An Introductory Guide to the Heterogeneous Agents Resources and toolKit

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Agenda for Today: A Flavor of HARK

1. "Microeconomic" models in HARK: the `AgentType` class
2. Example HARK model: consumption-saving with permanent and transitory shocks to labor income
3. An application: Fagereng, Holm, and Natvik
4. 30,000 foot view: What else is in HARK?

Heterogeneous Agents

Two dimensions of heterogeneity

- *Ex post* heterogeneity: Agents differ because a different sequence of events or shocks has happened to them
- *Ex ante* heterogeneity: Agents differ in objectives, preferences, expectations, etc before anything in the model “happens”

Microeconomic Models

Microeconomic models in HARK:

- Concern decision-making of one agent
- Discrete time
- Sequence of choices
- Possibly subject to risk
- Agents treat inputs to problem as *exogenous*

Key restriction

Model solution can be interpreted as iteration on sequence of “one period problems”, conditional on solution to subsequent period.

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- Conceptual (and operational) groupings of similar things.

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- Python is an *object-oriented* language: allows you to define your own data structures and objects as a *class*
- Conceptual (and operational) groupings of similar things.
- **Attributes**: What characteristics define one *instance*?
- **Methods**: What actions can instances of the class "do"?

Crash Course in Object-Oriented Programming (2/2)

- Consider a class called `Jar`:
  - A `Jar` is made of a `material`, has a `volume`, has `contents` (including names and volumes), and might be closed
  - Methods: `open`, `close`, `addTo`, `dump`, `pourInto`,...

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  - Just like `Jar`... but you can't `close` a `Can`
- Could have two classes with all those methods, but instead...

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- And a class called `Can`:
  - Just like `Jar`... but you can't `close` a `Can`
- Could have two classes with all those methods, but instead...
- Make a class called `Container` with all those methods
  - `Jar` is a subclass of `Container`, inherits its methods
  - `Can` also a subclass, but overwrites `close` to do nothing

HARK's microeconomic structure: `AgentType`

- General purpose class for representing economic agents
- Each model creates a subclass of `AgentType`
  - Includes model-specific attributes, functions, and methods...
  - ...And how to solve the "one period problem" for that model
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  - ...That lets different models "play nicely" together
- Complex models extend basic ones through class inheritance

Example Model: Basic Consumption-Saving

Consider a basic consumption-saving model: Formal model

- Discrete time, geometric discounting
- Agent chooses consumption vs saving
- CRRA utility from consumption
- Exogenous interest factor on assets
- Labor income received each period...
- ...Subject to (fully) permanent and transitory shocks
- Maybe a hard borrowing constraint

In HARK: class `IndShockConsumerType`

Defining An Agent's Problem (1/5)

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- Are those things the same in every period, or do they vary across periods?

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- Basic consumption-saving model:
  - `time_inv = ['CRRA', 'Rfree', 'DiscFac', 'BoroCnstArt', 'vFuncBool', 'CubicBool', 'aXtraGrid']`
  - `time_vary = ['IncomeDstn','LivPrb','PermGroFac']`

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- Basic consumption-saving model:
  - `CRRA = 3.2`
  - `DiscFac = 0.96`
  - `Rfree = 1.03`
  - `PermGroFac = [1.005,...,1.005,0.4,0.998,...,0.998]`
  - `LivPrb = [1,...,1,0.997,0.994,0.991,...,0]`
  - `IncomeDstn = [too much to put here]`

Defining An Agent's Problem (3/5)

- How many times does that sequence of periods happen?
- Just once? Ten times? Indefinitely?
  - `cycles = 1` : Sequence happens once, lifecycle model
  - `cycles = 40` : Sequence occurs 40 times in a loop
  - `cycles = 0` : Sequence repeats forever, infinite horizon

Defining An Agent's Problem (4/5)

- Must know *how* to solve a one period problem...
- ...*conditional* on solution to next period's problem...
- ...given values of time (in)variant parameters

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- Must know *how* to solve a one period problem...
- ...*conditional* on solution to next period's problem...
- ...given values of time (in)variant parameters
- Basic consumption-saving model:
  - `solveOnePeriod = solveConsIndShock`
  - Inputs are named in `time_vary` and `time_inv`...
  - ...plus `solution_next`, the output from previous call
  - Attributes of `solution_next`: `cFunc`, `vPfunc`, `vPPfunc`, etc

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How does the backward induction loop start?

