

Seven Bridges of Königsberg

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Königsberg aka Kalinigrad



Figure: (Source: Google Maps)

The Problem

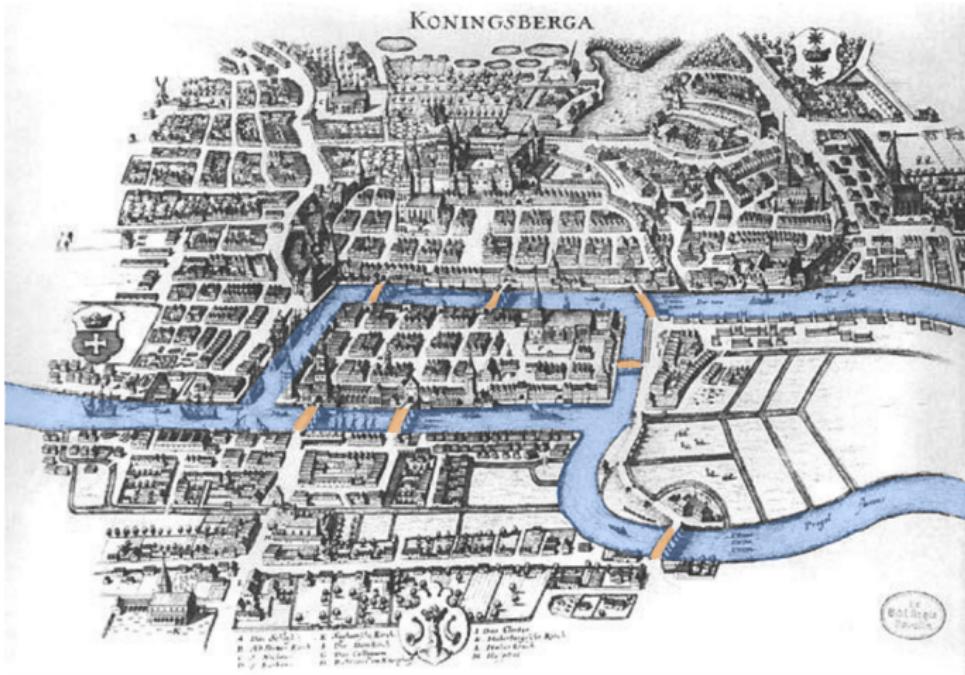


Figure: (Source: Merian-Erben, Wikimedia Commons)

