Hand in problem 3

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The secret key is 'Pyukumuku'.

Task 1

- For the state-space model of the form

$$s(k+1) = As(k) + w(k) \tag{1}$$
$$z(k) = Cs(k) + v(k) \tag{2}$$

- The matrices A, C and w(k) are given as follows:

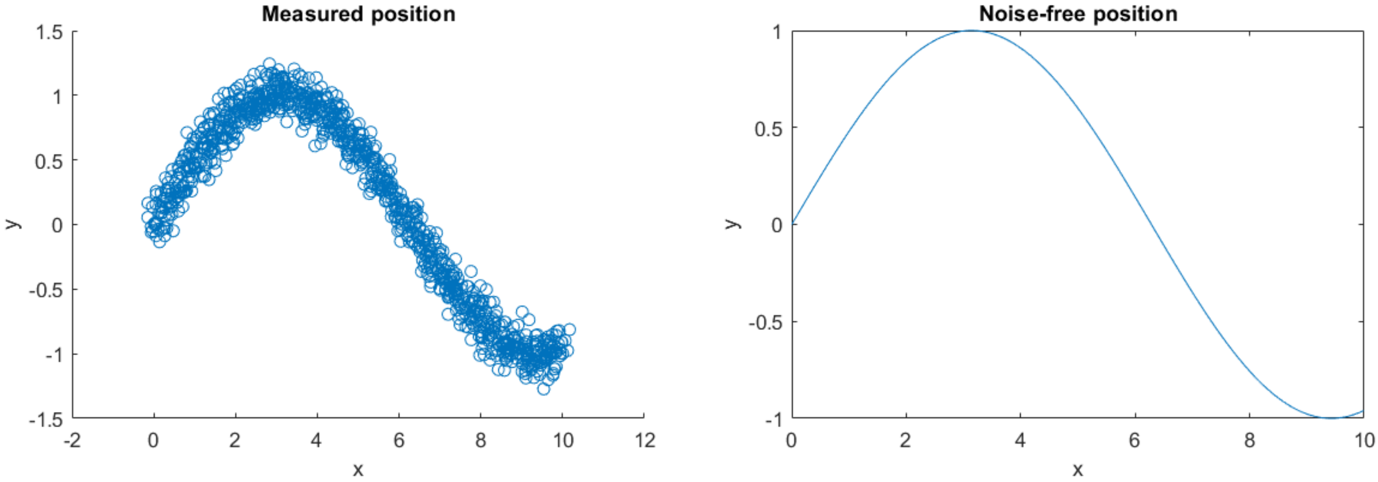
$$A = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{4}$$

- The process noise $w(k)$ and measurement noise $v(k)$ are zero mean random variables with a normal distribution and covariance matrices Q and R independent of the state variable $s(k)$.

$$\begin{bmatrix} w(k) \\ v(k) \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \right) \tag{5}$$

Task 2

- The measured and noise free positions are plotted as follows:



(a) Measured position

(b) Noise free position

Figure 1: Position data

Task 3

- The completed code for the Kalman filter is as follows in the figure 2.

```
for t=1:N
    % Filter update based on measurement
    % Xfilt(:,t) = Xpred(:,t) + ...
    Xfilt(:,t) = Xpred(:,t) + P*C'*inv(C*P*C'+R)*(Y(:,t)-C*Xpred(:,t));

    % Uncertainty update
    Pplus = P - P*C'*inv(C*P*C'+R)*C*P; %TODO: This line is missing som

    % Prediction
    Xpred(:,t+1) = A * Xfilt(:,t); %TODO: This line is missing some cod

    % Uncertainty propagation
    P = A*Pplus*A' + Q; %TODO: This line is missing some code!
end
```

Figure 2: Kalman Filter

Task 4

- The covariance matrix R is diagonal in nature and only affects the x and y position of the object. By calculating the variance of x and y positions, we get

