ENERGY INTENSITY, DIRECTED TECHNOLOGICAL CHANGE AND ECONOMIC GROWTH: WILL THE AUTONOMOUS DECLINE OF ENERGY INTENSITY CONTINUE?

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World and U.S. energy intensities have declined over the past century, falling approximately at an average rate of 1.2–1.5 percent a year. The decline has persisted through periods of stagnating or even falling energy prices, suggesting the decline is driven in a large part by autonomous factors, independent of price changes. In this paper, we use directed technical change theory to understand the autonomous decline in energy intensity and ask whether the decline will continue. We show in an economy with no state-dependence, where existing knowledge does not make R&D more profitable, energy intensity continues to decline, albeit at a slower rate than output growth, from profit driven energy augmenting innovation. However, in an economy with complete state-depdence, energy intensity eventually stops declining because labor augmenting innovation crowds out energy augmenting innovation. In either case, energy intensity never declines faster than output grows, and so energy use always increases, as long as the extraction cost of energy stays constant.

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The latest version of this paper and the Online Appendix can be found at `https://github.com/akshayshanker/Energy`.

1. INTRODUCTION

Why does energy intensity fall as an economy grows? Why does energy intensity fall slower than the rate of output growth? Will energy intensity continue to fall in the absence of sustained energy price increases? In this paper, we use a model of endogenous growth with directed technical change to answer these questions.

The past century has seen a persistent decline in energy intensity both globally and in many individual countries. Figure 1 shows U.S. energy intensity and the U.S. real energy price from 1900 to 2015. While the energy price has fluctuated, we find no clear increasing trend.[1] And even during periods of falling prices, particularly between 1980 and 2000, energy intensity continued to decline. Figure 2 shows energy intensity and per capita GDP for 100 countries between 1971 and 2010 and Figure 3 for selected countries between 1900 and 2010 – per capita GDP growth is associated with a proportional decline in energy intensity. Csereklyei et al. (2016) find that the elasticity of energy intensity with respect to per capita GDP is -.3 and that there is conditional and unconditional beta convergence and sigma convergence of energy intensity.

Energy economists have suspected autonomous factors *tied to growth*, rather than increasing prices, have played a role in the world-wide decline of energy intensity.[2] Indeed, climate policy models,[3] and projections of future energy intensity, for example by the International Energy Association (IEA) (IEA, 2016) and the U.S. Energy Information Administration (EIA, 2017), both assume the autonomous decline of energy intensity will continue and will be more rapid than in the past (Stern, 2017).

However, despite energy intensity being "central to the achievement of a range of policy goals, including energy security, economic growth and environmental sustainability" (IEA, 2017) and despite the well-established literature incorporating endogenous technical change in models estimating the costs of climate policy (Goulder and Schneider, 1999; Nordhaus, 2002; Jakeman et al., 2004; Popp, 2004), there has been no formal analytical study of what, in terms of economic incentives,

[1]See the Online Appendix, Section E.2, for trend tests.

[2]The climate policy modeling literature refers to the autonomous decline of energy intensity as Autonomous Energy Efficiency Improvement (AEEI). Though extensively debated, estimates of AEEI for the U.S. range from .5% to 2% per year (See Williams et al. (1990), Löschel (2002), Stern (2004), Sue Wing and Eckaus (2007) and Webster et al. (2008)). Newell et al. (1999), who use micro-level data on efficiency of air-conditioners, find price increases induce technical change but "autonomous drives of energy efficiency explain up to 62% of total changes in energy efficiency". On the other hand, Kaufmann (2004) using a co-integration analysis finds no left-over deterministic trend.

[3]See Popp et al. (2010) for an overview of the use of AEEI in models with exogenous technical change. Some models with induced technical change also incorporate and exogenous efficiency trend, for example, Popp (2004).

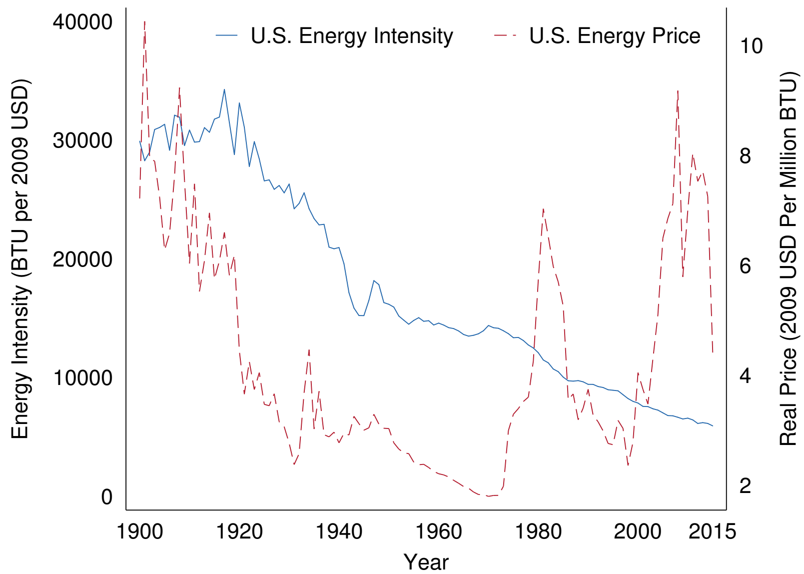

FIGURE 1.— U.S. energy intensity (including traditional fuels) and real price index of primary energy between 1900 and 2015. Energy intensity fell at an average rate of 1.2 percent per year between 1900 and 1950 and 1.5 percent per year between 1950 and 2015. See the Online Appendix, section E.1, for a detailed description of the data sources.

drives autonomous declines in energy intensity and whether the autonomous decline will continue. We undertake such an analysis in this paper.

We use a model of directed technical change (Acemoglu, 1998, 2002), where innovation can augment labor or energy, in a Schumpetarian endogenous growth model with quality improvements. The quality improvement model can be traced to Aghion and Howitt (1992), however, our treatment with a large number of firms most closely follows Acemoglu (2009), Chapter 14.[4] Following established estimates in the literature (see Stern and Kander (2012) and also Van der Werf (2008)), we assume the elasticity of substitution between energy and labor services is less than one.

[4]The structure of our modeling is also similar to Acemoglu and Cao (2015), except Acemoglu and Cao (2015) also incorporate radical innovations while we incorporate directed technical change.

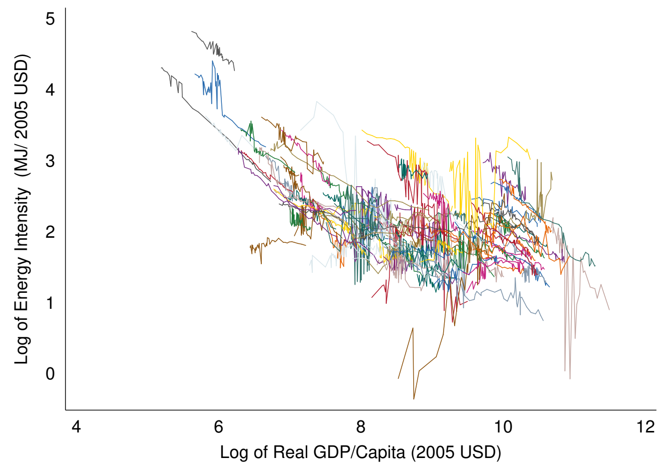

FIGURE 2.— Natural log of energy intensity and natural log of output for a cross section of countries. Csereklyei et al. (2016) describe the sources of the data.

1.1. *Main Findings*

Our first finding is that falling energy intensity without increasing energy extraction costs is *inconsistent* with balanced growth.[5] Rather, an autonomous decline of energy intensity implies a falling cost share of energy and a non-constant, but possibly converging growth rate of output. Since we can no longer use balanced growth paths (BGP), as in the text-book treatment of directed technical change, we proceed to formalize asymptotic behaviour.

Motivated by evidence (Bloom et al., 2017) for new innovations being more difficult to find as technologies advance, we begin with an economy without state dependence. Under no state dependence, the existing level of knowledge does not improve the profitability of research. We show asymptotic convergence to a growth path where energy intensity falls at a constant rate due to investment in energy augmenting technologies. Consistent with the data, energy intensity declines slower than output grows, and we express the asymptotic elasticity between energy intensity and output as a function of only the elasticity of substitution between energy and labor.

To understand incentives to undertake energy augmenting research, recall that profits of monopolistically competitive entrepreneurs are the product of the price a new product can command and the number of units of the new product that can be sold, the market size. [6] As energy use increases alongside growing output (and

[5]As defined by Hassler et al. (2016) – see Subsection 2.3.

[6]See Acemoglu (2009), ch. 15 for a detailed explanation of these market size and price effects.

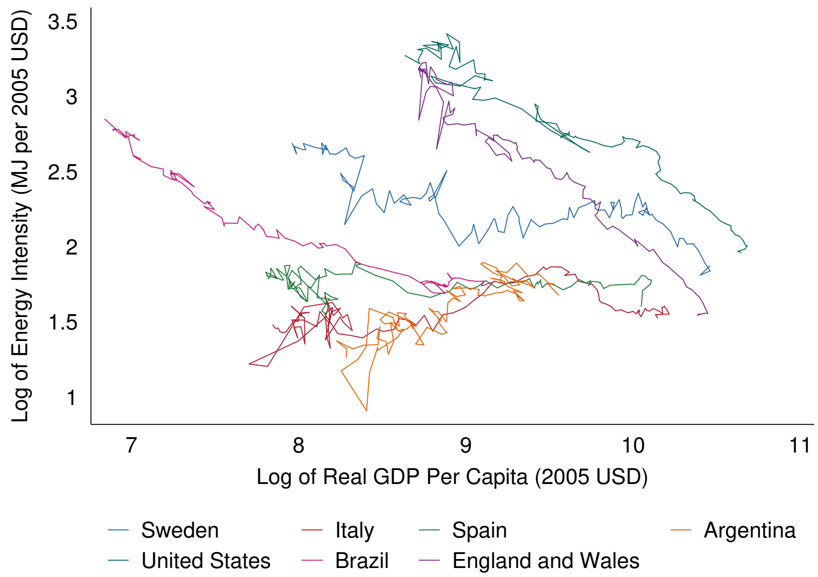

FIGURE 3.— Energy intensity and real GDP per capita for selected countries between 1900 and 2010.

energy extraction cost stay constant) the price of energy services falls, reducing profitability and incentives for energy augmenting research. However, the market size of energy increases, maintaining incentives to conduct energy augmenting research.

Nonetheless, the rate of decline of energy intensity is slower than the rate of output growth because energy augmenting technologies advance at a slower rate than labor augmenting technologies. While the market size effect of increasing energy use is sufficient to maintain energy augmenting innovation, the price effect of falling energy prices relative to the labor wage (Figure 4) is stronger than the market size effect, of greater per capita energy use, because the elasticity of substitution is less than one. The stronger price effect means greater incentives to undertake labor augmenting research and in turn a faster rate of labor saving innovation.

This model can also explain convergence of energy intensities conditional on output. Two countries starting with similar GDP per-capita but where one country has a higher energy intensity will both converge to the same energy intensity - GDP per-capita relationship.

We also examine an economy with complete state-dependence.[7] We find an econ-

[7]To clarify terminology, throughout the paper, we use the term complete state-dependence to refer to state-dependence in the extreme form where the elasticity of existing knowledge to producing

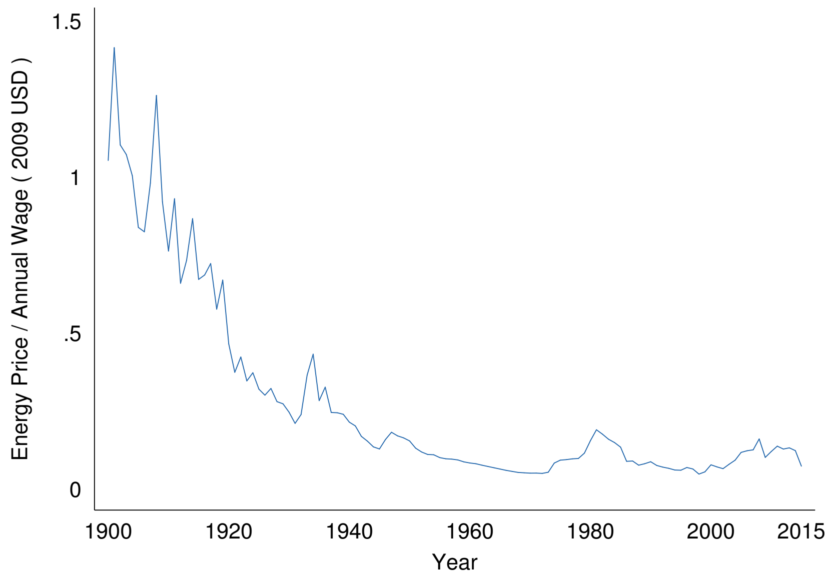

FIGURE 4.— The ratio of U.S. real energy price to real annual wage per employee shows a declining trend, creating stronger incentives for labor saving innovation.

omy with complete state dependence possesses a BGP. While autonomous reductions in energy intensity can occur outside a BGP, in the absence of price changes energy intensity eventually converges to a strictly positive constant. The reason is that state-dependence compounds the stronger incentive to undertake labor augmenting research arising from the price effect. The relative incentives to undertake labor-augmenting research continue to grow until they crowd out all energy-augmenting research, leading to an eventual end to the autonomous decline in energy intensity.

While the model without complete state dependence accounts for the observed trends in energy intensity, we cannot rule out a model with state dependence. If we assume sufficiently many countries, including the U.S., begin with energy intensities away from the long-run BGP, then a state dependence model can account for the observed falls in energy intensity associated with growth. One piece of evidence in favor of a complete state dependence economy is the relatively high elasticity of energy intensity with respect to output in the cross-section for high-income countries — as income increases across high-income countries energy intensity falls at a relatively slow rate. It turns out that a high elasticity can only be supported in a model with a persistent autonomous decline in energy intensity if the elasticity of substitution is higher than previous estimates in the literature (see

new knowledge is one. When the elasticity is less than one, then we have limited state dependence.

Stern and Kander (2012) or Hassler et al. (2016), we discuss these issues in detail in Section 3).

1.2. *Related Literature*

Our paper complements Hassler et al. (2016) and André and Smulders (2014) who also analyze models with endogenous energy- and labor-augmenting technical change. Hassler et al. (2016) provide a calibrated and tractable framework to forecast the role of energy intensity improvements in generating possibilities of growth where energy is scarce. André and Smulders (2014) uses a directed technical change model to account for stylized facts in U.S. energy use. However, both authors do not explicitly analyze the role and potential for autonomous drivers of falling energy intensity to persist along a growth path. In particular, both papers also only study innovation under state dependence and use a Hotelling rule featuring an exponentially increasing energy price, which becomes the driver of long-run reductions in energy intensity.

