

An Introduction to the Heterogeneous Agents Resources and toolKit

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Minicourse

“Hands-On Heterogeneous Agent Macroeconomics”

Goethe University and SAFE

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Agenda: A Flavor of HARK

1. “Microeconomic” models in HARK: the `AgentType` class

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 - ▶ Consumption with permanent and transitory shocks to income

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2. Example HARK model
 - ▶ Consumption with permanent and transitory shocks to income
3. 30,000 foot view: What else is in HARK?

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Model solution can be constructed as iteration on sequence of “one period problems,” conditional on solution to subsequent period.

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- ▶ *Ex ante* heterogeneity: Agents differ in objectives, preferences, expectations, etc before anything “happens” to them
 - ▶ Some people are more risk averse than others, e.g.

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 - ▶ e.g., calculate ‘impatience’ implied by its parameter values

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- ▶ Complex models extend basic ones through "class inheritance"

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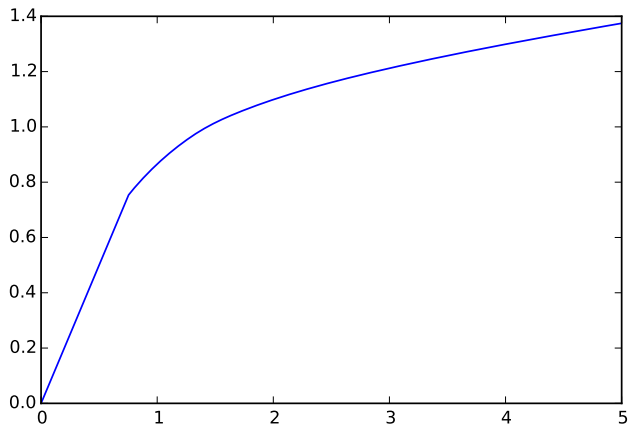
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 - ▶ Mathematical Details: [Formal model](#)

Buffer Stock Model Consumption Function



Horizontal Axis: "Money"; Vertical Axis: "Spending"

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- ▶ Much room for improvement: endogenous labor supply (e.g.)

Other Consumption-Saving Models in HARK

- ▶ TractableBufferStock: Highly specialized idiosync shocks

And even more to come...

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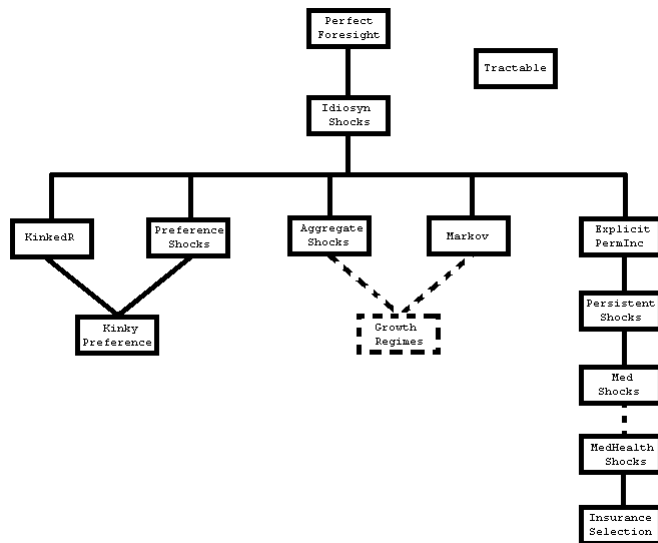
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- ▶ MedHealthShock:* Medical shocks plus discrete health states
- ▶ DynInsSel:* ...plus choice over medical insurance contracts

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Consumption-Saving Model Tree



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- ▶ Roadmap [Link](#)

Planned Future of Additions to HARK

1. Endogenous labor supply models [Link](#)

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2. Durable goods models [Link](#)
3. Various bits, large and small [Link](#)

The Future of HARK: Incorporating Labor (1/4)

