We Finally Have a Believable Life Cycle Model

Christopher Carroll[1]
Johns Hopkins University
Econ-ARK

Alan Lujan
Johns Hopkins University
Econ-ARK

Wednesday 15th May, 2024

Abstract

The 'life cycle model' of optimal saving for retirement is familiar to anyone who has taken an introductory economics class. When hiring a financial advisor, it is plausible to imagine that people interpret the advisor's job as being just to tailor optimal life-cycle-model choices to their particular circumstances. But academics and financial industry experts have long known that the advice about both saving and portfolio choice provided by rigorous academic life-cycle models is deeply problematic – for example, such models imply that retirees should plan to run their wealth down to zero or some small amount and then (optimally!) live pension-check to pension-check (at least approximately). This paper makes the case that recent developments in the economics literature, when combined with the kinds of feedback that advisors get from their clients, can finally generate advice that is both mathematically optimal and intuitively plausible.

Keywords

1 Introduction

Franco Modigliani and Richard Brumberg (1954)[1] were the first to propose that it might be possible understand consumer financial choices as reflecting optimal responses to the realities of the path of income and of spending needs over the lifetime. An enormous academic literature has followed their pioneering work, but only very recently have academic models of mathematically optimal behavior gotten within spitting range of matching either the choices that real people actually make, or the choices that financial advisors recommend to them.

Over the past couple of decades, three things have changed in the academic literature that have all contributed to making the models' predictions more believable.

[1] Correspondence to: ccarroll@llorracc.org

[1] **?**

The first is that the rapid advance of computational capacity has finally made it possible to construct credible answers to the question "what saving and portfolio choices are actually optimal?" in a real world that is extremely complex. In particular, the incorporation of realistic descriptions of the uncertainties people face (about their own income, stock returns, interest rates, health expenditures, mortality, and more) makes computation of the optimal solution of the problem astonishingly difficult. Much harder, say, than the computation of optimal trajectories for spacecraft; comparable perhaps to the computational difficulty of figuring out how to drive a car roughly as well as a human (another problem where realistic computational solutions have only recently become available).

A second development has been a new openness by academic economists to the idea that people's beliefs and preferences can be probed by *asking them* about their beliefs and preferences. The traditional academic approach had been to attribute to the optimal decisionmaker beliefs based on economists' perceptions of the relevant facts (like the rate of return, and riskiness, of the stock market). It turns out that economists' beliefs differ substantially from the beliefs that most people actually hold, and it seems reasonable to suppose that the decisions people make reflect their own beliefs rather than the beliefs of economists. Specifically with respect to stock returns, Mateo **?** has shown that even college-educated people systematically have held beliefs about stock market returns that are pessimistic compared to the returns the market has historically delivered. He argues that this explains why, historically, people have been less eager to invest in stocks than models calibrated with economists' more optimistic expectations. He argues that the behavior of college-educated people over most of their lives is reasonably consistent with rational decisionmaking (given their beliefs).

In fact, the failure of retired households to draw down their wealth substantially as they age – the 'drawdown failure' – has been a challenge to the life cycle model for many years (cf. **?** for an early statement). The most common attempt to explain this in the academic literature has been to postulate a 'bequest motive,' typically interpreted as a desire to leave a legacy to one's heirs. But our analysis below confirms the thrust of the existing literature which finds that a bequest motive does not seem to be important for the median college-educated household, for whom the drawdown failure is very substantial. (We focus on college educated households partly because a growing body of literature -including **?**'s work cited above – finds that the behavior of the college-educated population comes much closer to matching the predications of optimizing models than the behavior of people with less education).

The model section of the paper examines a direct and simple solution, which is to assume that people have an intrinsic desire to hold wealth, which seems to be distinct from the consequences those wealth holdings have for their ability to consume in the future. The main substantive/mathematical point of this paper (in the "Model Results" section) is to show that a model with 'wealth in the utility function' explains observed post-retirement behavior of both wealth-holding and portfolio choice better than the standard model (with or without a bequest motive).

This provides an attractive resolution to the awkward fact that the financial industry's advice has, until now, had no rigorous underpinning but tradition. Our resolution takes the satisfying form that perhaps financial advisors had a better idea all along what constituted advice consistent with people's true preferences.