Current situation

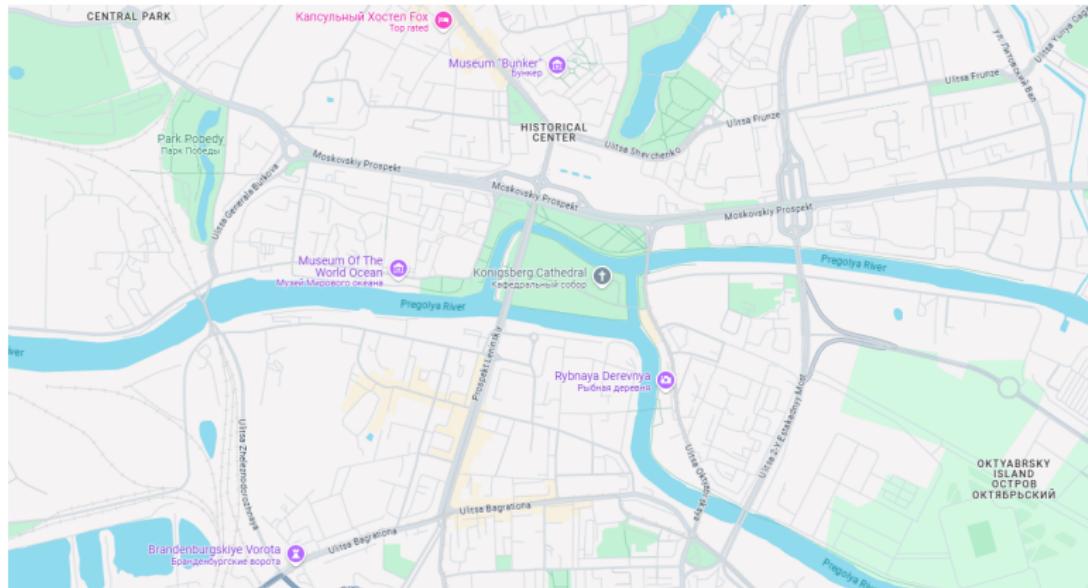
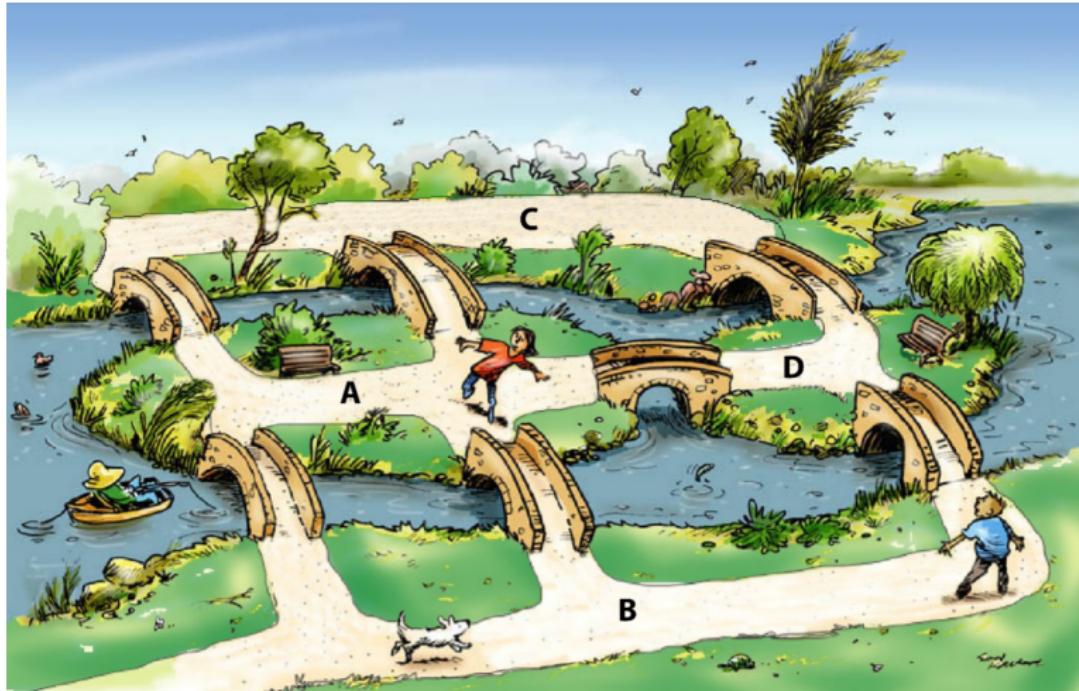


Figure: (Source: Google Maps)

A new problem!

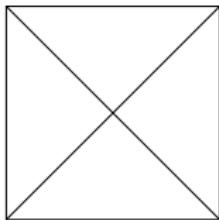


Is it really?

1. Can you find the required path?
2. Is this picture same as the one you saw of Königsberg Bridges?
3. Can you try to simplify the picture?

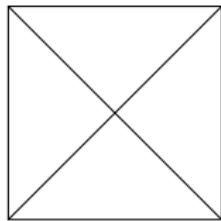
Another probelm!

Can you draw this picture without lifting the pencil?



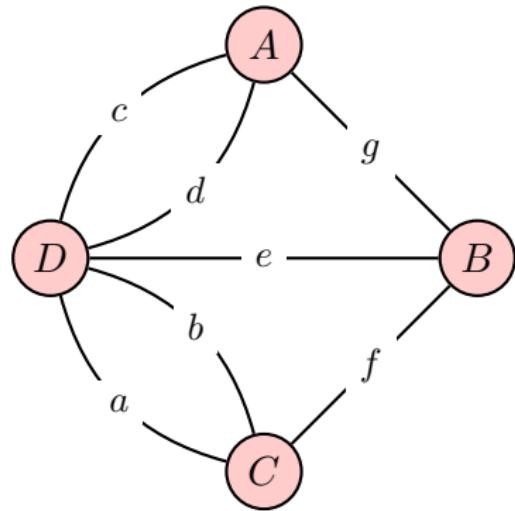
Another problem!

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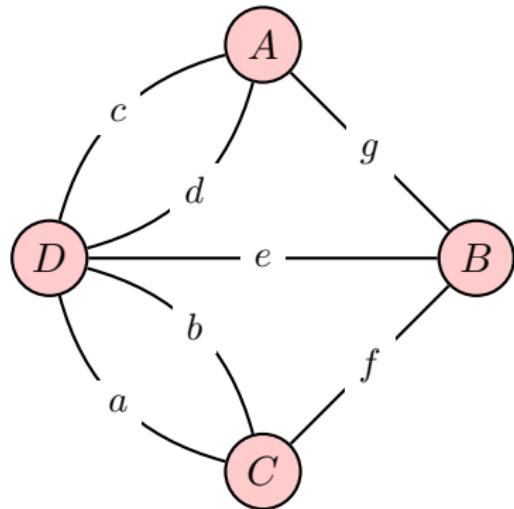


Is this somehow related to the Königsberg problem?