Our key finding that a model with complete state dependence cannot feature a constant growth rate of energy-augmenting technical change along an asymptotic growth path is analogous to the main finding of Acemoglu (2003). Acemoglu (2003) shows how under state dependence, long-run technical change can only be labor-augmenting, rather than capital-augmenting for interest rates to remain constant. Our model under complete state dependence is similar to Acemoglu's, except with capital replaced by energy. A specification with limited or no state dependence is ruled out by Acemoglu precisely because it would lead to a fall in capital intensity given constant interest rates — just as energy intensity falls with constant energy prices in the model presented here with no state dependence.

Finally, our paper relates to empirical work by Sue Wing (2008) showing that most of the decline in U.S. energy intensity before the 1970s oil price shock can be attributed to structural change towards sectors with lower energy intensity, while falls in intensity during the period of increasing prices can be attributed to within sector declines in intensity. Mckibbin et al. (2004) take a similar view, arguing that the relationship between output and energy use depends on changes in the shares of different sectors in an economy, rather than just on an energy efficiency trend within sectors. These arguments may at first seem inconsistent with our argument, where structural change implies an increase in the quantity ratio of energy services to output. However, energy-augmenting technical change in models with high aggregation such as ours should be seen as any change in the economy that allows more output per unit of energy, including technical change leading to structural change,[8] than strictly as 'energy efficiency' within specific industries. The results

[8]The definition of energy services in an aggregated directed technical change model does not

of Sue Wing (2008) also point to an important difference between price induced technological change and autonomous technological change, whereas in our aggregated model, both are captured by the same variable.

2. SCHUMPETERIAN GROWTH MODEL OF ENERGY AND DIRECTED TECHNICAL CHANGE

2.1. *The Model Economy*

Consider a continuous time economy where t, with $t \in \mathbb{R}_+$, indexes time. Assume a risk neutral representative consumer, so preferences over a time path for consumption are[9]

$$\int_{t=0}^{\infty} e^{-\rho t} C(t)\, \mathrm{d}t \tag{1}$$

where ρ is the discount factor and $C(t)$ is consumption at time t. The consumer faces a standard dynamic optimization problem and chooses a path of consumption and assets, $\{a(t)\}_{t=0}^{\infty}$, given a path of interest rates, $\{r(t)\}_{t=0}^{\infty}$ and wage rates $\{w_L(t)\}_{t=0}^{\infty}$ to maximize (1). For any t, the resource constraint for the economy is

$$Z(t) + C(t) + X(t) + \kappa(t)E(t) \leqslant Y(t) \tag{2}$$

where $Z(t)$ is the total level of R&D, $X(t)$ is total expenditure on machine varieties, $E(t)$ energy use and $\kappa(t)$ is the energy extraction cost, which is exogenous. We use an exogenous extraction cost as our focus is on autonomous endogenous technical change — we are interested in the response of technological change *given a path of prices*.[10]

At each t, final goods $Y(t)$ are produced using two intermediate goods: energy services $Y_E(t)$ and labor services $Y_L(t)$:

$$Y(t) = \left(\gamma_E Y_E(t)^{\frac{\epsilon-1}{\epsilon}} + \gamma_L Y_L(t)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

map to any particular sector in the economy, but may resemble a notion broader than "useful work" within an economy, which has shown an increasing trend, see Warr et al. (2010).

[9]Linear utility simplifies analysis of the dynamics in the model. Aghion and Howitt (1992) makes the same assumption as does Acemoglu (2003) when analyzing dynamics. Under a constant relative risk aversion (CRRA) utility function, the main results of our paper hold conditional on the assumption that any equilibrium path is an asymptotic balanced growth path (ABGP). However, we were unable to verify that every equilibrium path is an ABGP in the case of CRRA utility.

[10]Some other researchers e.g. André and Smulders (2014) assume the price of energy follows a Hotelling rule. However, observed energy prices are inconsistent with a simple Hotelling rule which would imply an exponentially growing price (Hamilton, 2009).

where ϵ is the elasticity of substitution and we assume $\epsilon < 1$. Competitive firms produce intermediate goods using a continuum of machines indexed by i, where $i \in [0, 1]$. The intermediate production functions are

$$Y_E(t) = (1-\alpha)^{-1} E(t)^{\alpha} \int_0^1 q_E(i,t)^{\alpha} x_E(i,t \mid q_E(i,t))^{1-\alpha} \, \mathrm{d}i \tag{4}$$

and

$$Y_L(t) = (1-\alpha)^{-1} L(t)^{\alpha} \int_0^1 q_L(i,t)^{\alpha} x_L\left(i,t \mid q_L(i,t)\right)^{1-\alpha} \, \mathrm{d}i \tag{5}$$

Let j index the sectors E and L. In the equations above, $q_j(i,t)$ denotes the highest quality of machine i at time t in sector j and $x_j(i,t \mid q_j(i,t))$ is the quantity of machine type i in sector j with quality $q_j(i,t)$.

We assume labor supply is fixed with $L(t) = L$ and $L > 0$ for all t. The energy sector uses a fuel $E(t)$, which can be extracted at a fixed cost $\kappa(t)$.

The price of final output is normalized to one and all prices will be stated in terms of the final good price. We assume machines of all varieties have a constant production cost $(1-\alpha)$, and monopolists who own patents for the varieties make and sell machines to the intermediate producers.

Let $p_j^x(i,t| \, q)$ denote the price of machine i in sector j at time t with quality q. And let $p_j(t)$ denote the price of the intermediate good $Y_j(t)$.

Turning to machine qualities, we assume a quality ladder for highest quality machines as follows

$$q_j(i,t) = \lambda^{n_j(i,t)} q_j(i,0), \qquad j \in \{E, L\}, i \in [0,1], \, t \in \mathbb{R}_+ \tag{6}$$

where $q_j(0,i)$ is the quality of machine i at time 0 and $n_j(i,t)$ equals the random number of incremental innovations on the machine variety up to time t. The arrival of a new innovation improves the machine quality by a factor λ.

New entrants engage in research to improve machine varieties. New entrants who have a successful innovation own a perpetual patent on the machine variety, however, once a new variety has been invented, the improved quality captures the whole market for the variety — Schumpeterian creative destruction. The following assumption[11] ensures the firm with the highest quality machine can charge the unconstrained monopoly price:

[11] See Acemoglu (2009) Sections 12.3.3 and 14.1.2 for discussion.

ASSUMPTION 1 $\lambda \geqslant \left(\frac{1}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}}$.

Final goods are used to undertake R&D which can be directed towards energy or labor machine improvements. A prospective entrant expending $z_j(i,t)$ units of R&D to improve the equality of machine i in sector j at time t generates a flow rate of machine improvement equal to $\eta_j b_j(i,t)$, where η_j is an exogenous parameter and[12]

$$b_j(i,t) := \frac{z_j(i,t)}{q_j(i,t)}$$

The denominator implies that higher quality machines are more difficult to improve upon, cancelling out state dependence embedded in the quality ladder.

Total R&D expenditure will be

$$Z(t) = \int_0^1 z_E(i,t)\,\mathrm{d}i + \int_0^1 z_L(i,t)\,\mathrm{d}i \tag{7}$$

2.2. *Equilibrium Characterization*

Objects in the model economy can be arranged in a tuple $\mathscr{E}$, where

$$\mathscr{E} := ((\Omega, \mathscr{F}, \mathbb{P}), q_E, q_L, v_E, v_L, p_E^x, p_L^x, x_L, x_E, z_E, z_L, C, X, E, Z, r, w, a)$$

and

1. the tuple $(\Omega, \mathscr{F}, \mathbb{P})$ is a probability space
2. the path of leading machine qualities, q_E and q_L, are $(\mathbb{R}_+^{[0,1]})^{\mathbb{R}_+}$ valued random variables defined on $(\Omega, \mathscr{F}, \mathbb{P})$
3. the functions $p_E^x \colon [0,1] \times \mathbb{R}_+^2 \to \mathbb{R}_+$ and $p_L^x \colon [0,1] \times \mathbb{R}_+^2 \to \mathbb{R}_+$ are paths of monopolist prices
4. the functions $p_E \colon \mathbb{R}_+ \to \mathbb{R}_+$ and $p_L \colon \mathbb{R}_+ \to \mathbb{R}_+$ are intermediate goods prices
5. the functions $x_E \colon [0,1] \times \mathbb{R}_+^2 \to \mathbb{R}_+$ and $x_L \colon [0,1] \times \mathbb{R}_+^2 \to \mathbb{R}_+$ are machine quantities

[12] Recall the flow rate tells us the probability of a machine improvement occurring during a small period of time Δt given R&D expenditure $z_j(i,t)$ is $\frac{\eta_j z_j(i,t)}{q_j(i,t)}\Delta t$. Formally,

$$\mathbb{P}\{n_j(t+\Delta t, i) - n_j(t,i) = 1\} = \frac{\eta_j z_j(i,t)}{q_j(i,t)}\Delta t + o(\Delta t), \qquad j \in \{E, L\}$$

6. paths for consumption, total expenditure on machines, energy, total R&D and energy that satisfy $C, X, Z, E \in \mathscr{C}^1(\mathbb{R}_+)$ where $\mathscr{C}^1(\mathbb{R}_+)$ is the space of all real valued continuously differentiable functions on $\mathbb{R}_+$
7. the functions $z_E \colon \mathbb{R}_+ \to \mathbb{R}_+$ and, $z_L \colon \mathbb{R}_+ \to \mathbb{R}_+$ are paths for R&D
8. the functions $v_E \colon [0,1] \times \mathbb{R}^2_+ \to \mathbb{R}_+$ and $v_L \colon [0,1] \times \mathbb{R}^2_+ \to \mathbb{R}_+$ are paths for the value functions
9. the functions $r \colon \mathbb{R}_+ \to \mathbb{R}_+$, $w \colon \mathbb{R}_+ \to \mathbb{R}_+$ and $a \colon \mathbb{R}_+ \to \mathbb{R}_+$ are paths for the interest rate, wage rate and assets.

Section A.1 of the Appendix gives a definition of equilibrium; we now turn to documenting the familiar static equilibrium conditions, given technology.

Profit maximization by final goods producers gives

$$\gamma_E Y(t)^{\frac{1}{\epsilon}} Y_E(t)^{-\frac{1}{\epsilon}} = p_E(t), \qquad \gamma_L Y(t)^{\frac{1}{\epsilon}} Y_L(t)^{-\frac{1}{\epsilon}} = p_L(t) \tag{8}$$

At each t, profit maximization by the intermediate goods producers gives machine demands for the highest quality machine of type i

$$x_E(i,t \,|\, q_E(i,t)) = p_E(t)^{-\frac{1}{\alpha}} p^x_E(i,t \,|\, q_E(i,t))^{\frac{1}{\alpha}} q_E(i,t) E(t) \tag{9}$$

$$x_L(i,t \,|\, q_L(i,t)) = p_L(t)^{-\frac{1}{\alpha}} p^x_L(i,t \,|\, q_L(i,t))^{\frac{1}{\alpha}} q_L(i,t) L \tag{10}$$

and first order conditions for energy and labor

$$\alpha(1-\alpha)^{-1} p_E(t) E(t)^{\alpha-1} \int_0^1 q_E(i,t)^{\alpha} x_E(i,t \,|\, q_L(i,t))^{1-\alpha} \,\mathrm{d}i = \kappa(t) \tag{11}$$

$$\alpha(1-\alpha)^{-1} p_L(t) L^{\alpha-1} \int_0^1 q_L(i,t)^{\alpha} x_L(i,t \,|\, q_L(i,t))^{1-\alpha} \,\mathrm{d}i = w_L(t) \tag{12}$$

Since the cost of machine varieties is $1-\alpha$ and by Assumption 1, monopolists who own the highest quality machine production technologies set $p^x_j(i,t \,|\, q_j(i,t)) = 1$, machine demands for the highest quality machine of type i are then

$$x_E(i,t \,|\, q_E(i,t)) = p_E(t)^{\frac{1}{\alpha}} q_E(i,t) E(t), \qquad x_L(i,t \,|\, q_L(i,t)) = p_L(t)^{\frac{1}{\alpha}} q_L(i,t) L \tag{13}$$

Profits for monopolists who own the leading edge machines are

$$\pi_E(i,t \,|\, q_E(i,t)) = \alpha p_E(t)^{\frac{1}{\alpha}} q_E(i,t) E(t), \qquad \pi_L(i,t \,|\, q_L(i,t)) = \alpha p_L(t)^{\frac{1}{\alpha}} q_L(i,t) L \tag{14}$$

Owners of lower equality machines receive zero profits.

