Chalmers University of Technology

SSY130 - Applied Signal Processing

Project I
1B - Interpolation, Modulation, Demodulation and Decimation

Project Report 1B: Group 5

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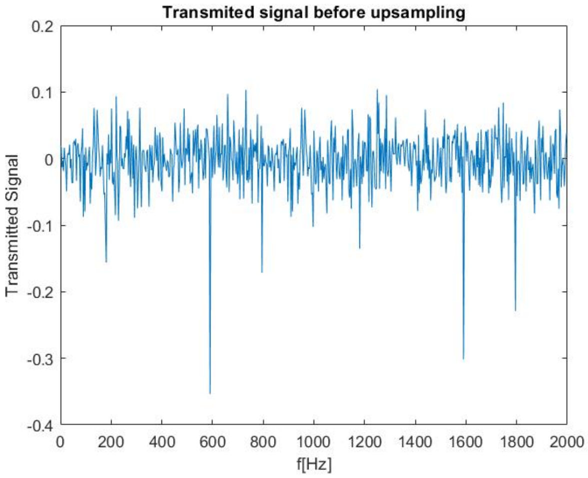
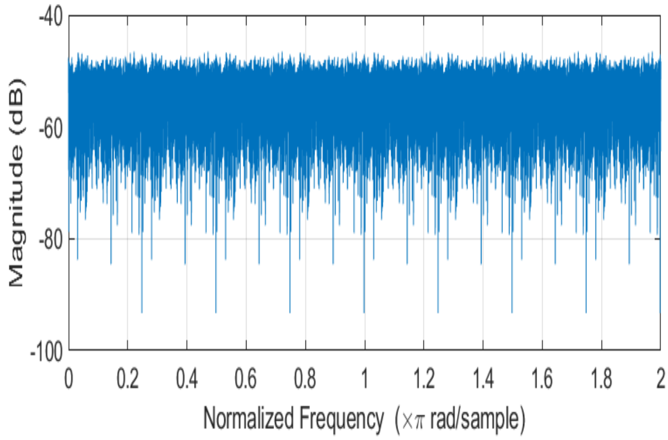
Question 1

Given that the sampling frequency is $16kHz$ and the up-sampling factor given is 8, therefore the bandwidth of the transmitted audio signal is $16/8 = 2kHz$ (Ref fig.1a).



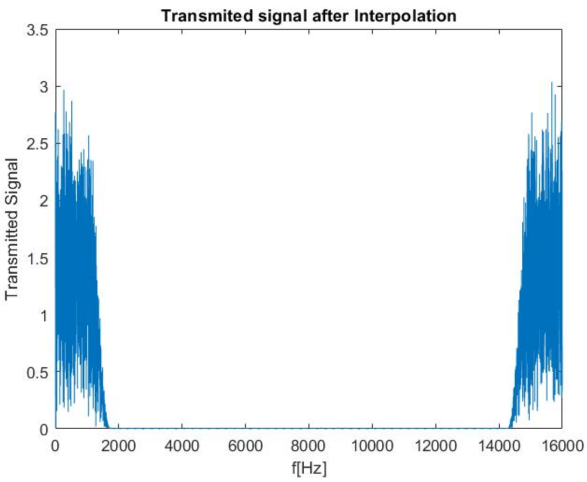
(a) Transmitted audio signal



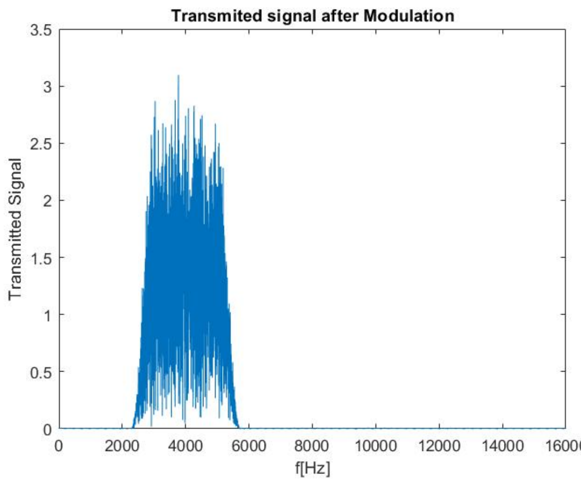
(b) Final sampling frequency

Figure 1: Interpolation

Now the signal after interpolation (Ref fig.2a), is passed through a modulation block, where we modulate the audio signal at 4kHz. After modulation the signal lies between $4 \pm (2/2)kHz = 4 \pm 1kHz$ that is 3KHz to 5KHz. Since the FIR filter used in the interpolation is not ideal,this leads to some part of the signal being outside 3KHz and 5KHz (Ref fig.2b).



