Chalmers University of Technology

SSY130 - Applied Signal Processing

Project I
1A - Baseband Communication

Project Report 1 A: Group 5

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Scenario 1

The ideal (trivial) channel h_1 is used, no noise is added (SNR = Inf, i.e. w(n) = 0) and cyclic prefix N_{cp} is set to zero, implying that y(n) = z(n).

1a

In this case $N_{cp} = 0$. The below given equation1 has information of signal data - S(k) and channel signal H(k) (kronecker delta). When y(n) passes through the $OFDM^{-1}$ block it undergoes discrete Fourier transform and gives the output as R(k) as mentioned in equation 2, where channel signal H(k) will be converted into unit step function of gain 1 in frequency domain and the received signal S(k) to frequency domain accordingly.

$$y(n) = z(n) * h(n) + w(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k)S(k)e^{j2\pi kn/N} + W(n) \tag{1}$$

$$R(k) = \sum_{n=0}^{N-1} y(n)e^{-j2\pi kn/N} = H(k)S(k) + N(k) \tag{2}$$

After computing R(k), we pass the signal through the equalization block where we use equation3 to retrieve the original signal.

$$\hat{S}(k) = R(k)/H(k) \tag{3}$$

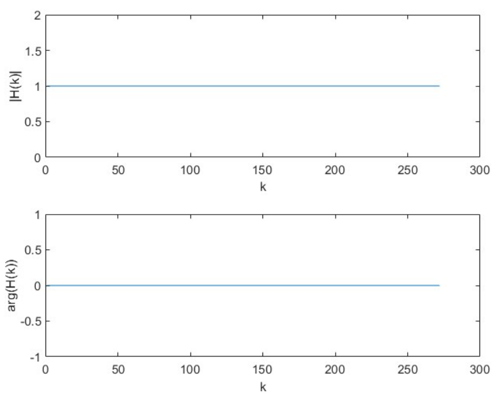
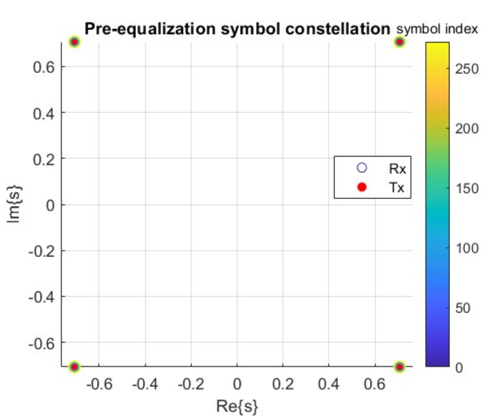
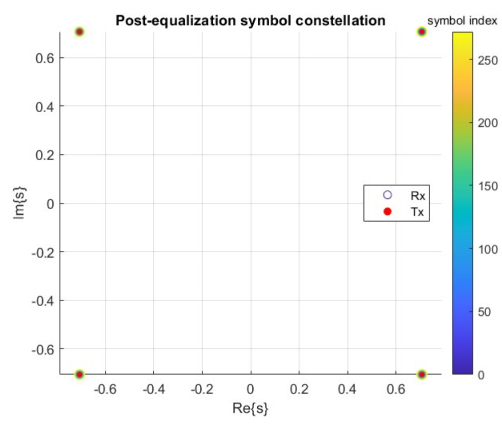


Figure 1: Channel $H_1(k)$ response

Since the channel has a gain of only '1', the signals before and after equalization remain the same. The transmitted and received symbols are identical due to the absence of noise and synchronization errors, with EVM = $3.01e^{-16} \approx 0$ and BER = 0. This is conclusive as there are no bit errors occurring and the error vector (comparing symbols at transmitter and receiver) corresponds to negligible. This can be seen through figure 2.



