An Introduction to the Heterogeneous Agents Resources and toolKit

Chris Carroll, Matt White

Minicourse

"Hands-On Heterogeneous Agent Macroeconomics"

Goethe University and SAFE

2018-05-28

Agenda: A Flavor of HARK

1. "Microeconomic" models in HARK: the `AgentType` class

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3. 30,000 foot view: What else is in HARK?

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Model solution can be constructed as iteration on sequence of "one period problems," conditional on solution to subsequent period.

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- *Ex ante* heterogeneity: Agents differ in objectives, preferences, expectations, etc before anything "happens" to them
  - Some people are more risk averse than others, e.g.

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(For more, see WhyPython)

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    - e.g., calculate 'impatience' implied by its parameter values

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- Complex models extend basic ones through "class inheritance"

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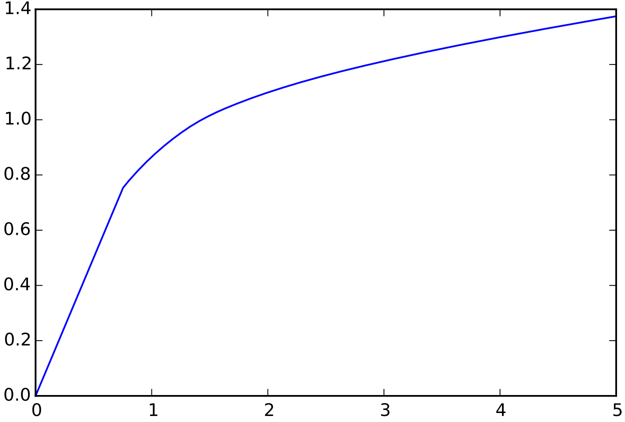
  - Mathematical Details: Formal model

Buffer Stock Model Consumption Function



Horizontal Axis: "Money"; Vertical Axis: "Spending"

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- Much room for improvement: endogenous labor supply (e.g.)

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And even more to come...

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- `MedHealthShock`:* Medical shocks plus discrete health states
- `DynInsSel`:* ...plus choice over medical insurance contracts

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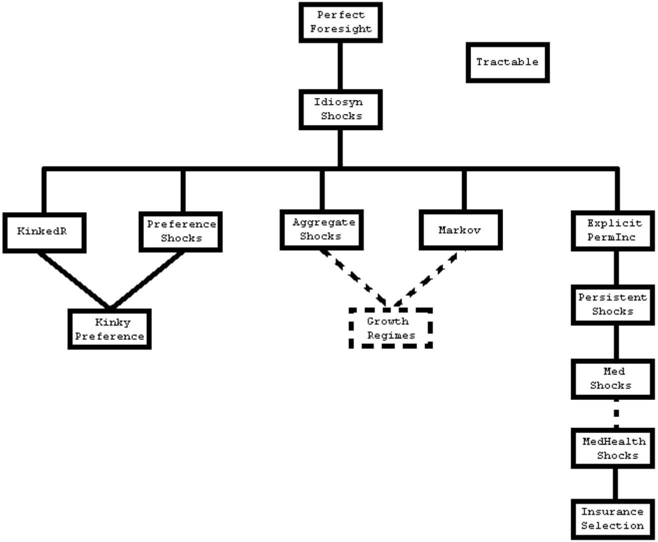
Consumption-Saving Model Tree



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The Future of HARK: Incorporating Labor (1/4)

Model of labor supply on intensive margin:

$$u(c,\ell) = ((1-\ell)^{\alpha} c)^{1-\rho}/(1-\rho),$$

$$v_t(b_t,\theta_t) = \max_{c_t,\ell_t} u(c_t,\ell_t) + \beta \cancel{D}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(b_{t+1},\theta_{t+1}) \right] \text{ s.t.}$$

$$y_t = \ell_t \theta_t, \qquad \ell_t \in [0,1],$$

$$a_t = m_t + y_t - c_t, \qquad a_t \geq \underline{a},$$

$$b_{t+1} = R/(\Gamma_t \psi_{t+1}) a_t,$$

$$\psi_{t+1} \sim F_{\psi t+1}(\psi), \qquad \theta_{t+1} \sim F_{\theta t+1}(\theta), \quad \mathbb{E}[\psi_{t+1}] = 1.$$

