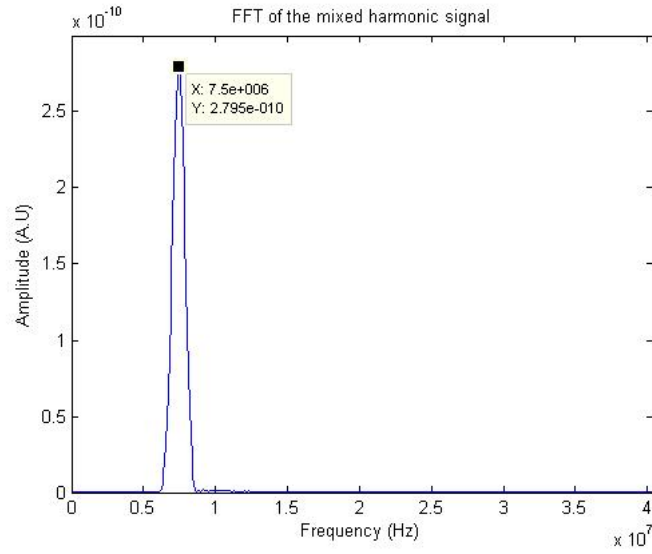


Figure 3.11: Fourier Transform of the New Wave Generated

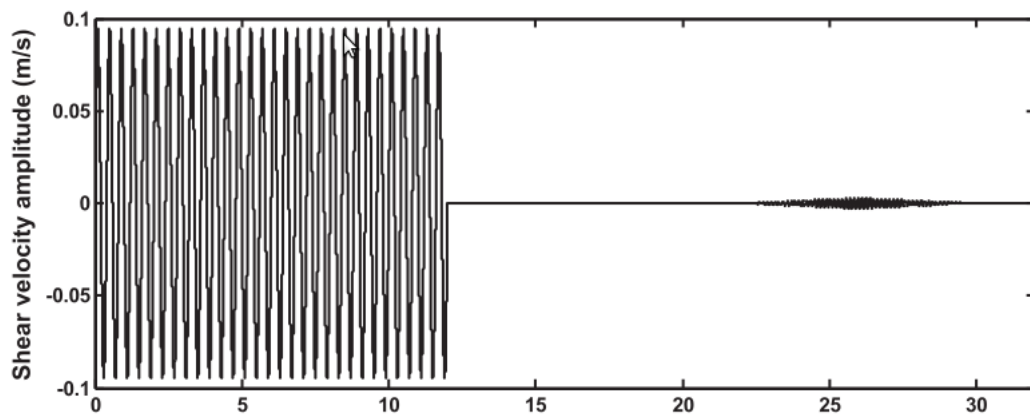


Figure 3.12: Simulation Results From DE solver[8]

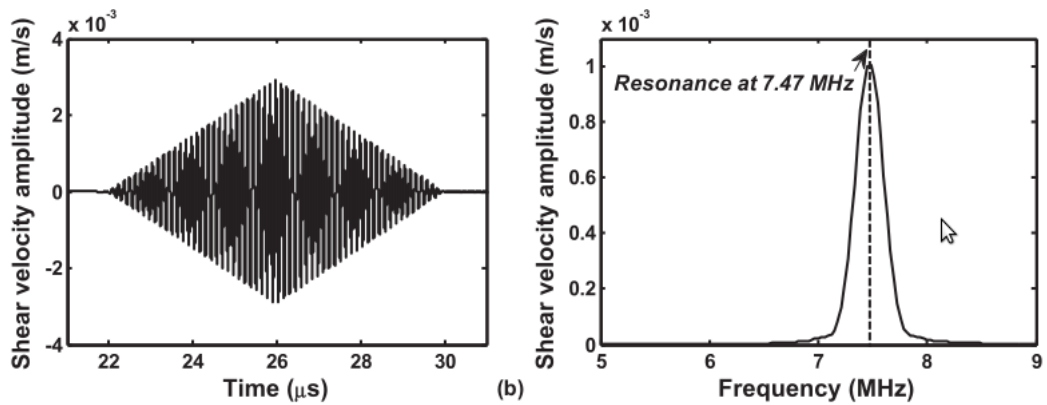


Figure 3.13: Zoomed solution and Fourier Transform of the solution from DE solver [8]