(a) Pre-equalization Constellation Plot



(b) Post-equalization Constellation Plot

Figure 2: Constellation Plot

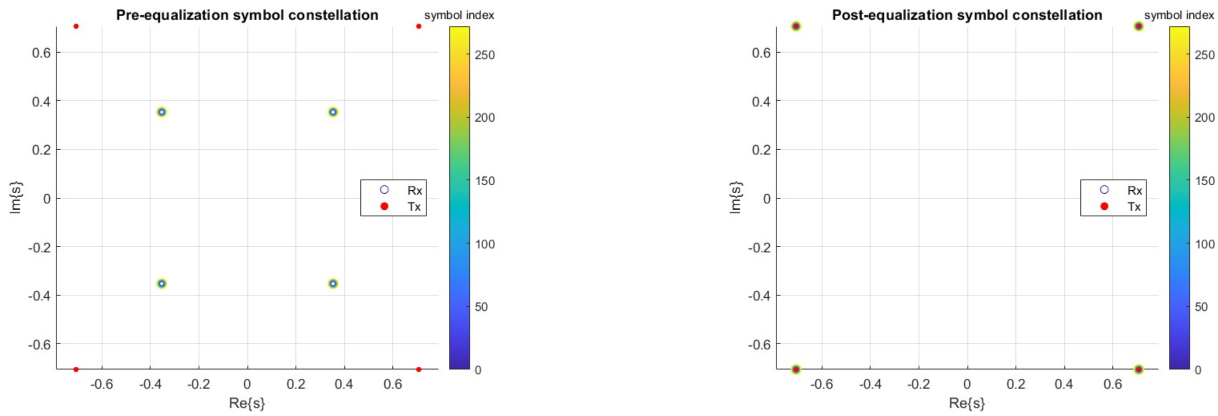
1b

The Cyclic prefix method is used to ensure that there is no interference of OFDM symbols, when passed through the physical channel(Where the signal undergo convolution- refer equation1. This is done by adding a guard interval at the beginning of the window. The guard interval is the later 'N_{cp}' terms of the transmitted window itself. The cyclic prefix number (N_{cp}) has to be equal to or greater than the length of discrete impulse response -1 (N_h-1) of the channel to avoid the symbol interference.
For receiving a near-zero EVM, the cyclic prefix must satisfy the condition $N_{cp} \geq N_h - 1$. Thus, in our case, N_{cp} must be greater than or equal to 59 (60 - 1). The transmitted signal carrying N_{cp} will result in the reciever signal to be stationary at n=0. Hence, y(n) will be identical to z(n).

1c

For channel h_2: EVM $= 3.01e^{-}16$, $\alpha = 0.5$ and BER=0. With h_2, we observed an amplitude shift in pre-equalization diagrams by 0.5(Rx scaled down by 0.5) and in post-equalization after dividing R(k) with h_2 the transmitted and received signal match. This is shown through equation4 and proved in figure3.

$$Y(k) = DTFT[z(n) * \alpha\delta(n)] = \sum_{n=0}^{N-1} \alpha\delta(n) * s(k)e^{-j2\pi kn/N} = \alpha S(k) \tag{4}$$