Define $Q_E :\ = \int_0^1 q_E(i,t)\,\mathrm{d}i$ and $Q_L :\ = \int_0^1 q_L(i,t)\,\mathrm{d}i$ as the average qualities of leading machines. Intermediate output, using (4) and (5) with (13) becomes

$$(15)\qquad Y_E(t) = (1-\alpha)^{-1}E(t)p_E(t)^{\frac{1-\alpha}{\alpha}}Q_E(t), \qquad Y_L(t) = (1-\alpha)^{-1}Lp_L(t)^{\frac{1-\alpha}{\alpha}}Q_L(t)$$

Next, using the intermediate demand conditions, (8), we can write the ratio of prices as

$$\frac{p_E(t)}{p_L(t)} = \gamma\left(\frac{Y_E(t)}{Y_L(t)}\right)^{-\frac{1}{\epsilon}} = \gamma\left(\frac{E(t)}{L}\right)^{-\frac{1}{\epsilon}}\left(\frac{p_E(t)}{p_L(t)}\right)^{-\frac{1-\alpha}{\alpha\epsilon}}\left(\frac{Q_E(t)}{Q_L(t)}\right)^{-\frac{1}{\epsilon}}$$

where $\gamma = \frac{\gamma_E}{\gamma_L}$. And solving for the ratio of prices gives

$$(16)\qquad \frac{p_E(t)}{p_L(t)} = \gamma^{\frac{\alpha\epsilon}{\sigma}}\left(\frac{E(t)Q_E(t)}{LQ_L(t)}\right)^{-\frac{\alpha}{\sigma}}$$

where $\sigma = 1 + \alpha(\epsilon - 1)$ is the elasticity of substitution between the factors of production. Using (15), we can also derive a ratio of intermediate goods

$$(17)\qquad \frac{Y_E(t)}{Y_L(t)} = \left(\frac{E(t)Q_E(t)}{LQ_L(t)}\right)^{\frac{\epsilon\alpha}{\sigma}}\gamma^{\frac{\epsilon(1-\alpha)}{\sigma}}$$

Next, insert (8) into (15), use the expression for final output [13] and the ratio of intermediate goods above (17) to derive

$$(18)\qquad Y(t) = (1-\alpha)^{-1}\left[\gamma_E^{\frac{\epsilon}{\sigma}}\left(E(t)Q_E(t)\right)^{\frac{\sigma-1}{\sigma}} + \gamma_L^{\frac{\epsilon}{\sigma}}\left(LQ_L(t)\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

We now derive a condition characterizing energy intensity. Use (11) and the definition of $Y_E(t)$ to arrive at

$$(19)\qquad \alpha\theta_E(t)\frac{Y(t)}{E(t)} = \kappa(t)$$

[13]Note $Y(t) = \left(1 + \gamma^{-1}\left(\frac{Y_L(t)}{Y_E(t)}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} Y_E(t)\gamma_E$.

where

$$\theta_E(t) := \frac{p_E(t)Y_E(t)}{Y(t)} = \left(1 + \gamma^{-\frac{\epsilon}{\sigma}} \left(\frac{E(t)Q_E(t)}{Q_L(t)L}\right)^{\frac{1-\sigma}{\sigma}}\right)^{-1} \tag{20}$$

is the cost share of energy services in final production. The second equality results from using (8) and (17). Use (20) along with (18) to write (19) as

$$\gamma_E^{\frac{\epsilon}{\sigma(\sigma-1)}} \alpha(1-\alpha)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y(t)}{E(t)}\right)^{\frac{1}{\sigma}} Q_E(t)^{\frac{\sigma-1}{\sigma}} = \kappa(t) \tag{21}$$

Equation (21) tells us that energy-augmenting technical change is energy saving. Appendix B gives further comparative static results on how energy augmenting technologies affect energy use.

Now we turn to the dynamic aspects of equilibrium concerning technology choice. Since consumers are risk neutral, the interest rate in the economy will be ρ for all t. Let $v_j(i, t \mid \lambda q)$ denote the value of a successful innovation of machine i in sector j with quality q at time t. We have[14]

$$v_j(i, t \mid \lambda q) = \mathbb{E}_t \int_{s=t}^{T_j(i,t)} e^{-\rho s} \pi_j(i, s \mid \lambda q)\, \mathrm{d}s$$

where $T_j(i, t)$ is the random stopping time after which a new innovation replaces the incumbent. Free entry and exit implies[15]

$$\eta_j \frac{v_j(i, t \mid \lambda q)}{q} \leqslant 1, \qquad j \in \{E, L\}, t \in \mathbb{R}_+ \tag{22}$$

holds for all t and $i \in [0, 1]$ and the inequality is an equality of $z_j(i, t) > 0$. Assuming the value function is differentiable implies $\dot{v}_j(i, t \mid q) = 0$. The Hamilton Jacobi

[14]The random variable $T_j(i, t)$ has a distribution:

$$\mathbb{P}(T_j(i, t) \geqslant t + s) = e^{-\int_t^s \eta_j b_j(i, s_1)\, \mathrm{d}s_1}$$

[15]Recall that expenditure of $z_j(i, t)$ units of final good on R&D generates a flow rate of $\eta_j \frac{z_j(i,t)}{q_j(i,t)}$. Since the price of the final good is normalised to one the value of spending one unit of the final good on research is $\frac{\eta_j v_j(i,t \mid \lambda q)}{q(i,t)} - 1$, which should not be strictly positive.

Bellman (HJB) equation[16] for $v_j(t, i \mid q)$ is

$$\text{(23)} \qquad \dot{v}_j(i, t \mid q) = \big(\rho + \eta_j b_j(i, t)\big)\, v_j(i, t \mid q) - \pi_j(i, t \mid q)$$

Immediately giving, using (22) and (14),

$$\text{(24)} \qquad \eta_E b_E(i, t) + \rho \geqslant \eta_E \lambda \alpha p_E(t)^{\frac{1}{\alpha}} E(t), \qquad \eta_L b_L(i, t) + \rho \geqslant \eta_L \lambda \alpha p_L(t)^{\frac{1}{\alpha}} L$$

where, once again, the inequality is an equality of $z_j(i, t) > 0$. Note the above equation implies $b_j(i, t)$ is no longer conditioned on the machine variety — there is a common rate of innovation for all machines in a sector. Let $b_j(t) := \int_0^1 b_j(i, t)\,\mathrm{d}i$ for $j \in \{E, L\}$.

Finally, for each i, $q_j(i, t)$, is a random process. However, at each t, the average machine quality $Q_j(t)$ for each sector j will be deterministic,[17] determined by the innovation rate b_j as follows[18]

$$\text{(25)} \qquad \dot{Q}_j(t) = (\lambda - 1)\eta_j b_j(t) Q_j(t), \qquad\qquad j \in \{E, L\}$$

2.3. *Main Theoretical Results*

An equilibrium growth path is a balanced growth path if output and consumption both grow at a constant rate g, energy grows at a constant rate g_E, and Q_E and Q_L grow at constant rates g_{Q_E} and g_{Q_L} respectively. For any path, $\varphi(t)$, we will use $\hat{\varphi}(t)$ to refer to the growth rate of $\varphi(t)$ at time t, that is $\hat{\varphi}(t) = \frac{\dot{\varphi}(t)}{\varphi(t)}$, and we use $\varphi(t) \to \bar{\varphi}(t)$ to mean $\lim\limits_{t \to \infty} \varphi(t) = \bar{\varphi}(t)$ for a path $\bar{\varphi}(t)$.[19]

PROPOSITION 2.1 *Let an economy $\mathscr{E}$ be an equilibrium. If $\hat{Q}_E(t) > c$ for all t, where $c > 0$, and $\dot{\kappa}(t) = 0$, then $\hat{Y}(t)$, $\hat{E}(t)$, $\hat{Q}_L(t)$ and $\hat{Q}_E(t)$ cannot be constant.*

PROOF: By (18), $Y(t) = \tilde{F}(Q_E(t)E(t), Q_L(t)L)$ where $\tilde{F}$ is a homogeneous of degree one production function. Using Euler's Theorem, $g_{Q_E} + g_E = g_{Q_L}$ must hold for g, g_E, g_{Q_E} and g_{Q_L} to be constant (See Claim D.1 in the Online Appendix, Section

[16] See Acemoglu (2009), Equation (14.13).

[17] The average quality will equal $\mathbb{E}q(i, t)$ with probability one (Sun and Zhang, 2009), this is refered to as no aggregate uncertainty (NAU) and in this paper we view NAU as an assumption without providing a formal proof.

[18] Acemoglu and Cao (2015) (Footnote 23) provide a concise derivation of Equation (25).

[19] We say $\varphi(t) \to \bar{\varphi}(t)$ if and only if for every $\epsilon > 0$, there exists T such that for all $t > T$, $|\varphi(t) - \bar{\varphi}(t)| < \epsilon$.

D.0.1). However, $g_{Q_E} + g_E = g_{Q_L}$ yields a contradiction, since Equation (20) implies $\theta_E(t)$ is constant, which in turn implies $\hat{Q}_E(t) = 0$ and $E(t)/Y(t)$ is constant by (21) and (37).

Q.E.D.

Thus a BGP cannot feature autonomous energy intensity improvements. We study the possibility of autonomous energy intensity improvements along *asymptotic* balanced growth paths (ABGP).[20]

DEFINITION 2.1 An equilibrium economy $\mathscr{E}$ is an asymptotic balanced growth path (ABGP) if $\hat{Y}(t) \to g$, $\hat{C}(t) \to g$, $\hat{E}(t) \to g_E$, $\hat{Q}_E(t) \to g_{Q_E}$, $\hat{Q}_L(t) \to g_{Q_L}$, $b_E(t) \to b_E^\star$ and $b_L(t) \to b_L^\star$, where all limits are real valued.

ASSUMPTION 2 *The economy $\mathscr{E}$ satisfies*

$$\lambda \eta_L \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} (1-\theta_E(0))^{\frac{1}{1-\sigma}} L > \rho > (\lambda - 1)\eta_L \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} L \tag{26}$$

Note by (20), we have $\alpha p_L(t)^{\frac{1}{\alpha}} L = \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} (1-\theta_E(t))^{\frac{1}{1-\sigma}} L$ for each t. Thus, by the free entry and exit condition and (24), the first inequality of the assumption above ensures the growth rate of labor augmenting technologies and output is positive. The second inequality ensures corporate assets do not grow faster than the discount rate, ensuring the transversality condition holds and (1) remains finite.

We now show an ABGP features autonomous energy intensity improvements.

THEOREM 2.1 *If $\mathscr{E}$ is an ABGP, satisfies assumption 1- 2 and $\dot{\kappa}(t) = 0$ for all t, then*

1. $\frac{\hat{Q}_E(t)}{\hat{Q}_L(t)} \to \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$
2. $\theta_E(t) \to 0$
3. $\hat{Y}(t) \to \hat{Q}_L(t) \to g$ *with* $g = (\lambda-1)(\lambda \eta_L \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} L - \rho)$
4. $\hat{E}(t) - \hat{Y}(t) \to \hat{Y}(t) \frac{(1-\theta_E(t))(\sigma-1)}{2-\theta_E(t)-\sigma}$.

The proof of the theorem is in the Appendix. Part 1. tells us the growth rate of energy-augmenting technologies converges to a rate lower than the growth rate of labor-augmenting technologies, since we have assumed $\sigma < 1$. Part 2. tells us the cost share of energy converges to zero. Part 3 tell us the growth of output converges to the growth rate of labor-augmenting technologies, because the cost share of labor converges to 100% of output (Part 2.).

[20]The definition of BGP and ABGP varies. Hassler et al. (2016) define a BGP in the same way as us. However, Acemoglu (2003) uses the term BGP to refer to what we define at ABGP. The distinction matters in our model since BGPs cannot feature autonomous intensity improvements while ABGPs can.

Part 4. tells us energy intensity declines, but at a slower rate than output growth. Energy intensity declines because of energy-augmenting technical change (recall equation (21)). And the rate of decline is slower than the rate of output growth since energy-augmenting technologies grow slower than labor augmenting technologies (because of the price effect discussed at equation (34)). On the other hand, labor services become an increasingly important contributor to output as the share of energy services falls to zero; thus output growth converges to the rate of growth of labor augmenting technologies (see decomposition of output growth at equation (41) in the Appendix).

The following result is an implication of Theorem 2.1, Part 2, and says if initially there are no energy augmenting technical advances (and energy intensity does not fall), then there exists some time T after which energy augmenting technologies will begin to advance.

COROLLARY 2.1 *Let $\mathscr{E}$ be an ABGP. If $\dot{\kappa} = 0$ and*

$$\lambda \eta_E \alpha \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(0)^{\frac{1}{1-\sigma}} E(0) - \rho < 0 \tag{27}$$

then there exists $T \geqslant 0$ such that for all $t > T$,

$$\eta_E b_E(t) = \eta_E \alpha \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(t)^{\frac{1}{1-\sigma}} E(t) - \rho > 0$$

Figure 5 shows ABGPs (in bold), assuming $\kappa(t)$ remains constant. To understand the direction of ABGPs intuitively, note output is growing and $b_E(t) > 0$, hence energy intensity must be falling. And we verified in Theorem 2.1 that the elasticity of E/Y with respect to Y must converge to $\frac{\sigma-1}{2-\sigma}$.

Below, we show that, for a given level of output, energy intensity converges for all ABGPs (conditional convergence).

PROPOSITION 2.2 *If $\mathscr{E}$ is an ABGP, then*

$$\frac{E(t)}{Y(t)} \to G(Y(t))$$

where $G \colon \mathbb{R}_{++} \to \mathbb{R}_{++}$ is a monotone decreasing function satisfying $\lim_{x\to\infty} G(x) \to 0$.

The Online Appendix, Section D.1, studies the transitional dynamics of ABGPs in detail, in particular why ABGPs starting below $G(Y)$ in Figure 5 must cross $G(Y)$ as they converge. The Online Appendix also shows that any equilibrium path in an ABGP.

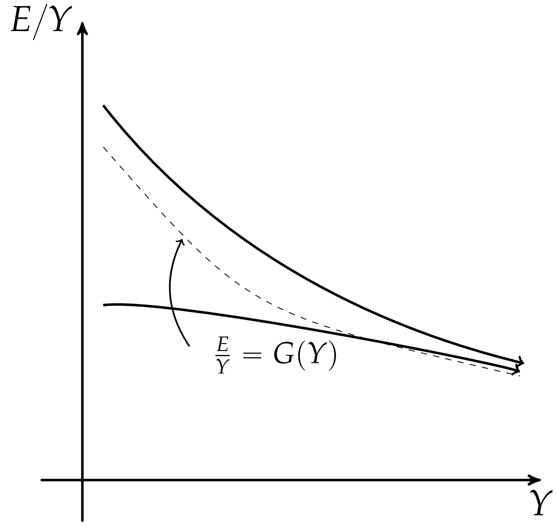

FIGURE 5.— ABGPs for a economy with no state dependence (in bold). All ABGPs feature declining energy intensity, but must converge to the dotted curve.

For conditional convergence of energy to take place, ABGPs starting with higher energy intensity must experience a faster decline in energy intensity relative to output. For the next result, let $\{E, Y, Q_E, Q_L, \theta_E\}$ and $\{\tilde{E}, \tilde{Y}, \tilde{Q}_E, \tilde{Q}_L, \tilde{\theta}_E\}$ be two equilibrium paths.

CLAIM 2.1 If $\tilde{Y}(t) = Y(t)$ and $\tilde{E}(t) > E(t)$, then

1. $\frac{\hat{\tilde{Q}}_E(t)}{\hat{\tilde{Q}}_L(t)} > \frac{\hat{Q}_E(t)}{\hat{Q}_L(t)}$
2. $\hat{\tilde{Q}}_E(t) > \hat{Q}_E(t)$
3. $\frac{\hat{\tilde{E}}(t)-\hat{\tilde{Y}}(t)}{\hat{\tilde{Y}}(t)} < \frac{\hat{E}(t)-\hat{Y}(t)}{\hat{Y}(t)}$

The Online Appendix, Section D, gives a proof of the this claim.

