Seven Bridges of Königsberg

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Königsberg aka Kalinigrad

Figure: (Source: Google Maps)

The Probelm

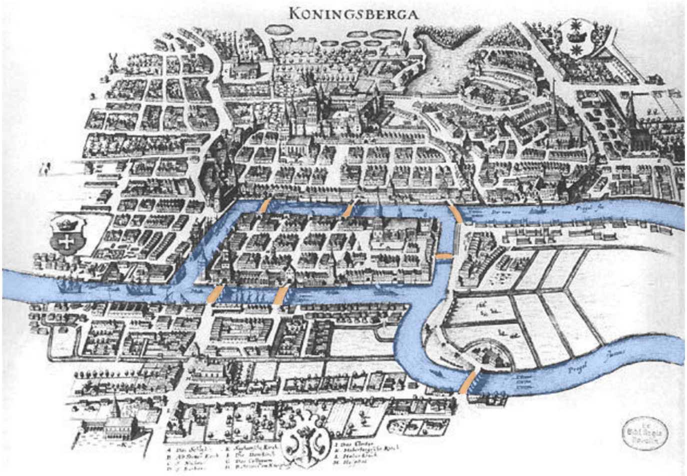


Figure: (Source: Merian-Erben, Wikimedia Commons)

Current situation

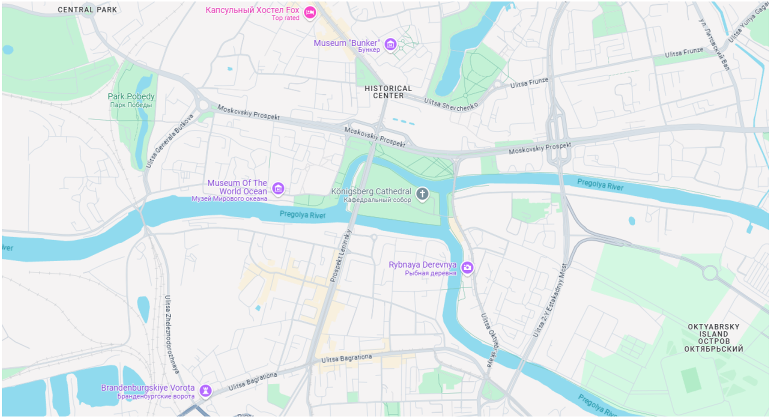
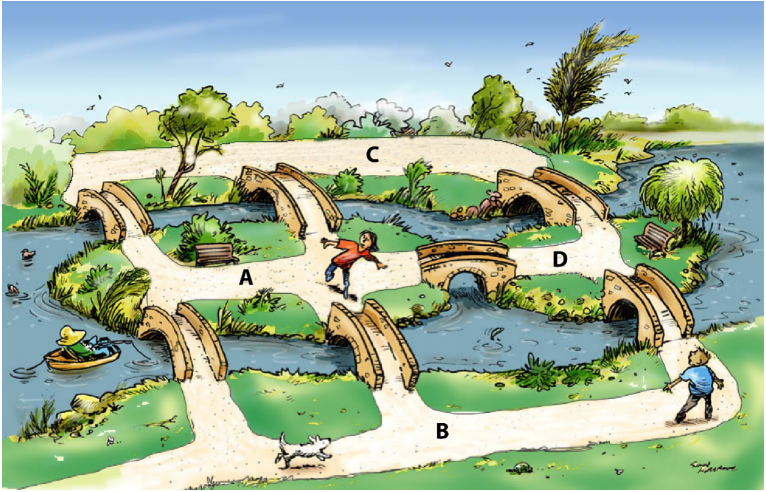


Figure: (Source: Google Maps)

A new problem!

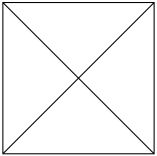


Is it really?

1. Can you find the required path?
2. Is this picture same as the one you saw of Königsberg Bridges?
3. Can you try to simplify the picture?

Another probelm!

Can you draw this picture without lifting the pencil?



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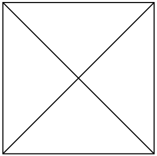
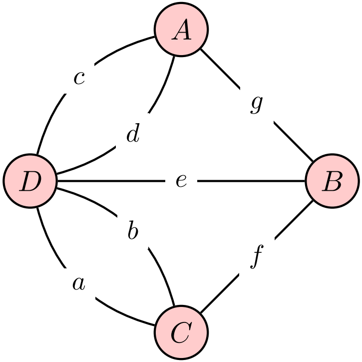
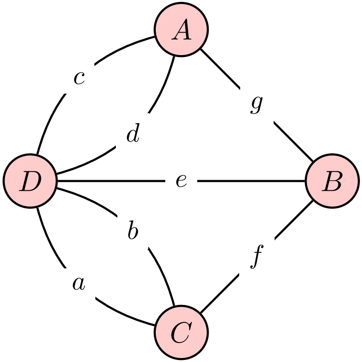


Is this somehow related to the Königsberg problem?

A Graph



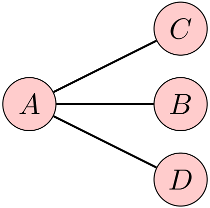
A Graph



Now can you try to trace the lines without lifting your pencil?