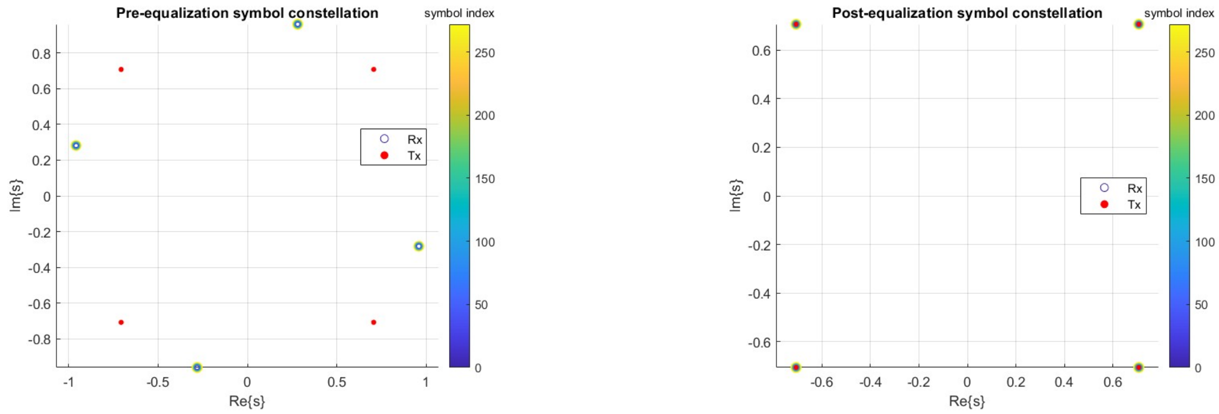
(a) Pre-equalization Constellation Plot

(b) Post-equalization Constellation Plot

Figure 3: Constellation Plot

With h_3: EVM=$3.25e^{-16}$, $\alpha = \cos 0.5 + j \sin 0.5$ and BER=0. The pre-equalization diagrams are rotated counter-clockwise by 0.5 radians, this signifies a Phase Shift. We can conclude this effect shown on figure4a through equation5 and post-equalization the phase shift is nullified by dividing R(k) by H(k) matching the transmitted signal (figure4b).

$$Y(k) = DTFT[z(n) * \alpha\delta(n)] = \sum_{n=0}^{N-1} e^{0.5j}\delta(n) * S(k)e^{-j2\pi kn/N} = S(k)e^{0.5j} \tag{5}$$



(a) Pre-equalization Constellation Plot

(b) Post-equalization Constellation Plot

Figure 4: Constellation Plot

1d

A delay in time domain $x(n-k)$ is $X(\omega)e^{-j\Delta t\omega k}$ in frequency domain.
A synchronization error is a time delay, with the expression $e^{-j2\pi n_{se}f/f_s}$ in frequency domain. For synchronization error $n_s = \pm 1$, the exponential of e will be 0 or a multiple of 2π when $f \to 0$ and $f \to f_s$ respectively. Thus, only the first and last parts of the message are intact after transmission over the channel, as in figure 5a.
Similarly, for $n_s = \pm 2$, the exponential of e will be 0 or a multiple of 2π when $f \to 0$, $f \to f_s$ and $f \to f_s/2$. Thus, the first, middle and last parts of the message are intact after transmission over the channel, as in figure 5b. The same logic can be expanded to higher values of n_s.

```
Transmitted: 'Alice: Would you tell me, please, which way I ought to go from here?'
Recieved:    'Alice: W_t62_5_066_40_E_E_E_E_E_m here?'
```

(a) With synchronisation error = 1

```
Transmitted: 'Alice: Would you tell me, please, which way I ought to go from here?'
Recieved:    'Alict_5_62_E_IE_e, whic~_8_,_5_E_E_re?'
```

(b) With synchronisation error = 2

Figure 5: Transmitted data vs Received data

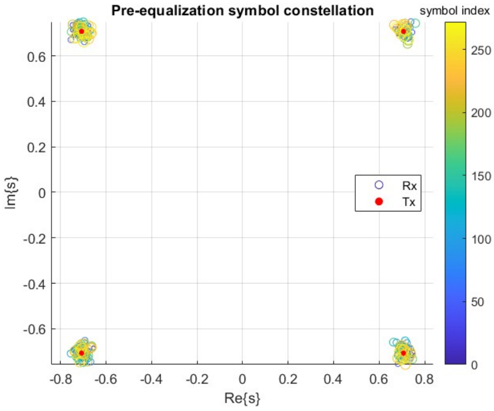
Since, the message is essentially lost between the start and the end, the values of EVM and BER are very large as shown in table1 below.