2.4. *State Dependence Economy*

The main difference between the complete state dependence and no state dependence economy detailed above are innovation incentives. Assume a constant unit measure of scientists who undertake research directed towards energy or labor machine improvements. If a scientist works to improve the quality of machine i in sector j, the flow rate of machine improvement is η_j. We assume prospective entrants can only choose to work in a sector j instead of choosing the specific machine variety i to improve. Once an entrant chooses j, they are randomly allocated to a machine variety i with no congestion, such that each variety i has at most one scientist allocated to it at t. Let s_j denote the measure of scientists working to improve machines in sector j. The flow rate of innovations in sector j will be $s_j\eta_j$.[21]

To characterize incentives to innovate under state dependence, let

$$V_j(t) := \int_0^1 v_j(i,t \mid q_j(i,t))\,\mathrm{d}i\,, \qquad j \in \{E, L\}\,, t \in \mathbb{R} \tag{28}$$

and let

$$\Pi_j(t) = \int_0^1 \pi_j(i,t \mid q_j(i,t))\,\mathrm{d}i\,, \qquad j \in \{E, L\}\,, t \in \mathbb{R}_+ \tag{29}$$

The free entry and exit conditions under state dependence are

$$\begin{aligned} V_E(t) &\geqslant V_L(t), \qquad \text{if} \quad s_E > 0 \\ V_L(t) &\geqslant V_E(t), \qquad \text{if} \quad s_L > 0 \end{aligned} \tag{30}$$

The definition of an economy and equilibrium are identical to the no state dependence case, with b_j replaced by s_j and the free entry and exit conditions specified by (30) instead of (22). We use $\mathscr{E}_S$ to denote a complete state dependence economy.

The first state dependence result tells us sustained autonomous energy intensity improvements are not possible.

THEOREM 2.2 *If $\mathscr{E}_S$ is an ABGP and $\dot{\kappa}(t) = 0$, then $\hat{Y}(t) \to \hat{Q}_L(t) \to g$ where $g = (\lambda - 1)\eta_L$, $\theta_E(t) \to \theta_E^\star$ where $\theta_E^\star > 0$ and $\hat{E}(t) - \hat{Y}(t) \to 0$.*

Moreover, if the energy extraction costs are low enough or energy-augmenting technologies are advanced enough, then there exists a BGP with no energy-augmenting research.

PROPOSITION 2.3 *Let $\mathscr{E}_S$ be an equilibrium. If $\dot{\kappa}(t) = 0$ for all t, then there exists a balanced growth path with $\eta_L s_L(t) = \eta_L$ and $s_E(t) = 0$ for all t if and only if*

$$\left(\frac{\kappa}{Q_E(0)}\right)^{1-\sigma} \leqslant \frac{\left(\alpha(1-\alpha)\right)^{1-\sigma} \gamma^{\frac{\epsilon}{1-\sigma}}}{\gamma^{\frac{2\epsilon}{\sigma-1}} \frac{\rho+\eta_L}{\rho-\eta_L} + 1}, \qquad t \geqslant 0 \tag{31}$$

[21]The random allocation of scientists across machine varieties ensures all varieties experience innovation in equilibrium. Acemoglu et al. (2012) make a similar assumption in a discrete time model.

Combining (37) and (19) with (31) implies

$$\frac{E(0)}{Y(0)} \leqslant \frac{\alpha\kappa^{-1}\gamma_L^{\frac{\epsilon}{\sigma-1}}}{\gamma^{\frac{2\epsilon}{\sigma-1}}\frac{\rho+\eta_L}{\rho-\eta_L}+1} := \phi \tag{32}$$

must hold at time 0 for for a BGP to exist. Noting (36), if Q_E is constant along the BGP, then θ_E remains constant, and thus, by (19), $\frac{E}{Y}$ remains constant and satisfies (32) along the BGP.

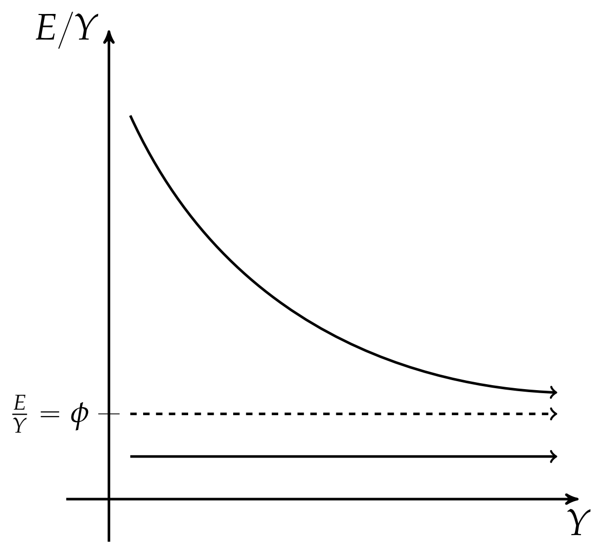

FIGURE 6.— ABGPs for an economy with state dependence. All ABGPs must converge to a BGP where energy intensity is constant.

Turning now to convergence under state dependence, consider Figure 6, which shows ABGPs for a state dependence economy with a constant real energy price. The dotted line is the BGP where $\frac{E}{Y} = \phi$, where ϕ is implied by Equation (32). By Proposition 2.3, any economy with energy intensity below ϕ has a unique BGP with no energy-augmenting technical change. However, again by Proposition 2.3, if energy intensity is strictly greater than ϕ, energy-augmenting technologies must advance. And by Theorem 2.2, Q_E and hence energy intensity must converge to a constant.

3. DISCUSSION

3.1. *Why Energy Augmenting Technologies Advance Slower than Labor Augmenting Technologies and Stop Advancing Under state dependence*

First, let us see why as an economy grows and energy augmenting technologies advance, there are continued incentives to undertake energy augmenting research in the no state dependence economy. Incentives to innovate (Equation (24)) are the product of the price effect and market size effect:

$$\eta_E b_E(t) = \eta_E \lambda \overbrace{\alpha p_E(t)^{\frac{1}{\alpha}}}^{\text{Price effect}} \underbrace{E(t)}_{\text{Market size effect}} - \rho \tag{33}$$

and, by equation by (20) and noting (52) in the Appendix, $\alpha p_E(t)^{\frac{1}{\alpha}} = \alpha \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(t)^{\frac{1}{1-\sigma}}$. Thus along an ABGP, if the price of energy is constant and energy use expands, the price of energy services falls.At the same time, the market size of E increases, sustaining incentives to innovate. By contrast, suppose E was constant, then along a BGP, Q_E grows at the same rate as Y and the price of energy services stays constant.

However, the relative incentive to innovate favors labor augmenting research. With innovation in both sectors, by (24),

$$\frac{\rho + \eta_E b_E(t)}{\rho + \eta_L b_L(t)} = \frac{\eta_E}{\eta_L} \overbrace{\left(\frac{p_E(t)}{p_L(t)}\right)^{\frac{1}{\alpha}}}^{\text{Price effect}} \underbrace{\frac{E(t)}{L}}_{\text{Market size effect}} = \frac{\eta_E}{\eta_L} \left(\frac{Q_E(t)}{Q_L(t)}\right)^{-\frac{1}{\sigma}} \left(\frac{E(t)}{L(t)}\right)^{\frac{\sigma-1}{\sigma}} \tag{34}$$

As both technologies advance, energy use grows,[22] the market size effect increases profitability for energy-augmenting research and the price effect decreases profitability. If $\sigma < 1$, then the price effect is stronger leading to an overall fall in the profitability of energy augmenting research.

The fall in the relative profitability of energy augmenting research means more R&D resources are allocated to labor augmenting research and the growth rate of labor augmenting technologies will be faster. When there are no state dependence, the fall in the ratio $\frac{Q_E(t)}{Q_L(t)}$ is sufficient to eventually balance relative profits, so the

[22] In this statement, we presume labor augmenting technologies grow at least as fast as energy augmenting technologies. Then by the first order condition for energy use, Equation (21), energy use expands. However, if output grows sufficiently slower than energy augmenting technologies advance, energy use can fall. In Appendix B, we give the exact conditions under which energy use may fall when labor augmenting technologies do not advance.

left hand side of the above equation converges to a constant and innovation occurs in both sectors.

The exact relative rates of innovation under a rational expectations equilibrium in the case with state dependence are more difficult to characterize using current profit ratios. Nonetheless, we now consider the current profit ratios to gain intuition about the result we proved in Section 2.4. Suppose that for innovation to occur in both sectors, we must have

$$\begin{aligned}\frac{\rho + \eta_E s_E(t)}{\rho + \eta_L s_L(t)} &= \frac{\eta_E \Pi_E(t)}{\eta_L \Pi_L(t)} \\ &= \frac{\eta_E}{\eta_L} \overbrace{\left(\frac{p_E(t)}{p_L(t)}\right)^{\frac{1}{\alpha}}}^{\text{Price effect}} \underbrace{\frac{E(t)}{L}}_{\text{Market size effect}} \frac{\overbrace{Q_E(t)}^{\text{State dep.}}}{Q_L(t)} = \frac{\eta_E}{\eta_L}\left(\frac{Q_E(t)}{Q_L(t)}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{E(t)}{L(t)}\right)^{\frac{\sigma-1}{\sigma}}\end{aligned} \tag{35}$$

As energy use grows, once again, the price effect leads to a fall in relative profitability to do energy research. However, now, as $\frac{Q_E(t)}{Q_L(t)}$ falls, the state dependence further increases the profitability of labor-augmenting research. Whether the relative profits for energy- and labor-augmenting research remain in a constant ratio or whether relative profits for energy keep falling depends on how fast energy use grows relative to the fall in technology ratiio. Use (20) to write

$$\left(\frac{Q_E(t)E(t)}{Q_L(t)L}\right)^{\frac{\sigma-1}{\sigma}} = \frac{\theta_E(t)}{1-\theta_E(t)}\gamma^{\frac{-\epsilon}{\sigma}} \tag{36}$$

Now use (19) and (21) to arrive at

$$\theta_E(t) = (\alpha(1-\alpha))^{\sigma-1}\,\kappa(t)^{1-\sigma}\gamma_E^{\frac{\epsilon}{\sigma-1}} Q_E(t)^{\sigma-1} \tag{37}$$

Thus, if $Q(t) \to \infty$, then we must have $\frac{\rho+\eta_E s_E(t)}{\rho+\eta_L s_L(t)} \to 0$, which cannot hold since $s_L \leqslant s$. Continued energy augmenting technological advancement cannot now occur along any ABGP since incentives to invest in labor augmenting technologies increase at such high a rate from state dependence increasingly favouring labour augmenting technologies, that investment in energy augmenting technologies must keep decreasing for the no arbitrage condition to hold.

3.2. *The Possibility of Limited State Dependence and Population Growth*

Aghion et al. (2016) find evidence of state dependence among firms in the auto industry who undertake 'clean' versus 'dirty' innovation. However, state can be of a limited form where the elasticity of existing knowledge to the productivity of new innovations is less than one. Indeed, Aghion et al. (2016) find an elasticity of the creation of new knowledge with respect to the existing level of knowledge of approximately .3. We briefly discuss the implications of such a specification. Suppose an innovation possibilities frontier of the form

$$\dot{Q}_j(t) = (\lambda - 1)\eta_j s_j(t) Q_j(t)^{\psi} \tag{38}$$

where $\psi \in (0, 1)$. Such a formulation requires that s_j grows through population growth for sustained per capita growth to be possible, similar to Jones (1995). And such a model would imply a relative current profit ratio of the form:

$$\begin{aligned}\frac{\rho + \eta_E s_E(t)}{\rho + \eta_L s_L(t)} &= \frac{\eta_E}{\eta_L}\frac{\Pi_E(t)}{\Pi_L(t)} \\ &= \frac{\eta_E}{\eta_L}\left(\frac{p_E(t)}{p_L(t)}\right)^{\frac{1}{\alpha}}\frac{E(t)}{L}\left(\frac{Q_E(t)}{Q_L(t)}\right)^{\psi} = \frac{\eta_E}{\eta_L}\left(\frac{Q_E(t)E(t)}{Q_L(t)L(t)}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Q_E(t)}{Q_L(t)}\right)^{\psi-1}\end{aligned}$$

When $Q_E(t) \to \infty$, $\theta_E(t) \to 0$ and $\left(\frac{Q_E(t)E(t)}{Q_L(t)L(t)}\right)^{\frac{\sigma-1}{\sigma}} \to 0$. And since $\left(\frac{Q_E(t)}{Q_L(t)}\right)^{\psi-1} \to \infty$, an ABGP where the relative profitability of labor and energy augmenting technologies remain constant is possible. [23]

3.3. *Complete State Dependence or Not?*

The estimation of Aghion et al. (2016) along with the argument of Bloom et al. (2017) that innovations are harder and harder to make are both more consistent without complete state dependence. In this section, we revisit the observed trends of energy intensity and discuss whether they can be explained by the no state dependence economy or a complete state dependence economy.

First we solve Equation (21) for Q_E. In discrete time, adding a stationary — but probably serially correlated — error term, we have:

$$Q_{E,t} + u_{E,t} = A_E \kappa_t^{\frac{\sigma}{\sigma-1}}\left(\frac{E_t}{Y_t}\right)^{\frac{1}{\sigma-1}} \tag{39}$$

[23] In a model with population growth where scientists engage in R&D, using a similar process to the proof for Theorem 2.1, we can show $\frac{\hat{Q}_E(t)}{\hat{Q}_L(t)} \to \frac{2-\psi}{1-\psi-\sigma}$ along an ABGP if we assume relative profitability is given by (35). However, we were unable to show a rational expectations equilibrium implied ABGPs in this case. Thus our study in this paper focused on the lab equipment model with no population growth.

where $A_E = \gamma_E^{-\frac{\epsilon}{(\sigma-1)^2}} \alpha^{-\frac{\sigma}{\sigma-1}} (1-\alpha)^{-1}$.[24] We compute the RHS of (39) using annual U.S. data from 1900 to 2015 as described in the Online Appendix and assuming that $\sigma = .5$ and apply the Hodrick-Prescott filter to obtain the estimate of $Q_{E,t}$ plotted in Figure 7.[25]

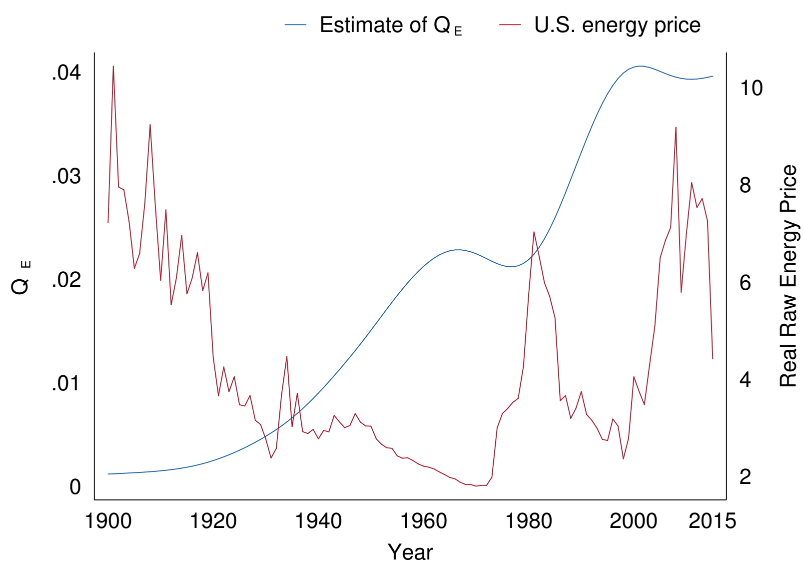

FIGURE 7.— Smoothed estimate of $Q_E(t)$, estimated assuming $\sigma = .5$, and the U.S. energy price.

