

# An Introduction to the Heterogeneous Agents Resources and toolKit

Chris Carroll, Matt White

“Hands-On Heterogeneous Agent Macroeconomics”  
Minicourse at Open Source Macro Bootcamp, Chicago  
July-August 2018

hi

# Agenda: A Flavor of HARK

1. “Microeconomic” models in HARK: the `AgentType` class

# Agenda: A Flavor of HARK

1. “Microeconomic” models in HARK: the `AgentType` class
2. Example HARK model

# Agenda: A Flavor of HARK

1. “Microeconomic” models in HARK: the `AgentType` class
2. Example HARK model
  - ▶ Consumption with permanent and transitory shocks to income

# Agenda: A Flavor of HARK

1. “Microeconomic” models in HARK: the `AgentType` class
2. Example HARK model
  - ▶ Consumption with permanent and transitory shocks to income
3. 30,000 foot view: What else is in HARK?

# Microeconomic Models in HARK

# Microeconomic Models in HARK

- ▶ Concern decision-making of one agent



# Microeconomic Models in HARK

- ▶ Concern decision-making of one agent
- ▶ Discrete time

# Microeconomic Models in HARK

- ▶ Concern decision-making of one agent
- ▶ Discrete time
- ▶ Sequence of choices

# Microeconomic Models in HARK

- ▶ Concern decision-making of one agent
- ▶ Discrete time
- ▶ Sequence of choices
  - ▶ Household: Consumption, labor supply, portfolio choice, etc

# Microeconomic Models in HARK

- ▶ Concern decision-making of one agent
- ▶ Discrete time
- ▶ Sequence of choices
  - ▶ Household: Consumption, labor supply, portfolio choice, etc
  - ▶ Firm: Investment, Employment, R&D, ...

# Microeconomic Models in HARK

- ▶ Concern decision-making of one agent
- ▶ Discrete time
- ▶ Sequence of choices
  - ▶ Household: Consumption, labor supply, portfolio choice, etc
  - ▶ Firm: Investment, Employment, R&D, ...
- ▶ Agents treat inputs to problem as *exogenous*

# Microeconomic Models in HARK

- ▶ Concern decision-making of one agent
- ▶ Discrete time
- ▶ Sequence of choices
  - ▶ Household: Consumption, labor supply, portfolio choice, etc
  - ▶ Firm: Investment, Employment, R&D, ...
- ▶ Agents treat inputs to problem as *exogenous*

Key restriction: Essentially, Bellman equation

# Microeconomic Models in HARK

- ▶ Concern decision-making of one agent
- ▶ Discrete time
- ▶ Sequence of choices
  - ▶ Household: Consumption, labor supply, portfolio choice, etc
  - ▶ Firm: Investment, Employment, R&D, ...
- ▶ Agents treat inputs to problem as *exogenous*

Key restriction: Essentially, Bellman equation

*Model solution can be constructed as iteration on sequence of “one period problems,” conditional on solution to subsequent period.*

## Two kinds of heterogeneity

- ▶ *Ex post* heterogeneity: Agents differ because a different sequence of events or shocks has happened to them



## Two kinds of heterogeneity

- ▶ *Ex post* heterogeneity: Agents differ because a different sequence of events or shocks has happened to them
  - ▶ Luck of the draw

# Two kinds of heterogeneity

- ▶ *Ex post* heterogeneity: Agents differ because a different sequence of events or shocks has happened to them
  - ▶ Luck of the draw
- ▶ *Ex ante* heterogeneity: Agents differ in objectives, preferences, expectations, etc before anything “happens” to them

# Two kinds of heterogeneity

- ▶ *Ex post* heterogeneity: Agents differ because a different sequence of events or shocks has happened to them
  - ▶ Luck of the draw
- ▶ *Ex ante* heterogeneity: Agents differ in objectives, preferences, expectations, etc before anything “happens” to them
  - ▶ Some people are more risk averse than others, e.g.

# HARK's “Master Class”: AgentType

- ▶ General purpose class for representing economic agents

# HARK's “Master Class”: AgentType

- ▶ General purpose class for representing economic agents

# HARK's “Master Class”: AgentType

- ▶ General purpose class for representing economic agents
- ▶ Each model creates a subclass of AgentType

# HARK's “Master Class”: `AgentType`

- ▶ General purpose class for representing economic agents
- ▶ Each model creates a subclass of `AgentType`
  - ▶ e.g. `PerfForesightConsumerType` is an `AgentType` subclass

# HARK's “Master Class”: `AgentType`

- ▶ General purpose class for representing economic agents
- ▶ Each model creates a subclass of `AgentType`
  - ▶ e.g. `PerfForesightConsumerType` is an `AgentType` subclass
  - ▶ Includes attributes, functions, and methods...