Table 1: EVM and BER values for different N_{cp} values

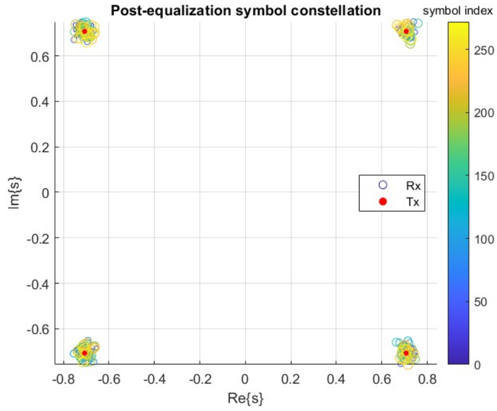
n_{se}	EVM	BER
1	1.41	0.496
-1	1.41	0.5
2	1.41	0.493
-2	1.41	0.496

1e

We now investigate the effect of adding noise during the message transmission by varying the effective signal to noise ratio (SNR). With SNR=30dB, the transmitted and received messages are nearly intact as shown in figure6, since SNR is a log function inversely proportional to noise.



(a) Pre-equalization Constellation Plot

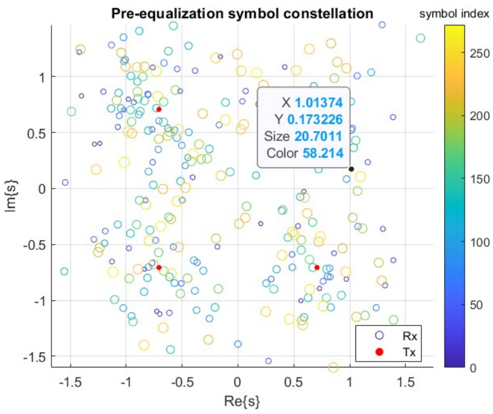


(b) Post-equalization Constellation Plot

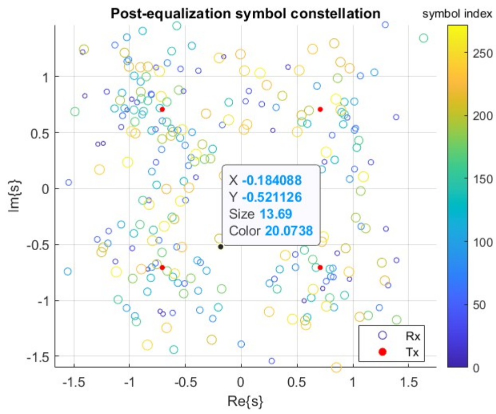
Figure 6: Constellation Plot for SNR = 30

For SNR=5dB, the added noise overpowers symbols to move to incorrect quadrants of frequency (figure7), resulting in the following message at the receiver,

```
Transmitted: 'Alice: Would you tell me, please, which way I ought to go from here?'
Recieved:    'Ilice: Wo}ld y/u uell }e,0pleasul`whiSh way(I neght to_gm!n_oo her%?'
EVM: 0.516, BER: 0.0404
```



(a) Pre-equalization Constellation Plot



(b) Post-equalization Constellation Plot

Figure 7: Constellation Plot for SNR = 5

Scenario 2

Channel h_4 (the low-pass filter) and set the cyclic prefix to 60.

2a

As the index of the signal increases, the amplitude and phase of the received signal change based on the value of $|H(k)|$ and $arg(H(k))$ as in figure 9a .

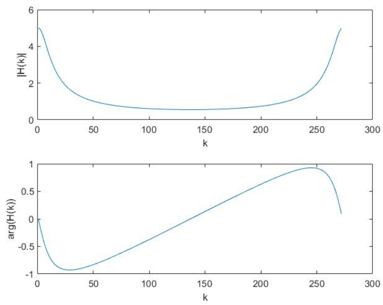
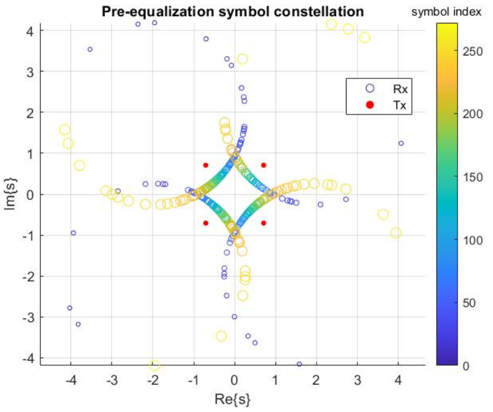
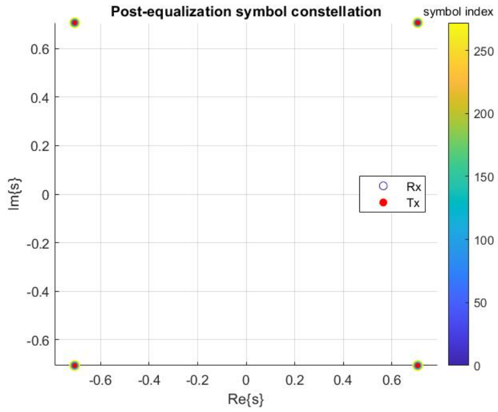


