ONLINE APPENDIX FOR ENERGY INTENSITY, DIRECTED TECHNOLOGICAL CHANGE AND ECONOMIC GROWTH: WILL THE AUTONOMOUS DECLINE OF ENERGY INTENSITY CONTINUE?

Akshay Shanker & David Stern

PROOF OF CLAIM 2.1: We will use the growth equations and notation developed in the proof of Theorem 2.1. First, write the no-arbitrage conditions as follows

$$
\begin{aligned}
\frac{\eta_E \tilde{z}_E}{\eta_L \tilde{z}_L} &= \frac{\eta_E \alpha \kappa(t)^{-1} \gamma_E^{\frac{\epsilon}{\sigma-1}} \tilde{\theta}_E(t)^{\frac{2-\sigma}{1-\sigma}} \tilde{Y}(t) - \rho}{\eta_L \gamma_L^{\frac{\epsilon}{\sigma-1}} \left(1 - \tilde{\theta}_E(t)\right)^{\frac{1}{1-\sigma}} L - \rho} \\
&> \frac{\eta_E \alpha \kappa(t)^{-1} \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(t)^{\frac{2-\sigma}{1-\sigma}} Y(t) - \rho}{\eta_L \gamma_L^{\frac{\epsilon}{\sigma-1}} \left(1 - \theta_E(t)\right)^{\frac{1}{1-\sigma}} L - \rho} \\
&= \frac{\eta_E z_E}{\eta_L z_L}
\end{aligned}
$$

where the equality uses (51) and (77). The inequality follows from noting the RHS is decreasing in θ_E holding and since $\tilde{Y}(t) = Y(t)$. This serves to prove part 1. of the claim.

To show part 2. of the claim, note

$$
\begin{aligned}
\eta_E \tilde{z}_E &= \eta_E \alpha \kappa(t)^{-1} \gamma_E^{\frac{\epsilon}{\sigma-1}} \tilde{\theta}_E(t)^{\frac{2-\sigma}{1-\sigma}} \tilde{Y}(t) - \rho \\
&> \eta_E \alpha \kappa(t)^{-1} \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(t)^{\frac{2-\sigma}{1-\sigma}} - \rho = \eta_E z_E
\end{aligned}
$$

As such $\hat{\tilde{Q}}_E(t) > \hat{Q}_E(t)$. Next, taking time derivatives of Equation (21), we have

$$
\begin{aligned}
\frac{\hat{\tilde{E}}(t)}{\hat{\tilde{Y}}(t)} &= (1-\sigma) \frac{\hat{\tilde{Q}}(t)_E}{\hat{\tilde{Y}}(t)} + 1 \\
&= \frac{(\sigma - 1)\, \hat{\tilde{Q}}(t)_E}{\frac{1+\hat{\theta}_E(\sigma-1)}{1-\hat{\theta}_E} \hat{\tilde{Q}}(t)_E - \hat{\tilde{B}}(t)} \\
&= \left(\frac{\frac{1+\hat{\theta}_E(\sigma-1)}{1-\hat{\theta}_E(t)} \hat{\tilde{Q}}(t)_E - \hat{\tilde{B}}(t)}{(\sigma-1)\, \hat{\tilde{Q}}_E(t)} \right)^{-1} + 1 \\
&= \left(\frac{1 + \tilde{\theta}(t)_E\, (\sigma - 1)}{\left(1 - \tilde{\theta}(t)_E\right) (\sigma - 1)} + (1-\sigma)^{-1} \left(1 - \frac{\tilde{z}_L(t)}{\tilde{z}_E(t)} \right) \right)^{-1} + 1 \\
&< \left(\frac{1 + \theta_E(t)\, (\sigma - 1)}{\left(1 - \theta_E(t)\right) (\sigma - 1)} + (1-\sigma)^{-1} \left(1 - \frac{z_L(t)}{z_E(t)} \right) \right)^{-1} + 1 \\
&= \frac{\hat{E}(t)}{\hat{Y}(t)}
\end{aligned}
$$

where the second equality use (42) along with (44) to derive

$$\hat{Y}(t) = \frac{1+\theta_E(t)(\sigma-1)}{1-\theta_E(t)}\hat{Q}_E(t) - \hat{B}(t) \tag{1}$$

thus we have shown $\frac{\hat{\hat{E}}(t)}{\hat{\hat{Y}}(t)} < \frac{\hat{E}(t)}{\hat{Y}(t)}$, which directly implies part 3 of the claim.