A Graph



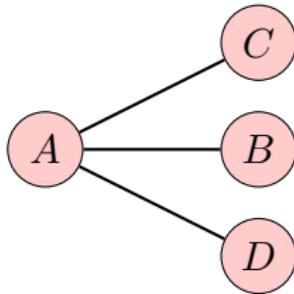
A Graph



Now can you try to trace the lines without lifting your pencil?

Graph Theory 101

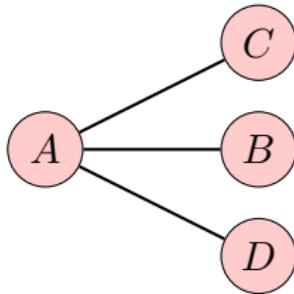
A graph is made of a set of points and lines connecting them. These points are called **nodes** or **vertices**. The lines are called **edges**¹. Degree is the number of edges connected to a node.



¹The terminology is inconsistent between fields

Graph Theory 101

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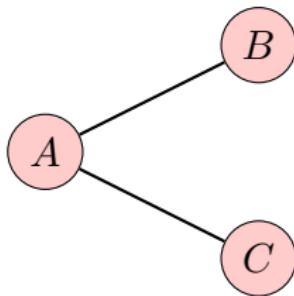
$$d(A) = 3$$

$$d(p) = 1, \quad p = B, C, D$$

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Graph Theory 101

A graph is made of a set of points and lines connecting them. These points are called **nodes** or **vertices**. The lines are called **edges**². Degree is the number of edges connected to a node.

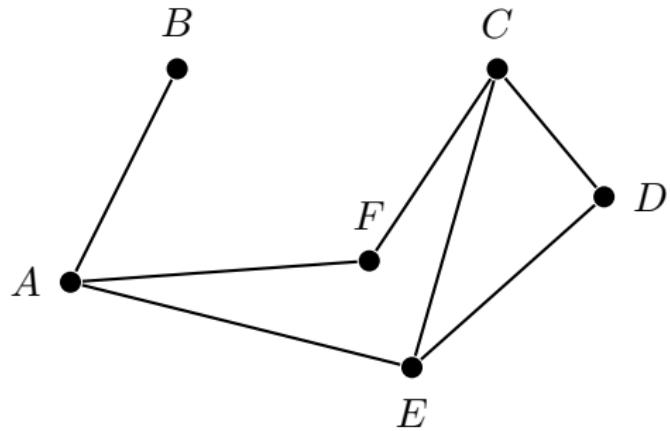


$$d(A) = 2$$

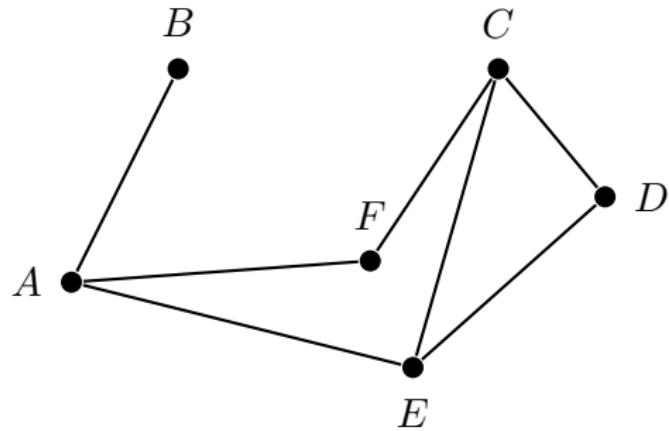
$$d(p) = 1, \quad p = B, C$$

²The terminology is inconsistent between fields

Find the degrees

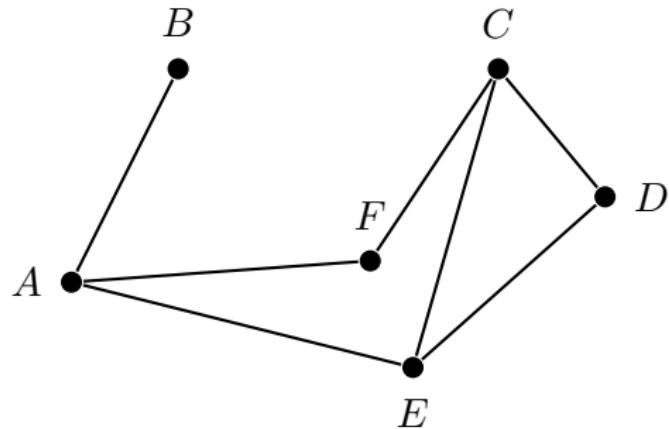


Find the degrees



Node	Degree
A	3
B	1
C	3
D	2
E	3
F	2

Find the degrees



Node	Degree
A	3
B	1
C	3
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Draw a graph and try to work out the degrees!