The Future of HARK: Incorporating Labor (2/4)

Model of labor supply on extensive margin:

$$u(c, \ell) = c^{1-\rho}/(1-\rho) - \alpha\ell,$$

$$v_t(b_t, \theta_t, \ell_{t-1}) = \max_{c_t, \ell_t} u(c_t, \ell_t) + \beta \cancel{D}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(b_{t+1}, \theta_{t+1}, \ell_t) \right]$$

$$y_t = \ell_t \theta_t, \qquad \ell_t \in \{0, \ell_{t-1}\},$$

$$a_t = m_t + y_t - c_t, \quad a_t \geq \underline{a},$$

$$b_{t+1} = R/(\Gamma_t \psi_{t+1}) a_t,$$

$$\psi_{t+1} \sim F_{\psi t+1}(\psi), \quad \theta_{t+1} \sim F_{\theta t+1}(\theta), \;\; \mathbb{E}[\psi_{t+1}] = 1.$$

The Future of HARK: Incorporating Labor (3/4)

Model of endogenous employment search:

$$u(c,s) = ((1-s)^{\alpha} c)^{1-\rho}/(1-\rho),$$

$$v_t(m_t,e_t) = \max_{c_t,s_t} u(c_t,s_t) + \beta \cancel{D}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(m_{t+1},e_{t+1}) \right] \text{ s.t.}$$

$$a_t = m_t - c_t, \quad a_t \geq \underline{a}, \quad s_t \in [0,1],$$

$$m_{t+1} = R/(\Gamma_t^e \psi_{t+1}) a_t + \theta_t e_{t+1} + \underline{b}(1-e_{t+1}),$$

$$\text{Prob}(e_{t+1}=1 | e_t=0) = s_t, \qquad \text{Prob}(e_{t+1}=0 | e_t=1) = \mho,$$

$$\psi_{t+1} \sim F^e_{\psi t+1}(\psi), \quad \theta_{t+1} \sim F_{\theta t+1}(\theta), \quad \mathbb{E}[\psi_t] = 1.$$

The Future of HARK: Incorporating Labor (4/4)

Applications of `Market` for labor models:

- Non-trivial calculation of $L_t = \int_0^1 \ell_{it} p_{it} \theta_{it} di$ for Cobb-Douglas

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- Disutility of employment search and probability of job loss depend on labor market slackness
- Can look at behavior in response to change in SS, etc

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The Future of HARK: Durable Goods (1/3)

General durable goods model:

$$u(c, d) = (c^{\alpha}, d^{1-\alpha})^{1-\rho}/(1-\rho).$$

$$v_t(m_t, d_t) = \max_{c_t, i_t} u(c_t, d_t) + \beta \cancel{D}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(m_{t+1}, d_{t+1}) \right] \text{ s.t.}$$

$$a_t = m_t - c_t, \qquad a_t \geq \underline{a},$$

$$D_t = d_t + g(i_t), \qquad d_{t+1} = (1-\delta_{t+1})D_t, \qquad \delta_{t+1} \sim F_{\delta}(\delta),$$

$$m_{t+1} = R/(\Gamma_t \psi_{t+1}) a_t + \theta_{t+1},$$

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The Future of HARK: Durable Goods (2/3)

Variations of durable goods model require different solvers:

- Easiest case: $g(i_t)$ is concave, $i_t \in \mathrm{R}$. Every end-of-period state (a_t, D_t) associated with *some* beginning of period state.
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- Even harder: $g(i_t) = \widehat{g}(i_t) + K\mathbf{1}(i_t \neq 0)$, with $\widehat{g}(0) = 0$ and $\widehat{g}(\cdot)$ concave. Must check $i_t = 0$ soln everywhere, discont.

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- Just ugh: $g(i_t) = \pi i_t + K\mathbf{1}(i_t \neq 0)$, $i_t \geq 0$.

The Future of HARK: Durable Goods (3/3)

Applications for `Market` with durable goods:

- Endogenous pricing of durable good: housing market

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The Future of HARK: Durable Goods (3/3)

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- Some specifications overlap with health models

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The Future of HARK: Small To-Do Items

Contributions that would get your feet wet in HARK:

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- Bequest motives: warm glow, other?