Q.E.D.

APPENDIX E: TRANSITIONAL DYNAMICS FOR THE NO SCALE EFFECTS REGIME

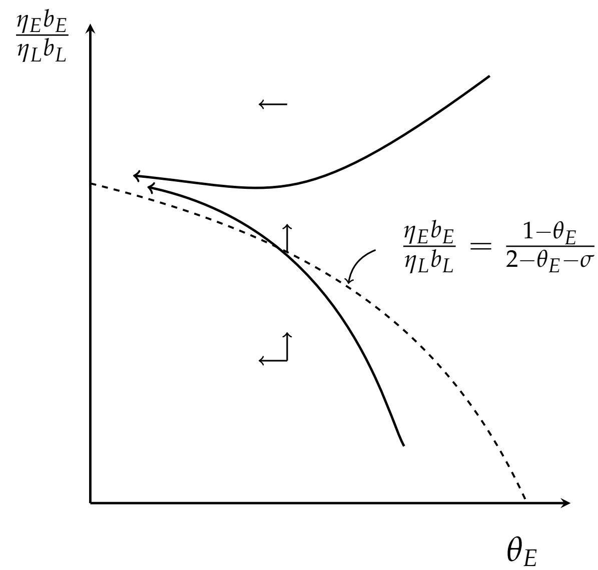

FIGURE E.1.— ABGPs for a regime with no scale effects. All ABGPs must converge to a BGP where energy intensity is zero.

Figure E.1 shows the dynamics for an equilibrium with no scale effects from two possible initial conditions. First, if $\frac{\eta_E b_E(0)}{\eta_L b_L(0)} \leqslant \frac{1-\theta_E(0)}{2-\theta_E(0)-\sigma}$, then Claim E.1 confirms $\frac{\eta_E b_E}{\eta_L b_L}$ rises till $\frac{\eta_E b_E(t)}{\eta_L b_L(t)} > \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$. Second, if $\frac{\eta_E b_E(t)}{\eta_L b_L(t)} > \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$, we are not able to verify exact conditions for the direction of movement of $\frac{\eta_E b_E(t)}{\eta_L b_L(t)}$. However, proposition (E.1) confirms any equilibrium is an asymptotic growth path, which, along with Theorem 2.1 confirms $\lim\limits_{t\to\infty} \frac{\eta_E b_E(t)}{\eta_L b_L(t)} = \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$, implying the paths shown by figure E.1.

CLAIM E.1 If $\mathscr{E}$ is an equilibrium with no scale effects, then there exists T such that for all $t > T$, $\frac{\eta_E b_E(t)}{\eta_L b_L(t)} \geqslant \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$.

PROOF: If $\frac{\eta_E b_E(t)}{\eta_L b_L(t)} \leqslant \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$ for some t, then $\frac{\hat{Q}_E(t)}{\hat{Q}_L(t)} \leqslant \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$. Recall the definition of $\Gamma(t)$ at equation (45), and note (46) holds along the equilibrium path. Recalling Equation (48) and using $\frac{\hat{Q}_E(t)}{\hat{Q}_L(t)} \leqslant \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$, we have

$$\hat{\Gamma}(t) = \frac{\sigma - 1}{1-\theta_E(t)}\hat{Q}_E(t) - \hat{B}(t) \geqslant 0 \tag{2}$$