## **CHAPTER 4**

### **Sensitivity Analysis**



## **CHAPTER 5**

### **Inverse Model**

## **CHAPTER 6**

### **Summary and Future Work**

# APPENDIX A

## Appendix

### A.1 Solver Code

```
1 import numpy as np
2 import scipy as sp
3 import defaults as df
4 from math import sin, pi, cos
5 from matplotlib.pyplot import imshow, plot, show, draw, pause, clim,
    ↪ figure
6 import sys
7 from constants import *
8 transverse = []
9 longi = []
10
11 _transverse = 0
12 _longi = 0
13 #from matplotlib import figure
14 class Solver:
15     _transverse = 0
16     _longi = 0
17
18     Simulation = None
19     Location = None
20     Width = None
21     #Create a Movie Variable to calculate number of movies and plots
    ↪ , to bring them up when necessary. add arguments to put it
    ↪ in grid, instead of what's happening here. This is
    ↪ hardcoded waste.
```

```

22     def putMovie(self , pauseTime):
23         data = np.reshape(self.Simulation.Grid[:, :, 0, 1], (self.
                ↪ Simulation.ElementSpan[0], self.Simulation.ElementSpan
                ↪ [1]))          #          #
24         figure("Wave_Movie_Transverse")
25         imshow(data)
26         clim([-1e-8, 1e-8])
27         draw()
28         pause(pauseTime)
29
30         data = np.reshape(self.Simulation.Grid[:, :, 1, 1], (self.
                ↪ Simulation.ElementSpan[0], self.Simulation.ElementSpan
                ↪ [1]))          #          #
31         figure("Wave_Movie_Longitudinal")
32         imshow(data)
33         clim([-1e-8, 1e-8])
34         draw()
35         pause(pauseTime)
36
37
38
39     def putSource(self , i , frequency , index , waveType = 0):
40         #Multiply with gaussian to remove edge effects.
41
42         #Adding default Source
43
44         #waveType 0 - Transverse. 1 - Longitudinal
45
46         X_S = round(self.Location[0])
47         Y_S = slice(round(self.Simulation.ElementSpan[0]/2) - round(
                ↪ self.Width[1]/2) , round(self.Simulation.ElementSpan
                ↪ [0]/2) + round(self.Width[1]/2))
48

```

```

49      #Raised Cosine Pulse.
50
51      self.Simulation.Grid[Y_S,index,waveType,2] = (1-cos(2*pi*
           ↪ frequency*i*self.Simulation.Dt/self.Simulation.Pulses))
           ↪ *cos(2*pi*frequency*i*self.Simulation.Dt)*1e-8
52      return (1-cos(2*pi*frequency*i*self.Simulation.Dt/self.
           ↪ Simulation.Pulses))*cos(2*pi*frequency*i*self.
           ↪ Simulation.Dt)*1e-8
53
54      #Sine Pulse. Trying to recreate the paper
55      #self.Simulation.Grid[Y_S,index,waveType,2] = sin(2*pi*
           ↪ frequency*i*self.Simulation.Dt)*1e-8
56
57      #print self.Simulation.Grid[X_S, round(self.Simulation.
           ↪ ElementSpan[1]/2) - round(self.Width[0]/2) + 1,0,2]
58
59      #Line Sources only, currently. Multiple Sources must be
           ↪ accounted for, Must think of a matrix solution. So much
           ↪ fight for something that might not even work. Pain.
60      def setSource(self, Location = None, Width = None, Theta = None)
           ↪ :
61
62      if Location is None:
63          self.Location = [df.LOCATION*self.Simulation.Dimensions
           ↪ [0]/self.Simulation.Dx]
64      else :
65          self.Location.append(Location*self.Simulation.Dimensions
           ↪ [0]/self.Simulation.Dx)
66
67      if Width is None:
68          self.Width = [(df.WIDTH/self.Simulation.Dx)*D for D in
           ↪ self.Simulation.Dimensions]
69      else :