Some more examples



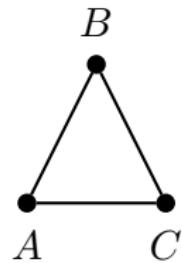
(1)



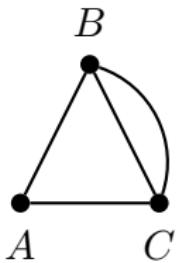
(2)



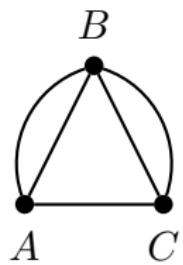
(3)



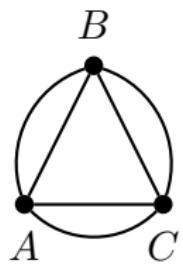
(4)



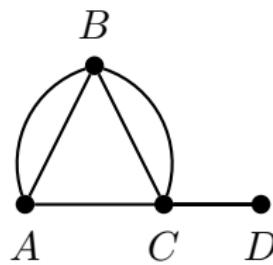
(5)



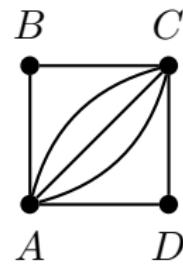
(6)



(7)

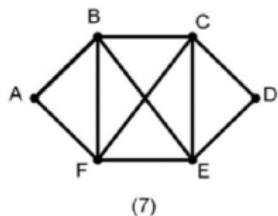
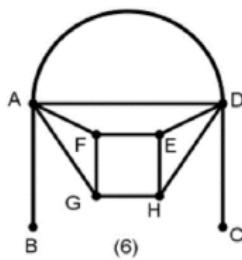
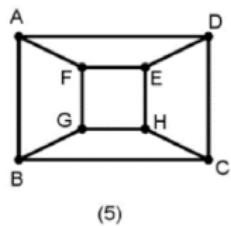
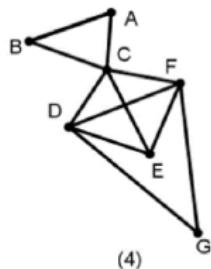
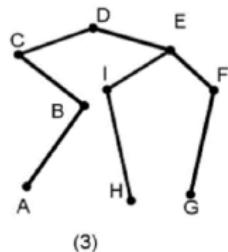
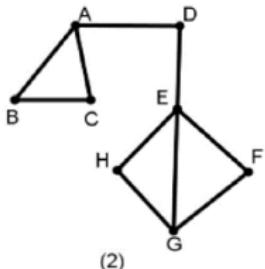
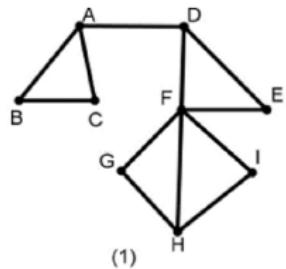


(8)



(9)

Some more examples



Do you see any patterns?

- ▶ Fill up the tables
- ▶ What feature of a graph makes it traceable?
- ▶ What pattern do you see for the graphs where the starting and ending point of the path is the same vertex?
- ▶ What about the graphs where starting and ending points are different?

Observations

An **Eulerian path** is a trail in a finite graph which visits every edge exactly once. Similarly, an **Eulerian cycle** is a Eulerian trail which starts and ends on the same vertex.