Now take growth rates across $\Gamma(t)$ as defined at Equation (45) and use Equation (2) to arrive at

$$\frac{\eta_E \dot{b}_E(t)}{\eta_L \dot{b}_L(t)} \geqslant \frac{\rho + \eta_E b_E(t)}{\rho + \eta_L b_L(t)} \tag{3}$$

which in turn implies

$$\frac{\hat{b}_E(t)}{\hat{b}_L(t)} \geqslant \frac{\eta_L b_L(t)}{\eta_E b_E(t)}\frac{\rho + \eta_E b_E(t)}{\rho + \eta_L b_L(t)} > 1 \tag{4}$$

where the strict inequality holds since $\frac{\eta_L b_L(t)}{\eta_E b_E(t)} > 1$. Thus if $\frac{\eta_E b_E(t)}{\eta_L b_L(t)} \leqslant \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$, then $\frac{\eta_E b_E(t)}{\eta_L b_L(t)}$ rises till $\frac{\eta_E b_E(t)}{\eta_L b_L(t)} > \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$.

Moreover, if $\frac{\eta_E b_E(T)}{\eta_L b_L(T)} \geqslant \frac{1-\theta_E(T)}{2-\theta_E(T)-\sigma}$ for some T, then for any $t > T$, $\frac{\eta_E b_E(t)}{\eta_L b_L(t)} \geqslant \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$. To see why, suppose for some $t' > T$, we have $\frac{\eta_E b_E(t')}{\eta_L b_L(t')} < \frac{1-\theta_E(t')}{2-\theta_E(t')-\sigma}$, then there must be some $t'' \leqslant t'$ such that $\frac{\eta_E b_E(t'')}{\eta_L b_L(t'')} = \frac{1-\theta_E(t'')}{2-\theta_E(t'')-\sigma}$ and $\frac{\eta_E b_E(t'')}{\eta_L b_L(t'')}$ is decreasing. However, at t'', (2) must then hold, which in turn implies (3) holds and $\frac{\eta_E b_E(t'')}{\eta_L b_L(t'')}$ cannot be decreasing.

Q.E.D.

PROPOSITION E.1 *If $\mathscr{E}$ is a no scale effects equilibrium and $\alpha\gamma_L^{\frac{\epsilon}{\sigma-1}}(1-\theta_E(0))^{\frac{1}{1-\sigma}} L - \rho > 0$, then $\mathscr{E}$ is an ABGP.*

PROOF: To show $\mathscr{E}$ is an ABGP, we show $\lim_{t\to\infty} b_E(t) = b_E^\star$ and $\lim_{t\to\infty} b_L(t) = b_L^\star$.

Before commencing the proof, note that since Q_E is non-decreasing, by Equation (38), $\theta_E(t) \leqslant \theta_E(0)$ for all $t \in \mathbb{R}_+$. Moreover, use (25) and (52) to write equilibrium innovation in the labour sector as

$$\rho + \eta_L b_L(t) = \alpha\gamma_L^{\frac{\epsilon}{\sigma-1}}(1-\theta_E(t))^{\frac{1}{1-\sigma}} L \tag{5}$$

implying $b_L(t) \in [\alpha\gamma_L^{\frac{\epsilon}{\sigma-1}}(1-\theta_E(0))^{\frac{1}{1-\sigma}}L-\rho, \alpha\gamma_L^{\frac{\epsilon}{\sigma-1}}L-\rho]$ for all t.

Next, by Claim E.1, there exists T such that for all $t > T$, $\frac{\eta_E b_E(t)}{\eta_L b_L(t)} \geqslant \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$. Thus, we have

$$(6) \qquad b_E(t) \geqslant \frac{\alpha\gamma_L^{\frac{\epsilon}{\sigma-1}}(1-\theta_E(0))^{\frac{1}{1-\sigma}}L-\rho}{\eta_E}\frac{1}{2-\sigma} : = \underline{b}_E, \qquad t \geqslant T$$

Since $b_E(t) \geqslant \underline{b}_E > 0$ for all t, $\lim_{t\to\infty} Q_E(t) = \infty$ and by equation (38), $\lim_{t\to\infty}\theta_E(t) = 0$, which in turn implies by Equation (5) that $\lim_{t\to\infty}\eta_L b_L(t) = \alpha\gamma_L^{\frac{\epsilon}{\sigma-1}}L-\rho$. Let $b_L^\star := \lim_{t\to\infty} b_L(t) = \alpha\gamma_L^{\frac{\epsilon}{\sigma-1}}L-\rho$.