Graph Theory 101

A graph is made of a set of points and lines connecting them. These points are called **nodes** or **vertices**. The lines are called **edges**[1]. Degree is the number of edges connected to a node.



[1] The terminology is inconsistent between fields

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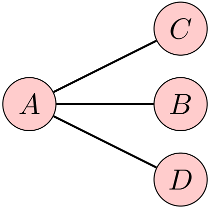

$$d(A) = 3$$
$$d(p) = 1, \qquad p = B, C, D$$

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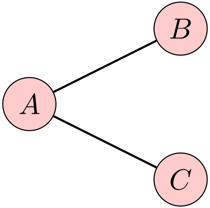
Graph Theory 101

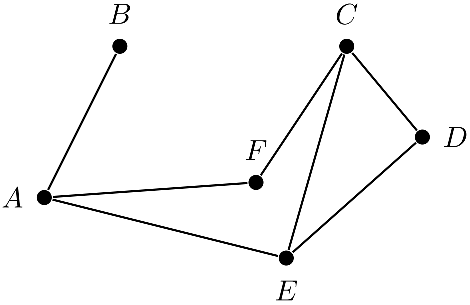
A graph is made of a set of points and lines connecting them. These points are called **nodes** or **vertices**. The lines are called **edges**[2]. Degree is the number of edges connected to a node.



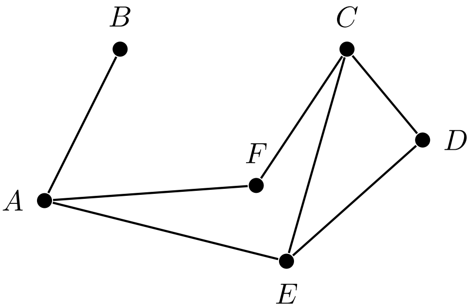
$$d(A) = 2$$
$$d(p) = 1, \qquad p = B, C$$

[2]The terminology is inconsistent between fields

Find the degrees

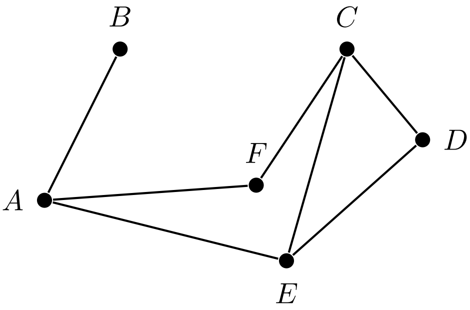


Find the degrees



Node	Degree
A	3
B	1
C	3
D	2
E	3
F	2

Find the degrees



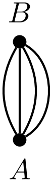
Node	Degree
A	3
B	1
C	3
D	2
E	3
F	2

Draw a graph and try to work out the degrees!

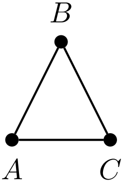
Some more examples

(1)

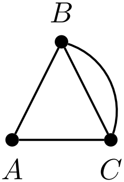
(2)



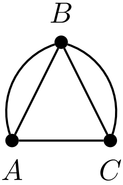
(3)



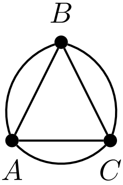
(4)



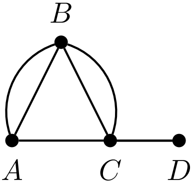
(5)



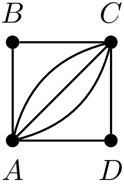
(6)



(7)

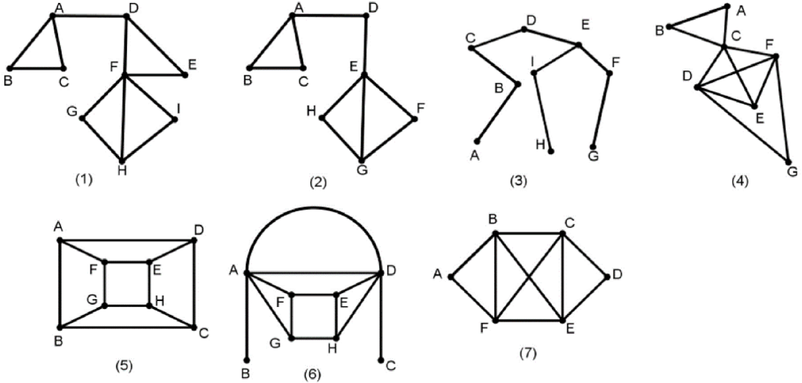


(8)



(9)

Some more examples



Do you see any patterns?

- Fill up the tables
- What feature of a graph makes it traceable?
- What pattern do you see for the graphs where the starting and ending point of the path is the same vertex?
- What about the graphs where starting and ending points are different?

Observations

An **Eulerian path** is a trail in a finite graph which visits every edge exactly once. Similarly, an **Eulerian cycle** is a Eulerian trail which starts and ends on the same vertex.

