

# Linear control system design SSY285

## Assignment M1: Dynamic model of DC motor with flywheel

### Problem

Consider the system shown in Figure 1 which consists of an electric DC motor (having separate magnetization) that drives a flywheel and that is influenced by external torque.

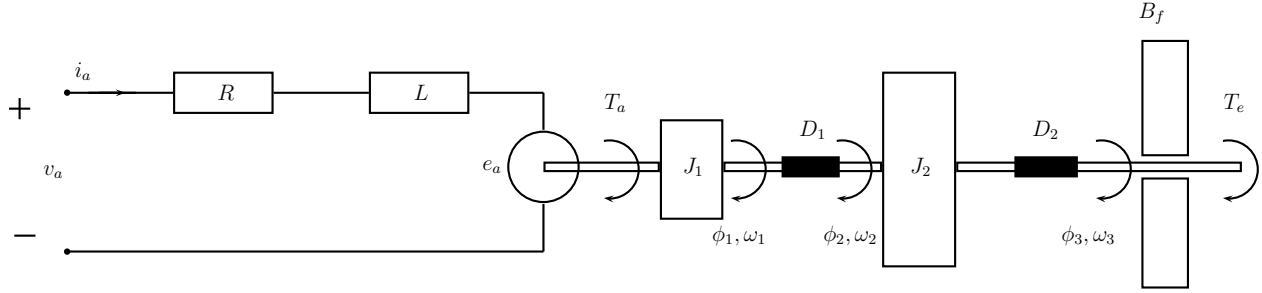


Figure 1: DC motor with flywheel

The system is characterized by the following variables: the external voltage applied to the rotor  $v_a$ , the rotor current  $i_a$ , the induced rotor voltage  $e_a$ , the torque produced by rotor  $T_a$ , and the external torque applied to flywheel axis  $T_e$ . Additionally, the axis is characterised by angles  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  and angular velocities  $\omega_1$ ,  $\omega_2$ ,  $\omega_3$ .

The system is also characterized by electrical parameters:  $L$  is the rotor inductance and  $R$  is the rotor resistance. Furthermore,  $K_E$  is the coefficient which relates induced voltage  $e_a$ , to rotor velocity  $\omega_1$ , and  $K_T$  is the rotor torque constant (driving torque to rotor current  $i_a$ ). There are also mechanical parameters:  $J_1$  is the rotor inertia and  $J_2$  is the flywheel inertia. Both  $D_1$  and  $D_2$  represent the flexibility of the axis (modelled as torsion springs) on each side of flywheel. Finally,  $B_f$  denotes dynamic (linear) friction proportional to the angular velocity  $\omega_3$ .

### Questions

- Given the relations  $e_a = K_E \cdot \omega_1$  and  $T_a = K_T \cdot i_a$ , formulate a mathematical model for this system, using basic laws of electricity and mechanics. How many linear differential equations and of what order are needed to describe this system completely? Which are the system inputs in this case?
- Assume that the inductance is very small,  $L \approx 0$ , then choose state variables to compose a state vector  $x(t)$ , and inputs for vector  $u(t)$  of the underlying system. Pay attention to select the state and control inputs such that you formulate a continuous time state equation under the form of  $\dot{x}(t) = Ax(t) + Bu(t)$ . What are the matrices A and B?
- The output  $y(t)$  of the system in a state space model is related to state and input, as  $y(t) = Cx(t) + Du(t)$ . Give the matrices C and D for the following two cases: **(1)**  $y_1(t) = \phi_2$ ,  $y_2(t) = \omega_2$  and **(2)**  $y_1(t) = i_a$ ,  $y_2(t) = \omega_3$ .

d) Assume the following parameter values:

$$R = 1\Omega, \quad K_E = 10^{-1}Vs/rad, \quad K_T = 10^{-1}Nm/A, \quad J_1 = 10^{-5}kgm^2, \quad J_2 = 4 \cdot 10^{-5}kgm^2 \\ B_f = 2 \cdot 10^{-3}Nms, \quad D_1 = 20Nm/rad, \quad D_2 = 2Nm/rad$$

Calculate (with **Matlab**) the eigenvalues of the A-matrix. Is the system input-output stable for both cases **(1)** and **(2)** of c)? Investigate if pole-zero cancellation occurs.

- e) Assume that the external torque is zero. Give the transfer function from the input, applied rotor voltage, to the output, defined in case **(2)** of subproblem c). Calculate the poles and transmission zeros in this case. Is the system of minimum phase?
- f) Suppose that in the initial state, the applied rotor voltage and the external torque are all zero. Then the applied rotor voltage is stepwise changed from 0V to 10V. Suppose also that once all angular velocities have reached the steady state, suddenly an external (velocity reducing) torque of 0.1Nm is applied stepwise. Investigate by simulation, how the state vector components evolve in time. Is it reasonable that the angles are increasing unboundedly? Plot also the outputs of for both cases in subproblem c) as a function of time.

**Pre-approval of solution is mandatory before submission (by TA in tutorial session)**