Theorems

Sundry Transformations

1. Division Folding
   - If $x \in \mathbb{Z} \wedge a > 0 \wedge b > 0$
     - $(x/a)/b = x/(a * b)$

Bounds

1. Bounds Computation
   - If $x \in [a, b] \wedge y \in [c, d]$, then
     - $x + y \in [a + c, b + d]$
     - $x - y \in [a - d, b - c]$
2. Expression Reduction using bounds
   - If $x \in [a, b]$ then
     - $x/k \rightarrow ?$ (constant or zero?)
     - $x\%k \rightarrow ?$ (x or x + some constant?)

Iterators

- Let I be an iterator $(start, end, step)$
- $x \in I$ if $(start <= x <= end) \wedge (step|(x - start))$
- $k^{th}Iter : (k : Z)(I : (start, end, step)) \rightarrow Z := start + k \times step$

1. Modulo Simplification
   - If $x \in I(start, end, step)$
     - $(c|step) \implies x\%c = start\%c$
2. Iterator Replacement
   - If $x \in I(start, end, step)$
     - $x + c \in$ I(start + c, end + c, step)
     - $\forall k \in Z, k^{th}Iter\ k\ I(start+c, end+c, step) = k^{th}Iter\ k\ I(start, end, step)$
   - If $(c|step) \wedge x \in I(start, end, step)$
     - $x/c \in I(start/c, end/c, step/c)$
     - $\forall k \in Z, k^{th}Iter\ k\ I(start/c, end/c, step/c) k^{th}Iter\ k\ I(start, end, step)$