2 Literature Review

See **?** for a comprehensive review of the literature.

2.1 Uncertainty Matters

Literature finding traditional LC models work pretty well during working life

2.2 'Drawdown failure'

Literature documenting failure to draw down wealth. Hurd (late 80s), etc

2.3 Portfolio Choice

Lit on portfolio choice requiring huge RRA

3 Models

3.1 The Baseline Academic Model

3.1.1 The Life Cycle Portfolio ('LCP') Model

We begin by describing the consumer's optimal consumption/saving problem over the life cycle when they have no access to a risky asset (like the stock market) that earns a higher (expected) rate of return than the safe asset. After we have finished describing the plain life cycle model we will augment it to add optimal portfolio choice between safe and risky assets.

In each period, a consumer's flow of utility depends on how much they consume. We assume that the utility function is of the standard Constant Relative Risk Aversion form:

$$\mathrm{u}(c) = \frac{c^{1-\rho}}{1-\rho} \tag{1}$$

but of course the consumer is smart enough to realize that preserving some resources for the future is a good idea; this is why all wealth is not consumed immediately.

We follow a tradition dating back to **?** in assuming that a consumer's financial circumstances depend chiefly on two variables. $\mathbf{p}_t$ is the consumer's permanent income (roughly, the income they would normally expect to receive in the absence of surprises like winning the lottery or a temporary layoff), while $\mathbf{m}_t$ is total market resources (the sum of financial assets and current income – think of this as the pool of resources that can be immediately spent; 'money' in the colloquial sense of 'how much money does grandma have?').

The 'value' of having a given amount of market resources $\mathbf{m}_t$ right now, and of knowing your current permanent income level to be $\mathbf{p}_t$, is determined by the utility you will experience from consumption today, as well as the utility you expect to experience in the future. Any future period matters to you only to the extent that you expect to survive to that period.

In formal mathematical terms, the consumer's objective is to maximize present discounted utility from consumption over a life cycle that ends no later than date T (often set to age 120):

$$\mathbf{v}_t(\mathbf{m}_t, \mathbf{p}_t) = \max_{\{\mathrm{c}\}_t^T} \ \mathrm{u}(\mathbf{c}_t) + \mathbb{E}_t \left[\sum_{n=1}^{T-t} \mathcal{L}_t^{t+n} \beta^n \mathrm{u}(\mathbf{c}_{t+n}) \right] \tag{2}$$

$$ \tag{3} $$

$$\mathcal{L}_t^{t+n} : \text{probability to } \mathcal{L}\text{ive until age } t+n \text{ given you are alive at age } t \tag{4}$$

$$\bullet\, \mathcal{L}_{120}^{121} = 0.0 \text{ says that a 120 year old has zero probability of living to 121} \tag{5}$$

$$\bullet\, \mathcal{L}_{80}^{90} = 0.3 \text{ says that an 80 year old has a 30 percent chance of reaching 90} \tag{6}$$

$$\beta : \text{time discount factor (captures degree of present bias)} \tag{7}$$

$$\bullet \text{ captures age-varying spending needs (mainly, adjusts for family size)} \tag{8}$$

$$\bullet \text{ rate of'present bias'; } \beth = 1 \text{ corresponds to no present bias} \tag{9}$$

We use standard calibrations for mortality by age from actuarial mortality tables used by the Social Security administration. We set the 'pure' rate of time preference to $\beta = 1$ because that means that the optimal choice is to care exactly as much your future self as much as your present self (conditional on surviving into the future).

One of the fundamental discoveries of the past 40 years or so is the extent to which optimal choice is profoundly altered by the presence of uncertainty. **?** proposed a simple formulation that remains an excellent description of annual income shocks even today. Friedman said that there are two components to income: A 'permanent' component that is roughly what a person would expect to earn in a 'normal' year (say, their annual salary), and a 'transitory' component that reflects events like unemployment spells or lottery winnings; these make a given year's actual income deviate from its expected value.