Model of labor supply on intensive margin:

$$u(c, \ell) = ((1 - \ell)^\alpha c)^{1-\rho} / (1 - \rho),$$

$$v_t(b_t, \theta_t) = \max_{c_t, \ell_t} u(c_t, \ell_t) + \beta \mathbb{E}_t [(\psi_{t+1} \Gamma_t)^{1-\rho} v_{t+1}(b_{t+1}, \theta_{t+1})] \quad \text{s.t.}$$

$$y_t = \ell_t \theta_t, \quad \ell_t \in [0, 1],$$

$$a_t = m_t + y_t - c_t, \quad a_t \geq \underline{a},$$

$$b_{t+1} = R/(\Gamma_t \psi_{t+1}) a_t,$$

$$\psi_{t+1} \sim F_{\psi_{t+1}}(\psi), \quad \theta_{t+1} \sim F_{\theta_{t+1}}(\theta), \quad \mathbb{E}[\psi_{t+1}] = 1.$$

The Future of HARK: Incorporating Labor (2/4)

Model of labor supply on extensive margin:

$$u(c, \ell) = c^{1-\rho}/(1-\rho) - \alpha\ell,$$

$$v_t(b_t, \theta_t, \ell_{t-1}) = \max_{c_t, \ell_t} u(c_t, \ell_t) + \beta \mathbb{E}_t [(\psi_{t+1} \Gamma_t)^{1-\rho} v_{t+1}(b_{t+1}, \theta_{t+1}, \ell_t)]$$

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The Future of HARK: Incorporating Labor (3/4)

Model of endogenous employment search:

$$u(c, s) = ((1 - s)^\alpha c)^{1-\rho} / (1 - \rho),$$

$$v_t(m_t, e_t) = \max_{c_t, s_t} u(c_t, s_t) + \beta \mathbb{E}_t [(\psi_{t+1} \Gamma_t)^{1-\rho} v_{t+1}(m_{t+1}, e_{t+1})] \quad \text{s.t.}$$

$$a_t = m_t - c_t, \quad a_t \geq \underline{a}, \quad s_t \in [0, 1],$$

$$m_{t+1} = R/(\Gamma_t^e \psi_{t+1}) a_t + \theta_t e_{t+1} + \underline{b}(1 - e_{t+1}),$$

$$\text{Prob}(e_{t+1} = 1 | e_t = 0) = s_t, \quad \text{Prob}(e_{t+1} = 0 | e_t = 1) = \mathbb{U},$$

$$\psi_{t+1} \sim F_{\psi_{t+1}}^e(\psi), \quad \theta_{t+1} \sim F_{\theta_{t+1}}(\theta), \quad \mathbb{E}[\psi_t] = 1.$$

The Future of HARK: Incorporating Labor (4/4)

Applications of Market for labor models:

- ▶ Non-trivial calculation of $L_t = \int_0^1 \ell_{it} p_{it} \theta_{it} di$ for Cobb-Douglas

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- ▶ Disutility of employment search and probability of job loss depend on labor market slackness
- ▶ Can look at behavior in response to change in SS, etc

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The Future of HARK: Durable Goods (1/3)

General durable goods model:

$$u(c, d) = (c^\alpha, d^{1-\alpha})^{1-\rho} / (1 - \rho).$$

$$v_t(m_t, d_t) = \max_{c_t, i_t} u(c_t, d_t) + \beta \mathbb{E}_t [(\psi_{t+1} \Gamma_t)^{1-\rho} v_{t+1}(m_{t+1}, d_{t+1})] \quad \text{s.t.}$$

$$a_t = m_t - c_t, \quad a_t \geq \underline{a},$$

$$D_t = d_t + g(i_t), \quad d_{t+1} = (1 - \delta_{t+1})D_t, \quad \delta_{t+1} \sim F_\delta(\delta),$$

$$m_{t+1} = R / (\Gamma_t \psi_{t+1}) a_t + \theta_{t+1},$$

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The Future of HARK: Durable Goods (2/3)

Variations of durable goods model require different solvers:

- ▶ Easiest case: $g(i_t)$ is concave, $i_t \in \mathbb{R}$. Every end-of-period state (a_t, D_t) associated with *some* beginning of period state.
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- ▶ Just ugly: $g(i_t) = \pi i_t + K\mathbf{1}(i_t \neq 0)$, $i_t \geq 0$.