Figure 8: Channel H_4 response

The pre-equalization symbols undergo phase shift from 0 degrees to -57 degrees, and then to +57 degrees according to the symbol index and amplitude shift by 0.58 also according to the symbol index as shown in 8.



(a) Pre-equalization Constellation Plot

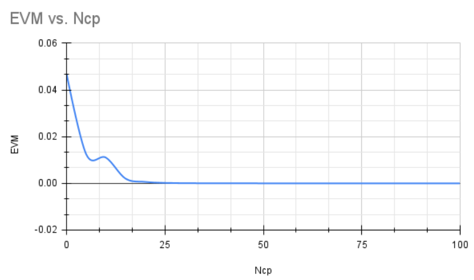


(b) Post-equalization Constellation Plot

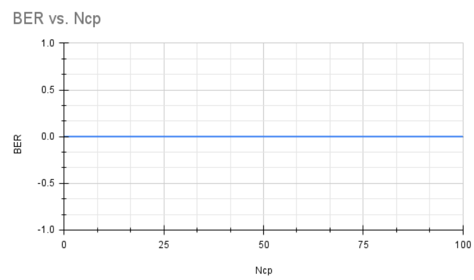
Figure 9: Constellation Plot

2b

When the system is configured in this manner, EVM=$1e^{-15}$ and BER=0. We notice that for all $N_{cp} \geq 59$ EVM remains close to zero. On the other hand, if we choose $N_{cp} < 59$, EVM increases as we decrease N_{cp} as in figure 10a. As a result, we conclude that the magic number is $N_{cp} \geq 59$, which is exactly the size of the channel impulse response minus one, resulting in $N_{cp} \geq N_h - 1$.
BER is always zero because there is no noise nor sync error as in figure 10b.



(a) EVM vs N_{cp}



(b) BER vs N_{cp}

Figure 10: EVM and BER vs N_{cp}

2c

The N_{cp} value has significant effect on EVM and BER values for given channel h'_4. If the $N_{cp} \geq N_h - 1 = 59$ we observe EVM and BER error is zero. As we keep reducing the N_{cp} values below 59, we start observing the error only in EVM but BER=0. When $N_{cp} \leq 30$ non-zero values can be observed for both BER and EVM as shown in table 2. The channel response is shown in figure 11.
At $N_{cp} \geq 59$, EVM error is less than machine tolerance and BER = 0.
At $N_{cp} = 30$, BER = 0 and the message is intact.

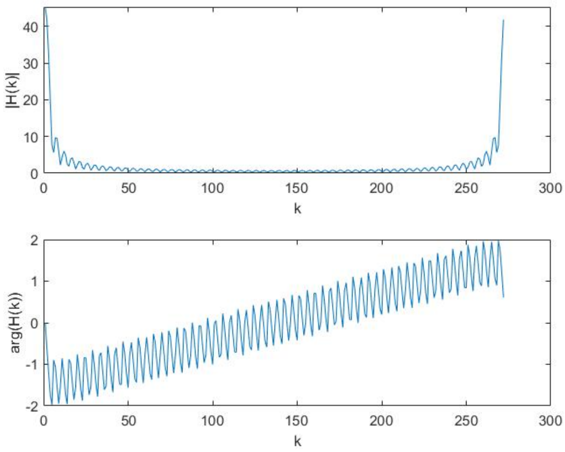


Figure 11: Channel H_4' response

Table 2: EVM and BER values for different N_{cp} values

N_{cp}	EVM	BER
0	0.699	0.068
20	0.471	0.0129
29	0.415	0.00551
30	0.352	0
40	0.285	0
59	$4.025e^{-15}$	0