(a) Audio Signal Before Modulation

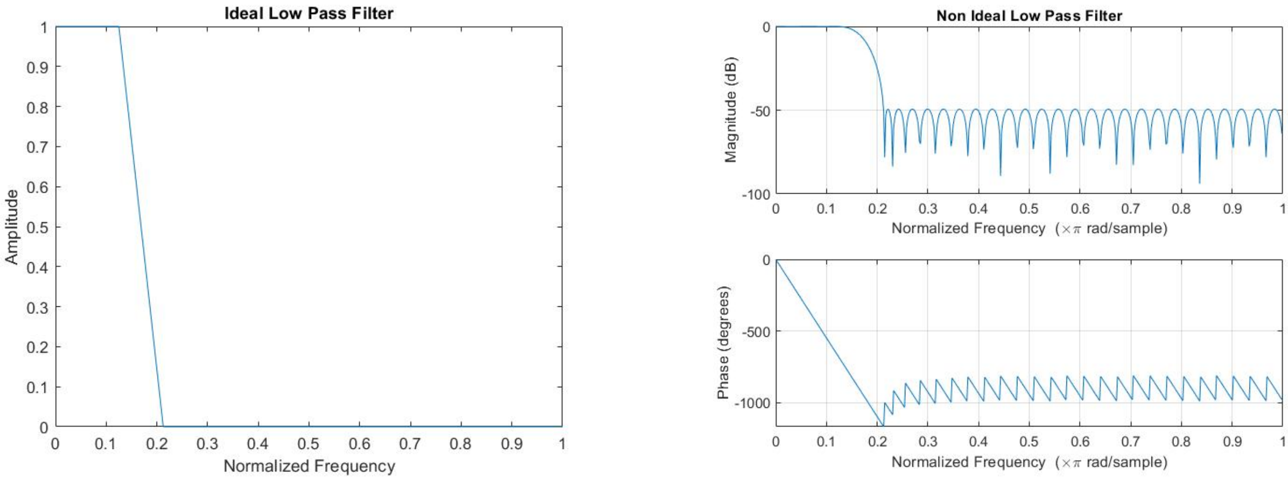


(b) Audio Signal Before Modulation

Figure 2: Modulation

Question 2

Non-zero EVM values are always obtained even with snr=inf condition as a result of the signal being filtered through a non-ideal LP-filters in the interpolation and decimation processes. The original audio signal has band width of 2KHz (refer the figure 1a). The LP filter used in the interpolation process is not ideal and has transition band and ripples in the stop band as shown in Figure3b



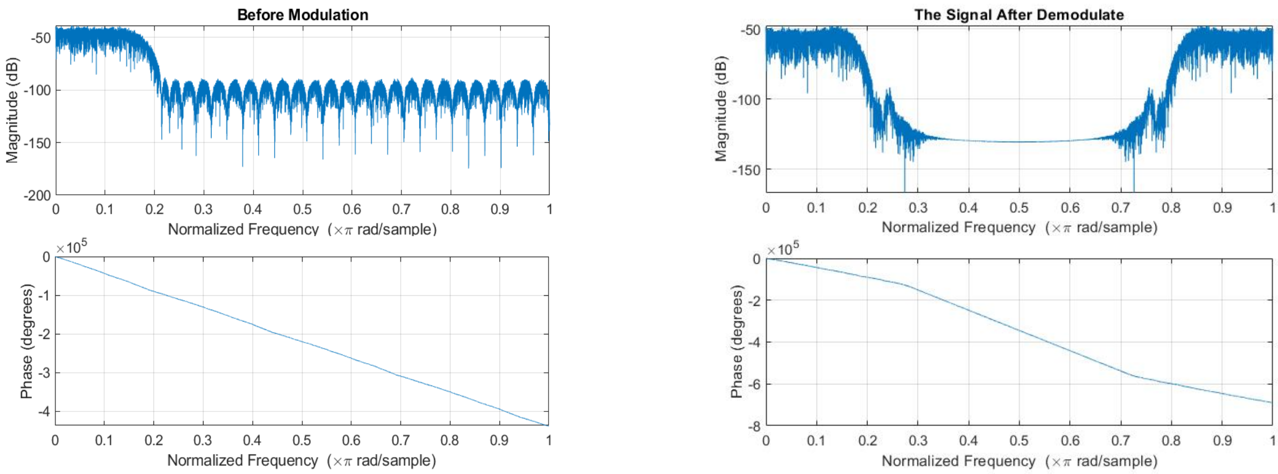
(a) Ideal Low Pass Filter *(b) Non-Ideal Low Pass Filter*