To meld Friedmanian uncertainty with a Modlglianian life cycle, we need one more definition, whose purpose is to capture the predictable patterns that (noncapital) income follows over the lifetime (income starts low, rises with age and experience, and falls at retirement to the level of any regular pension payments):

$$\Gamma_{t+1} : \text{typical life cycle permanent income growth factor by age} \tag{10}$$

The typical life cycle pattern is altered, in any particular consumer's case, by 'permanent shocks' which we represent with the variable ψ. At any given age, actual permanent growth can deviate from the average experience of others of the same age in either a positive direction ($\psi > 1$ would correspond to an unexpected promotion or a switch to a higher-paying job) or a negative direction ($\psi < 1$ might be the result of a failure to be promoted or a change to a lower paying job).

This gives us the following description of the dynamics of permanent income $\mathbf{p}$:

$$\mathbf{p}_{t+1} = \mathbf{p}_t \Gamma_{t+1} \psi_{t+1} \tag{11}$$

$$\mathbb{E}_t[\mathbf{p}_{t+1}] = \mathbf{p}_t \Gamma_{t+1} \tag{12}$$

where the second line follows from the first because the expected value of the permanent shock is $\mathbb{E}_t[\psi] = 1$.

The transitory shock to income has two modes. In unemployment spells, the consumer earns no income; we assume that such spells occur with probability $\wp$.If the consumer remains employed, we will assume that the income shocks are lognormally distributed:[$\hat{\text{b}}$etterunemp]

$$\xi_s = \begin{cases} 0 & \text{with probability } \wp > 0 \\ \xi_s/\wp & \text{with probability } (1-\wp) \end{cases} \tag{13}$$

[$\hat{\text{b}}$etterunemp] It is straightforward to extend the model to allow for a more realistic treatment of unemployment, for example by taking account of the existence of an unemployment insurance system; such an adjustment does not change the substantive conclusions we are interested in.

It is conventional to assume that shocks to permanent income and to the transitory income of the employed are lognormally distributed:

$$\log \psi_s \sim \mathcal{N}(-\sigma^2_{[\psi,t]}/2, \sigma^2_{[\psi,t]}) \tag{14}$$

$$\log \xi_s \sim \mathcal{N}(-\sigma^2_{[\xi,t]}/2, \sigma^2_{[\xi,t]}) \tag{15}$$

which, together with the other assumptions, guarantee that the expected value of the transitory and of the permanent shocks are both 1: $\mathbb{E}_t[\psi_{t+1}] = \mathbb{E}_t[\theta_{t+1}] = 1$. (We use standard calibrations of both of these shock processes.)

Under the assumptions we have made about the structure of the utility function (homotheticity) and budget constraint (linearity and geometric returns), it is possible to recast the problem entirely in terms of *ratios* of the model variables to permanent income $\mathbf{p}$. So, for example, italic $c = \mathbf{c}/\mathbf{p}$ is the ratio of the (boldface) level of consumption to the level of permanent income $\mathbf{p}$ (see **?** for the math).

Another way to make the problem easier to understand is to combine several of the multiplicative terms into portmanteau variables. Defining boldface $\boldsymbol{\beta}_{t+1}$ as

$$\boldsymbol{\beta}_{t+1} = \beta(\psi_{t+1}\Gamma_{t+1})^{1-\rho} \tag{16}$$

Under the assumptions we have made, it turns out that the consumer's problem can be expressed more simply by realizing that it boils down to a 'now versus later' problem. All the consumer really needs to know about the future is summarized by the value they will expect as a consequence of ending the current period with a certain ratio of assets to permanent income, $a = \mathbf{a}/\mathbf{p}$. We can represent the value of ending the period with assets of a using the Gothic variant of the letter v:

$$\mathfrak{v}_t(a_t) = \mathbb{E}_t[\boldsymbol{\beta}_{t+1}\mathrm{v}_{t+1}(m_{t+1})] \tag{17}$$

Finally we are ready to add portfolio choice to the problem. Suppose the consumer can invest in a risky asset that earns rate of return $\log \mathbf{R} \sim \mathcal{N}(\mathsf{r} + \varphi - \sigma^2/2, \sigma^2)$. That is, we make the conventional assumption that the risky asset is distributed lognormally with an expected equity premium of φ.