# HARK's “Master Class”: AgentType

- ▶ General purpose class for representing economic agents
- ▶ Each model creates a subclass of AgentType
  - ▶ e.g. PerfForesightConsumerType is an AgentType subclass
  - ▶ Includes attributes, functions, and methods...
    - ▶ All AgentType subclasses have a solve() method

# HARK's “Master Class”: AgentType

- ▶ General purpose class for representing economic agents
- ▶ Each model creates a subclass of AgentType
  - ▶ e.g. PerfForesightConsumerType is an AgentType subclass
  - ▶ Includes attributes, functions, and methods...
    - ▶ All AgentType subclasses have a solve() method
  - ▶ Common structure  $\Rightarrow$  different models “play nicely” together

# HARK's “Master Class”: AgentType

- ▶ General purpose class for representing economic agents
- ▶ Each model creates a subclass of AgentType
  - ▶ e.g. PerfForesightConsumerType is an AgentType subclass
  - ▶ Includes attributes, functions, and methods...
    - ▶ All AgentType subclasses have a solve() method
  - ▶ Common structure  $\Rightarrow$  different models “play nicely” together
  - ▶ Even though guts of solve() method differ for each subclass

# HARK's “Master Class”: AgentType

- ▶ General purpose class for representing economic agents
- ▶ Each model creates a subclass of AgentType
  - ▶ e.g. PerfForesightConsumerType is an AgentType subclass
  - ▶ Includes attributes, functions, and methods...
    - ▶ All AgentType subclasses have a solve() method
  - ▶ Common structure  $\Rightarrow$  different models “play nicely” together
  - ▶ Even though guts of solve() method differ for each subclass
  - ▶ Much easier to compare and exchange models

# HARK's “Master Class”: AgentType

- ▶ General purpose class for representing economic agents
- ▶ Each model creates a subclass of AgentType
  - ▶ e.g. PerfForesightConsumerType is an AgentType subclass
  - ▶ Includes attributes, functions, and methods...
    - ▶ All AgentType subclasses have a solve() method
  - ▶ Common structure  $\Rightarrow$  different models “play nicely” together
  - ▶ Even though guts of solve() method differ for each subclass
  - ▶ Much easier to compare and exchange models
- ▶ Complex models extend basic ones through “class inheritance”

# Workhorse: Buffer Stock Consumption Model

Class IndShockConsumerType

- ▶ Inherits attributes of PerfForesightConsumerType

# Workhorse: Buffer Stock Consumption Model

Class IndShockConsumerType

- ▶ Inherits attributes of PerfForesightConsumerType
  - ▶ Geometric discounting  $\beta$  per period

# Workhorse: Buffer Stock Consumption Model

## Class IndShockConsumerType

- ▶ Inherits attributes of PerfForesightConsumerType
  - ▶ Geometric discounting  $\beta$  per period
  - ▶ One choice: How much to consume vs save



# Workhorse: Buffer Stock Consumption Model

## Class IndShockConsumerType

- ▶ Inherits attributes of PerfForesightConsumerType
  - ▶ Geometric discounting  $\beta$  per period
  - ▶ One choice: How much to consume vs save
  - ▶ CRRA utility from consumption

# Workhorse: Buffer Stock Consumption Model

## Class IndShockConsumerType

- ▶ Inherits attributes of PerfForesightConsumerType
  - ▶ Geometric discounting  $\beta$  per period
  - ▶ One choice: How much to consume vs save
  - ▶ CRRA utility from consumption
  - ▶ Exogenous interest factor for asset returns

# Workhorse: Buffer Stock Consumption Model

## Class IndShockConsumerType

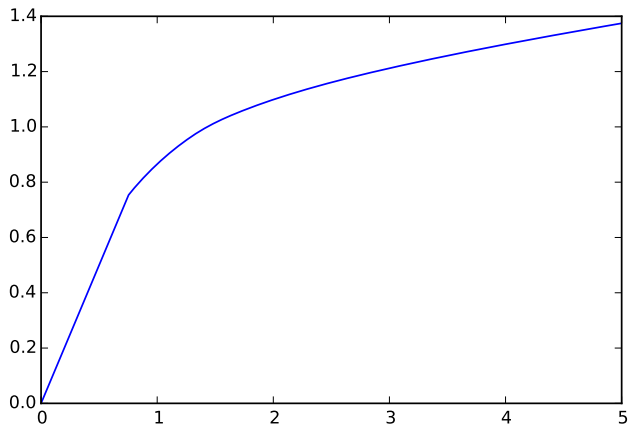
- ▶ Inherits attributes of PerfForesightConsumerType
  - ▶ Geometric discounting  $\beta$  per period
  - ▶ One choice: How much to consume vs save
  - ▶ CRRA utility from consumption
  - ▶ Exogenous interest factor for asset returns
- ▶ Adds assumptions about income uncertainty and constraints

# Workhorse: Buffer Stock Consumption Model

## Class IndShockConsumerType

- ▶ Inherits attributes of PerfForesightConsumerType
  - ▶ Geometric discounting  $\beta$  per period
  - ▶ One choice: How much to consume vs save
  - ▶ CRRA utility from consumption
  - ▶ Exogenous interest factor for asset returns
- ▶ Adds assumptions about income uncertainty and constraints
  - ▶ Mathematical Details: [Formal model](#)

# Buffer Stock Model Consumption Function



Horizontal Axis: "Money"; Vertical Axis: "Spending"