```

```

70         self.Width.append((Width/self.Simulation.Dx)*D for D in
           ↪ self.Simulation.Dimensions)
71
72     if Theta is None:
73         self.Location = [df.THETA]
74     else:
75         self.Location.append(Theta)
76
77
78
79     def Solve(self):
80         #First Equation We'll be solving will be the standard wave
           ↪ equation.
81         self.setSource()
82         #Setting the Source first. Now, let's solve the DE like a
           ↪ boss
83         #
84         _X = slice(0,self.Simulation.ElementSpan[0]-2)
85         X = slice(1,self.Simulation.ElementSpan[0]-1)
86         X_ = slice(2,self.Simulation.ElementSpan[0])
87
88         _Y = slice(0,self.Simulation.ElementSpan[1]-2)
89         Y = slice(1,self.Simulation.ElementSpan[1]-1)
90         Y_ = slice(2,self.Simulation.ElementSpan[1])
91
92         #_X indicates previous X coordinate and X_ indicts the one
           ↪ after
93         r_var = round(self.Simulation.Time/self.Simulation.Dt)
94         #print "Total Iterations are " , r_var
95         c_t2 = pow(self.Simulation.MaterialProperties.WaveVelocityT
           ↪ ,2)
96         c_l2 = pow(self.Simulation.MaterialProperties.WaveVelocityL
           ↪ ,2)

```

```

97
98     # sdata = sp.zeros((r_var,1))
99     for i in range(1,int(r_var)):
100         dv_y = (self.Simulation.Grid[X,Y_,1,1] - self.Simulation
101             ↪ .Grid[X,_Y,1,1])/(2*self.Simulation.Dx)
102         d2v_y = (self.Simulation.Grid[X,Y_,1,1] - 2*self.
103             ↪ Simulation.Grid[X,Y,1,1] + self.Simulation.Grid[X,
104             ↪ _Y,1,1])/pow(self.Simulation.Dx,2)
105         du_y = (self.Simulation.Grid[X,Y_,0,1] - self.Simulation
106             ↪ .Grid[X,_Y,0,1])/(2*self.Simulation.Dx)
107         d2u_y = (self.Simulation.Grid[X,Y_,0,1] - 2*self.
108             ↪ Simulation.Grid[X,Y,0,1] + self.Simulation.Grid[X,
109             ↪ _Y,0,1])/pow(self.Simulation.Dx,2)
110
111         #Solving for Displacements in the X directio
112
113         self.Simulation.Grid[X,Y,0,2] = 2*self.Simulation.Grid[X
114             ↪ ,Y,0,1] - self.Simulation.Grid[X,Y,0,0] + pow(self.
115             ↪ Simulation.Dt,2)*(c_t2*d2u_y + self.Simulation.
116             ↪ MaterialProperties.BetaT*c_t2*(dv_y*d2u_y + du_y*
117             ↪ d2v_y))
118
119         self.Simulation.Grid[X,Y,1,2] = 2*self.Simulation.Grid[X
120             ↪ ,Y,1,1] - self.Simulation.Grid[X,Y,1,0] + pow(self.
121             ↪ Simulation.Dt,2)*(c_l2*d2v_y + self.Simulation.
122             ↪ MaterialProperties.BetaL*c_l2*dv_y*d2v_y + self.
123             ↪ Simulation.MaterialProperties.BetaT*c_t2*du_y*d2u_y
124             ↪ )
125
126         self.Simulation.SourceSignal[i,0] = sum(self.Simulation.
127             ↪ Grid[:, -2,0,2])/self.Simulation.Grid.shape[1]
128
129
130