We now turn to show $\lim_{t\to\infty} b_E(t) = b_E^\star$ for some $b_E \in \mathbb{R}_{++}$. Recall the definition of $\Gamma(t)$ at equation (45), and note (46) holds along the equilibrium path. Recalling Equation (48) and using $\frac{\hat{Q}_E(t)}{\hat{Q}_L(t)} \geqslant \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$, we have

$$\hat{\Gamma}(t) = \frac{\sigma-1}{1-\theta_E(t)}\hat{Q}_E(t) - \hat{B}(t) \leqslant 0, \qquad t \geqslant T$$

Since $\hat{\Gamma}(t) = \frac{\dot{\Gamma}(t)}{\Gamma(t)}$ and $\Gamma(t) \geqslant 0$, $\Gamma(t)$ must be decreasing for all $t > T$. However,

$$\Gamma(t) = \frac{\rho + b_E(t)}{\rho + b_L(t)} \geqslant \frac{\rho + \underline{b}_E}{\alpha\gamma_L^{\frac{\epsilon}{\sigma-1}}L} > 0, \qquad t \geqslant T$$

implying $\lim_{t\to\infty}\Gamma(t) > 0$. Let $\bar{\Gamma} := \lim_{t\to\infty}\Gamma(t)$ and use the properties of limits of functions to conclude,

$$\lim_{t\to\infty} b_E(t) = \lim_{t\to\infty}\left(\Gamma(t)(b_L(t)+\rho)-\rho\right) = \left(\bar{\Gamma}(b_L^\star+\rho)-\rho\right) =: b_E^\star$$

where $b_E^\star \in \mathbb{R}_{++}$, thus completing the proof.

Q.E.D.

APPENDIX F: DATA SOURCES AND FURTHER DATA ANALYSIS

F.1. *U.S. Time Series Data Sources*

F.1.1. *GDP, Implicit Price Deflator, Employment, and Wages*

Data on GDP and wages from 1929 to 2015 are sourced from the U.S. Bureau of Economic Analysis (BEA) National Income and Product Accounts (NIPA). GDP data is real GDP in chained 2009 dollars. We extended the GDP series to 1900 using the growth rates from estimates of the GNP from the *Historical Statistics of the United States: Colonial Times to 1970* (HS70) Annual data on GNP is available from 1889. Total wages from 1929 to 2015 are given by "compensation of employees" and deflated to 2009 using the GDP implicit price deflator. Page 236 in the latter source provides estimates of the labor share of national income for years prior to 1929. Page 224 has some estimates of the national income prior to 1929. Prior to 1919 these are for 5-year averages. For years prior to 1919 we use these averages to obtain the ratio of national income to GNP for each 5-year period and then estimated annual national income by multiplying the ratio by annual GNP. We then multiplied average labor compensation shares in the national income to estimate labor compensation back to 1900. *Historical Statistics of the United States: Millennial Edition* gives employment from 1890 to 1990. We extend this forward to 2015 using the growth rate of total employment in the NIPA. We can then compute labor productivity and annual wage series per employee from 1900 on.

F.1.2. *Primary Energy Consumption and Heat Rates*

We use data from the U.S. Energy Information Administration (EIA) website for 1949-2015 for consumption of the following energy carriers: Coal, natural gas, petroleum, nuclear electricity, hydroelectricity, geothermal energy, solar energy, wind energy, and biomass, in quadrillion BTU. The energy totals given for the non-combustible renewables and nuclear power are the equivalent quantity of fossil fuels that would be needed to generate the same amount of electricity. We used these heat rates, which are supplied by EIA to convert the price of electricity to a price per BTU of primary energy. Earlier energy quantity data were obtained from the HS70 and Appendix D of the EIA's Monthly Energy Review. These data also include animal feed, which was a large source of energy in 1900.