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- Fix/improve aggregate shocks model: handle life cycle models
- Fix/generalize `ExplicitPermInc` models: `PermGroFunc`
- Portfolio allocation models; eventually: asset pricing
- Advanced features on more solvers: cubic spline interpolation
- Various numeric methods detached from particular models

The Future of HARK: Heavy Lifting

If you're feeling ambitious or are comfortable with HARK:

- Incorporate `opencl4py` with basic consumption-saving model. "Repack" model inputs into memory buffers, pass to OpenCL solver. OpenCL simulator: easier, big gains for some models.

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- Generalized Markov solver: make "solution schema" so that Markov state can be added to any correctly specified solver
- Models of firm creation / bankruptcy / investment / hiring

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Example Model: Basic Consumption-Saving

Consumption-saving model with idiosyncratic permanent and transitory shocks to income (normalized format):

$$u(c) = c^{1-\rho}/(1-\rho).$$

$$v_t(m_t) = \max_{c_t} u(c_t) + \beta \cancel{D}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_{t+1})^{1-\rho} v_{t+1}(m_{t+1}) \right] \text{ s.t.}$$

$$a_t = m_t - c_t, \quad a_t \geq \underline{a},$$

$$m_{t+1} = R/(\Gamma_{t+1}\psi_{t+1})a_t + \theta_{t+1},$$

$$\psi_{t+1} \sim F_{\psi t+1}(\psi), \quad \theta_{t+1} \sim F_{\theta t+1}(\theta), \quad \mathbb{E}[\psi_t] = 1.$$

Example Model: Basic Consumption-Saving

Model solution in two lines:

$$\text{FOC: } u'(c_t) = R\beta \cancel{D}\mathbb{E}_t\left[(\psi_{t+1}\Gamma_{t+1})^{-\rho} v'_{t+1}(m_{t+1})\right],$$

$$\text{EC: } v'_t(m_t) = u'(c_t).$$

Will use endogenous grid method:

$$\mathfrak{v}'_t(a_t) \equiv R\beta\mathbb{E}_t\left[(\psi_{t+1}\Gamma_{t+1})^{-\rho} v'_{t+1}(m_{t+1})|a_t\right],$$

$$c_t = \mathfrak{v}'_t(a_t)^{-\rho}, \qquad m_t = a_t + c_t \text{ (for exogenous set of } \{a_t\}).$$

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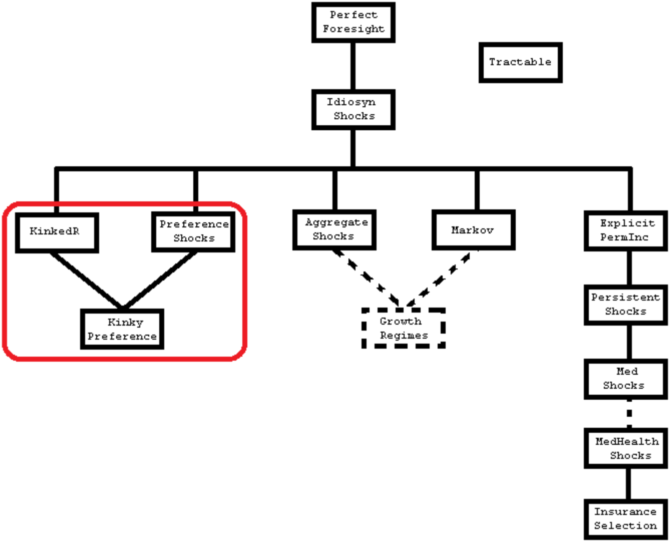
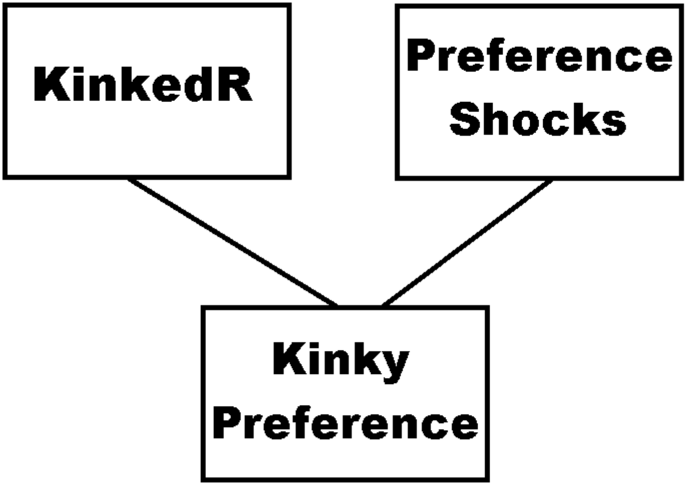
Consumption-Saving Model Tree