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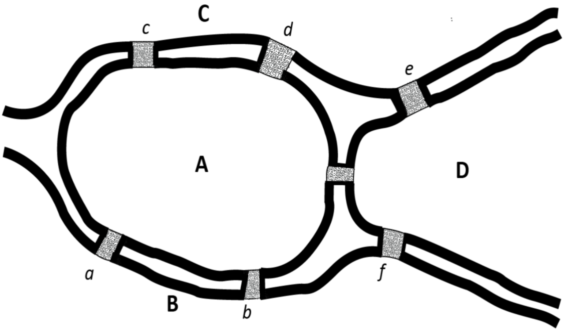
An **Eulerian path** is a trail in a finite graph which visits every edge exactly once. Similarly, an **Eulerian cycle** is a Eulerian trail which starts and ends on the same vertex.

Conditions

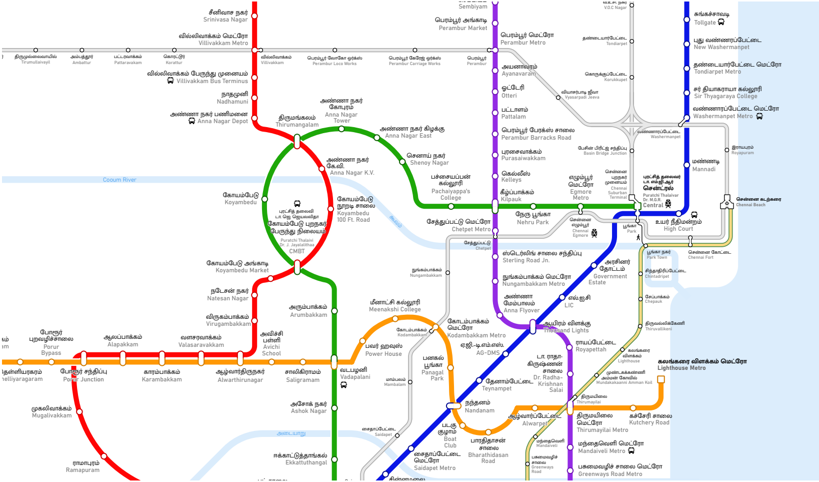
- For an Eulerian path a graph must have two vertices with odd degrees.
- For an Eulerian cycle, a graph must have all even degree vertices.

Euler enters!



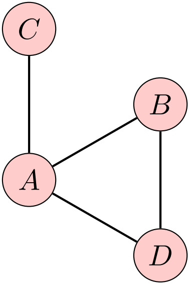
Can you map this to the map we have seen?

Try this out



Sembiyam
Srinivasa Nagar
Perambur Market
V.O.C Nagar
Tollgate
Villivakkam Metro
Perambur Metro
Tondiarpet
New Washermanpet
Tirumullaivayil
Ambattur
Pattaravakam
Korattur
Villivakkam
Perambur Loco Works
Perambur Carriage Works
Perambur
Tondiarpet Metro
Villivakkam Bus Terminus
Ayanavaram
Korukkupet
Sir Thyagaraya College
Nadhamuni
Otteri
Vyasarpadi Jeeva
Anna Nagar Tower
Washermanpet Metro
Anna Nagar Depot
Pattalam
Thirumangalam
Perambur Barracks Road
Anna Nagar East
Washermanpet
Basin Bridge Junction
Royapuram
Shenoy Nagar
Purasaiwakkam
Anna Nagar K.V.
Mannadi
Coourn River
Kelleys
Pachaiyappa's College
Egmore Metro
Chennai Suburban Terminal
Koyambedu
Kilpauk
Puratchi Thalaivar Dr. M.G.R. Central
Chennai Beach
Koyambedu 100 Ft. Road
Nehru Park
Chetpet Metro
Chennai Egmore
High Court
Park
Puratchi Thalaivi Dr. J. Jayalalithaa CMBT
Chetpet
Park Town
Chennai Fort
Sterling Road Jn.
Koyambedu Market
Government Estate
Nungambakkam
Chintadripet
Nungambakkam Metro
Natesan Nagar
Arumbakkam
Anna Flyover
LIC
Chepauk
Meenakshi College
Virugambakkam
Kodambakkam
Thousand Lights
Thiruvallikeni
Kodambakkam Metro
Avichi School
Porur Bypass
Alapakkam
Valasaravakkam
Power House
AG-DMS
Royapettah
Lighthouse Metro
Lighthouse
Panagal Park
Dr. Radha-Krishnan Salai
Porur Junction
Karambakkam
Alwarthirunagar
Saligramam
Vadapalani
Mambalam
Mundakakaanni Amman Koil
Teynampet
Nandanam
Thirumayilai
Mugalivakkam
Ashok Nagar
Alwarpet
Thirumayilai Metro
Kutchery Road
Saidapet
Boat Club
Mandaiveli
Mandaiveli Metro
Bharathidasan Road
Ramapuram
Ekkattuthangal
Saidapet Metro
Greenways Road
Greenways Road Metro

How to represent a graph



Adjacency Matrix

$$
\begin{array}{c c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}
\end{array}
$$

Edge List

$$\{(A, B), (A, C), (A, D), (B, D)\}$$