```

```

114
115     self.Simulation.SData[i,0] = sum(self.Simulation.Grid
    ↪ [:,1,1,2])/self.Simulation.Grid.shape[1]
116
117     #self.Simulation.SData[i,0] = sum(self.Simulation.Grid
    ↪ [:,1,0,2])/self.Simulation.Grid.shape[1]
118
119     #print self.Simulation.Grid[15,15,0,2]
120
121     #Boundary COnditions. Making the ends soft reflections.
    ↪ Let's see how that works out.
122
123     '''
124     if self.Simulation.Mixing is not True:
125         self.Simulation.Grid[-1,:,0,2] = self.Simulation.
    ↪ Grid[-2,:,0,2]
126     else:
127         # self.Simulation.Grid[:,0,1,2] = self.Simulation.
    ↪ Grid[:,1,1,2]
128         # self.Simulation.Grid[:,0,0,2] = self.Simulation.
    ↪ Grid[:,1,0,2]
129         self.Simulation.Grid[:, -1,0,2] = self.Simulation.
    ↪ Grid[:, -2,0,2]
130         # self.Simulation.Grid[:, -2,1,2] = self.Simulation.
    ↪ Grid[:, -1,1,2]
131
132         #Updates go Here
133         '''
134
135     if(i <= round(self.Simulation.Pulses*(1.0/(self.
    ↪ Simulation.WaveProperties.Frequency)))/self.
    ↪ Simulation.Dt)):
136         transverse.append(self.putSource(i,self.Simulation.

```



```

    ↪ WaveProperties.Frequency, 0, TRANSVERSE))
137     else:
138         if (self._transverse == 0):
139             plot(transverse)
140             show()
141             self._transverse = 1
142             self.Simulation.Grid[:,1,0,2] = self.Simulation.Grid
    ↪[:,0,0,2]
143             self.Simulation.Grid[:,−2,0,2] = self.Simulation.
    ↪Grid[:,−1,0,2]
144             #self.Simulation.Grid[:,0,1,2] = self.Simulation.
    ↪Grid[:,1,1,2]
145
146
147     if self.Simulation.Mixing == True:
148
149         if (i <= round(self.Simulation.Pulses*(1.0/(0.997*4*
    ↪self.Simulation.WaveProperties.Frequency))/self
    ↪.Simulation.Dt)):
150             longi.append(self.putSource(i,0.997*4*self.
    ↪Simulation.WaveProperties.Frequency,−1,
    ↪LONGITUDINAL))
151     else:
152         if (self._longi == 0):
153             plot(longi)
154             show()
155             self._longi = 1
156             self.Simulation.Grid[:,−1,1,2] = self.Simulation
    ↪.Grid[:,−2,1,2]
157             self.Simulation.Grid[:,0,1,2] = self.Simulation.
    ↪Grid[:,1,1,2]
158             #self.Simulation.Grid[:,−1,0,2] = self.
    ↪Simulation.Grid[:,−2,0,2]

```

```

159
160         self.Simulation.Grid[:, :, 1, 0] = self.Simulation.Grid
           ↪[:, :, 1, 1]
161         self.Simulation.Grid[:, :, 1, 1] = self.Simulation.Grid
           ↪[:, :, 1, 2]
162
163         self.Simulation.Grid[:, :, 0, 0] = self.Simulation.Grid
           ↪[:, :, 0, 1]
164         self.Simulation.Grid[:, :, 0, 1] = self.Simulation.Grid
           ↪[:, :, 0, 2]
165     #         print i
166         if i%round(0.05*r_var) == 0:
167             #print i
168             if self.Simulation.ViewMovie == True:
169                 self.putMovie(0.01)
170                 sys.stdout.write('=='*int(round(i/round(0.1*r_var))))
           ↪)
171
172             #p.plot.show()
173         #print self.Simulation.MaterialProperties.BetaL, self.
           ↪Simulation.MaterialProperties.BetaT, self.Simulation.
           ↪MaterialProperties.WaveVelocityL, self.Simulation.Dt
174
175     '''
176         figure("Source Signal")
177         plot(self.Simulation.SourceSignal)
178
179         pause(0.01)
180         figure("Non Linear Signal")
181         plot(self.Simulation.SData)
182         show()
183     '''
184

```

```

185 #         np.save("TotalSignal", self.Simulation.SourceSignal)
186 #         np.save("LinSignal", sdata)
187 def __init__(self, Simulation = None):
188     if Simulation is None:
189         raise ValueError("Simulation_Cannot_be_None._Please_
           ↳ Initialize_a_New_Simulation_to_proceed")
190     else:
191         self.Simulation = Simulation
192         self.Solve()
193
194 if __name__ == "__main__":
195     raise Exception("Cannot_run_file_as_a_standalone_file._Please_
           ↳ run_through_proper_initialized_channels")