The Future of HARK: Durable Goods (3/3)

Applications for Market with durable goods:

- ▶ Endogenous pricing of durable good: housing market

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The Future of HARK: Durable Goods (3/3)

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The Future of HARK: Durable Goods (3/3)

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- ▶ Endogenous pricing of durable good: housing market
- ▶ Dynamics of demand for durables after an aggregate shock
- ▶ Some specifications overlap with health models

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The Future of HARK: Small To-Do Items

Contributions that would get your feet wet in HARK:

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- ▶ Portfolio allocation models; eventually: asset pricing
- ▶ Advanced features on more solvers: cubic spline interpolation
- ▶ Various numeric methods detached from particular models

The Future of HARK: Heavy Lifting

If you're feeling ambitious or are comfortable with HARK:

- ▶ Incorporate `opencl4py` with basic consumption-saving model.
“Repack” model inputs into memory buffers, pass to OpenCL solver. OpenCL simulator: easier, big gains for some models.

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- ▶ Generalized Markov solver: make “solution schema” so that Markov state can be added to any correctly specified solver
- ▶ Models of firm creation / bankruptcy / investment / hiring

Example Model: Basic Consumption-Saving

Consumption-saving model with idiosyncratic permanent and transitory shocks to income (normalized format):

$$u(c) = c^{1-\rho}/(1-\rho).$$

$$v_t(m_t) = \max_{c_t} u(c_t) + \beta \mathbb{E}_t [(\psi_{t+1} \Gamma_{t+1})^{1-\rho} v_{t+1}(m_{t+1})] \quad \text{s.t.}$$

$$a_t = m_t - c_t, \quad a_t \geq \underline{a},$$

$$m_{t+1} = R/(\Gamma_{t+1} \psi_{t+1}) a_t + \theta_{t+1},$$

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Example Model: Basic Consumption-Saving

Model solution in two lines:

$$\text{FOC: } u'(c_t) = R\beta \mathbb{E}_t [(\psi_{t+1}\Gamma_{t+1})^{-\rho} v'_{t+1}(m_{t+1})] ,$$

$$\text{EC: } v'_t(m_t) = u'(c_t).$$

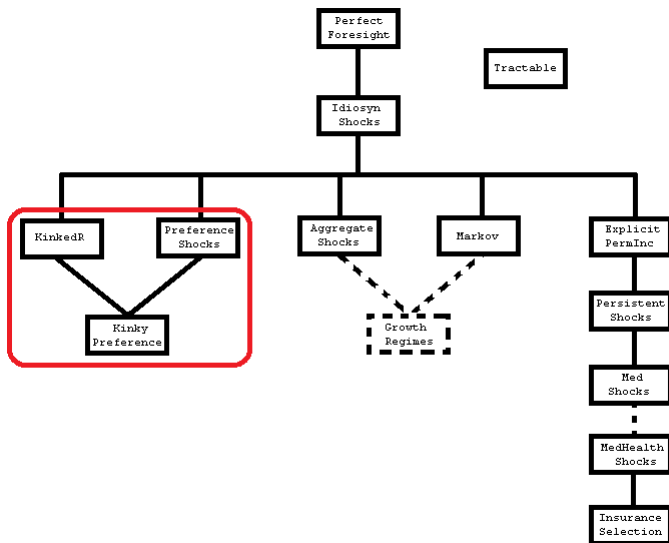
Will use endogenous grid method:

$$v'_t(a_t) \equiv R\beta \mathbb{E}_t [(\psi_{t+1}\Gamma_{t+1})^{-\rho} v'_{t+1}(m_{t+1})|a_t] ,$$