Figure 3: Low Pass Filter

When the signal is passed through the LP filter, undesired frequencies are also passed as a result of the transition band and the ripples in the stop band in the filter. These undesired frequencies affects the received signal which always gives non-zero EVM values.

Question 3

- The **modulation and demodulation** blocks do not contribute to channel H because the modulation effect is completely removed during demodulation.
- Modulation involves shifting the absolute position of a signal in the frequency domain. Each sample is multiplied by a complex exponential with frequency f_{cm}, the center modulation frequency(Ref fig.4).



(a) Signal after modulation *(b) Signal after demodulation*

Figure 4: Modulation and Demodulation

- H estimated is mainly affected by the process of **interpolation and decimation.**
- As described in the previous question, low-pass filters applied during interpolation and decimation are not ideal and are the source of most interference in the received signals.
- By using only the **real part of the pilot signal**, the magnitude is divided by 2 (shown in the next question), the channel estimation is also averaged. However, this effect is compensated during equalization phase.

Question 4

The modulated signal, $z_m(n)$ and it's frequency response $Z_m(\omega)$ are given as

$$z_m(n) = z_i(n)e^{j2\pi n f_{cm}/f_s}, \ \ n = 0 \pm 1, \pm 2, ... \tag{1}$$
$$Z_m(\omega) = Z_i(\omega - \omega_s f_{cm}/f_s) \tag{2}$$

If the real part of the modulated signal is given as $z_{mr} = Re(z_m)$, then its frequency response is given as

$$z_{mr} = Re(z_m) = Re(z_i(n)e^{j2\pi n f_{cm}/f_s}) \tag{3}$$
$$Z_{mr}(\omega) = \frac{1}{2}(Z_i(\omega - \omega_s f_{cm}/f_s) + Z_i^*(-\omega - \omega_s f_{cm}/f_s)) \tag{4}$$

- Therefore, from equation (4) we see that by only taking the real part of the modulated signal, the information in the transmitted signal is not lost but only mirrored along ω_s (The magnitude of the signal is divided by 2).
- The following demodulation and decimation processes remove the copy.
- Choosing an appropriate modulation frequency f_{cm} will ensure that information is not lost on the received(demodulated) signal containing only real values.
- If the modulation frequency f_{cm} is close to the sampling frequency f_s, then the complex conjugate Z_i^* will cause aliasing after demodulation.

Question 5

While passing the signal through the Interpolation and Decimation blocks, we pass the signal through Low-pass filters. Here are the effects of various properties of the filter,

- Passband ripple : Large pass band ripple would cause signal amplitude fluctuations in the time domain, resulting in ripples in the transmitted signal. Under both stages, interpolation and decimation, the ripples' effect can be considered less important as the pilot is used for equalization and as H estimation occurs over the rippled signal, the effect is cancelled out on the data signal.
- Stopband attenuation : The stopband attenuation plays a major role in the EVM deviation based on its size. By a small stopband attenuation, we can expect increasing EVM deviation. Although this impacts signals at both stages, the effect caused at interpolation can be cancelled at the decimation stage. Hence, a more important property at decimation than interpolation.
- Transition band width : The transition band width is most important as the smaller the width, the more clear and window cut signal can be received. This property is important in both stages as smaller widths can avoid unnecessary signals to be transmitted and adding in as noise. This can easily cause distortions if not taken care of.
- Phase linearity : The linear phase filter guarantees that the relative phase of each frequency component in the signal does not vary, which implies that the delay of each frequency component in the signal remains constant after passing through the linear phase filter. The estimation of H will result in the same value irrespective of the delay as of the compensation of cyclic prefix. This can be considered more important during the decimation stage than the interpolation.