```

## A.2 Problem Formulation Code

```
1 from data import waveProperties , materialProperties
2 import numpy as np
3 import scipy as sp
4 import matplotlib as mp
5 import defaults as df
6 import sys
7 from solver import Solver as sl
8 import scipy.io as sio
9 from matplotlib.pyplot import plot , figure
10
11
12 #
    ↪ #####
    ↪
13 #Rules of code: Class elements always begin with a capital letter.
    ↪ Defaults are always allcaps. Arguments to functions to mimic
    ↪ class members.
14 #
    ↪ #####
    ↪
15
16 class simulation:
17
18     def save(self , filename):
19         sio.savemat(filename , {"SData": self.SData , "SourceSignal":
            ↪ self.SourceSignal })
20
21     def setMixing(self , val):
22         self.Mixing = val
23
24     def setStep(self , Dx):
```

```

25         #Courant Condition check
26         return (Dx/self.MaterialProperties.WaveVelocityL)/2
27
28     def setMesh(self):
29
30         if self.Mesh == 0:
31             return (float)(self.WaveProperties.WaveLength/8.0)
32         elif self.Mesh == 1:
33             return (float)(self.WaveProperties.WaveLength/12.0)
34         elif self.Mesh == 2:
35             return (float)(self.WaveProperties.WaveLength/64.0)
36         elif self.Mesh == 3:
37             return (float)(self.WaveProperties.WaveLength/128.0)
38
39         #Time is of type float; Dimensions is a list of floats.
40
41
42     def setParam(self, paramName, value):
43
44         if paramName == 'l':
45             self.MaterialProperties.l = value
46             #self.MaterialProperties.BetaT = (self.
47                 ↪ MaterialProperties.Lambda + 2*self.
48                 ↪ MaterialProperties.Mu)/self.MaterialProperties.Mu +
49                 ↪ self.MaterialProperties.m/self.MaterialProperties.
50                 ↪ Mu
51
52             self.MaterialProperties.refreshParams()
53         if paramName == 'm':
54             self.MaterialProperties.m = value
55             self.MaterialProperties.refreshParams()
56
57         if paramName == 'BetaT':
58             self.MaterialProperties.BetaT = value

```

```

54
55     def getParam(self , paramName):
56
57         if paramName == 'l':
58             return self.MaterialProperties.l
59         if paramName == 'm':
60             return self.MaterialProperties.m
61         if paramName == 'BetaT':
62             return self.MaterialProperties.BetaT
63
64         return 0
65
66
67     def __init__(self , MaterialProperties = None, WaveProperties =
        ↪ None, Reflections = None, Dimensions = None, WaveGuide =
        ↪ None, Mesh = None, Pulses = None):
68
69         if MaterialProperties is None:
70             self.MaterialProperties = materialProperties()
71         else:
72             self.MaterialProperties = MaterialProperties
73
74         if WaveProperties is None:
75             self.WaveProperties = waveProperties()
76         else:
77             self.WaveProperties = WaveProperties
78
79         if Reflections is None:
80             self.Reflections = df.REFLECTIONS
81         else:
82             self.Reflections = Reflections
83
84         if Dimensions is None:

```

```

85         self.Dimensions = df.DIMENSIONS
86     else:
87         self.Dimensions = Dimensions
88
89     if WaveGuide is None:
90         self.WaveGuide = df.WAVEGUIDE
91     else:
92         self.WaveGuide = WaveGuide
93
94     if Mesh is None:
95         self.Mesh = df.MESH
96     else:
97         self.Mesh = Mesh
98
99     if Pulses is None:
100         self.Pulses = df.PULSES
101     else:
102         self.Pulses = Pulses
103
104     self.Time = 2*self.Reflections*self.Dimensions[1]/self.
        ↳ MaterialProperties.WaveVelocityL
105
106     #1D, 2D or 3D
107     self.DimensionCount = len(self.Dimensions)
108     ##         self.WaveProperties.WaveVelocity = self.MaterialProperties
        ↳ .WaveVelocity
109     self.WaveProperties.WaveLength = (float) (self.
        ↳ MaterialProperties.WaveVelocityL/self.WaveProperties.
        ↳ Frequency)
110     self.Mixing = False
111     self.Dx = self.setMesh()
112     self.Dt = self.setStep(self.Dx)
113