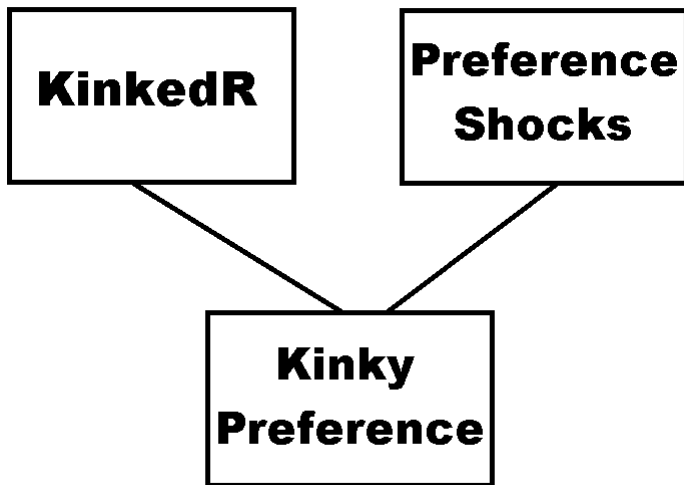
$$c_t = v'_t(a_t)^{-\rho}, \quad m_t = a_t + c_t \text{ (for exogenous set of } \{a_t\} \text{)}.$$

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Consumption-Saving Model Tree



Consumption-Saving Model Tree



Kinked R: Costly Borrowing (1/3)

Make one small adjustment to idiosyncratic income shocks model:
interest rate on borrowing is higher than rate on saving.

$$\begin{aligned}u(c) &= \frac{c^{1-\rho}}{1-\rho}, \\v(m_t) &= \max_{c_t} u(c_t) + \beta \mathbb{E}_{t+1} [v_{t+1}(m_{t+1})], \\a_t &= m_t - c_t, \quad a_t \geq \underline{a}, \\m_{t+1} &= R/(\Gamma_{t+1}\psi_{t+1})a_t + \theta_{t+1}, \\\theta_{t+1} &\sim F_{\theta_{t+1}}, \quad \psi_{t+1} \sim F_{\psi_{t+1}}, \quad \mathbb{E}[\psi_{t+1}] = 1, \\R &= \begin{cases} R_{\text{boro}} & \text{if } a_t < 0 \\ R_{\text{save}} & \text{if } a_t > 0 \end{cases}, \quad R_{\text{boro}} \geq R_{\text{save}}.\end{aligned}$$

Kinked R: Costly Borrowing (2/3)

ConsKinkedRsolver inherits from ConsIndShockSolver

Additions to `__init__` method:

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Additions to `prepareToCalcEndOfPrdvP`:

- ▶ Four lines to use correct value of R for each value of a_t

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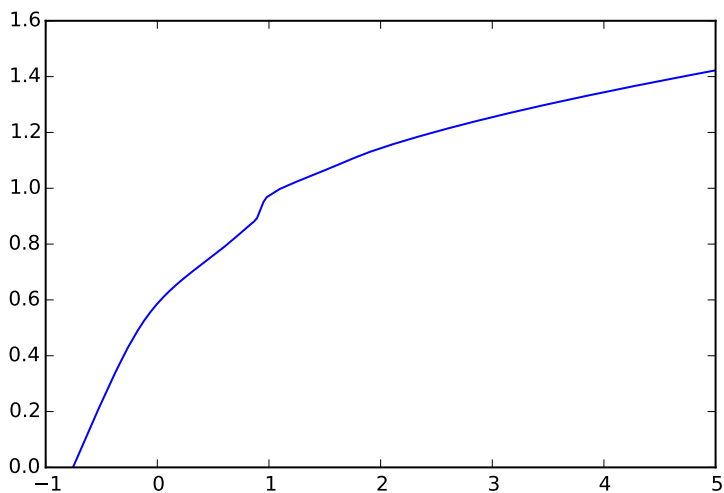
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Additions to `prepareToCalcEndOfPrdvP`:

- ▶ Four lines to use correct value of R for each value of a_t
- ▶ One line to apply that change to calculation of m_{t+1}
- ▶ Three lines to recalculate minimum MPC and human wealth

Kinked R: Costly Borrowing (3/3)



Marginal Utility Shocks (1/4)

