

Planned Future of Additions to HARK

1. Endogenous labor supply models [Link](#)
2. Durable goods models [Link](#)
3. Various bits, large and small [Link](#)

The Future of HARK: Incorporating Labor (1/4)

Model of labor supply on intensive margin:

$$u(c, \ell) = ((1 - \ell)^\alpha c)^{1-\rho} / (1 - \rho),$$

$$v_t(b_t, \theta_t) = \max_{c_t, \ell_t} u(c_t, \ell_t) + \beta \mathbb{E}_t [(\psi_{t+1} \Gamma_t)^{1-\rho} v_{t+1}(b_{t+1}, \theta_{t+1})] \quad \text{s.t.}$$

$$y_t = \ell_t \theta_t, \quad \ell_t \in [0, 1],$$

$$a_t = m_t + y_t - c_t, \quad a_t \geq \underline{a},$$

$$b_{t+1} = R/(\Gamma_t \psi_{t+1}) a_t,$$

$$\psi_{t+1} \sim F_{\psi_{t+1}}(\psi), \quad \theta_{t+1} \sim F_{\theta_{t+1}}(\theta), \quad \mathbb{E}[\psi_{t+1}] = 1.$$

The Future of HARK: Incorporating Labor (2/4)

Model of labor supply on extensive margin:

$$u(c, \ell) = c^{1-\rho}/(1-\rho) - \alpha\ell,$$

$$v_t(b_t, \theta_t, \ell_{t-1}) = \max_{c_t, \ell_t} u(c_t, \ell_t) + \beta \mathbb{E}_t [(\psi_{t+1} \Gamma_t)^{1-\rho} v_{t+1}(b_{t+1}, \theta_{t+1}, \ell_t)]$$

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The Future of HARK: Incorporating Labor (3/4)

Model of endogenous employment search:

$$u(c, s) = ((1 - s)^\alpha c)^{1-\rho} / (1 - \rho),$$

$$v_t(m_t, e_t) = \max_{c_t, s_t} u(c_t, s_t) + \beta \mathbb{E}_t [(\psi_{t+1} \Gamma_t)^{1-\rho} v_{t+1}(m_{t+1}, e_{t+1})] \quad \text{s.t.}$$

$$a_t = m_t - c_t, \quad a_t \geq \underline{a}, \quad s_t \in [0, 1],$$

$$m_{t+1} = R/(\Gamma_t^e \psi_{t+1}) a_t + \theta_t e_{t+1} + \underline{b}(1 - e_{t+1}),$$

$$\text{Prob}(e_{t+1} = 1 | e_t = 0) = s_t, \quad \text{Prob}(e_{t+1} = 0 | e_t = 1) = \mathbb{U},$$

$$\psi_{t+1} \sim F_{\psi_{t+1}}^e(\psi), \quad \theta_{t+1} \sim F_{\theta_{t+1}}(\theta), \quad \mathbb{E}[\psi_t] = 1.$$

The Future of HARK: Incorporating Labor (4/4)

Applications of Market for labor models:

- ▶ Non-trivial calculation of $L_t = \int_0^1 \ell_{it} p_{it} \theta_{it} di$ for Cobb-Douglas
- ▶ Disutility of employment search and probability of job loss depend on labor market slackness
- ▶ Can look at behavior in response to change in SS, etc

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The Future of HARK: Durable Goods (1/3)

General durable goods model:

$$u(c, d) = (c^\alpha, d^{1-\alpha})^{1-\rho} / (1 - \rho).$$

$$v_t(m_t, d_t) = \max_{c_t, i_t} u(c_t, d_t) + \beta \mathbb{E}_t [(\psi_{t+1} \Gamma_t)^{1-\rho} v_{t+1}(m_{t+1}, d_{t+1})] \quad \text{s.t.}$$

$$a_t = m_t - c_t, \quad a_t \geq \underline{a},$$

$$D_t = d_t + g(i_t), \quad d_{t+1} = (1 - \delta_{t+1})D_t, \quad \delta_{t+1} \sim F_\delta(\delta),$$

$$m_{t+1} = R / (\Gamma_t \psi_{t+1}) a_t + \theta_{t+1},$$

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The Future of HARK: Durable Goods (2/3)

Variations of durable goods model require different solvers:

- ▶ Easiest case: $g(i_t)$ is concave, $i_t \in \mathbb{R}$. Every end-of-period state (a_t, D_t) associated with *some* beginning of period state.
- ▶ Slightly harder: $i_t \geq 0$, must handle constraint

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- ▶ Somewhat harder: $g(i_t) = \pi i_t$. One locus in (a_t, D_t) space is optimal; each point on optimal (a_t, D_t) locus associated with locus in (m_t, d_t) space.

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- ▶ Just ugly: $g(i_t) = \pi i_t + K\mathbf{1}(i_t \neq 0)$, $i_t \geq 0$.

The Future of HARK: Durable Goods (3/3)

Applications for Market with durable goods:

- ▶ Endogenous pricing of durable good: housing market
- ▶ Dynamics of demand for durables after an aggregate shock
- ▶ Some specifications overlap with health models

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The Future of HARK: Small To-Do Items

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- ▶ Advanced features on more solvers: cubic spline interpolation
- ▶ Various numeric methods detached from particular models

The Future of HARK: Heavy Lifting

If you're feeling ambitious or are comfortable with HARK:

- ▶ Incorporate `opencl4py` with basic consumption-saving model.
“Repack” model inputs into memory buffers, pass to OpenCL solver. OpenCL simulator: easier, big gains for some models.

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- ▶ Models of firm creation / bankruptcy / investment / hiring