The portfolio return the consumer earns will depend on the share of their assets they invest in the risky versus the safe asset. Calling the share ς, the portfolio-weighted rate of return will be

$$\mathcal{R}_{t+1} = \mathsf{R} + (\mathbf{R}_{t+1} - \mathsf{R})\varsigma \tag{18}$$

and the consumer is assumed to make the optimal choice of portfolio share:

$$\mathfrak{v}_t(a) = \max_{\varsigma} \quad \mathbb{E}_t[\boldsymbol{\beta}_{t+1}\mathrm{v}_{t+1}(\mathcal{R}_{t+1}a + \theta_{t+1}) \tag{19}$$

The consumer's objective in the consumption stage of the problem can now be expressed in Bellman form as:

$$\mathrm{v}_t(m_t) = \max_{\{c_t\}} \ \mathrm{u}(c_t) + \mathcal{L}_{t+1}\mathfrak{v}_t(a_t) \tag{20}$$

$$\text{s.t.} \tag{21}$$

$$a_t = m_t - c_t \tag{22}$$

object	meaning
m, c, a	market resources, consumption, and end-of-period assets, normalized by permanent income
v	the normalized value function
$\mathcal{L}_{t+1} \equiv \mathcal{L}_t^{t+1}$	probability a person alive at date t survives to date $t+1$

since a measures available market resources that are unspent, this formulation makes it crystal clear that the consumer faces a tradeoff between the utility of consumption today and the expected value of preserving assets $a = m - c$ for the future.[2]

We calibrate the model to include two kinds of uncertainty after retirement.

First, we incorporate estimates from **?** of the size of shocks to medical expenditures for retirees; a perfectly rational reason not to run down your wealth, or not to run it down too far, is a fear of large medical expenses that you want to be able to meet. Such uncertainty has the potential to deter the drawdown of wealth; see **?** for an argument that it is the principal explanation for the 'drawdown failure.' While such effects are present in our model, our model estimation results below will find that the model still predicts much more drawdown of wealth than the data show.

Second, we assume that there are 'ordinary' expenditure shocks in retirement that are of similar magnitude to income shocks during working life (following recent estimates from **?**). Again, in principle, the presence of such shocks provides a precautionary motive to draw down wealth more slowly.

3.1.2 The LCP model with 'Warm Glow' Bequests

The LCP model sketched above assumes that the only reason to hold wealth is to spend it later – which means that eventually an age must come at which the consumer begins to spend their wealth down. As the literature has demonstrated, and as we will confirm below using data from SCF's from 1995 to 2022, the path of the median wealth ratio after retirement does not look anything like what that model predicts.

Of course, the model can make no sense at all of the behavior of the very rich. Bill Gates, for example, has chosen to allocate a large portion of his lifetime wealth to the Bill and Melinda Gates foundation; and even with his so-far-uncontributed wealth, he shows no sign of drawing it down remaining wealth over his remaining lifetime.

But for a substantial fraction of retirees, the 'drawdown' phase seems never to come (or at best the drawdown is modest). This is the 'drawdown failure' mentioned in the introduction.

From the mathematical point of view, it is clear that some other motive for holding onto wealth must be added to the framework if it is to explain these facts. A natural candidate is a bequest motive: The idea that people take pleasure in the thought of leaving something to their heirs.

This can be accommodated very simply by adding another term to the sources of utility: the value the consumer places on the bequest, which we will denote as $e(a)$ (think of this as the utility they experience from the thought of leaving an *e*state).

Defining the probability of passing away as the probability of not $\mathcal{L}$iving to the next period,

$$\mathcal{L} = (1 - \mathcal{L}) \tag{23}$$

the flow of utility that the consumer receives now includes both their utility from consumption *and* the pleasure they take from the thought that, if they pass away before next period (which happens with probability $\mathcal{L}$), their assets will pass to their heirs.

The consumer's new value function is therefore just

$$\mathrm{v}_t(m_t) = \max_{c_t} \overbrace{\mathrm{u}(c_t)}^{\text{present}} + \overbrace{\underbrace{\mathcal{L}_{t+1}\mathfrak{v}(a_t)}_{\text{live}} + \underbrace{\mathcal{L}_{t+1}e(a_t)}_{\text{die}}}^{\text{future}}. \tag{24}$$

[2]The normalization for value function involves more than just division by $\mathbf{p}$; see **?** for details.