Consider another small modification to IndShockModel:

- Multiplicative (idiosyncratic) shocks to utility each period.

$$\begin{aligned}u(c; \eta) &= \eta \frac{c^{1-\rho}}{1-\rho}, & \eta_t &\sim F_\eta, \\v(m_t, \eta_t) &= \max_{c_t} u(c_t; \eta_t) + \beta \mathbb{E}_{t+1}[v_{t+1}(m_{t+1})], \\a_t &= m_t - c_t, & a_t &\geq \underline{a}, \\m_{t+1} &= R/(\Gamma_{t+1}\psi_{t+1})a_t + \theta_{t+1}, \\ \theta_{t+1} &\sim F_{\theta_{t+1}}, & \psi_{t+1} &\sim F_{\psi_{t+1}}, \quad \mathbb{E}[\psi_{t+1}] = 1.\end{aligned}$$

Marginal Utility Shocks (1/4)

Consider another small modification to IndShockModel:

- ▶ Multiplicative (idiosyncratic) shocks to utility each period.
- ▶ Consumption “more valuable” in some periods than others.

$$\begin{aligned}u(c; \eta) &= \eta \frac{c^{1-\rho}}{1-\rho}, & \eta_t &\sim F_\eta, \\v(m_t, \eta_t) &= \max_{c_t} u(c_t; \eta_t) + \beta \mathbb{E}_{t+1}[v_{t+1}(m_{t+1})], \\a_t &= m_t - c_t, & a_t &\geq \underline{a}, \\m_{t+1} &= R/(\Gamma_{t+1}\psi_{t+1})a_t + \theta_{t+1}, \\ \theta_{t+1} &\sim F_{\theta_{t+1}}, & \psi_{t+1} &\sim F_{\psi_{t+1}}, \quad \mathbb{E}[\psi_{t+1}] = 1.\end{aligned}$$

Marginal Utility Shocks (2/4)

New input PrefShkDstn is constructed:

- ▶ PrefShkStd: Standard deviation of (log) pref shocks

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- ▶ PrefShkTailCount: Discrete shocks in “augmented tail”

Marginal Utility Shocks (3/4)

`ConsPrefShockSolver` inherits from `ConsIndShockSolver`

Additions to `__init__` method:

Marginal Utility Shocks (3/4)

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Marginal Utility Shocks (3/4)

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Additions to `__init__` method:

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- ▶ 8 lines: Values of c_t and m_t for each η_t in `PrefShkDstn`

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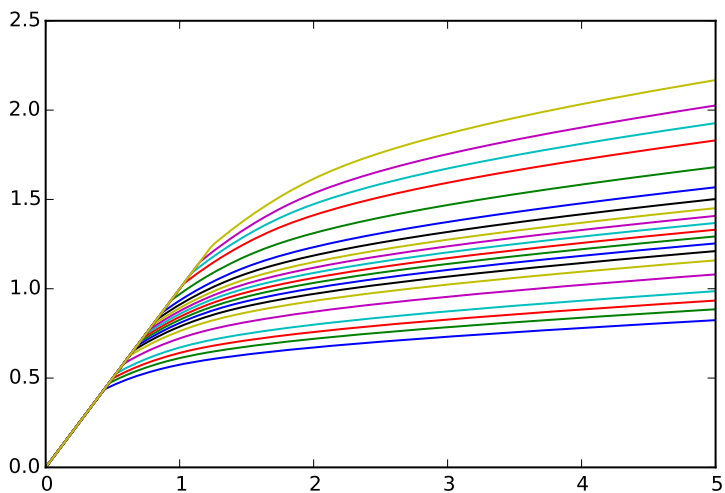
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- ▶ 8 lines: Values of c_t and m_t for each η_t in `PrefShkDstn`

Replace `usePointsForInterpolation`

- ▶ 6 lines: Construct `cFunc` as a `LinearInterpOnInterp1D`
- ▶ 6 lines: Make `vPfunc` by integrating marginal utility across η_t

Marginal Utility Shocks (4/4)



Combination Inheritance: “Kinky Preferences” (1/4)

Combine those two extensions to `IndShockModel`:

- ▶ Borrowing has higher interest rate than saving...