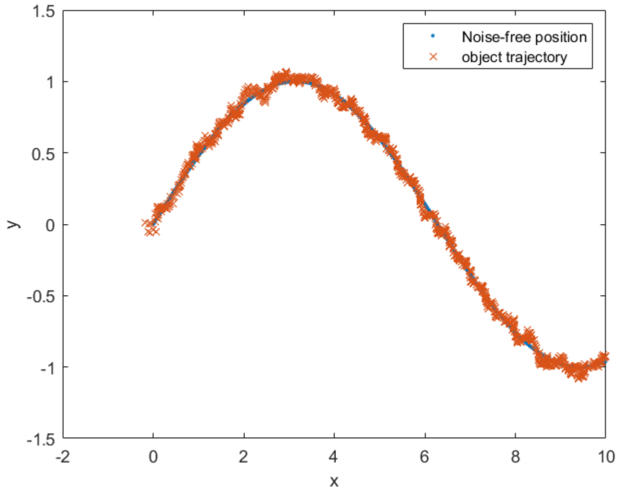
$$R \approx \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} = \begin{bmatrix} 9.6054e^{-3} & 0 \\ 0 & 9.4060e^{-3} \end{bmatrix} \tag{6}$$

- Since the Q matrix should be selected to model possible speed changes, it is of the following form.

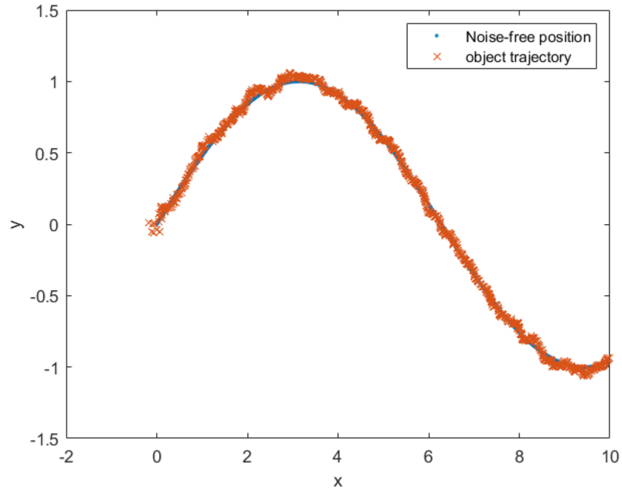
$$Q = \gamma * \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

- By changing the value of γ, different behaviour of Kalman filter was observed as shown in figure 3.
- With $\gamma = 1e^{-4}$, the best results were found. Therefore the covariance matrix Q was chosen as

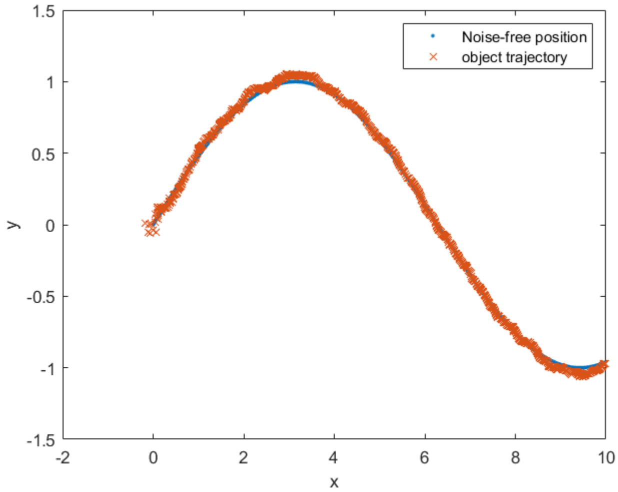
$$Q = 1e^{-4} * \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$