The literature has commonly used a 'warm glow utility from bequests' motive of the form:

$$e(a) = \alpha \frac{(a + \underline{a})^{1-\rho}}{1-\rho} \tag{25}$$

where the ρ coefficient is the same as in the utility function for consumption. (see, e.g., **?**).

3.2 Wealth in the Utility Function

3.2.1 Why Do the Rich Save So Much?

The historian Fredrick Cople **?**'s chronicle of the behavior of the richest Americans since the Revolution contains a feast of quotations from the richest Americans, articulating a host of motivations for wealth accumulation; very few of these mention anything resembling the bequest motive as formulated in the academic life cycle literature. (Andrew Carnegie was most explicit: 'I would rather leave my son a curse than the almighty dollar.') This is one of those places where economists' new openness to the idea of taking seriously what people say about their motivations has bite. While it is not unreasonable to be sceptical about taking such quotations at face value, **?** shows that essentially all of the motivations articulated (wealth brings power; wealth allows philanthropy; wealth is a way of 'keeping score'; and more) can be captured in a mathematical formulation in which wealth enters the utility function directly.

This perspective is also consonant with the views of Max **?**, who argued that the 'spirit of capitalism' was a value system in which it was intrinsically virtuous to accumulate wealth.

The **?** has for many years asked respondents a question about their motivations for saving. While respondents' answers are fairly heterogeneous, the SCF has a suggested aggregation of the many different answers into categories that correspond approximately to some of the motivations that the academic literature has considered. The category that best matches the 'bequest' motivation is 'Family' (which includes 'to help the kids out' and 'to leave an estate' but also includes saving for 'weddings and other cermonies' and 'to have children/a family.')

The table below presents the responses to this question for college-educated households older than age 70 from the 1995 to the 2022 waves of the SCF:

Reason	Proportion	Explanation
'Family'	0.06	Bequests; weddings, bar mitzvahs, etc
'Retirement'	0.27	
'Liquidity/The Future'	0.40	
'Purchases'	0.13	cars, vacation homes, etc
'Cannot save'	0.06	
Other	0.08	

bequests were really a primary motivation for saving for most (college-educated) people, it would be surprising for them to mention this motivation so rarely.

Given these (and other) objections to the bequest motive, and given the problems of a model without a bequest motive, it seems natural to consider alternative modifications to the framework.

3.2.2 Mathematical Specification

The most general way we economists have of incorporating people's motivations into our models of behavior is simply to assume that the decisionmaker directly values something – in this case, wealth. The next question is how best to incorporate the item in the utility function to study any particular question. **?**, for example, proposed a utility function specifically designed to capture saving behavior as wealth approached infinity, and accomplishing that goal required some mathematical structure that delivered the desired results but was unwieldy (and not obviously necessary for explaining the behavior of the bottom 99 percent, whose wealth does not approach infinity).[3]

Money in the Utility Function

It turns out that there is a literature in macroeconomics, pioneered by Miguel **?**, that has long included 'money' in the utility function of the representative agent in one form or another.

A well-known paper by **?** proposed a specific utility function designed to capture the stability of the ratio of money to GDP, and Rotemberg along with James Poterba estimated this model on U.S. data in **?**.

The structure of their utility function is

$$\mathrm{u}(c, \ell) = \frac{\left(c^{1-\delta}\ell^{\delta}\right)^{1-\rho}}{1-\rho} \tag{26}$$

where ℓ captures the the ℓiquidity services provided by money-holding.

To be clear, the aim of that literature was to explain the holding of ℓ defined as dollar cash holdings, to study questions like the 'velocity' of money and the role of money supply and money demand in determining interest rates – not to explain saving behavior.

Wealth In the Utility Function: Cobb-Douglas Form

But for the question of how to incorporate wealth in the utility function, **?** proposed a mathematically identical formulation,

$$\mathrm{u}(c, a) = \frac{\left(c^{1-\delta}a^{\delta}\right)^{1-\rho}}{1-\rho} \tag{27}$$

where a takes the place of ℓ in the Rotemberg-Poterba utility function.[4] The Cobb-Douglas functional form for the TRP utility function is commonly used in other contexts, but does not seem to have been explored as a formulation for how to put a direct wealth-holding motive in the utility function.

The upshot is that if we credit the proposition that the ownership of wealth yields utility, then there is good precedent for the functional form of **?**.