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Combine those two extensions to `IndShockModel`:

- ▶ Borrowing has higher interest rate than saving...
- ▶ ...and there are shocks to marginal utility
- ▶ HARK makes this pretty easy

Combination Inheritance: “Kinky Preferences” (2/4)

$$\begin{aligned}u(c, \eta) &= \eta \frac{c^{1-\rho}}{1-\rho}, & \eta_t &\sim F_\eta, \\v(m_t, \eta_t) &= \max_{c_t} u(c_t) + \beta \mathbb{E}[v_{t+1}(m_{t+1})], \\a_t &= m_t - c_t, & a_t &\geq \underline{a}, \\m_{t+1} &= R/(\Gamma_{t+1}\psi_{t+1})a_t + \theta_{t+1}, \\ \theta_{t+1} &\sim F_{\theta_{t+1}}, & \psi_{t+1} &\sim F_{\psi_{t+1}}, \quad \mathbb{E}[\psi_{t+1}] = 1, \\R &= \begin{cases} R_{boro} & \text{if } a_t < 0 \\ R_{save} & \text{if } a_t > 0 \end{cases}, & R_{boro} &\geq R_{save}.\end{aligned}$$

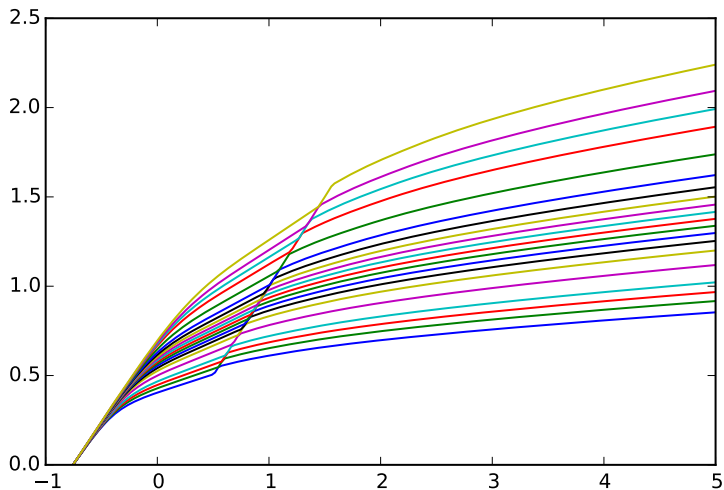
Combination Inheritance: “Kinky Preferences” (3/4)

ConsKinkyPrefSolver inherits from two parent classes. Entirety of the code for the solver:

```
class ConsKinkyPrefSolver(ConsPrefShockSolver,ConsKinkedRsolver):  
    def __init__(self,solution_next,...):  
        ConsKinkedRsolver.__init__(self,solution_next,...)  
        self.PrefShkPrbs = PrefShkDstn[0]  
        self.PrefShkVals = PrefShkDstn[1]
```

Combination Inheritance: “Kinky Preferences” (4/4)

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Macroeconomics in HARK (1/5)

- ▶ Some inputs to micro models are exogenous to each agent...
- ▶ ...But endogenous to collective whole of agents
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- ▶ Equilibrium: consistency between what agents *believe* the endogenous objects are and what values / dynamic processes *actually occur* when agents act on those beliefs
- ▶ Fixed point in the space of beliefs
- ▶ Need a representation of beliefs about endogenous objects
- ▶ And a rule for how agents form beliefs from observing history

Macroeconomics in HARK (2/5)

“The computational algorithm has two key features. First, it is based on bounded rationality in the sense that we endow agents with boundedly rational perceptions of how the aggregate state evolves... Second, we use solution by simulation, which works as follows: (i) given the boundedly rational perceptions, we solve the individuals' problems using standard dynamic programming methods; (ii) we draw individual and aggregate shocks over time for a large number of individuals; (iii) ...we generate a time series for all aggregates; and finally (iv) we compare the perceptions about the aggregates to those in the actual simulations, and these perceptions are then updated. We think this approach... can be productive for other applications.”