Before 1950, the growth rate of Q_E was positive, while prices declined, suggesting an autonomous decline of energy intensity. After 1960, and before 1980, the growth rate of Q_E declines to under zero by the early 1970s, before rising and falling again in a lagged response to prices.

Both the economy without state dependence and economy with state dependence can be consistent with the path of Q_E in the U.S.. If we suppose the U.S. economy

[24]As A_E only changes the value of $Q_{E,t}$ without affecting its growth rate — and A_L similarly affects $Q_{L,t}$, we set $A_E = A_L = 1$ in the following.

[25]See the Online Appendix, Section E.4, for raw and smoothed estimates of Q_E for different values of σ. Hassler et al. (2016) assume there is no error in their equivalent of (39). As a result, they find that only very low values of σ are compatible with a reasonable time path for $Q_{E,t}$. But given our model is very simple, energy intensity likely adjusts slowly to its equilibrium value; and variables are measured with error — the addition of an error term seems more reasonable to us than not. However, values above .8, even with smoothing, give unreasonable jumps and declines in line with the observation made by Hassler et al. (2016). As such, we maintain a reasonable value of σ to be between 0 and .5.

was sufficiently energy inefficient in the early 1900s, there could have been autonomous improvements in Q_E under a state dependence economy (recall Proposition 2.3). The growth of $Q_E(t)$ does decline below zero just before the price shocks of the 1970s and in the early 2000s, however, this does not imply a model with no autonomous energy efficiency improvements along an ABGP. Both complete state dependence and no state dependence predict such a decline, where the decline in the growth rate of $Q_E(t)$ is associated with a fall in energy prices. To see why, for a given level of $Q_E(t)$ and $Q_L(t)$, when prices fall, energy consumption increases, and recalling (34) and (35), energy-augmenting technical change becomes relatively less profitable due to the stronger price effect.

Both the no state dependence and complete state dependence economy can also be consistent with paths of energy intensity across countries. First, both economies result in convergence of energy intensities across countries given the level of output — consistent with conditional convergence shown by Csereklyei et al. (2016) and the increasingly 'tighter' relationship between output per capita and energy intensity seen in Figure 2 and Figure 3.

Second, both models can explain why many countries experience no declines in energy intensity. Figure 3 shows that some countries, such as Italy and Spain, with low energy intensities have close to constant paths of energy intensity. Under no state dependence, initially, energy intensity may be so low given output such that (27) holds and no energy augmenting innovation occurs. On the other hand, under scale effects, countries showing constant energy intensity through time may be on their long-run BGP, to which all countries will converge to.

Perhaps one piece of evidence in favor of the state dependence scenario is the low absolute elasticity of energy intensity to output through time and in the cross-section for countries with low energy intensity. Figure 9 presents scatter plots of the natural log of energy intensity against the natural log of GDP per capita in PPP dollars at ten year intervals between 1971 and 2010. The fitted regression lines in the figure are cubic polynomials; we compared the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC) for a linear model and cubic model, and found the cubic has the lowest score on both criteria. The regression estimates are presented in Table 3.3. Using these coefficients we can estimate the elasticity of energy intensity to output in the cross-section (Table 3.3). There is a negative relationship between energy intensity and output, however, for higher levels of output, the fall in energy intensity associated with an increase in output is less. For example, in 2010, for countries at a GDP per capita of 2,000 USD, a 1 percent increase in output is associated with a .5 percent fall in energy intensity, while for countries with a GDP per capita of 30,000, a 1 percent increase in output is associated with .06 percent fall in energy intensity. At least in the cross section of countries, the elasticity of energy intensity to output is not constant and becomes

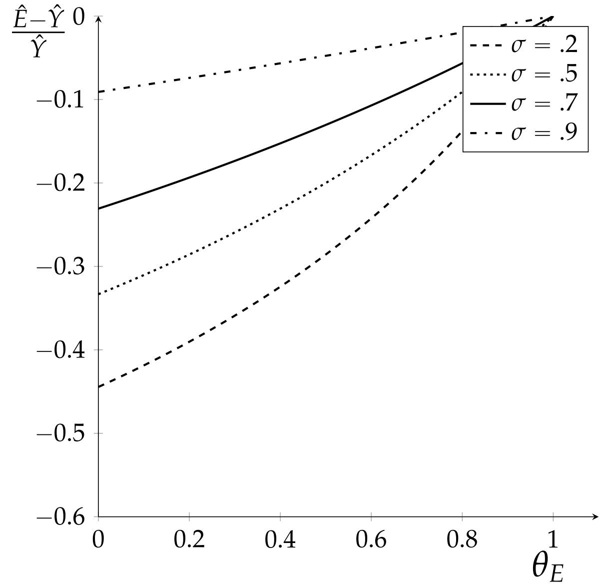

FIGURE 8.— Elasticity of Energy Intensity to Output along the 'stable path', all ABGPs in the model with no state dependence converge to this 'stable path'.

close to zero among richer countries.

Consider the elasticity of energy intensity to output towards which ABGPs converge to, given by Theorem 2.1,

$$\hat{E} - \hat{Y} = \hat{Y} \frac{(\sigma - 1)\,(1 - \theta_E(t))}{2 - \theta_E(t) - \sigma}$$

With constant prices, the low in absolute value elasticities seen in the cross-section of energy intensity to output can only be predicted along the ABGP in the no state dependence model if σ is approximately .9 (see Figure 8). However, such elasticities of substitution are higher than the estimates in the literature, which range from close to zero (Hassler et al., 2016) to .68 (Stern and Kander, 2012). An elasticity of substitution of .9 is also not consistent with a reasonable evolution of $Q_E(t)$. As Figure E.5 in the Online Appendix suggests, even with smoothing applied, a high σ implies Q_E rises and falls almost by 1000% between 1970 and 2015; it is difficult to attribute Q_E in this case to technical change.

Finally, note the shape of the relationship between $\theta_E(t)$ and the elasticity of energy intensity to output in Figure 8: the elasticity is *more negative* for lower levels of energy intensity. Figure 3.3 shows the elasticity of the average annual rate of energy intensity decline with respect to average annual output growth for the 100 countries between 1971 and 2010. The mean elasticity is -.28 and the negative rela-

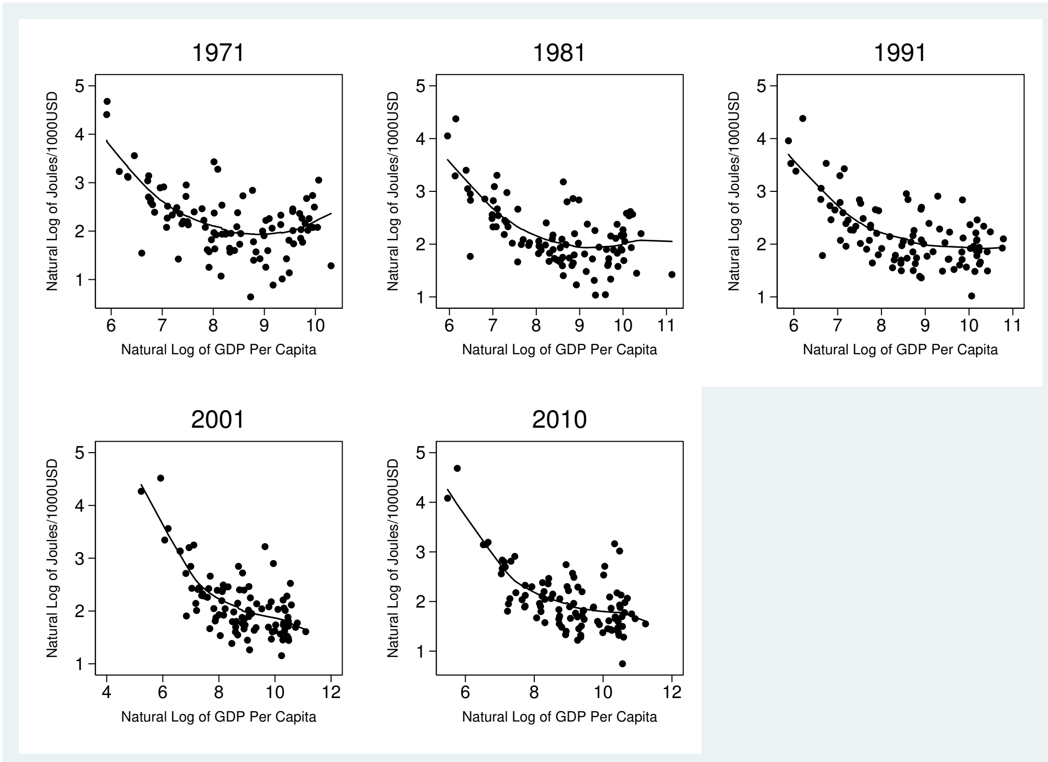

FIGURE 9.— Natural log of energy intensity and natural log of output for a cross section of countries. Csereklyei et al. (2016) describe the sources of the data.

tionship between elasticity and energy intensity in 1970 is statistically significant. Therefore, the elasticity is more negative in countries with higher energy intensity.

Thus, for no (or incomplete) state dependence to be consistent with the data, the cross section of countries countries *cannot lie along the dotted line in Figure 5*. In light of claim 2.1, we require dispersion around between countries such that poorer countries lie north of the dotted line and richer countries to lie south of the dotted line in Figure 5. Such a configuration is not inconsistent with the conditional convergence of energy intensity noted by Csereklyei et al. (2016); thus we see the higher elasticity of energy intensity to output among countries with lower energy intensity as only very mild evidence in favour of a complete state dependence economy.

	1971	1981	1991	2001	2010
Ln(Y)	-11.28	-12.25**	-9.413*	-8.740**	-9.215**
	(6.947)	(4.368)	(4.274)	(2.744)	(3.073)
$Ln(Y)^2$	1.111	1.280*	0.960	0.906**	0.931*
	(0.865)	(0.525)	(0.517)	(0.335)	(0.368)
$Ln(Y)^3$	-0.0354	-0.0443*	-0.0326	-0.0316*	-0.0315*
	(0.0355)	(0.0208)	(0.0206)	(0.0134)	(0.0145)
Constant	39.19*	40.84***	32.67**	30.22***	32.34***
	(18.40)	(11.97)	(11.62)	(7.385)	(8.432)
Observations	98	99	99	99	99

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

TABLE I

CUBIC POLYNOMIAL REGRESSION TABLE

USD/ capita	1971	1981	1971	2001	2010
2,000	-0.527	-0.470	-0.470	-0.444	-0.522
10,000	0.176	0.055	-0.026	-0.093	-0.082
20,000	0.310	0.068	0.010	-0.093	-0.043
30,000	0.320	0.060	-0.013	-0.135	-0.063

TABLE II

ELASTICITY OF ENERGY INTENSITY TO OUTPUT IN THE CROSS-SECTION

4. CONCLUSION

We develop an endogenous model of energy and economic growth with directed technical change to understand the economic drivers and future potential for autonomous energy efficiency improvements. Our analysis focused on understanding the trade-offs firms face between undertaking energy-augmenting and non-energy-augmenting research. Our key finding was that unless energy prices rise, there is a relatively stronger incentive to undertake labor augmenting research, since the price of labor rises relative to the price of energy. Whether or not labor augmenting research completely crowds out energy augmenting research depends on whether or not there is complete state dependence; with complete state dependence effects, the stronger incentive for labor-augmenting research is compounded and autonomous energy efficiency improvements are eventually crowded out. On the other hand, in a model without complete state dependence, autonomous energy efficiency improvements continue at a constant rate relative to output, albeit

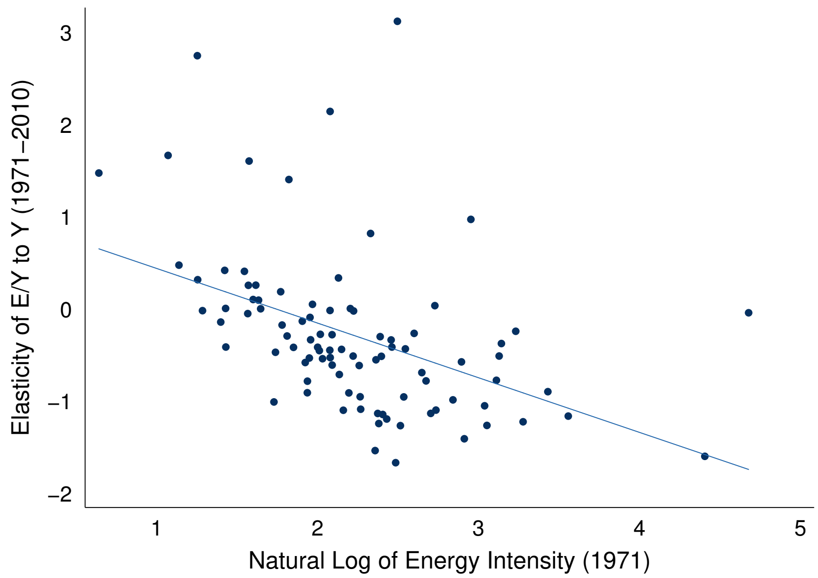

FIGURE 10.— Elasticity of average rate of decline in energy intensity to average rate of output growth between 1971 and 2010. The line of best fit has a slope coefficient of -0.59 (std. err. 0.12).

at a slower rate than labor augmenting technical change. On the other hand, countries that initially have high energy intensity will initially see more rapid energy augmenting technical change and over time labor augmenting technical change will become increasingly relatively important. This explains and generalizes the findings of Stern and Kander (2012) for Sweden.