```

```

114         #print self.Dx
115         ##List of elementsb
116         self.ElementSpan = [round(X/self.Dx) for X in self.
            ↪ Dimensions]
117
118         #Append Dimensions
119         self.ElementSpan.append(3)
120         #Append Times
121         self.ElementSpan.append(3)
122
123         self.Grid = sp.zeros(tuple(self.ElementSpan), float)
124         self.NLGrid = sp.zeros(tuple(self.ElementSpan), float)
125         self.SourceSignal = sp.zeros((round(self.Time/self.Dt),1))
126         self.SData = sp.zeros((round(self.Time/self.Dt),1))
127         self.ViewMovie = False
128         self.viewPlot = True
129
130 def __init__():
131     args = sys.argv
132     args = [arg.replace('—',',') for arg in args]
133     names = []
134     sim = simulation()
135     print sim.Dt, sim.Dx, sim.MaterialProperties.WaveVelocityL, sim.
        ↪ WaveProperties.WaveLength
136     if 'mixing' in args:
137         sim.setMixing(True)
138     if 'movie' in args:
139         sim.ViewMovie = True
140     solution = sl(sim)
141
142     if 'noplot' in args:
143         pass
144     else:

```



```

145         figure(5)
146         plot(sim.SData)
147
148     if 'save' in args:
149         try:
150             ind = args.index('savenames')
151             names.append(args[ind+1])
152             names.append(args[ind+2])
153         except:
154             print "Using Default File names to save data"
155             names.append("TotalSignal")
156             names.append("NLinSignal")
157         sio.savemat(names[0],{names[0]:sim.SourceSignal})
158         sio.savemat(names[1],{names[1]:sim.SData})
159
160
161
162 if __name__ == "__main__":
163     __init__()

```

### A.3 Material Data Setting Code and Constants

```
1 import defaults as df
2 from math import sqrt
3
4
5 ## These classes are created to create a default set of elements. I
   ↪ will implement a file reader to get element data later.
   ↪ createing a new object of this type ensures that we get a nice
   ↪ default simulation. Let's hope this works. Solver is yet to be
   ↪ implemented. Sigh
6
7 class waveProperties:
8     def __init__(self, Frequency = None):
9         if Frequency is None:
10             self.Frequency = df.FREQUENCY
11         else:
12             self.Frequency = Frequency
13
14         self.WaveLength = None
15
16
17 class materialProperties:
18     def __init__(self, Mu = None, K = None, Rho = None, A = None, B
   ↪ = None, C = None, l = None, m = None, Lambda = None):
19
20         ##Initialize All defaults if none.
21
22         if Mu is None:
23             self.Mu = df.MU
24         else:
25             self.Mu = Mu
26
```

```

27         if K is None:
28             self.K = df.K
29         else:
30             self.K = K
31
32         if Rho is None:
33             self.Rho = df.RHO
34         else:
35             self.Rho = Rho
36
37         if A is None:
38             self.A = df.A
39         else:
40             self.A = A
41
42         if B is None:
43             self.B = df.B
44         else:
45             self.B = B
46
47         if C is None:
48             self.C = df.C
49         else:
50             self.C = C
51
52         if l is None:
53             self.l = df.l
54         else:
55             self.l = l
56
57         if m is None:
58             self.m = df.m
59         else:

```

```

60         self.m = m
61
62     if Lambda is None:
63         self.Lambda = df.Lambda
64     else:
65         self.Lambda = Lambda
66
67     self.WaveVelocityL = sqrt((self.Lambda + (2*self.Mu))/self.
        ↪ Rho)
68     self.WaveVelocityT = sqrt(self.Mu/self.Rho)
69     self.BetaL = 3 + 2*(self.l + 2*self.m)/(self.Lambda + 2*self
        ↪ .Mu)
70     self.BetaT = (self.Lambda + 2*self.Mu)/self.Mu + self.m/self
        ↪ .Mu
71
72     def refreshParams(self):
73
74
75         self.WaveVelocityL = sqrt((self.Lambda + (2*self.Mu))/self.
        ↪ Rho)
76         self.WaveVelocityT = sqrt(self.Mu/self.Rho)
77         self.BetaL = 3 + 2*(self.l + 2*self.m)/(self.Lambda + 2*self
        ↪ .Mu)
78         self.BetaT = (self.Lambda + 2*self.Mu)/self.Mu + self.m/self
        ↪ .Mu
79
80
81     class waveGuide:
82
83     def __init__(self, Boundary = None):
84         if Boundary is None:
85             self.Boundary = df.BOUNDARY
86         else:

```

```
87         self.Boundary = Boundary
88
89     ## Boundary Legend
90     ## 0 – All reflecting
91     ## 1 – Sides Reflecting Ends PML
92     ## 2 – Sides PML Ends Reflecting
93     ## 3 – Everything PML

```

```
1 LONGITUDINAL = 1
2 TRANSVERSE = 0
```

## A.4 Code for Automating Simulations

```
1 from formulation import simulation
2 from solver import Solver as sl
3
4 #Limit of L and M in terms of percentages. How do we combine this?
   → We need to run experiments, check correlations and all. Let's
   → see if it has any effect:w
5
6 __LIMIT = 10
7 __STEP = 1
8 for percent in range(-int(round(__LIMIT)), int(round(__LIMIT))+1,
   → __STEP):
9     sim = simulation()
10    oldl = sim.getParam('BetaT')
11    print percent/100.0
12    newl = oldl*(1 + (percent/100.0))
13    print oldl, newl
14    sim.setParam('BetaT', newl)
15    sim.setMixing(True)
16    sl(sim)
17    sim.save("%d.mat"%percent)
18
19 '''
20 for percent in range(-int(round(__LIMIT)), int(round(__LIMIT))+1,
   → __STEP):
21    sim = simulation()
22    oldl = sim.getParam('m')
23    print percent/100.0
24    newl = oldl*(1 + (percent/100.0))
25    print oldl, newl
26    sim.setParam('m', newl)
27    sim.setMixing(True)
```

```
28     sl(sim)
29     sim.save("Simulation_Save_m-%d_percent.mat"%percent)
30     '''
```

## A.5 Code to Analyse Sensitivity

```
1 import numpy as np
2 import scipy.io as sp
3 from matplotlib import pyplot as plt
4 import os
5
6 __DIR = "../data/sensitivity/tentoten"
7 __TOTALLENGTH = 2048
8 __STARTINDEX = 4900
9 __ENDINDEX = 5600
10 __PADDING = __TOTALLENGTH - (__ENDINDEX - __STARTINDEX)
11 __FILE = "amplitude_BetaT1010.txt"
12 files = [os.path.join(__DIR, f) for f in os.listdir(__DIR)]
13
14 fi = open(__FILE, 'w+')
15
16
17 def fft(signal):
18     fftsignal = np.zeros(__TOTALLENGTH)
19     #fftsignal[0:(__TOTALLENGTH - __PADDING)] = signal[__STARTINDEX:
20         ↪ __ENDINDEX]
21     fftsignal_2 = signal[__STARTINDEX:__ENDINDEX]
22     ftp = abs(np.fft.fft(fftsignal_2))
23     plot = plt.plot(ftp)
24     return plot
25
26 def ampcalc(data):
27     return abs(min(data) - max(data))
28
29 for f in files:
30     print f.split('/')
31     datafile = sp.loadmat(f)
```



```
31     fftplot = fft(datafile['SourceSignal'])
32     amplitude = ampcalc(datafile['SourceSignal'])
33     plt.savefig("%s.png"%f.split('/')[4])
34     plt.close()
35     fi.write('%s_%.25f\n'%(f.split('/')[4], amplitude))
```