–Per Krusell and Tony Smith (2006)

Macroeconomics in HARK (3/5)

HARK operationalizes K-S method with a farming metaphor:

1. Solve agents' microeconomic problem for some beliefs

Market's method solve loops on this process until convergence.

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 - ▶ reap: Collect some individual variables from the agents

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3. Use history to update beliefs about endogenous objects

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- ▶ `millRule`: Function that transforms ind \rightarrow agg variables
- ▶ `calcDynamics`: Function that transforms history into beliefs

Macroeconomics in HARK (5/5)

Extra methods of a Market-compatible AgentType:

- ▶ `marketAction`: What agents *do* to generate `reap_vars`.
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- ▶ `reset`: How to initialize for a new history: reset states

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Consumption-Saving with Aggregate Productivity Shocks

$$v_t(m_t, M_t) = \max_{c_t} u(c_t) + \beta \mathbb{E}_{t+1} [v_{t+1}(m_{t+1}, M_{t+1})],$$

$$a_t = m_t - c_t, \quad a_t \geq 0,$$

$$m_{t+1} = \frac{R_{t+1} a_t}{\Gamma_{t+1} \psi_{t+1} \Psi_{t+1}} + W_{t+1} \theta_{t+1} \Theta_{t+1} \ell,$$

$$A_t = \mathbf{A}(M_t), \quad k_{t+1} = (1 - \delta) A_t / (\Psi_{t+1} \ell),$$

$$R_{t+1} = \mathbf{R}(k_{t+1} / \Theta_{t+1}), \quad W_{t+1} = \mathbf{W}(k_{t+1} / \Theta_{t+1}),$$

$$M_{t+1} = R_{t+1} k_{t+1} + W_{t+1} \Theta_{t+1} \ell$$

$$\theta_{t+1} \sim F_{\theta_{t+1}}, \quad \psi_{t+1} \sim F_{\psi_{t+1}}, \quad \mathbb{E}[\psi_{t+1}] = 1,$$

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Some totally new inputs for an `AggShockConsumerType`:

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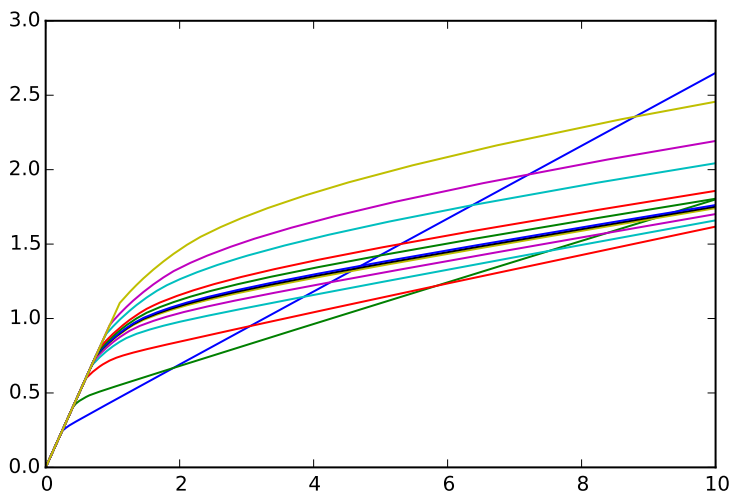
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- ▶ `IncomeDstn` has five elements: probs, idio shocks, agg shocks

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CobbDouglasEconomy is a subclass of Market:

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Loop that process for (say) 1000 periods

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- ▶ Distribute new **A** to consumers as Afunc

Other Applications of Market

Krusell and Smith were right: method is applicable to other topics

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- ▶ Papageorge et al risky sex framework: probability of contracting HIV from risky sex act depends on HIV infection rate and risky sex choices of the population
- ▶ Agent-to-agent interaction: could sow a permutation of what is reaped: imperfect knowledge, contagion of information, moves closer to “agent-based modeling”

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