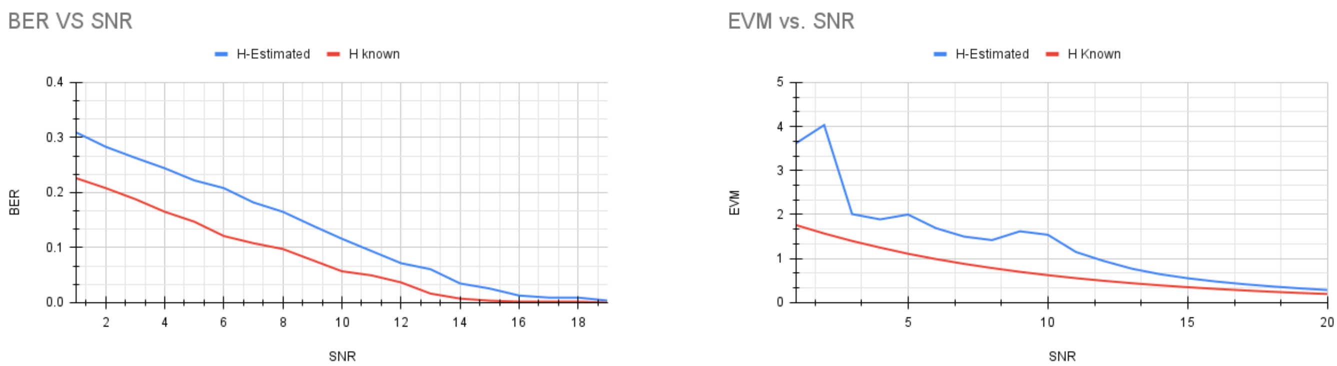
Scenario 3

3a

Since the pilot data is used to estimate the channel impulse response, the synchronization error is also captured during this estimation. The estimated channel response is also along with the synchronization error, thus while equalization, the error is nullified and the equalization happens more efficiently compared to the 'synchronization error in known channel' scenario. Therefore when a nonzero synchronization error is present, the unknown channel scenario performs better compared to the known channel scenario.

3b

For the low pass channel h_4 we observed that the EVM value nearly reaches zero($1.05e^{-15}$) for the N_{cp} value of 59 or more, and the similar behaviour was observed for channel-known scenario. From the above result we have came to the conclusion that the N_{cp} value needs to be larger than the length of the channel impulse response in order to achieve zero EVM.
Keeping the N_{cp} value constant at 60, the behaviour EVM and BER for different values of SNR was studied for both known-channel and unknown-channel scenarios in figure 12a and figure 12b.



(a) BER vs SNR *(b) EVM vs SNR*

Figure 12: BVR and EVM comparison for known and estimated H.

From figure 12a and figure 12b results we can observe that when the SNR value reduces the EVM and BER values increases. From the above observation we infer that the unknown-channel scenario is more sensitive to noise compared to the known-channel scenario.
The reason for unknown channel being more sensitive to noise is the added extra noise terms present in the denominator of the estimated channel 8,compared to the known-channel 7noise shown in