F.1.3. *Energy Prices*

Using a combination of documents and databases on the EIA website we assembled fossil fuel production prices from the earliest available date to 2015. Oil wellhead prices are available in the online interactive data from 1859 to 2015. The natural gas wellhead price is available from 1922 but discontinued after 2012. For 2013-15 we use Henry Hub spot prices available on this page:

`http://www.eia.gov/dnav/ng/hist/rngwhhdA.htm`

Coal prices were available for 1949-2011 from the 2011 Annual Energy Review. For 2012-2015 we used the Annual Coal Reports. Biomass prices for 1970 to 2014 are available from this webpage:

http://www.eia.gov/state/seds/data.cfm?incfile=/state/seds/sep_prices/total/pr_tot_US.html&sid=US

Electricity prices are available from the EIA from 1960 to 2015. We use the industrial electricity price as a proxy for the wholesale price of electricity. Earlier energy prices were obtained from the HS70. For electricity prices we used the price series for "large users" and assumed the nominal price per million BTU was constant prior to 1917. For the price of animal feed we used the price per bushel of oats received by farmers from HS70. We assumed 32lbs of oats per bushel and 12MJ per kg of feed (Pagan, 1998). We use the price of lumber from HS70 to project back the cost of biomass energy for years before 1970. We use the growth rates of the price of oil to project natural gas prices back to 1900. In computing the total value of energy we multiply the electricity produced by nuclear and non-combustible renewables by the price of electricity. We then compute energy productivity and price series for raw BTU of primary energy and for quality-adjusted energy use.

F.2. *U.S. Fuel Prices*

Figure F.1 shows the real price in 2009 US Dollars per BTU of each of the individual fuels. Even though each of these prices has increased absolutely over time, it is unclear whether these represent systematic trends or not. The aggregate energy price F.3 rises faster than most of the component prices because there is a positive correlation between price movements and cost shares.

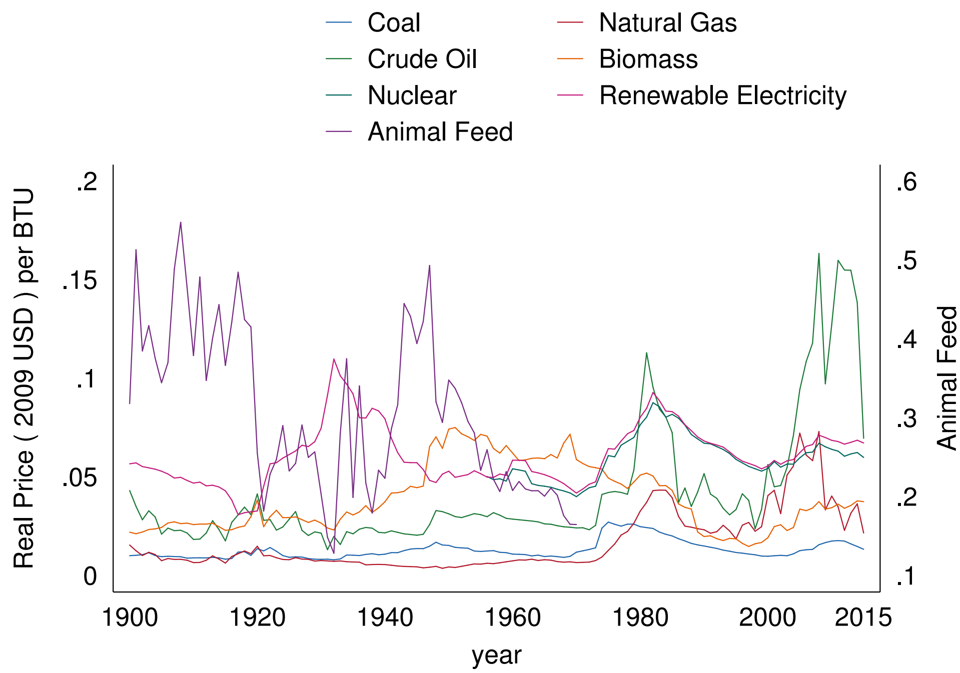

FIGURE F.1.— Real U.S. prices of energy carriers

Figure F.2 and Figure F.3 shows energy intensity and aggregate U.S. fuel price with and without animal feed. Animal feed was a significant part of the fuel mix prior to the 1930s, after which the two different price and energy intensity series converge. Note a clear positive trend in the aggregate price without animal feed. As noted above, there is a positive correlation between cost shares and changes in energy prices. In particular, oil price movements dominate the aggregate price in later decades.