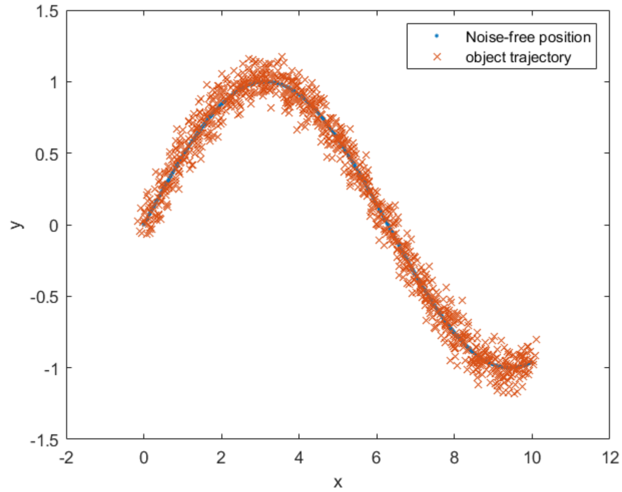
(a) Filter behaviour with $\gamma = 1e^{-2}$



(b) Filter behaviour with $\gamma = 1e^{-3}$



(c) Filter behaviour with $\gamma = 1e^{-4}$

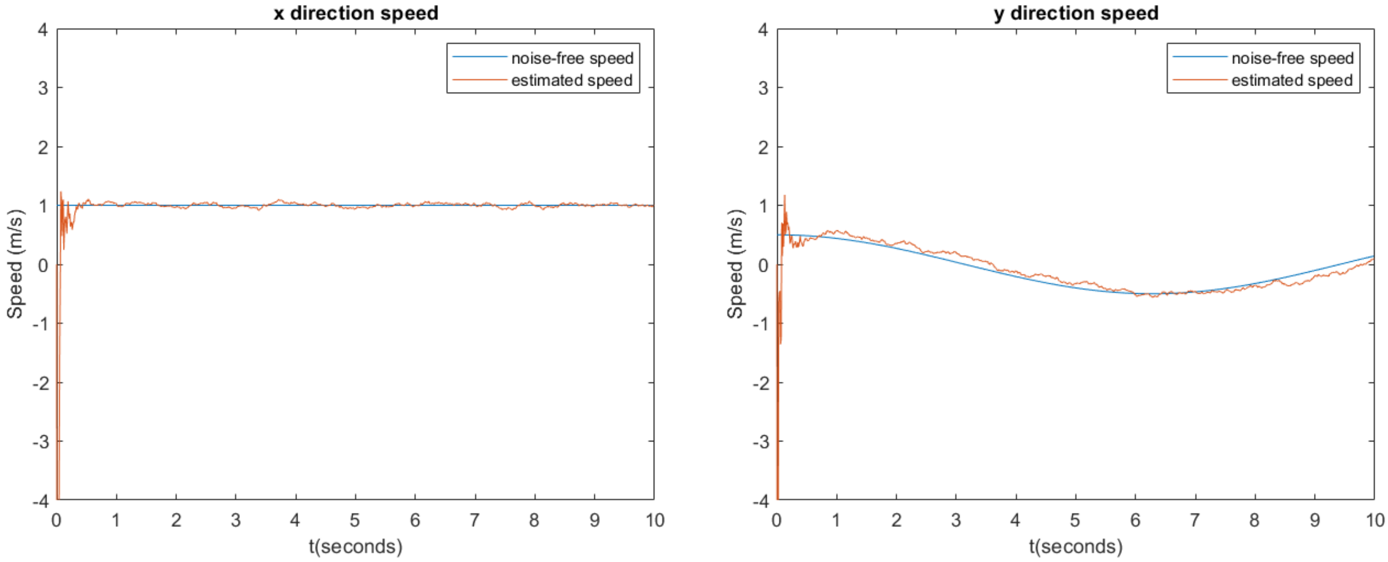


(d) Filter behaviour with $\gamma = 1e^{+2}$

Figure 3: Kalman Filter behaviour

- The estimated speed of the object in x and y direction is plotted in figure 4.

- The value of γ chosen in the previous section also had an impact of how fast the speed was estimated. Best compromise on speed versus accuracy was arrived with $\gamma = 1e^{-4}$.



(a) Speed in x direction

(b) Speed in y direction

Figure 4: Speed of the object