Existing empirical evidence (Aghion et al., 2016; Bloom et al., 2017) suggests an innovations possibility frontier without complete state dependence. However, we were unable to conclusively determine whether a model with complete state dependence or not more accurately accounts for past trends. This is because under both models, one, if energy intensity is high enough, autonomous energy efficiency improvements take place, and two, fluctuating prices have led to notable fluctuations in the estimate of energy-augmenting technology, which prevents us from determining whether energy intensity is converging to a constant or not. Some mild evidence in favor of the complete state dependence economy is the fact that the fall in energy intensity relative to output in the cross-section of high income countries is small, suggesting energy intensities in the cross-section converge to a constant.

In any case, with constant energy prices, energy intensity never declines faster than final output grows, implying that energy use increases as long as the economy grows. Furthermore, we should not expect the decline in energy intensity to be more rapid in the future than it was in the past unless the cost of extracting energy rises or policy very significantly changes innovation incentives. Autonomos improvements in energy efficiency are likely, therefore, to have a somewhat limited role in mitigating climate change.

APPENDIX A: TECHNICAL APPENDIX

A.1. *Definition of Equilibrium*

DEFINITION A.1 An economy $\mathscr{E}$ is an equilibriun if:

1. no aggregate uncertainty holds and for $j \in \{E, L\}$ and $i \in [0,1]$, the processes $(q_j(i,t))_{t=0}^{\infty}$ satisfy (6), where the arrival rate of innovations is identical across i in a sector $j \in \{E, L\}$: given by b_j for the no state dependence economy or s_j for the complete state dependence economy
2. at each $t \in \mathbb{R}_+$ and $i \in [0,1]$ given $p_E(t)$ and $p_L(t)$, monopolists who own blueprints of machine type i with quality q pick prices, $p_E^x(i,t \mid q)$ and $p_L^x(i,t \mid q)$, to maximize profits
3. at each t and for each draw from the random variable q_E and q_L, given $p_E(t)$, $p_E^x(t)$, $p_L^x(t)$ and $p_L(t)$, intermediate goods producers choose $E(t)$, $L(t)$ and $x_E(i,t,q_E(i,t))$ and $x_L(i,t,q_L(i,t))$ to maximize profits
4. final goods markets producers maximize profits given $p_E(t)$ and $p_L(t)$ and the market for intermediate goods clears
5. given r and w, consumers choose a to maximize their inter-temporal utility
6. the corporate asset market clears, that is $a(t) = V_L(t) + V_E(t)$, where V_L and V_E are defined by (28)
7. given q_E and q_L, the value functions satisfy

$$v_j(i,t \mid q_j(i,t)) = \mathbb{E}_t \int_{s=t}^{T_j(i,t)} e^{-\rho s} \pi_j(i,s \mid q_j(i,t))\, \mathrm{d}s, \qquad j \in \{E, L\}, \mathbb{P} - a.e.$$

   where $\pi(i,s \mid q)$ is given in terms of the paths of prices, energy use and a quality q by (14)[26]
8. without state dependence, $s_E(t) = s_L(t) = 0$ and the free entry and exit condition

$$\eta_j \frac{v_j(i,t \mid q(i,t))}{q(i,t)} = 1, \qquad j \in \{E, L\}, \mathbb{P} - a.e.$$

   holds
9. with state dependence, $Z_E(t) = Z_L(t) = 0$ and the free entry and exit, (30), holds
10. the resource constraint (2) holds and $L(t) = L$ for all t.

[26]Recall expectations here are taken over T, where T is the random stopping time after which variety i in sector j experiences an innovation and the incumbent experiences zero profits thereafter. The distribution of T is pinned down entirely by the paths b_j and s_j for $j \in \{E, L\}$

A.2. *Proofs for no state dependence Economy*

To prepare for the proof of the main result, we derive growth equations that hold in equilibrium. Take time derivatives of $\theta_E(t)$, defined by (20), to write

$$\hat{\theta}_E(t) = \frac{\sigma-1}{\sigma}(1-\theta_E(t))\left(\hat{Q}_E(t)+\hat{E}(t)-\hat{Q}_L(t)\right) \tag{40}$$

Similarly, take time derivatives of $Y(t)$, defined by (18), to write

$$\hat{Y}(t) = (1-\theta_E(t))\,\hat{Q}_L(t)+\theta_E(t)\left(\hat{Q}_E(t)+\hat{E}(t)\right) \tag{41}$$

Now, note from (19), we have $\hat{\theta} = \hat{E} - \hat{Y}$. Use this expression in Equation (40) to write

$$\hat{E}(t) = \frac{\sigma-1}{\sigma}(1-\theta_E(t))(\hat{B}(t)+\hat{E}(t))+\hat{Y}(t) \tag{42}$$

Next, combine (42) with (41) to write

$$\hat{E}(t) = \frac{\sigma}{1-\theta_E(t)}\hat{Q}_E(t)-\hat{B}(t) \tag{43}$$

Let $\Gamma\colon \mathbb{R}_+ \to \mathbb{R}_+$ define the profitability ratio between energy and labor-augmenting research:

$$\Gamma(t) = \frac{\rho+\eta_E b_E(t)}{\rho+\eta_L b_L(t)}, \qquad t \in \mathbb{R}_+ \tag{44}$$

If innovation occurs in both sectors at time t, no arbitrage requires

$$\Gamma(t) = \left(\frac{Q_E(t)}{Q_L(t)}\right)^{-\frac{1}{\sigma}}\left(\frac{E(t)}{L(t)}\right)^{\frac{\sigma-1}{\sigma}} \tag{45}$$

And thus,

$$\hat{\Gamma}(t) = \frac{\sigma-1}{\sigma}\hat{E}(t)-\hat{B}(t)\left(\frac{1}{\sigma}\right) \tag{46}$$

Now combine (43) with (46),

$$
\begin{aligned}
\hat{\Gamma}(t) &= \frac{\sigma-1}{\sigma}\hat{E} - \frac{\hat{B}(t)}{\sigma} \\
&= \frac{\sigma-1}{\sigma}\left(\frac{\sigma}{1-\theta_E(t)}\hat{Q}_E(t) - \hat{B}(t)\right) - \frac{\hat{B}(t)}{\sigma} \qquad (47)\\
&= \frac{\sigma-1}{1-\theta_E(t)}\hat{Q}_E(t) - \hat{B}(t)
\end{aligned}
$$

PROPOSITION A.1 *Let $\mathscr{E}$ be a no state dependence equilibrium and let assumptions 1-2 hold. If $b_E(0) = 0$, then there exists $t' > 0$ such that $b_E(t') > 0$.*

PROOF: Suppose by contradiction $b_E(t) = 0$ for all $t \in \mathbb{R}_+$. Then Q_E is constant, and by (37), θ_E is constant. By Assumption 2 and the free entry-exit condition (24), b_L is strictly positive implying $Q_L(t) \to \infty$ and by (43), $E(t) \to \infty$. Since, using (20), we can write

$$\alpha p_E(t)^{\frac{1}{\alpha}} E(t) = \alpha \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(t)^{\frac{1}{1-\sigma}} E(t)$$

we must have $\alpha p_E(t)^{\frac{1}{\alpha}} E(t) \to \infty$. However, the free entry-exit conditions (24) are now violated because we assumed $b_E(t) = 0$ for all t, yielding a contraction.

Q.E.D.

PROOF OF THEOREM 2.1: First we verify there exists some t' such that for all $t > t'$, $b_E(t) > 0$ and $b_L(t) > 0$. By proposition A.1, there exists t' such that $b_E(t') > 0$ and $b_L(t') > 0$. Then by Claim D.2 in the Online Appendix, there exists $T > t'$ such that for all $t > T$

$$\frac{\eta_E b_E(t)}{\eta_L b_L(t)} \geqslant \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}$$

By Assumption 2 and the free entry-exit conditions (24), b_L is bounded below by a strictly positive constant $\lambda \eta_L \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} (1-\theta_E(0))^{\frac{1}{1-\sigma}} L - \rho$. Moreover, by (37), $\theta_E(t) \leqslant \theta(0)$ for all t, and, as such,

$$\eta_E b_E(t) \geqslant \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma}\eta_L b_L(t) \geqslant \frac{1-\theta_E(0)}{2-\sigma}(\lambda \eta_L \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} (1-\theta_E(0))^{\frac{1}{1-\sigma}} L - \rho) > 0$$

Now using the free entry-exit conditions (24), the above equation implies (45) - (47) holds for all $t > T$.

Along an ABGP, since $\Gamma(t)$ converges to a positive constant, $\hat{\Gamma}(t) \to 0$. By (47), recalling $\hat{B}(t) = \hat{Q}_E(t) - \hat{Q}_L(t)$, we can establish part 1. of the theorem

$$\frac{\hat{Q}_E(t)}{\hat{Q}_L(t)} \to \frac{1-\theta_E(t)}{2-\theta_E(t)-\sigma} \tag{48}$$

Since $b_E(t)$ is bounded below by a strictly positive constant for all $t > T$, we must have $\hat{Q}_E(t)$ is strictly positive for all $t > T$. Accordingly, $\theta_E(t) \to 0$ by (37), giving part 2 of the theorem. To establish part 3, write

$$b_L(t) = \lambda \eta_L \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} \left(1-\theta_E(t)\right)^{\frac{1}{1-\sigma}} L - \rho \to \lambda \eta_L \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} L - \rho \tag{49}$$

Thus $\hat{Q}_L(t) \to (\lambda-1)(\lambda \eta_L \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} L - \rho)$. Now note by (41), $\hat{Y}(t) \to \hat{Q}_L(t)$, establishing part 3.

Finally, for part 4, recall (21) and take taking time derivatives, to arrive at

$$\hat{E}(t) - \hat{Y}(t) = (\sigma-1)\,\hat{Q}_E(t) \to \hat{Y}(t)\frac{\left(1-\theta_E(t)\right)(\sigma-1)}{2-\theta_E(t)-\sigma} \to \hat{Y}(t)\frac{\sigma-1}{2-\sigma}$$

Q.E.D.

Proof of Proposition 2.2: Along an asymptotic balanced growth path $b_E(t) \to b_E^\star$ and $b_L(t) \to b_E^\star$, where by Theorem 2.1, $b_E^\star > 0$ and $b_L^\star > 0$.

Recall

$$\begin{aligned} \eta_E \alpha p_E(t)^{\frac{1}{\alpha}} E(t) &= \eta_E \alpha \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(t)^{\frac{1}{1-\sigma}} E(t) \\ &= \eta_E \alpha \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(t)^{\frac{2-\sigma}{1-\sigma}} \frac{E(t)}{\theta_E(t)} \\ &= \eta_E \alpha^2 \kappa(t)^{-1} \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(t)^{\frac{2-\sigma}{1-\sigma}} Y(t) \end{aligned} \tag{50}$$

where the first equality is from Equation (54), the second equality follows from dividing through and multypying by $\theta_E(t)$ and the final equality follows Equation (19). Now, by Equation (48) in the proof of Theorem 2.1,

$$\frac{\hat{Q}_E(t)}{\hat{Q}_L(t)} \to \frac{1-\theta_E(t)}{2-\sigma-\theta_E(t)} \tag{51}$$

But since $\frac{\hat{Q}_E(t)}{\hat{Q}_L(t)} = \frac{\eta_E b_E(t)}{\eta_L b_L(t)}$, we must have

$$\frac{\eta_E b_E(t)}{\eta_L b_L(t)} \to \frac{1-\theta_E(t)}{2-\sigma-\theta_E(t)}$$
$$\implies \frac{\eta_E \alpha^2 \kappa(t)^{-1} \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(t)^{\frac{2-\sigma}{1-\sigma}} Y(t) - \rho}{\alpha \eta_L \gamma_L^{\frac{\epsilon}{\sigma-1}} (1-\theta_E(t))^{\frac{1}{1-\sigma}} L - \rho} \to \frac{1-\theta_E(t)}{2-\sigma-\theta_E(t)}$$

The implication follows from the no arbitrage conditions, Equation (50) and (53). The above implies

$$Y(t) \to \frac{\frac{1-\theta_E(t)}{2-\sigma-\theta_E(t)} \left(\alpha \eta_L \gamma_L^{\frac{\epsilon}{\sigma-1}} (1-\theta_E(t)^{\frac{1}{1-\sigma}} L - \rho \right) + \rho}{\eta_E \kappa(t)^{-1} \alpha^2 \gamma_E^{\frac{\epsilon}{1-\sigma}} \theta_E(t)^{\frac{2-\sigma}{1-\sigma}}} := H(\theta_E(t))$$

where $H\colon (0, \theta_E(0)) \to \mathbb{R}_+$. Note H is differentiable. By assumption, $\theta_E(0) + \sigma < 1$, which can be shown to imply H has a negative derivative and hence H is decreasing. Since H is decreasing, H is injective. Moreover, since $(0, \theta_E(0))$ is open, by the open mapping theorem, H has a continuous inverse, which we now denote as $G : \mathbb{R}_+ \to (0, \theta_E(0))$.

Finally, we show for any $\epsilon > 0$, there exists T such that for all $t > T$,$|G(Y(t)) - \theta_E(t)| < \epsilon$. There exists $\delta > 0$ such that $|G(x) - G(y)| < \epsilon$ for $|x-y| < \delta$. Moreover, there exists T such that for all $t > T$, we have $|Y(t) - H(\theta_E(t))| < \delta$, giving $|G(Y(t)) - \theta_E(t)| < \epsilon$. This establishes that $G(Y(t)) \to \theta_E(t)$ and in view of Equation (19), implies $\frac{E(t)}{Y(t)} \to G(Y(t))$.

Q.E.D.