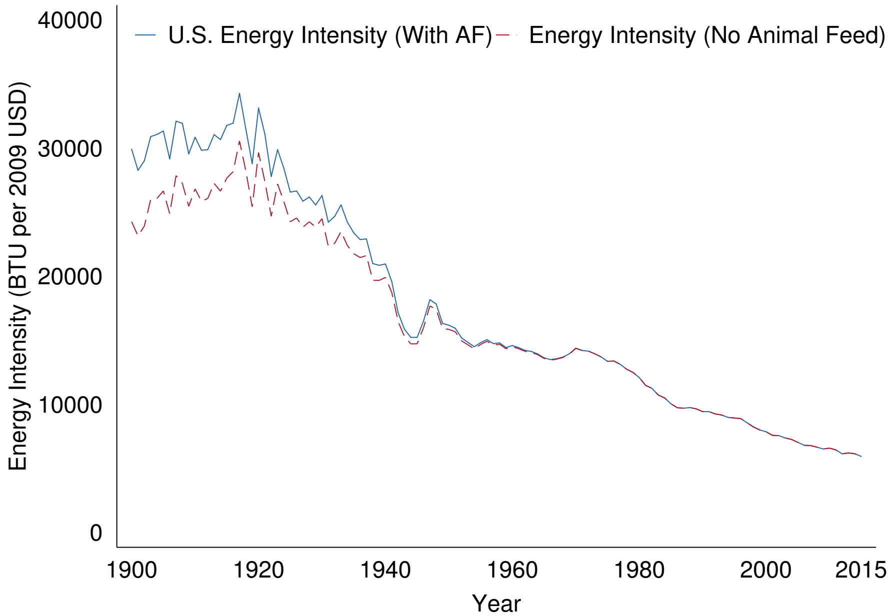

FIGURE F.2.— U.S. energy intensity with and without animal feed (AF)

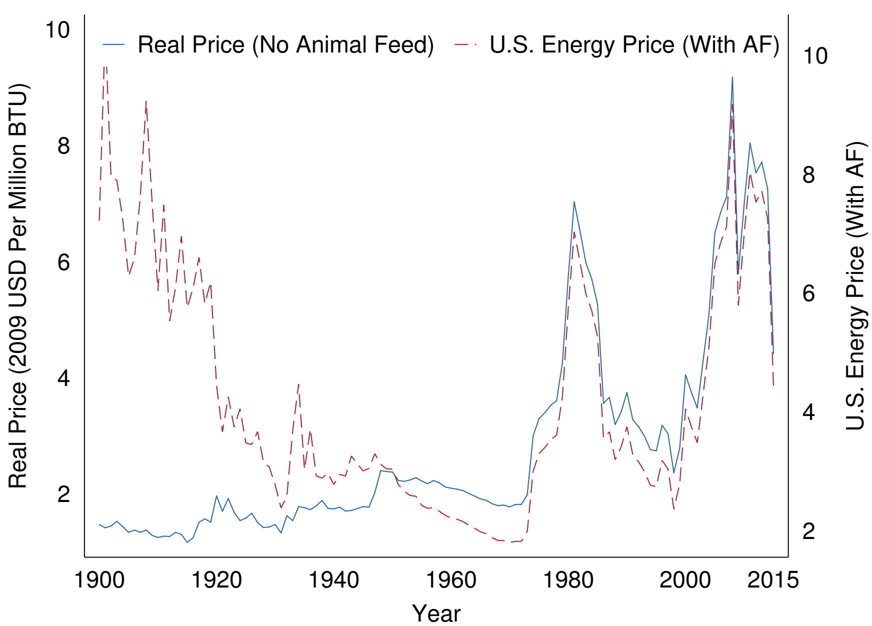

FIGURE F.3.— U.S. aggregate fuel price with and without animal feed (AF)