Consumption-Saving Model Tree



Kinked R: Costly Borrowing (1/3)

Make one small adjustment to idiosyncratic income shocks model: interest rate on borrowing is higher than rate on saving.

$$
\begin{aligned}
u(c) &= \frac{c^{1-\rho}}{1-\rho}, \\
v(m_t) &= \max_{c_t} u(c_t) + \beta \cancel{D}_{t+1} \mathbb{E}[v_{t+1}(m_{t+1})], \\
a_t &= m_t - c_t, \qquad a_t \geq \underline{a}, \\
m_{t+1} &= R/(\Gamma_{t+1}\psi_{t+1}) a_t + \theta_{t+1}, \\
\theta_{t+1} \sim F_{\theta t+1}, & \qquad \psi_{t+1} \sim F_{\psi t+1}, \quad \mathbb{E}[\psi_{t+1}] = 1, \\
R &= \begin{cases} R_{boro} & \text{if } a_t < 0 \\ R_{save} & \text{if } a_t > 0 \end{cases}, \qquad R_{boro} \geq R_{save}.
\end{aligned}
$$

Kinked R: Costly Borrowing (2/3)

`ConsKinkedRsolver` inherits from `ConsIndShockSolver`

Additions to `__init__` method:

Kinked R: Costly Borrowing (2/3)

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Additions to `__init__` method:

- Store new attributes `Rboro` and `Rsave`

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Additions to `__init__` method:

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Additions to `prepareToCalcEndOfPrdvP`:

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Additions to `__init__` method:

- Store new attributes `Rboro` and `Rsave`

Additions to `prepareToCalcEndOfPrdvP`:

- Four lines to use correct value of R for each value of a_t

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- Four lines to use correct value of R for each value of a_t
- One line to apply that change to calculation of m_{t+1}

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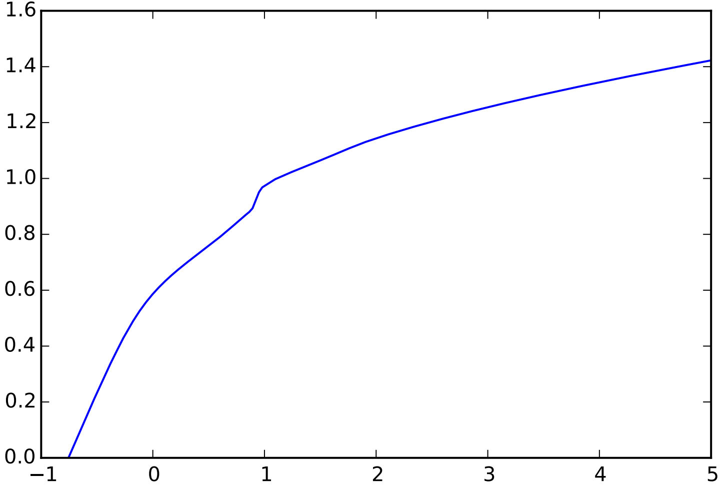
Additions to `__init__` method:

- Store new attributes `Rboro` and `Rsave`

Additions to `prepareToCalcEndOfPrdvP`:

- Four lines to use correct value of R for each value of a_t
- One line to apply that change to calculation of m_{t+1}
- Three lines to recalculate minimum MPC and human wealth

Kinked R: Costly Borrowing (3/3)



Marginal Utility Shocks (1/4)

Consider another small modification to `IndShockModel`:

- Multiplicative (idiosyncratic) shocks to utility each period.