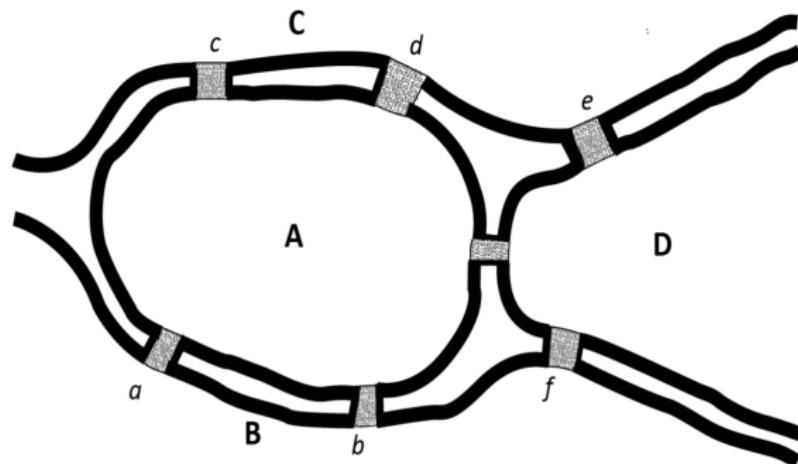
Observations

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Conditions

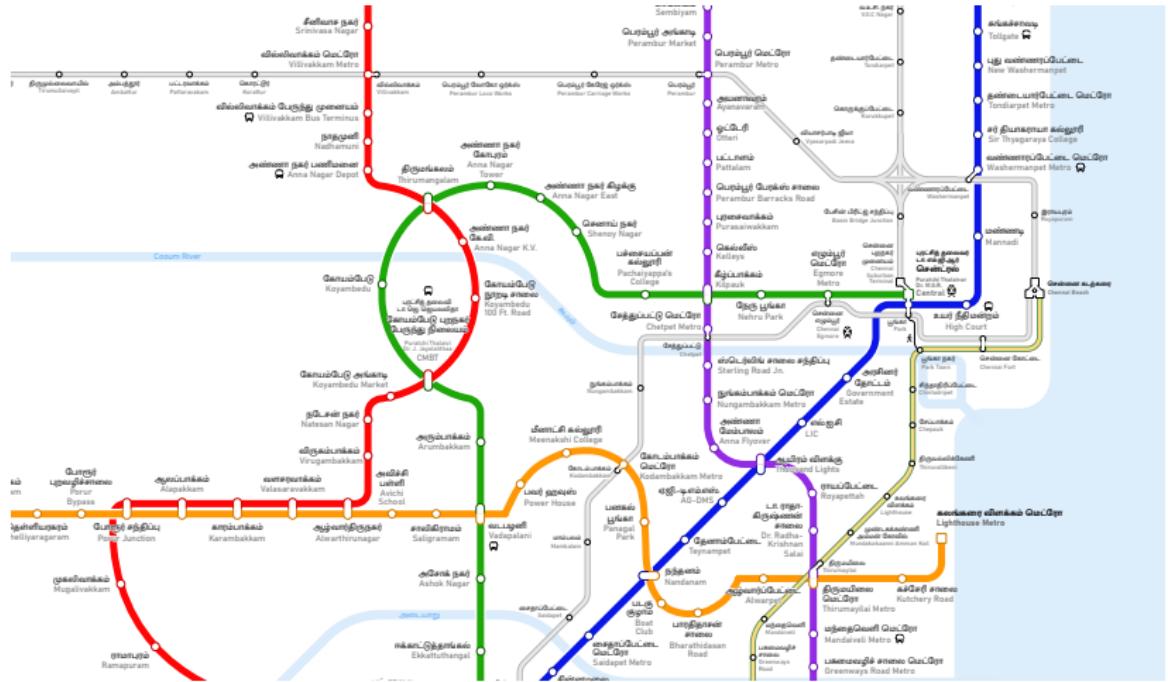
- ▶ For an Eulerian path a graph must have two vertices with odd degrees.
- ▶ For an Eulerian cycle, a graph must have all even degree vertices.

Euler enters!

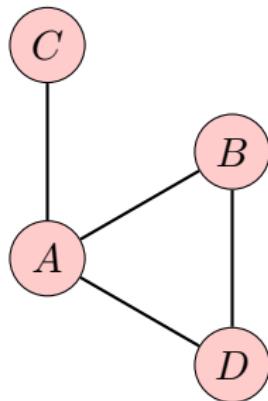


Can you map this to the map we have seen?

Try this out



How to represent a graph



Adjacency Matrix

$$\begin{pmatrix} & A & B & C & D \\ A & 0 & 1 & 1 & 1 \\ B & 1 & 0 & 0 & 0 \\ C & 1 & 0 & 0 & 0 \\ D & 1 & 1 & 0 & 0 \end{pmatrix}$$

Edge List

$$\{(A, B), (A, C), (A, D), (B, D)\}$$

Thank you

You can find the slides at

<https://akshaysanjeevk.github.io/resources/>

Reading Suggestions

- ▶ Harary, Frank. Graph theory (on Demand Printing of 02787). CRC Press, 2018.
- ▶ Newman, Mark, Networks, 2nd edn (Oxford, 2018; online edn, Oxford Academic, 18 Oct. 2018)