## A.6 Inverse Model Code

```
1 from sklearn.gaussian_process import GaussianProcess as GMM
2 #from sklearn.svm import SVR as GMM
3 import numpy as np
4 import matplotlib.pyplot as plt
5 import scipy as sp
6 FILE = 'data/sheet.csv'
7 dataset = np.vstack((set(map(tuple,np.genfromtxt(FILE, delimiter=',',)
    ↪ )))
8
9 def addNoise(snr):
10     signal = dataset[:, -1]
11     #print signal
12     signalstd = np.std(signal)
13     noisestd = signalstd/np.sqrt(snr)
14     noise = np.random.normal(0, noisestd, len(signal))
15     datasetnoisy = dataset
16     datasetnoisy[:, -1] = datasetnoisy[:, -1] + noise
17     return datasetnoisy
18
19 def ensemble(value, noise):
20
21     #mixture = GMM(C = 100)
22     mixture = GMM()
23     newdataset = addNoise(noise)
24     temp = np.copy(newdataset[:, -1])
25     #newdataset[:, -1] = newdataset[:, 2]
26     #newdataset[:, 2] = temp
27     #print newdataset[:, -1], newdataset[:, 3]
28     for ensemble in range(0, value):
29         np.random.shuffle(newdataset)
30         train = np.copy(newdataset[0:-10, :])
```

```

31     test = np.copy(newdataset[-10:-1,:])
32     test_pred = np.copy(test)
33     mixture.fit(newdataset[0:-10,0:-2],newdataset[0:-10,-1])
34     preds = mixture.predict(newdataset[-10:-1,0:-2])
35     test_pred[:, -1] = preds
36     errorabs = abs(dataset[-10:-1,-1]-preds)/(dataset
        ↪ [-10:-1,-1])
37     meanerrorabs = np.mean(errorabs)
38     stderrorabs = np.std(errorabs)
39     #print errorabs
40     print meanerrorabs , stderrorabs
41     #plt.plot(abs(dataset[-10:-1,-1]-preds))
42     #plt.ylim(-5e-12,5e-12)
43     #plt.scatter(dataset[-10:-1,0],dataset[-10:-1,-1])
44     #plt.plot(preds)
45     #plt.show()
46     np.savetxt('data/forward_train_%d_snr_%d.csv'%(ensemble,
        ↪ noise),train,delimiter=',')
47     np.savetxt('data/forward_test_%d_snr_%d.csv'%(ensemble,noise
        ↪ ),test,delimiter=',')
48     np.savetxt('data/forward_test_predict_%d_snr_%d.csv'%(
        ↪ ensemble,noise),test_pred,delimiter=',')
49     #sp.io.savemat('data/train_%d_snr_%d.mat'%(ensemble,noise),
        ↪ train)
50     #sp.io.savemat('data/test_%d_snr_%d.mat'%(ensemble,noise),
        ↪ test)
51     #sp.io.savemat('data/test_predict_%d_snr_%d.mat'%(ensemble,
        ↪ noise),test_pred)
52 for noise in range(2,20,2):
53     ensemble(1, noise)

```

## A.7 Defaults Code

```
1 FREQUENCY = 2.5e6
2 A = -3.1*(10^11)
3 B = 0
4 C = 0
5 BOUNDARY = 0 #Purely Reflecting
6 DIMENSIONS = [.010 , 0.030] #metres
7 MESH = 2 #0, 1, 2, 3 Coarse, Medium, fine and extrafine mesh l/8, l
   ↪ /12, l/64, l/128
8 MU = 2.68e10
9 Lambda = 5.43e10
10 K = 76e9
11 RHO = 2719
12 TIME = 1.5 #seconds
13 WAVEGUIDE = 1
14 LOCATION = 0.5
15 THETA = 0
16 WIDTH = 0.25
17 PULSES = 10
18 REFLECTIONS = 2
19 l = -38.75e10
20 m = -35.8e10
```

## **APPENDIX B**

### **Material Properties Used in Simulations**

## REFERENCES

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- [8] Liu Et, Al