$$
\begin{aligned}
u(c;\eta) &= \eta\frac{c^{1-\rho}}{1-\rho}, \qquad \eta_t \sim F_\eta, \\
v(m_t,\eta_t) &= \max_{c_t} u(c_t;\eta_t) + \beta \not{D}_{t+1}\mathbb{E}[v_{t+1}(m_{t+1})], \\
a_t &= m_t - c_t, \qquad a_t \geq \underline{a}, \\
m_{t+1} &= R/(\Gamma_{t+1}\psi_{t+1})a_t + \theta_{t+1}, \\
\theta_{t+1} \sim F_{\theta t+1}, &\qquad \psi_{t+1} \sim F_{\psi t+1}, \quad \mathbb{E}[\psi_{t+1}] = 1.
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$$

Marginal Utility Shocks (1/4)

Consider another small modification to `IndShockModel`:

- Multiplicative (idiosyncratic) shocks to utility each period.
- Consumption "more valuable" in some periods than others.

$$
\begin{aligned}
u(c;\eta) \quad &= \quad \eta \frac{c^{1-\rho}}{1-\rho}, \qquad \eta_t \sim F_\eta, \\
v(m_t,\eta_t) \quad &= \quad \max_{c_t} u(c_t;\eta_t) + \beta \not{D}_{t+1} \mathbb{E}[v_{t+1}(m_{t+1})], \\
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Marginal Utility Shocks (2/4)

New input `PrefShkDstn` is constructed:

- `PrefShkStd`: Standard deviation of (log) pref shocks

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- `PrefShkCount`: Number of discrete shocks in "body"
- `PrefShkTailCount`: Discrete shocks in "augmented tail"

Marginal Utility Shocks (3/4)

`ConsPrefShockSolver` inherits from `ConsIndShockSolver`

Additions to `__init__` method:

Marginal Utility Shocks (3/4)

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- 2 lines: Store preference shock distribution `PrefShkDstn`

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Replace `getPointsForInterpolation`

Marginal Utility Shocks (3/4)

`ConsPrefShockSolver` inherits from `ConsIndShockSolver`

Additions to `__init__` method:

- 2 lines: Store preference shock distribution `PrefShkDstn`

Replace `getPointsForInterpolation`

- 8 lines: Values of c_t and m_t for each η_t in `PrefShkDstn`

Marginal Utility Shocks (3/4)

`ConsPrefShockSolver` inherits from `ConsIndShockSolver`

Additions to `__init__` method:

- 2 lines: Store preference shock distribution `PrefShkDstn`

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Replace `usePointsForInterpolation`

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Additions to `__init__` method:

- 2 lines: Store preference shock distribution `PrefShkDstn`

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Replace `usePointsForInterpolation`

- 6 lines: Construct `cFunc` as a `LinearInterpOnInterp1D`

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Additions to `__init__` method:

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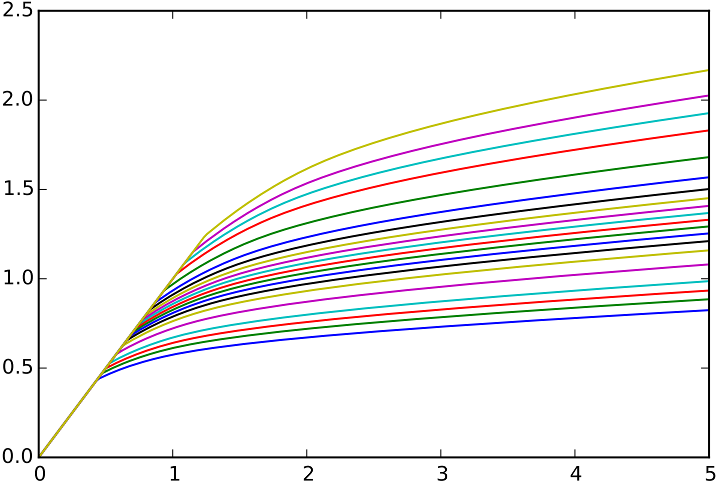
- 8 lines: Values of c_t and m_t for each η_t in `PrefShkDstn`

Replace `usePointsForInterpolation`

- 6 lines: Construct `cFunc` as a `LinearInterpOnInterp1D`
- 6 lines: Make `vPfunc` by integrating marginal utility across η_t

Marginal Utility Shocks (4/4)



Combination Inheritance: “Kinky Preferences” (1/4)

Combine those two extensions to `IndShockModel`:

- Borrowing has higher interest rate than saving...