Known H

$$r(k) = H(k)s(k) + n(k) \tag{6}$$

$$\hat{s}(k) = \frac{r(k)}{H(K)} = \frac{H(K)s(k) + n(k)}{H(k)} \tag{7}$$

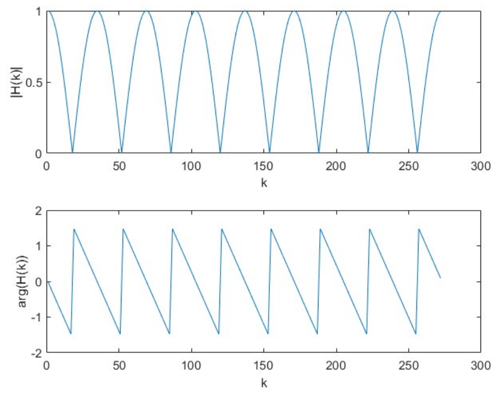
Estimated H

$$\hat{H}(k) = \frac{r_p(k)}{s_p(K)} = \frac{H(k)s_p(k) + n(k)}{s_p(k)} \tag{8}$$

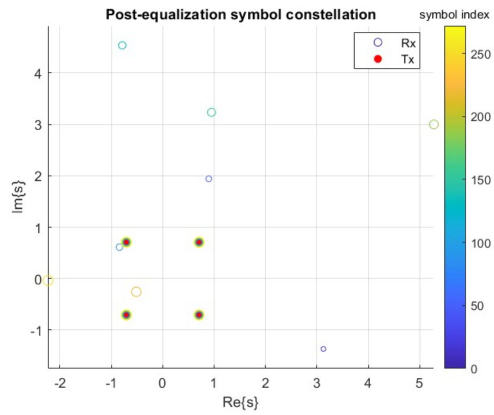
$$\hat{s}(k) = \frac{r_d(k)}{\hat{H}(K)} = \frac{H(K)s_d(k) + n(k)}{\hat{H}(k)} = \frac{(H(K)s_d(k) + n(k))s_p}{H(k)s_p(k) + n(k)} \tag{9}$$

3c

When multi-path channel h_5 is used with $N_{cp} \geq 59$, we always get non-zero BER values even though no sync_error or noise is present. This is because, there are instances where the channel response $|H_5|$ is equal to zero as presented in figure 13a. These instances results zeros in $H(k)s(k)$ and those data cannot be recovered in the equalization process while the rest of the data is recovered correctly as shown in the figure 13b.



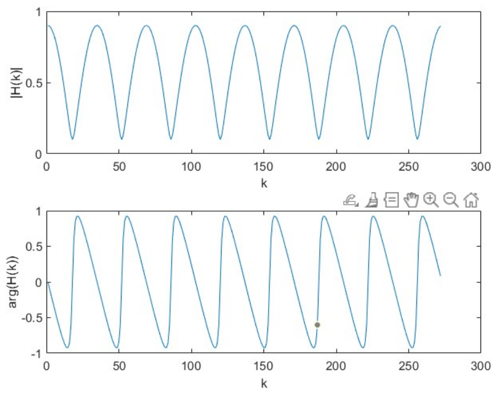
(a) Multi-path channel H_5 response



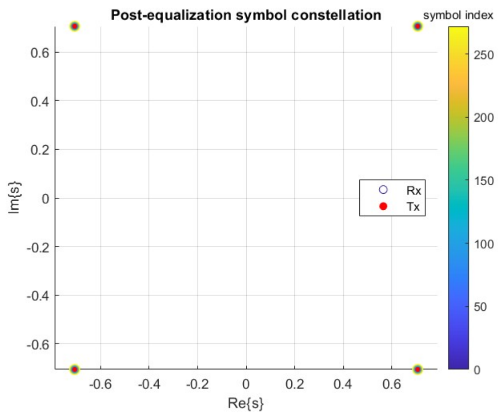
(b) Post-equalization constellation Plot for h_5

Figure 13: Channel H_5

When multi-path channel h_5' is used, BER=0 can be achieved under zero noise and zero synchronization error with adequate cyclic prefix value($N_{cp} \geq 59$) conditions. The reason for this can be observed in channel response of the h_5'. $|H_5'|$ is non-zero for the whole range as clearly seen in figure 14a. Hence the convolution process does not lead to any data loss and the equalization process results in correct message with BER=0.



(a) Multi-path channel H_5' response



(b) Post-equalization constellation plot for h_5'

Figure 14: Channel h_5'