# What Else Is In HARK or the Econ-ARK?

- ▶ General purpose tools for generating and representing distributions, interpolated functions, etc

# What Else Is In HARK or the Econ-ARK?

- ▶ General purpose tools for generating and representing distributions, interpolated functions, etc
- ▶ Tools for estimation / optimization (fairly sparse)

## What Else Is In HARK or the Econ-ARK?

- ▶ General purpose tools for generating and representing distributions, interpolated functions, etc
- ▶ Tools for estimation / optimization (fairly sparse)
- ▶ Framework for “macroeconomic” models: Market class



# What Else Is In HARK or the Econ-ARK?

- ▶ General purpose tools for generating and representing distributions, interpolated functions, etc
- ▶ Tools for estimation / optimization (fairly sparse)
- ▶ Framework for “macroeconomic” models: Market class
- ▶ Several extensions of basic consumption-saving model

# What Else Is In HARK or the Econ-ARK?

- ▶ General purpose tools for generating and representing distributions, interpolated functions, etc
- ▶ Tools for estimation / optimization (fairly sparse)
- ▶ Framework for “macroeconomic” models: Market class
- ▶ Several extensions of basic consumption-saving model
- ▶ Some small demonstration exercises

# What Else Is In HARK or the Econ-ARK?

- ▶ General purpose tools for generating and representing distributions, interpolated functions, etc
- ▶ Tools for estimation / optimization (fairly sparse)
- ▶ Framework for “macroeconomic” models: Market class
- ▶ Several extensions of basic consumption-saving model
- ▶ Some small demonstration exercises
- ▶ All results from several papers:

# What Else Is In HARK or the Econ-ARK?

- ▶ General purpose tools for generating and representing distributions, interpolated functions, etc
- ▶ Tools for estimation / optimization (fairly sparse)
- ▶ Framework for “macroeconomic” models: Market class
- ▶ Several extensions of basic consumption-saving model
- ▶ Some small demonstration exercises
- ▶ All results from several papers:
  - ▶ “The Distribution of Wealth and the Marginal Propensity to Consume” by Carroll, Slacalek, Tokuoka, and White (2017)

# What Else Is In HARK or the Econ-ARK?

- ▶ General purpose tools for generating and representing distributions, interpolated functions, etc
- ▶ Tools for estimation / optimization (fairly sparse)
- ▶ Framework for “macroeconomic” models: Market class
- ▶ Several extensions of basic consumption-saving model
- ▶ Some small demonstration exercises
- ▶ All results from several papers:
  - ▶ “The Distribution of Wealth and the Marginal Propensity to Consume” by Carroll, Slacalek, Tokuoka, and White (2017)
  - ▶ “Sticky Expectations and Consumption Dynamics” by Carroll, Crawley, Slacalek, Tokuoka, and White (2018)

# What Else Is In HARK or the Econ-ARK?

- ▶ General purpose tools for generating and representing distributions, interpolated functions, etc
- ▶ Tools for estimation / optimization (fairly sparse)
- ▶ Framework for “macroeconomic” models: Market class
- ▶ Several extensions of basic consumption-saving model
- ▶ Some small demonstration exercises
- ▶ All results from several papers:
  - ▶ “The Distribution of Wealth and the Marginal Propensity to Consume” by Carroll, Slacalek, Tokuoka, and White (2017)
  - ▶ “Sticky Expectations and Consumption Dynamics” by Carroll, Crawley, Slacalek, Tokuoka, and White (2018)
  - ▶ Several others are close

# What Else Is In HARK or the Econ-ARK?

- ▶ General purpose tools for generating and representing distributions, interpolated functions, etc
- ▶ Tools for estimation / optimization (fairly sparse)
- ▶ Framework for “macroeconomic” models: Market class
- ▶ Several extensions of basic consumption-saving model
- ▶ Some small demonstration exercises
- ▶ All results from several papers:
  - ▶ “The Distribution of Wealth and the Marginal Propensity to Consume” by Carroll, Slacalek, Tokuoka, and White (2017)
  - ▶ “Sticky Expectations and Consumption Dynamics” by Carroll, Crawley, Slacalek, Tokuoka, and White (2018)
  - ▶ Several others are close
- ▶ Much room for improvement: endogenous labor supply (e.g.)

# References I

- CARROLL, CHRISTOPHER D., EDMUND CRAWLEY, JIRI SLACALEK, KIICHI TOKUOKA, AND MATTHEW N. WHITE (2018): "Sticky Expectations and Consumption Dynamics," *Manuscript, Johns Hopkins University*.
- CARROLL, CHRISTOPHER D., JIRI SLACALEK, KIICHI TOKUOKA, AND MATTHEW N. WHITE (2017): "The Distribution of Wealth and the Marginal Propensity to Consume," *Quantitative Economics*, 8, 977–1020, At <http://econ.jhu.edu/people/ccarroll/papers/cstwMPC>.
- FAGERENG, ANDREAS, MARTIN B. HOLM, AND GISLE J. NATVIK (2017): "MPC Heterogeneity and Household Balance Sheets," discussion paper, Statistics Norway.