A.3. *Proofs for state dependence Economy*

We now prepare some preliminary notation before turning to the proof of theorem 2.2. Noting $p_L(t) = \gamma_L \left(\frac{Y(t)}{Y_L(t)}\right)^{\frac{1}{\epsilon}}$, we have

$$p_L(t) = \gamma_L \left(\frac{Y(t)}{Y_L(t)}\right)^{\frac{1}{\epsilon}} = \gamma_L \left(\gamma_L + \gamma_E \left(\frac{Y_E}{Y_L}\right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{1}{\epsilon-1}} \tag{52}$$
$$= \gamma_L^{\frac{\epsilon}{\epsilon-1}} \left(1 + \gamma^{\frac{\epsilon}{\sigma}} \left(\frac{Q_E(t)E(t)}{Q_L(t)L}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\epsilon-1}}$$

where the third equality uses (17). Using (20), we can then write

$$\alpha p_L(t)^{\frac{1}{\alpha}} L = \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} \left(1-\theta_E(t)\right)^{\frac{1}{1-\sigma}} L \tag{53}$$

similarly

$$\alpha p_E(t)^{\frac{1}{\alpha}} E(t) = \alpha \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E(t)^{\frac{1}{1-\sigma}} E(t) \tag{54}$$

Now define

$$J_L(t) := \mathbb{E}_t \int_t^{T_L(t)} e^{-\rho s} \alpha p_L(s)^{\frac{1}{\alpha}} L \, \mathrm{d}s, \quad J_E(t) := \mathbb{E}_t \int_t^{T_E(t)} e^{-\rho s} \alpha p_E(s)^{\frac{1}{\alpha}} E(s) \, \mathrm{d}s$$

where $T_j(t)$ is the random stopping time after which an incumbent is replaced by a new entrant. Note the distribution of $T_j(t)$ does not depend on the individual machine i, since each machine experiences a common innovation, hence replacement rate $s_j \eta_j$. Recalling the definition of $V_L(t)$ from (28),

$$\begin{aligned} V_L(t) = \int_0^1 v_L(i,t\,|\,q) \, \mathrm{d}i &= \int_0^1 \mathbb{E}_t \int_{s=t}^{T_L(t)} e^{-\rho s} \pi_L(s) \, \mathrm{d}s \, \mathrm{d}i \\ &= \int_0^1 q(i,t) \mathbb{E}_t \int_{s=t}^{T_L(t)} e^{-\rho s} \alpha p_L(s)^{\frac{1}{\alpha}} L \, \mathrm{d}s \, \mathrm{d}i \\ &= Q_L(t) \mathbb{E}_t \int_{s=t}^{T_L(t)} e^{-\rho s} \alpha p_L(s)^{\frac{1}{\alpha}} L \, \mathrm{d}s \\ &= Q_L(t) J_L(t) \end{aligned}$$

The HJB equation for $J_L(t)$ will be

$$\dot{J}_L(t) = \left(\rho + \eta_L s_L(t)\right) J_L(t) - \alpha p_L(t)^{\frac{1}{\alpha}} L \tag{55}$$

and similarly, the HJB equation for $J_E(t)$ becomes

$$\dot{J}_E(t) = \left(\rho + \eta_E s_E(t)\right) J_E(t) - \alpha p_E(t)^{\frac{1}{\alpha}} E(t) \tag{56}$$

LEMMA A.1 *If an equilibrium with state dependence is an asymptotic balanced growth path with* $s_E^\star > 0$ *and* $s_L^\star > 0$, *then* $\eta_L s_L^\star = \rho + 2\eta_E s_E^\star$.

PROOF: By Equation (37), since $Q_E(t) \to \infty$, $\theta_E(t) \to 0$. Now, if $\theta_E(t)_E \to 0$, using (53), we have

$$\alpha p_L(t)^{\frac{1}{\alpha}} L \to \alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} L \tag{57}$$

which implies $J_L(t)$ converges to a constant. Note both the terms on the RHS of the HJB equation for $J_L(t)$, Equation (55), converge, implying $\dot{J}_L(t)$ converges. But since $J_L(t)$ converges to a constant, we must have $\dot{J}_L(t) \to 0$. Once again by the HJB condition, Equation (55),

$$J_L(t) \to \frac{\alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} L}{\rho + s_E^\star} \tag{58}$$

Also note

$$\hat{J}_L(t) = \frac{\dot{J}_L(t)}{J_L(t)} \tag{59}$$

The denominator converges a positive constant, while numerator converges to zero, implying $\hat{J}_L(t) \to 0$. Next, from the free entry and exit condition, since $s_E^\star > 0$ and $s_L^\star > 0$, there exists $\bar{T}$ such that

$$J_E(t) = \frac{Q_L(t) J_L(t)}{Q_E(t)}, \qquad t > \bar{T} \tag{60}$$

Taking time derivatives, we have

$$\hat{J}_L(t) - \hat{J}_E(t) = \hat{Q}_E(t) - \hat{Q}_L(t), \qquad t > \bar{T} \tag{61}$$

Rearranging gives

$$\hat{J}_E(t) = \hat{Q}_L(t) - \hat{Q}_E(t) + \hat{J}_L(t) \to \eta_L s_L^\star - \eta_E s_E^\star \tag{62}$$

where $\hat{J}_L(t) \to 0$ by the argument proceeding Equation (59). Now use the above with the HJB condition for $J_E(t)$, Equation (56),

$$\frac{\alpha p_E(t)^{\frac{1}{\alpha}} E(t)}{J_E(t)} = \rho + \eta_E s_E(t) - \hat{J}_E(t) \to \rho + 2\eta_E s_E^\star - \eta_L s_L^\star \tag{63}$$

Note the limit is a (possibly zero) constant. Moreover, by the HJB condition for $J_L(t)$, Equation (55),

$$\frac{\alpha p_L(t)^{\frac{1}{\alpha}} L}{J_L} = \rho + \eta_L s_L^\star - \hat{J}_L(t) \to \rho + \eta_L s_L^\star \tag{64}$$

since $\hat{J}_E(t) \to 0$. As such,

$$\frac{\theta_E(t)}{1-\theta_E(t)}\gamma^{-\epsilon\sigma} = \left(\frac{Q_E(t)E(t)}{Q_L(t)L}\right)^{\frac{\sigma-1}{\sigma}}$$
$$= \frac{p_E^{\frac{1}{\alpha}} Q_E(t)E(t)}{p_L^{\frac{1}{\alpha}} Q_L(t)L} = \frac{p_E^{\frac{1}{\alpha}} J_L(t)E(t)}{p_L^{\frac{1}{\alpha}} J_E(t)L} \to \frac{\rho + 2\eta_E s_E^\star - \eta_L s_L^\star}{\rho + \eta_L s_L^\star}$$

where the first equality comes from (36), the second equality uses (16), the third uses the free entry and exit conditions (30) and convergence at the final step follows from (63) and (64) and noting the denominator converges to a strictly positive constant. Thus, if $\theta_E(t) \to 0$, then $\eta_L s_L^\star = \rho + 2\eta_E s_E^\star$.

Q.E.D.

CLAIM A.1 If an equilibrium with state dependence is an asymptotic balanced growth path with $s_E^\star > 0$ and $s_L^\star > 0$, then there exists $\bar{T}$ and M, with $M < \infty$ such that

$$\mathbb{E}_t \int_{s=t}^{T_E(t)} e^{\int_t^s \hat{E}(\bar{s}) - \eta_E s_E(\bar{s}) - \rho \, \mathrm{d}\bar{s}} \leqslant M < \infty, \qquad t > \bar{T}$$

PROOF: The distribution of $T_E(t)$ is

$$\mathbb{P}\left(T_E(t) \leqslant t+s\right) = 1 - e^{-\int_0^s \eta_E s_E(s_1+t)\,\mathrm{d}s_1}$$

and the density of $T_E(t) - t$ is $\eta_E s_E(s) e^{-\int_0^s \eta_E s_E(s_1+t)\mathrm{d}s_1}$. Allowing us to write

$$\mathbb{E}_t \int_{s=t}^{T_E(t)} e^{\int_t^s \hat{E}(\bar{s}) - \eta_E s_E(\bar{s}) - \rho \, \mathrm{d}\bar{s}} \, \mathrm{d}s = \mathbb{E}_t \int_0^{T_E(t)-t} e^{\int_0^s \hat{E}(\bar{s}+t) - \eta_E s_E(\bar{s}+t) - \rho \, \mathrm{d}\bar{s}} \, \mathrm{d}s$$
$$= \eta_E s_E(s) \int_0^\infty \int_0^T e^{\int_0^s \hat{E}(\bar{s}+t) - \eta_E s_E(\bar{s}+t) - \rho \, \mathrm{d}\bar{s}} \, \mathrm{d}s \, e^{-\int_0^T \eta_E s_E(s_1+t)\mathrm{d}s_1} \, \mathrm{d}T$$

Define

$$c := \lim_{t\to\infty} \{\hat{E}(t) - 2\eta_E s_E(t) - \rho\} \tag{65}$$

We now show $c < 0$. Taking growth rates across equation (21) gives $\hat{E}(t) - \hat{Y}(t) = (\sigma-1)\hat{Q}_E(t)$, and thus $\hat{E}(t) - \hat{Y}(t) \to \eta_E s_E^\star(\sigma - 1)$, and $\hat{E}(t) \to \eta_E s_E^\star(\sigma-1) + \eta_L s_L^\star$

since $\hat{Y}(t) \to \eta_L s_L^\star$ by (41). If $s_L^\star = 0$, then

$$c = \eta_E s_E^\star(\sigma - 1) + \eta_L s_L^\star - 2\eta_E s_E^\star - \rho = \eta_E s_E^\star(\sigma - 3) - \rho < 0 \tag{66}$$

On the other hand, if $s_L^\star > 0$, then by Lemma A.1, $\eta_L s_L^\star = \rho + 2\eta_E s_E^\star$

$$c = \eta_E s_E^\star(\sigma - 1) + \eta_L s_L^\star - \eta_E 2 s_E^\star - \rho = s_E^\star(\sigma - 1) < 0 \tag{67}$$

Next, note there exists $\epsilon > 0$ such that $c + \epsilon < 0$. Moreover, since $\hat{E}(t) - \eta_E s_E(t) - \rho \to \eta_E s_E^\star(\sigma - 2) + s_L^\star - \rho$, there exists $\bar{T}$ such that for all $t > \bar{T}$, we have

$$\hat{E}(t) - \eta_E s_E(t) - \rho < \eta_E s_E^\star(\sigma - 2) + \eta_L s_L^\star - \rho + \frac{\epsilon}{2} := c_1 \tag{68}$$

and

$$-\eta_E s_E(t) < -\eta_E s_E^\star + \frac{\epsilon}{2} := c_2 \tag{69}$$

The above two inequalities give

$$\begin{aligned} &\eta_E s_E(s) \int_0^\infty \int_0^T e^{\int_0^s \hat{E}(\bar{s}) - \eta_E s_E(\bar{s}) - \rho \, \mathrm{d}\bar{s}} \, \mathrm{d}s \, e^{-\int_0^T \eta_E s_E(s_1) \mathrm{d}s_1} \, \mathrm{d}T \\ &\leqslant \eta_E s_E(s) \int_0^\infty \int_0^T e^{\int_0^s c_1 \, \mathrm{d}\bar{s}} \, \mathrm{d}s \, e^{\int_0^T c_2 \mathrm{d}s_1} \, \mathrm{d}T = \int_0^\infty \frac{e^{T(c_1 + c_2)} - e^{Tc_2}}{c_1} \, \mathrm{d}T := M < \infty \end{aligned} \tag{70}$$

where the first inequality follows from monotonicity of the exponential function and noting (68) and (69). The second inequality comes from solving the inside integrals and the final inequality comes from noting $c_1 + c_2 = c + \epsilon < 0$.

Q.E.D.

PROOF OF THEOREM 2.2: If $s_E^\star > 0$, then there exists $\bar{T}$ such that for $t > \bar{T}$, the no arbitrage condition holds

$$\frac{J_E(t) Q_E(t)}{J_L(t) Q_L(t)} \geqslant 1, \qquad t > \bar{T} \tag{71}$$

to prove the theorem, we will show this condition cannot hold if $Q_E(t) \to \infty$ and

$E(t)/Y(t) \to 0$. From the definition of $J_E(t)$ and $J_L(t)$, we have

$$
\begin{aligned}
\frac{J_E(t)Q_E(t)}{J_L(t)Q_L(t)} &= \frac{Q_E(t)\mathbb{E}_t \int_{s=t}^{T_E(t)} \pi_E(s)^{\frac{1}{\alpha}} E(s) e^{-\rho s}\,\mathrm{d}s}{Q_L(t)J_L(t)} \\
&= \kappa(t)(1-\theta_E(t))^{1-\sigma} L \left(\frac{\theta_E(t)}{1-\theta_E(t)}\right)^{\frac{1}{1-\sigma}} \\
&\quad \times \frac{E(t)Q_E(t)}{LQ_L(t)} \frac{\mathbb{E}_t \int_{s=t}^{T(t)} e^{\int_{\bar{s}=t}^{s} \hat{E}(\bar{s}) - \eta_E s_E(\bar{s}) - \rho\,\mathrm{d}\bar{s}}\,\mathrm{d}s}{J_L(t)} \\
&= \kappa(t)\,(1-\theta_E(t))^{1-\sigma}\, L \left(\frac{\theta_E(t)}{1-\theta_E(t)}\right) \\
&\quad \times \frac{\mathbb{E}_t \int_{s=t}^{T(t)} e^{\int_{\bar{s}=t}^{s} \hat{E}(\bar{s}) - \eta_E s_E(\bar{s}) - \rho\,\mathrm{d}\bar{s}}\,\mathrm{d}s}{J_L(t)}
\end{aligned} \tag{72}
$$

where we have omitted constants in front of the RHS above for simplicity. The second equality above uses (54) and (37) to derive

$$
\begin{aligned}
\pi_E(s)^{\frac{1}{\alpha}} E(s) = \kappa(t)\theta_E(s)^{\frac{1}{1-\sigma}} E(s) &= Q_E(s)^{-1}E(s) \\
&= \kappa(t)Q_E(t)^{-1}E(t)e^{\int_{\bar{s}=t}^{s} \hat{E}(\bar{s}) - \eta_E s_E(\bar{s})\,\mathrm{d}\bar{s}} \\
&= \kappa(t)\theta_E(t)^{\frac{1}{1-\sigma}} E(t)e^{\int_{\bar{s}=t}^{s} \hat{E}(\bar{s}) - \eta_E s_E(\bar{s})\,\mathrm{d}\bar{s}}
\end{aligned}
$$

for $s > t$. Note once again, we have omitted constants in front of the RHS for simplicity. The third equality at (72) uses (36).

To complete the proof, by Claim A.1, $\mathbb{E}_t \int_{s=t}^{T} e^{\int_{\bar{s}=t}^{s} \hat{E}(\bar{s}) - \eta_E s_E(\bar{s}) - \rho\,\mathrm{d}\bar{s}}\,\mathrm{d}s < M$, where $M < \infty$. As such,

$$
\frac{J_E(t)Q_E(t)}{J_L(t)Q_L(t)} \leqslant \kappa(t)\,(1-\theta_E(t))^{\frac{1}{1-\sigma}}\, L \left(\frac{\theta_E(t)}{1-\theta_E(t)}\right) \frac{M}{J_L(t)}
$$

Recall $\theta_E(t) \to 0$ by (37) if $Q_E(t) \to \infty$ and $J_L(t)$ converges to a constant, $\frac{J_E(t)Q_E(t)}{J_L(t)Q_L(t)} \to 0$. However, this contradicts (71).

Q.E.D.

PROOF OF PROPOSITION 2.3: We first prove the *if* statement of the proposition.