To test for trends, we use the t_{DAN} Fomby and Vogelsang (2003) test (also known as the Dan-J test (Bunzel and Vogelsang, 2005)) based on a modified t-test on the slope parameter of the simple linear trend regression model:

$$y_{i,t} = \beta_{1,i} + \beta_{2,i} t_i + u_{i,t} \tag{7}$$

where t is a linear time trend, i indicates the location or sample period of the data, u, is a stochastic process that may or may not be stationary and and β_1 and β_2 are regression parameters to be estimated. Then the trend test statistic is given by:

$$t_{DAN} = \frac{\hat{\beta}_{2,i}}{\mathrm{se}\left(\hat{\beta}_{2,i}\right)} e^{-bJ}$$

where $\hat{\beta}_{2,i}$ is the estimate of the slope parameter and $\mathrm{se}\left(\hat{\beta}_{2,i}\right)$ its standard error, b is a parameter computed by Fomby and Vogelsang (2003), and

$$J = \frac{RSS_1 - RSS_4}{RSS_4}$$

where RSS_1 is the sum of squared residuals from (7), and RSS_4 is the sum of squared residuals from the following regression:

$$y_t = \sum_{i=0}^{9} \gamma_i t^i + v_t$$

The standard error $\mathrm{se}\left(\hat{\beta}_{2,i}\right)$ is computed as follows

$$\mathrm{se}\left(\hat{\beta}_{2,i}\right) = \sqrt{\hat{\sigma}^2 \left(\sum_{t=1}^{T} (t - \bar{t})^2\right)^{-1}}$$

with $\bar{t} = T^{-1} \sum_{t=1}^{T} t$, where T is the sample size and

$$\hat{\sigma}^2 = \hat{\gamma}_0 + 2 \sum_{j=1}^{T-1} \frac{\sin(j\pi/M)}{j\pi/M} \hat{\gamma}_j$$

where $\hat{\gamma}_j = T^{-1} \sum_{t=j+1}^{T} \hat{u}_t \hat{u}_{t-j}$ is a function of the estimated residuals $\hat{u}$ and $M = \max\{0.02T, 2\}$.

The recommended value of b and the critical values of t_{DAN} for a two-tailed test are as follows (Fomby and Vogelsang, 2003): $b = 2.466$, $t_{DAN} = 2.462$ at 1%; $b = 1.795$, $t_{DAN} = 2.052$ at 2.5%; $b = 1.322$, $t_{DAN} = 1.710$ at 5% and $b = 0.965$, $t_{DAN} = 1.329$ at the 10% significance level. Further values can be derived from the formulae in Bunzel and Vogelsang (2005). J can also be used in a left-tailed test of the null hypothesis that the errors in (7) contain a unit root autoregressive process or random walk. The null hypothesis is rejected for small values of the statistic. The critical values are 0.488 at 1%, 0.678 at 2.5%, and 0.908 at the 5% significance levels.

TABLE F.1

TREND TEST RESULTS FOR INDIVIDUAL PRICE SERIES - SIGNIFICANT TEST STATISTICS IN BOLD

Log real price	N	β	$t_{DAN}01$	$t_{DAN}025$	$t_{DAN}05$	$t_{DAN}10$	J
Coal	116	0.00454	0.05304	0.17444	0.40376	0.76072	1.77433
Natural Gas	116	0.01603	3.62E-04	0.00528	0.03497	0.1456	3.99518
Crude Oil	116	0.01168	0.06151	0.22571	0.56434	1.12699	1.93742
Biomass	116	0.00126	1.80E-10	7.36E-08	5.10E-06	1.25E-04	8.96129
Nuclear	60	0.0049	4.43E-24	1.67E-17	7.24E-13	2.29E-09	22.56978
Renewable	116	0.00231	0.00888	0.04083	0.11966	0.26939	2.27309
Animal Feed	71	-0.02972	-0.10661	-0.43794	-1.18566	**-2.51437**	2.10566

There is no significant trend in any of the series except animal feed, which has a significant negative trend at the 10% level. All have a unit root, so the series are not stationary, but apart from animal feed they do not have a significant drift either.