Question 6

In this question we will discuss how Conjugate multiplication method give similar result as Division method. We are using an example to prove that both methods gives the similar amplitude and phase.

$$R = cos(\omega_1) + jsin(\omega_1)$$
$$T = cos(\omega_2) + jsin(\omega_2)$$
$$\tilde{T} = cos(\omega_2) - jsin(\omega_2)$$

Division Method Computationally Costly:

$$H = \frac{R}{T} = \frac{cos(\omega_1) + jsin(\omega_1)}{cos(\omega_2) + jsin(\omega_2)} \tag{5}$$

solving the above equation:

$$H = cos(\omega_1 - \omega_2) + jsin(\omega_1 - \omega_2) \tag{6}$$

From the above equation we calculate the amplitude:

$$|H| = \sqrt{cos^2(\omega_1 - \omega_2) + sin^2(\omega_1 - \omega_2)} = 1 \tag{7}$$

Similarly phase shift is calculated

$$\tan\phi = \frac{imag}{Real} = \frac{\sin(\omega_1 - \omega_2)}{\cos(\omega_1 - \omega_2)} = \tan(\omega_1 - \omega_2) \tag{8}$$

Conjugate Multiplication Method
Now we will try to achieve the similar phase shift and amplitude using more cost effective way:
We are multiplying the conjugate of Transmitted signal with physical channel

$$\hat{H} = R\tilde{T} = (cos(\omega_1) + jsin(\omega_1))(cos(\omega_2) - jsin(\omega_2)) \tag{9}$$

solving the above equation:

$$\hat{H} = cos(\omega_1 - \omega_2) + jsin(\omega_1 - \omega_2) \tag{10}$$

From the above equation we calculate the amplitude:

$$|\hat{H}| = \sqrt{cos^2(\omega_1 - \omega_2) + sin^2(\omega_1 - \omega_2)} = 1 \tag{11}$$

As show above phase shift is calculated and it is $\tan(\omega_1 - \omega_2)$.
By comparing the division method and and conjugate multiplication method we came to conclusion that both the method gives same result.

Question 7

From eq(10)

$$\hat{H} = cos(\omega_1 - \omega_2) + jsin(\omega_1 - \omega_2) \tag{12}$$

$$\tilde{\hat{H}} = cos(\omega_1 - \omega_2) - jsin(\omega_1 - \omega_2) \tag{13}$$

Let's consider the equalized signal R_{eq}

$$\begin{aligned} R_{eq} &= R.\tilde{\hat{H}} && (14)\\ &= (cos(\omega_1) + jsin(\omega_1)).(cos(\omega_1 - \omega_2) - jsin(\omega_1 - \omega_2)) && (15)\\ &= cos(\omega_1 - \omega_1 + \omega_2) + jsin(\omega_1 - \omega_1 + \omega_2) && (16)\\ &= cos(\omega_2) + jsin(\omega_2) && (17) \end{aligned}$$

Phase of R_{eq}:

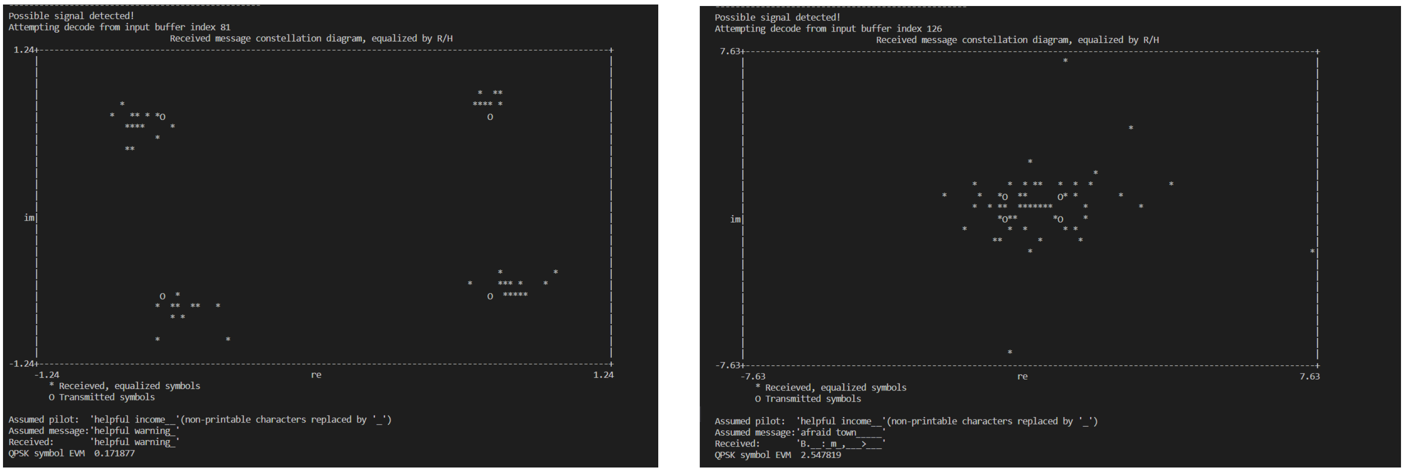
$$\begin{aligned} \tan(\phi) &= \frac{\sin(\omega_2)}{\cos(\omega_2)} && (18)\\ &= \tan(\omega_2) && (19) \end{aligned}$$

Which is the same as the phase of the transmitted signal T.

Question 8

- The low signal amplitudes are highly affected by the level of background noise as can be seen in figure 5. The EVM increases with increasing background noise level. The constellation plot as a consequence is spread out when noise is present.

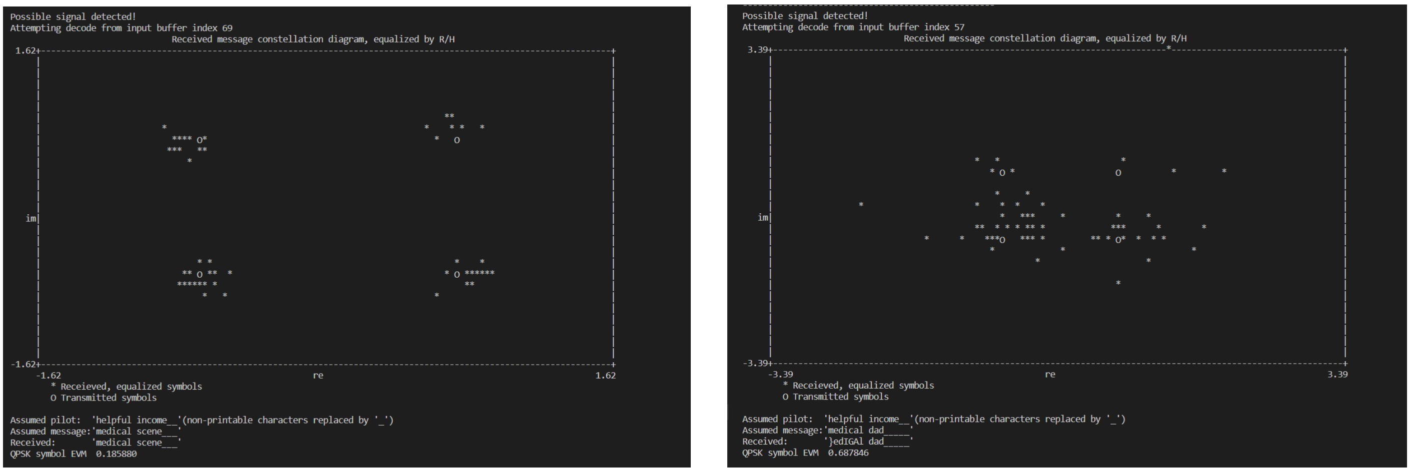
- The high signal amplitudes are also highly affected by the level of background noise as can be seen in figure 6. The EVM increases with increasing background noise level. The constellation plot as a consequence is spread out when noise is present.



(a) Constellation at low amplitude without background noise

(b) Constellation at low amplitude with background noise

Figure 5: Constellation at low amplitude



(a) Constellation at high amplitude without background noise

(b) Constellation at high amplitude with background noise

Figure 6: Constellation at high amplitude