Henceforth we will call this the Tzitzouris-Rotemberg-Poterba or 'TRP' utility function.

It is a simple matter to solve the revised problem with wealth in the utility function using the TRP utility specification. The revised value function of the problem is:

$$\mathrm{v}_t(m_t) = \max_{c_t}\ \mathrm{u}(c_t, a_t) + \mathcal{L}_{t+1}\mathfrak{v}_t(a_t) \tag{28}$$

[3]Specifically, a separable utility-from-wealth function was added to the maximizer's objective and with a coefficient of relative risk aversion smaller than that for the utility from consumption, which delivers the desired result.

[4]The question of whether a or m should be in the utility function is not very consequential; here we prefer a because assets after consumption are immune to considerations of whether the time period is a year, a quarter, a month, or a day.

The methods of solution are essentially the same as those for the model with a bequest.

(We are open to the possibility that wealth in the utility function is a reduced form for other motivations – indeed, that was the thesis of **?**. In particular, the fact that in our SCF table above, 'Liquidity/The Future' is the most popular answer among retirees for the most important reason to save might signal that the forms of uncertainty that we can measure – like the **?** calculations about nursing home expense risks – constitute only a fraction of the things that retirees might worry about. Maintaining a buffer stock of wealth to protect oneself against 'unknown unknowns' is quite possibly perfectly rational, and also nearly impossible to calibrate in a quantitative model in which we would need to have an accurate representation of people's beliefs about the magnitude, frequency, and persistence of 'unknown unknowns.' But if you knew those things, they would be, at best, 'known unknowns.')

4 Estimation and Calibration

4.0.1 Indirect Inference Described

Even if you knew all the parameters of the model (the consumer's coefficient of relative risk aversion; their time preference factor), solving an optimization problem that includes the many real-world complications described above (especially those due to uncertainty) is such a formidable problem that it only became possible about 25 years ago (and solving the models took days).

But of course we do not know the best values to choose for unobservable parameters like relative risk aversion and time preference rates. The solution to this problem that is now becoming standard is the method of 'indirect inference.' Essentially, this means specifying the structure of your model except for the values of parameters that you cannot measure well (like time preference and risk aversion), and asking a numerical search algorithm to seek the values of those parameters that lets the model fit the data as well as it is capable of doing. This requires the computer to solve the problem perhaps thousands of times, which is why indirect inference is only coming into its own now - when computer speeds have gotten fast enough to tackle the problem.

4.0.2 Indirect Inference Implemented

We are particularly interested in finding the optimal post-retirement choices, both for the rate of spending and for portfolio allocation between safe and risky assets.

The Method of Simulated Moments

The method of simulated moments consists of finding the parameters that make the model's simulated moments (statistics), like the median wealth and the median portfolio share, match the corresponding empirical facts as closely as possible.

Consider a real moment y_i where $i \in [1, N]$ and the corresponding simulated moment $\hat{y}_i(\theta)$, where θ is the vector of parameters that we are interested in estimating. By solving and simulating our structural model with different θ parameters, we can calculate the simulated moments $\hat{y}_i(\theta)$ for each parameter set. The method of simulated moments then consists of finding the parameter set θ that minimizes the distance between the simulated moments and the real moments. This is done by minimizing the following objective function:

$$\min_{\theta} \sum_{i=1}^{N} \left(\omega_i[y_i - \hat{y}_i(\theta)]\right)^2 \tag{29}$$

where ω_i is the weight of each moment in the objective function, representing the relative importance of each moment in the estimation process. For example, we might be more interested in matching the median wealth than the median portfolio share, so we would assign a higher weight to the former.

For our exercise, we are interested in matching the median wealth to income ratios throughout the life-cycle, and the median portfolio share of risky assets after retirement. Because aggregate age data can be noisy and subject to selection bias and measurement error, we will aggregate the data into 5-year age bins to smooth out the noise and reduce the impact of selection bias. Starting at age 25, we calculate the median wealth to income ratio as follows: Wealth is defined as the sum of all assets and liabilities, including financial assets, housing, vehicles, and debt. For income, we use the sum of all wages, salaries, social security, and retirement income, excluding capital gains and other non-recurring income. We then calculate the wealth to income ratio of every household in the age bin and remove households with an income of zero. The median wealth to income ratio is calculated from the remaining households. An important point is that in our structural model we hard-code retirement at age 65, whereas in the data we observe retirement at different ages, but predominantly between ages 60 and 70. Therefore, we avoid the data for ages 60 to 70 to prevent any bias in the estimation process, but keep the data for ages 70 and above to capture the behavior of retirees. Similarly, we calculate the median portfolio share of risky assets after retirement for ages 70 and above given by **?**.