We then tested for trends in the aggregate prices with the following results

Log real price	β	$t_{DAN}01$	$t_{DAN}025$	$t_{DAN}05$	$t_{DAN}10$	J
Raw Energy Price with AF	-2.00E-03	-6.64E-07	-3.25E-05	-5.04E-04	-0.004	5.79794
Raw Energy Price without AF	0.01375	0.64543	1.45564	**2.58247**	**3.98066**	1.21205

TABLE F.2

TREND TEST RESULTS FOR AGGREGATE PRICE SERIES - SIGNIFICANT TEST STATISTICS IN BOLD

For the data with animal feed there is no trend. For the series without animal feed there is a positive trend at the 5% significance level for the raw series.

APPENDIX G: TRENDS IN ENERGY INTENSITY — ENERGY INTENSITY DECLINES FASTER IN COUNTRIES WITH HIGHER ENERGY INTENSITY

Figure 2 shows the time series relationship between energy intensity and GDP per capita for key countries between 1971 and 2010. Figure G.4 shows the elasticity of the average annual rate of energy intensity decline with respect to average annual output growth for the 100 countries between 1971 and 2010. The mean elasticity is -.28 and the negative relationship between elasticity and energy intensity in 1970 is statistically significant.

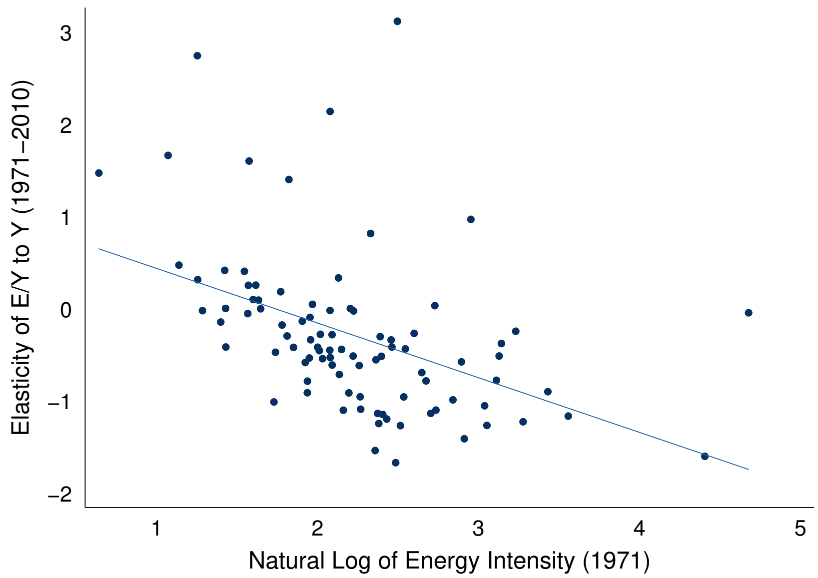

FIGURE G.4.— Elasticity of average rate of decline in energy intensity to average rate of output growth between 1971 and 2010. The line of best fit has a slope coefficient of -0.59 (std. err. 0.12).

REFERENCES

Bunzel, H. and Vogelsang, T. J. (2005). Powerful Trend Function Tests That Are Robust to Strong Serial Correlation, with an Application to the Prebisch: Singer Hypothesis. *Journal of Business & Economic Statistics*, 23(4):381–394.

Fomby, T. and Vogelsang, T. (2003). Test of Common Deterministic Trend Slopes Applied to Quarterly Global Temperature Data. In Fomby, T. and Hill, R., editors, *Maximum Likelihood Estimation of Misspecified Models: Twenty Years Later, Advances in Econometrics 17*. Elsevier.

Pagan, J. (1998). Carbohydrates in Equine Nutrition. In Pagan, J., editor, *Advances in Equine Nutrition*. Nottingham University Press, Nottingham.