1

Hand-in problem 1

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1.

For perfectly reconstructed continuous time signal, the sampling frequency should satisfy

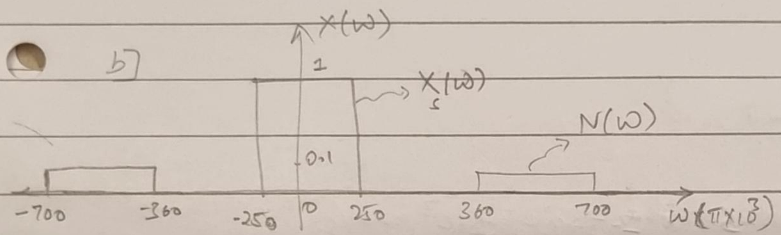
a) the Nyquist criterion

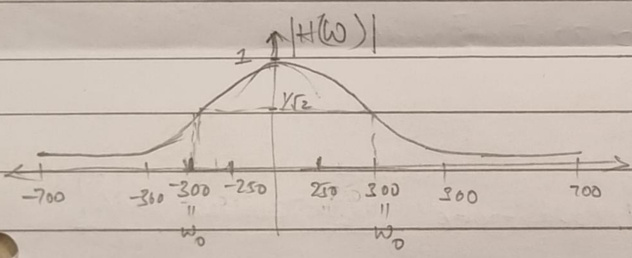
Sampling frequency, $\omega_s \geq 2\omega_{max}$ — ω_{max} is the max. signal frequency

$\omega_s \geq 2 \times 250\pi \times 10^3$

$\boxed{\omega_s \geq 500\pi \times 10^3 \text{ rad/s}}$

b)





Filtered x_s signal, $X_f(\omega) = 1 \times \frac{1}{1+j\omega/\omega_0} = \frac{1}{1+j\omega/\omega_0}$

Sampled x_f signal, $X_d(\omega) = \frac{1}{\Delta t} \times \sum_{k=-\infty}^{\infty} X_f(\omega + \omega_s k)$

at $\omega = 0$, $X_d(\omega) = \frac{1}{\Delta t} \sum_{k=-\infty}^{\infty} X_f(\omega_s k) = \frac{1}{\Delta t}$ $\boxed{X_d(\omega) = \frac{1}{\Delta t}}$

Filtered noise, $N_f(\omega) = 0.1 \times \frac{1}{1+j\omega/\omega_0} = \frac{0.1}{1+j\omega/\omega_0}$

Sample noise, $N_s(\omega) = \frac{1}{\Delta t} \sum_{k=-\infty}^{\infty} N_f(\omega + k\omega_s)$

0.030
0.05

②

Given condition.

$$\|N_s(\omega)\|_{\omega < 250\pi \times 10^3} < \frac{1}{20} \times \frac{1}{\Delta t}$$

$$\frac{1}{\Delta t}\frac{0.1}{\sqrt{1+\left(\frac{\omega+k\omega_s}{\omega_0}\right)^2}} < \frac{1}{\Delta t} \times \frac{1}{20} \Rightarrow 1+\left(\frac{\omega+k\omega_s}{\omega_0}\right)^2 > 4$$

$$\Rightarrow \frac{\omega+k\omega_s}{\omega_0} > \pm\sqrt{3}$$

at $\omega = 250\pi \times 10^3$ rad/s and $\omega_0 = 300\pi \times 10^3$ rad/s

$$k\omega_s > (\pm\sqrt{3} \times 300\pi - 250\pi) \times 10^3$$

at $k = -1$

$$\omega_s \begin{cases} < 10^3 \times (-\sqrt{3}\,300\pi + 250\pi) \simeq -270\pi \times 10^3 \text{ rad/s} \\ > 10^3 \times (\sqrt{3}\,300\pi + 250\pi) \simeq 770\pi \times 10^3 \text{ rad/s} \end{cases}$$

The only ω_s satisfying both conditions $\boxed{\omega_s > 770\pi \times 10^3 \text{ rad/s}}$

2)

The ZOH reconstructed signal is given as

$$X(\omega) = H_{ZOH}(\omega) \times X_d(\omega) \quad -(1)$$

where,

$$H_{ZOH}(\omega) = \Delta t \, e^{-j\pi\omega/\omega_s} \frac{\sin(\pi\omega/\omega_s)}{\pi\omega/\omega_s}$$

$$x_d(n) = \sin\left(2\pi n \frac{f_0}{f_s}\right) = \frac{e^{j2\pi n f_0/f_s} - e^{-j2\pi n f_0/f_s}}{2j}$$

; $\omega_0 = 2\pi f_0$

$\Delta t = 1/f_s$

$\omega_s = 2\pi f_s$

$f_0 = 5\,kHz$; $f_s = 30\,kHz$

$$X_d(\omega) = \sum_{k=-\infty}^{\infty} x_d(n) e^{-j\omega n \Delta t}$$

$$= \frac{1}{2j} \sum_{k=-\infty}^{\infty} \left(e^{jn\omega_0\Delta t} e^{-j\omega n\Delta t} - e^{-jn\omega_0\Delta t} e^{-j\omega n\Delta t} \right)$$

$$X_d(\omega) = \frac{\omega_s}{2j} \sum_{k=-\infty}^{\infty} \{ \delta(\omega + k\omega_s - \omega_0) - \delta(\omega + k\omega_s + \omega_0) \}$$

(1) =>

$$X(\omega) = \frac{\Delta t \, \omega_s}{2j} e^{-j\pi\omega/\omega_s} \frac{\sin(\pi\omega/\omega_s)}{\pi\omega/\omega_s} \times \sum_{k=-\infty}^{\infty} \{ \delta(\omega + k\omega_s - \omega_0) - \delta(\omega + k\omega_s + \omega_0) \}$$

Let $\omega_1 = -k\omega_s + \omega_0$ and $\omega_2 = -k\omega_s - \omega_0$

for

K	ω_1 (rad/s)	ω_2 (rad/s)	$X_1(\omega)$	$X_2(\omega)$
-3	5.96×10^5	5.34×10^5	0.1579	0.1765
-2	4.08×10^5	3.45×10^5	0.2308	0.2727
-1	2.2×10^5	1.57×10^5	0.4286	0.6
0	3.14×10^4	-3.14×10^4	3	3
1	-1.57×10^5	-2.2×10^5	0.6	0.4286
2	-3.45×10^5	-4.08×10^5	0.2727	0.2308
3	-5.34×10^5	-5.96×10^5	0.1765	0.1579

Fundamental freq. $f_0 = 5\,kHz$
magnitude = 3

Harmonics located at
K = -3, -2, -1, 1, 2, 3