Considering the selection of moments we have chosen, it is clear that there is an inbalance between the wealth to income moments and the portfolio share moments. There are more wealth to income moments than portfolio share moments, (12 to 5), and the portfolio share moments lie between 0 and 1, whereas the wealth to income ratios can be much larger. To account for this, we set the weights to normalize the wealth to income ratios by the highest ratio in the data, making them all lie between 0 and 1, and set the weights for the portfolio share moments to multiply by 12/5, so that the two sets of moments are equally weighted in the estimation process. This ensures that our estimation process puts even weight on the two sets of moments, despite the difference in scale and number of moments.

Having pinned down the moments we are interested in matching and their respective weights, we can now proceed to a discussion of estimating the parameters of our vaious models. We use the `Econ-ARK` project's `HARK` package to solve and estimate the models, and `estimagic` (**?**) to perform the estimation process. Our exercise consists of estimating 1 parameter (the constant coefficient of relative risk aversion (CRRA) parameter for the Life Cycle Portfolio Choice Model) up to 3 parameters (CRRA, the weight of the bequest motive, and the wealth-shifter of utility parameter for the `LCP+WarmGlow` model), so we develop a robust and efficient estimation process that can handle a varying number of parameters.

Our estimation process is computationally expensive, requiring the solving and simulation of the model given a parameter set many times. Because our simulated moments indeed require simulation, our moment generating functions $\hat{y}_i(\theta)$ have no analytical derivatives with respect to the parameters, so we must rely on numerical differentiation and clever optimization algorithms to find the optimal parameter set. We use the `tranquilo` algorithm (**?**), which stands for TrustRegion Adaptive Noise robust QuadratIc or Linear approximation Optimizer, to find the optimal parameter set. The `tranquilo` optimizer has many attractive features, such as being able to evaluate the function in parallel and estimate even noisy objective functions with many parameters, as well as being especially designed for least squares problems, such as the MSM.

4.0.3 Indirect Inference Results

5 Conclusion

To thoughtful academics, it has long been disturbing that the financial advice industry has paid so little attention to our hard work in constructing and solving impressively sophisticated dynamic stochastic optimization models of financial behavior. Those of us with a bit of humility have always suspected that the failure has been on our side: If all we could offer was models that produced risible advice like 'everyone should spend down their wealth to zero and live pension-check to pension-check,' while financial analysts' real world experience told them that such advice would not be welcomed by their advisees, then it was reasonable to disregard the academic literature.

The thesis of this paper, though, is that a confluence of factors has now finally brought us to a point where state-of-the-art mathematical/computational optimization models can provide advice that makes sense. Much more remains to be done to improve the models further; for example, a question of great practical importance that is now just at the edge of possibility of being computationally solved is to calculate the implications of nonfinancial (principally, housing) wealth for optimal financial choice. Because homeownership is such a complex phenomenon, the academic literature is only now reaching the point at which it may be possible to answer questions like "if I own a house, how should I modify my spending and portfolio plans to take that into account?"[5]

It would be a better world if financial advice could be justified as reflecting the mathematically optimal solution to a well-defined problem. Not only would academics have the satisfaction of knowing that they had finally come close to fulfilling the vision of Modligliani and Brumberg 70 years ago. Financial analysts could also sleep more soundly in the knowledge that the advice they were giving could be what many people probably think it already is: The adaptation to the client's particular circumstances of the advice that is the best that can be delivered by the latest high-tech computational optimization tools.

The time seems ripe for a much closer collaboration between academia and the financial industry in building this better world.

---

[5]We do know the *direction* of the effect. **?** shows that the addition of a new uncontrollable risk reduces the optimal choice of exposure to controllable risks like the stock market. But *by how much* one's stock exposure should be reduced because of house-price risk can only be answererd by solving a quantitatively plausible model.