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Combination Inheritance: “Kinky Preferences” (1/4)

Combine those two extensions to `IndShockModel`:

- Borrowing has higher interest rate than saving...
- ...and there are shocks to marginal utility
- HARK makes this pretty easy

Combination Inheritance: "Kinky Preferences" (2/4)

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\end{aligned}
$$

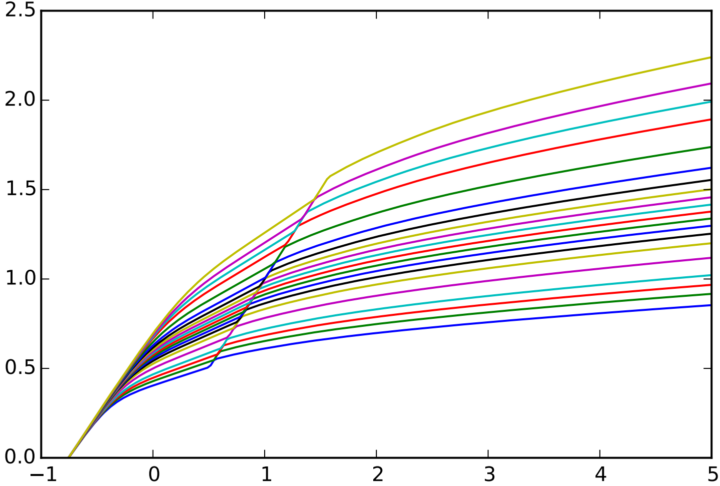
Combination Inheritance: "Kinky Preferences" (3/4)

`ConsKinkyPrefSolver` inherits from two parent classes. Entirety of the code for the solver:

```
class ConsKinkyPrefSolver(ConsPrefShockSolver,ConsKinkedRsolver):
    def __init__(self,solution_next,...):
        ConsKinkedRsolver.__init__(self,solution_next,...)
        self.PrefShkPrbs = PrefShkDstn[0]
        self.PrefShkVals = PrefShkDstn[1]
```

Combination Inheritance: “Kinky Preferences” (4/4)

Back



Macroeconomics in HARK (1/5)

- Some inputs to micro models are exogenous to each agent...
- ...But endogenous to collective whole of agents
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- Fixed point in the space of beliefs

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- Equilibrium: consistency between what agents *believe* the endogenous objects are and what values / dynamic processes *actually occur* when agents act on those beliefs
- Fixed point in the space of beliefs
- Need a representation of beliefs about endogenous objects
- And a rule for how agents form beliefs from observing history

Macroeconomics in HARK (2/5)

"The computational algorithm has two key features. First, it is based on bounded rationality in the sense that we endow agents with boundedly rational perceptions of how the aggregate state evolves... Second, we use solution by simulation, which works as follows: (i) given the boundedly rational perceptions, we solve the individuals' problems using standard dynamic programming methods; (ii) we draw individual and aggregate shocks over time for a large number of individuals; (iii) ...we generate a time series for all aggregates; and finally (iv) we compare the perceptions about the aggregates to those in the actual simulations, and these perceptions are then updated. We think this approach... can be productive for other applications."

–Per Krusell and Tony Smith (2006)

Macroeconomics in HARK (3/5)

HARK operationalizes K-S method with a farming metaphor:

1. Solve agents' microeconomic problem for some beliefs

`Market`'s method `solve` loops on this process until convergence.

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   - `store`: Record some information in a "history"

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1. Solve agents' microeconomic problem for some beliefs
2. Simulate many agents for many periods by looping on:
   - `sow`: Distribute current aggregate variables to agents
   - `cultivate`: Agents act according to their micro solution
   - `reap`: Collect some individual variables from the agents
   - `mill`: Generate aggregate variables from individual vars
   - `store`: Record some information in a "history"
3. Use history to update beliefs about endogenous objects

`Market`'s method `solve` loops on this process until convergence.