Part 1: If

Suppose (31) holds, we will show an allocation satisfying 1.- 4. of Definition A.1 with $\eta_L s_L(t) = \eta_L$ for all t is an equilibrium (satisfies 6. of Definition A.1) and is a

BGP. To show the allocation satisfies 6. of Definition A.1, it is sufficient to confirm the free entry and exit condition holds along the allocation path, that is,

$$\frac{J_L(t)Q_L(t)}{J_E(t)Q_E(t)} \geqslant 1, \qquad \forall\, t \geqslant 0 \tag{73}$$

Since $s_E = 0$, $Q_E(t) = Q_E(0)$ for all t and we must have, using Equation (37),

$$\theta_E(t) = \theta_E^\star := (\alpha(1-\alpha))^{\sigma-1}\,\kappa(t)^{1-\sigma}\gamma_E^{\frac{\epsilon}{\sigma-1}} Q_E(0)^{\sigma-1}, \qquad \forall\, t \geqslant 0 \tag{74}$$

By (53), $p_L(t)$ remains constant. As such,

$$J_L(t) = \frac{\alpha p_L(t)^{\frac{1}{\alpha}} L}{\rho + \eta_L} = \frac{\alpha \gamma_L^{\frac{\epsilon}{\sigma-1}} \left(1-\theta_E^\star\right)^{\frac{1}{1-\sigma}} L}{\rho+\eta_L}, \qquad \forall\, t \geqslant 0 \tag{75}$$

Since the replacement rate is zero for energy-augmenting innovations,

$$\begin{aligned} J_E(t) &= \int_t^\infty \alpha p_E(t)^{\frac{1}{\alpha}} E(t) e^{-\rho s}\,\mathrm{d}s \\ &= \alpha \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E^{\star\,\frac{1}{1-\sigma}} \int_0^\infty E(t) e^{-\rho s}\,\mathrm{d}s \\ &= \alpha \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E^{\star\,\frac{1}{1-\sigma}} E(t) \int_t^\infty e^{(\eta_L-\rho)s}\,\mathrm{d}s \\ &= \frac{\alpha \gamma_E^{\frac{\epsilon}{\sigma-1}} \theta_E^{\star\,\frac{1}{1-\sigma}} E(t)}{\rho-\eta_L} \end{aligned}$$

The second equality uses (54). For the third equality, note when $\hat{Q}_E(t) = 0$, $\hat{E}(t) = \eta_L = \hat{Y}(t)$ for all t by (21) and (41). The third equality follows from solving the integral.

We can now write

$$\begin{aligned} \frac{J_L(t)Q_L(t)}{J_E(t)Q_E(t)} &= \gamma^{\frac{-\epsilon}{\sigma-1}} \left(\frac{1-\theta_E^\star}{\theta_E^\star}\right)^{\frac{1}{1-\sigma}} \left(\frac{LQ_L(t)}{E(t)Q_E(t)}\right) \frac{\rho-\eta_L}{\rho+\eta_L} \\ &= \gamma^{\frac{-2\epsilon}{\sigma-1}} \left(\frac{1-\theta_E^\star}{\theta_E^\star}\right) \frac{\rho-\eta_L}{\rho+\eta_L} \\ &= \frac{J_L(0)Q_L(0)}{J_E(0)Q_E(0)} \end{aligned} \tag{76}$$

for all $t \geqslant 0$. The second equality follows from (36). The third equality follows from our observation that $\theta_E(t)$ is constant for all $t \geqslant 0$, given by (74).

Using algebra, and (74), we can verify

$$\gamma^{\frac{-2\epsilon}{\sigma-1}}\left(\frac{1-\theta_E^\star}{\theta_E^\star}\right)\frac{\rho-\eta_L}{\rho+\eta_L} \geqslant 1 \iff \left(\frac{\kappa}{Q_E(0)}\right)^{1-\sigma} \leqslant \frac{\left(\alpha(1-\alpha)\right)^{1-\sigma}\gamma^{\frac{\epsilon}{1-\sigma}}}{\gamma^{\frac{2\epsilon}{\sigma-1}}\frac{\rho+\eta_L}{\rho-\eta_L}+1} \tag{77}$$

And thus, if (31) holds, then $\frac{J_L(0)Q_L(0)}{J_E(0)Q_E(0)} \geqslant 1$, which by (76), implies (73). To confirm the equilibrium allocation is a BGP, note by (21) and (41), $\hat{Y}(t) = \eta_L$ and $\hat{E}(t) = \eta_L$ for all t. Moreover, $s_L(t) = 1$, $s_E(t) = 0$, $\hat{Q}_L(t) = \eta_L$ and $\hat{Q}_E(t) = 0$ for all t.

Now we turn to the *only if* statement of the proposition.

Part 2: Only If

Let an equilibrium allocation be a BGP with $\eta_L s_L(t) = \eta_L$. We show (31) must hold. Since $s_E(t) = 0$ for all t, (76) must hold along the growth path by the argument between proceeding Equation (73) above. Because the path is an equilibrium path, the free entry and exit conditions (73) must hold, and since (76) holds,

$$\gamma^{\frac{-2\epsilon}{\sigma-1}}\left(\frac{1-\theta_E^\star}{\theta_E^\star}\right)\frac{\rho-\eta_L}{\rho+\eta_L} \geqslant 1 \tag{78}$$

which, by (77), implies (31) holds. *Q.E.D.*

APPENDIX B: COMPARATIVE STATICS OF EQUILIBRIUM ENERGY USE AND TECHNOLOGY

In this section, we study the effect of energy augmenting technology on energy use. For a given level of output and real energy price, $\kappa(t)$, higher $Q_E(t)$ leads to lower energy use. Given a real energy price, we can decompose the effect of energy-augmenting technical change on energy use into the change in energy use given the level of output and the change in energy use as output increases given a level of energy intensity. Solve (21) for E and take the derivative with respect to Q_E to arrive at

$$\frac{\partial E}{\partial Q_E} = \Phi\,\overbrace{\frac{\partial Y}{\partial Q_E}Q_E^{\sigma-1}}^{\text{Output effect}} + \Phi\,\overbrace{(\sigma-1)YQ_E^{\sigma-2}}^{\text{Efficiency effect}} \tag{79}$$

where $\Phi = \gamma_E^{\frac{\epsilon}{\sigma-1}}\alpha^\sigma(1-\alpha)^{\sigma-1}$ and we have dropped the time index for simplicity. The efficiency effect is always negative, and the output is always positive. The following proposition tells us when increased energy efficiency results in a fall in energy use.

PROPOSITION B.1 *If* $\theta_E < (1-\sigma)(1-\alpha)^{\frac{1-\sigma}{\sigma}}$, *then* $\frac{\partial E}{\partial Q_E} < 0$.

PROOF: Evaluate the derivative on the right hand side of (79) and simplify the equation

$$\frac{\partial E}{\partial Q_E} = \Phi Q_E^{\sigma-2} Y \left(\gamma_E^{\frac{\epsilon}{\sigma}} Y^{\frac{1-\sigma}{\sigma}} \left(Q_E E\right)^{\frac{\sigma-1}{\sigma}} + \sigma - 1 \right) \tag{80}$$

Now note the expression for output (18) and θ_E (20) to arrive at the result.

Q.E.D.

When real energy prices are fixed, the result implies the rebound effect is less than 1 if the cost-share of energy is less than $(1-\sigma)(1-\alpha)^{\frac{1-\sigma}{\sigma}}$.[27]

REFERENCES

Acemoglu, D., 1998. Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality. The Quarterly Journal of Economics 113 (4), 1055–1089.
URL `https://academic.oup.com/qje/article-lookup/doi/10.1162/003355398555838`

Acemoglu, D., 2002. Directed Technical Change. The Review of Economic Studies 69 (4), 781–809.

Acemoglu, D., 2003. Labour and Capital Augmenting Technical Change. Journal of the European Economic Association 1 (1), 1–37.

Acemoglu, D., 2009. Introduction to Modern Economic Growth. Princeton University Press, Princeton, New Jersey.

Acemoglu, D., Aghion, P., Bursztyn, L., Hemous, D., feb 2012. The Environment and Directed Technical Change. American Economic Review 102 (1), 131–166.
URL `http://pubs.aeaweb.org/doi/abs/10.1257/aer.102.1.131`

Acemoglu, D., Cao, D., 2015. Innovation by entrants and incumbents. Journal of Economic Theory 157, 255–294.
URL `http://dx.doi.org/10.1016/j.jet.2015.01.001`

Aghion, P., Hemous, D., Martin, R., Reenen, J. V., Dechezleprêtre, A., 2016. Carbon Taxes, Path Dependency, and Directed Technical Change: Evidence from the Auto Industry. Journal of Political Economy 124 (1), 1–52.
URL `http://ideas.repec.org/p/fem/femwpa/2012.99.html`

Aghion, P., Howitt, P., 1992. A Model of Growth Through Creative Destruction. Econometrica 60 (2), 323–351.

André, F. J., Smulders, S., 2014. Fueling Growth When Oil Peaks: Directed Technological Change and the Limits to Efficiency. European Economic Review 69, 18–39.

Bloom, N., Jones, C. I., Van Reen, J., Webb, M., 2017. Are Ideas Getting Harder to Find? NBER Working Paper No. 23782.
URL `http://www.nber.org/papers/w23782`

[27] In a similar exogenous technical change setting, the rebound is less than 1 if $\theta_E < (1-\sigma)$ (Saunders, 2015).

Csereklyei, Z., Rubio- Varas, M., Stern, D., 2016. Energy and Economic Growth: The Stylized Facts Energy & The Economy. Energy Economics 37 (2).
EIA, 2017. Annual Energy Outlook. Tech. rep., U.S. Energy Information Admistration.
Goulder, L. H., Schneider, S. H., 1999. Induced Technological Change and The Attractiveness of CO2 Abatement Policies. Resource and Energy Economics 21 (3-4), 211–253.
URL http://www.sciencedirect.com/science/article/pii/S0928765599000044
Hamilton, J. D., 2009. Understanding Crude Oil Prices. The Energy Journal 30 (2).
Hassler, J., Krusell, P., Olovsson, C., 2016. Directed Technical Change as a Response to Natural-Resource Scarcity.
URL hassler-j.iies.su.se/PAPERS/estc.pdf{%}0A
IEA, 2016. World Energy Outlook 2016.
URL http://www.oecd-ilibrary.org/energy/world-energy-outlook{_}20725302
IEA, 2017. Market Report Series: Energy Efficiency 2O17. Tech. rep., International Energy Agency.
URL https://www.iea.org/publications/freepublications/publication/market-report-series-energy-efficiency-2017-.html
Jakeman, G., Hanslow, K., Hinchy, M., Fisher, B. S., Woffenden, K., 2004. Induced Innovations and Climate Change Policy. Energy Economics 26 (6), 937–960.
Jones, C., 1995. R&D-Based Models of Economic Growth. The Journal of Political Economy 103 (4), 759–784.
Kaufmann, R. K., 2004. The Mechanisms for Autonomous Energy Efficiency Increases : A Cointegration Analysis of the US Energy / GDP Ratio. The Energy Journal 25 (1), 63–86.
Löschel, A., 2002. Technological Change in Economic Models of Environmental Policy: A Survey. Ecological Economics 43 (2-3), 105–126.
Mckibbin, W. J., Pearce, D., Stegman, A., 2004. Long Run Projection For Climate Change Scenarios. Working Papers in International Economics (104).
Newell, R. G., Jaffe, A. B., Stavins, R. N., 1999. The Induced Innovation Hypothesis and Energy-Saving Technological Change. Quarterly Journal of Economics 114 (3), 941–975.
Nordhaus, W., 2002. Modeling Induced Innovation in Climate Change Policy. In: Grubler, A., Nakićenović, N., Nordhaus, W. (Eds.), Technological Change and the Environment. Resources for the Future Press, pp. 259–290.
Popp, D., jul 2004. ENTICE: endogenous technological change in the DICE model of global warming. Journal of Environmental Economics and Management 48 (1), 742–768.
URL http://linkinghub.elsevier.com/retrieve/pii/S0095069603001220
Popp, D., Newell, R. G., Jaffe, A. B., 2010. Energy, The Environment, and Technological Change. Vol. 2. Elsevier B.V.
URL http://dx.doi.org/10.1016/S0169-7218(10)02005-8
Saunders, H. D., 2015. Recent Evidence for Large Rebound: Elucidating the Drivers and their Implications for Climate Change Models. The Energy Journal 36 (1), 23–48.
URL http://www.scopus.com/inward/record.url?eid=2-s2.0-84919424574{&}partnerID=tZOtx3y1
Stern, D. I., 2004. Economic Growth and Energy. In: Cleveland, C. J. (Ed.), Encyclopedia of Energy. Vol. 2. Academic Press, San Diego, CA, pp. 35–51.
Stern, D. I., 2017. How accurate are energy intensity projections? Climatic Change 143 (3-4), 537–545.
Stern, D. I., Kander, A., 2012. The Role of Energy in the Industrial Revolution and Modern Economic Growth. The Energy journal 33 (3), 125–152.
URL http://cat.inist.fr/?aModele=afficheN{&}cpsidt=26029760
Sue Wing, I., 2008. Explaining The Declining Energy Intensity of the U.S. Economy. Resource and Energy Economics 30 (1), 21–49.
Sue Wing, I., Eckaus, R. S., 2007. The Implications of the Historical Decline in US Energy Intensity

for Long-Run CO2 Emission Projections. Energy Policy 35 (11), 5267–5286.

Sun, Y., Zhang, Y., 2009. Individual Risk and Lebesgue Extension Without Aggregate Uncertainty. Journal of Economic Theory 144 (1), 432–443.
URL http://dx.doi.org/10.1016/j.jet.2008.05.001

Van der Werf, E., 2008. Production Functions for Climate Policy Modeling: An Empirical Analysis. Energy Economics 30 (6), 2964–2979.

Warr, B., Ayres, R., Eisenmenger, N., Krausmann, F., Schandl, H., 2010. Energy use and economic development: A Comparative Analysis of Useful Work Supply in Austria, Japan, the United Kingdom and the US during 100 Years of Economic Growth. Ecological Economics 69 (10), 1904–1917.
URL http://dx.doi.org/10.1016/j.ecolecon.2010.03.021

Webster, M., Paltsev, S., Reilly, J., 2008. Autonomous efficiency improvement or income elasticity of energy demand: Does it matter? Energy Economics 30 (6), 2785–2798.

Williams, R. H., Lave, L. B., Perry, A. M., 1990. Low-Cost Strategies for Coping with CO Emission Limits (A Critique of CO Emission Limits: an Economic Cost Analysis for the USA by Alan Manne and Richard Richels). The Energy Journal 11 (4), 35–59.