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- `millRule`: Function that transforms ind $\longrightarrow$ agg variables
- `calcDynamics`: Function that transforms history into beliefs

Macroeconomics in HARK (5/5)

Extra methods of a `Market`-compatible `AgentType`:

- `marketAction`: What agents *do* to generate `reap_vars`. Often just simulate one period with `simOnePeriod`

Trivial to add more *ex ante* heterogeneity: just add more `AgentType` instances to `agents`!

Macroeconomics in HARK (5/5)

Extra methods of a `Market`-compatible `AgentType`:

- `marketAction`: What agents *do* to generate `reap_vars`. Often just simulate one period with `simOnePeriod`
- `reset`: How to initialize for a new history: reset states

Trivial to add more *ex ante* heterogeneity: just add more `AgentType` instances to `agents`!

Consumption-Saving with Aggregate Productivity Shocks

$$
\begin{aligned}
v_t(m_t, M_t) &= \max_{c_t} u(c_t) + \beta \cancel{D}_{t+1} \mathbb{E}[v_{t+1}(m_{t+1}, M_{t+1})], \\
a_t &= m_t - c_t, \qquad a_t \geq 0, \\
m_{t+1} &= \frac{R_{t+1} a_t}{\Gamma_{t+1} \psi_{t+1} \Psi_{t+1}} + W_{t+1} \theta_{t+1} \Theta_{t+1} \ell, \\
A_t = \mathbf{A}(M_t), & \qquad k_{t+1} = (1-\delta) A_t / (\Psi_{t+1} \ell), \\
R_{t+1} = \mathbf{R}(k_{t+1}/\Theta_{t+1}), & \qquad W_{t+1} = \mathbf{W}(k_{t+1}/\Theta_{t+1}), \\
M_{t+1} &= R_{t+1} k_{t+1} + W_{t+1} \Theta_{t+1} \ell \\
\theta_{t+1} \sim F_{\theta t+1}, & \qquad \psi_{t+1} \sim F_{\psi t+1}, \quad \mathbb{E}[\psi_{t+1}] = 1, \\
\Theta_{t+1} \sim F_{\Theta}, & \qquad \Psi_{t+1} \sim F_{\Psi}, \quad \mathbb{E}[\Psi_{t+1}] = \mathbb{E}[\Theta_{t+1}] = 1.
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Some totally new inputs for an `AggShockConsumerType`:

- `Rfunc` and `wFunc`: Factor payments as function of k_t

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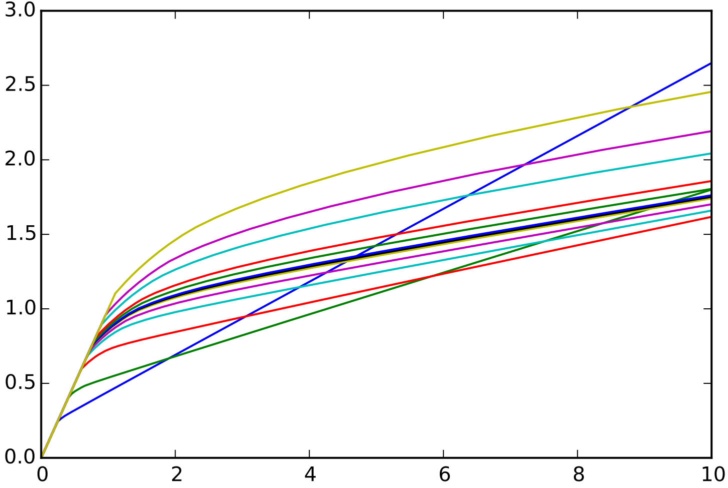
- `IncomeDstn` has five elements: probs, idio shocks, agg shocks

Consumption-Saving with Aggregate Productivity Shocks



Cobb-Douglas Economy in the `Market` Framework

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- `sow`: Distribute $(M_t, R_t, W_t, \Theta_t, \Psi_t)$ to consumers

Loop that process for (say) 1000 periods

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- Distribute new $\mathbf{A}$ to consumers as `Afunc`

Other Applications of `Market`

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- Agent-to-agent interaction: could `sow` a permutation of what is `reaped`: imperfect knowledge, contagion of information, moves closer to "agent-based modeling"

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References I

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