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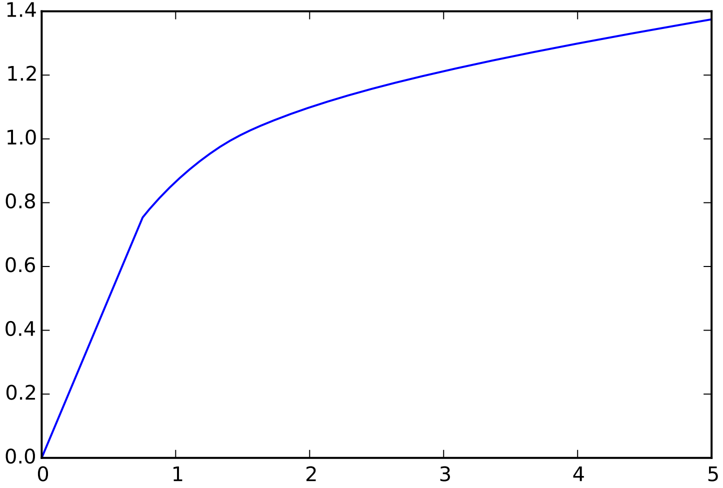
- Infinite horizon: Need an initial "guess" of the solution
- `solution_terminal` often can be found in closed form
- Created by class method `solveTerminal`
- Basic consumption-saving problem:
  - `solution_terminal.cFunc = consume everything`
  - `solution_terminal.vPfunc = u'(consume everything)`

Basic Model Consumption Function



Object-Oriented Solution Methods

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- "Parent" models are special cases of "child" models
- Solvers in HARK are objects that act (a lot) like functions
- Each model specifies a class for its solver
- Inherit solution method from parent solver...
- ...and add or change its methods / subroutines.

Small Extension: Costly Borrowing

- Suppose we want a small extension to basic model: interest rate when borrowing greater than interest rate when saving.
- `ConsKinkedRsolver` inherits from `ConsIndShockSolver`

Small Extension: Costly Borrowing

`ConsKinkedRsolver` inherits from `ConsIndShockSolver`

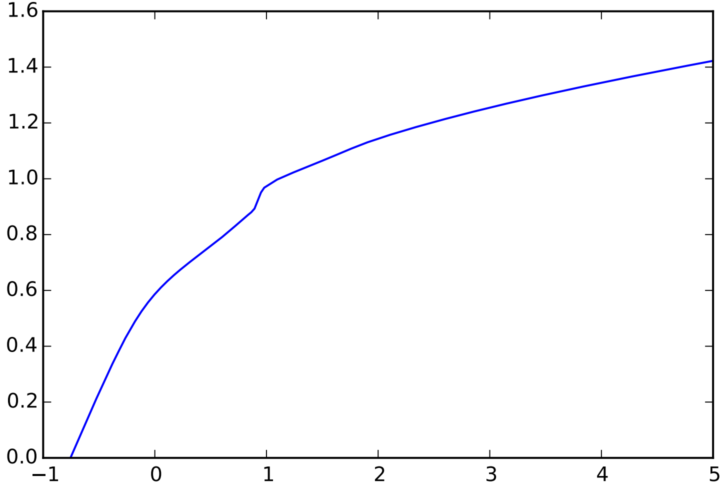
Additions to `__init__` method:

- Store new attributes `Rboro` and `Rsave`

Additions to `prepareToCalcEndOfPrdvP`:

- Four lines to use correct value of R for each value of a_t
- One line to apply that change to calculation of m_{t+1}
- Three lines to recalculate minimum MPC and human wealth

Small Extension: Costly Borrowing



An Application: Fagereng et al, Table 9

- Fagereng et al report on household's consumption response to lottery winnings by quartile of prize size and deposits
- MPC universally declines with prize size and liquidity
- Is Table 9 consistent with a single-asset consumption-saving model? How do we answer this in HARK?

An Application: Fagereng et al, Table 9

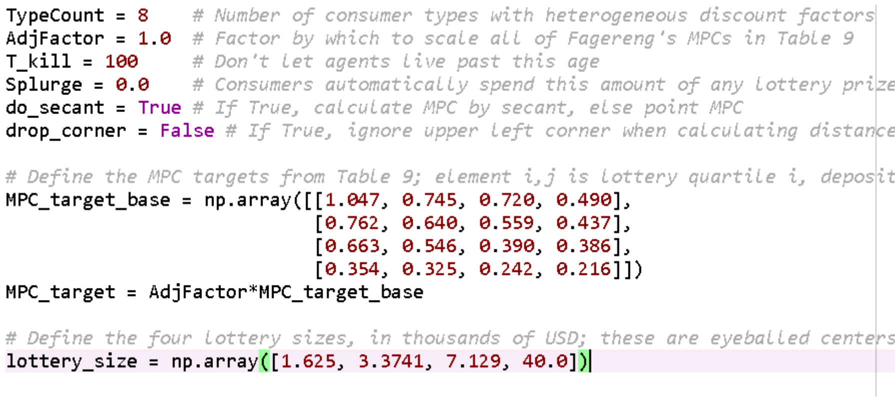
Import HARK tools, model, and parameters

(Importing basic packages omitted here)

```
from HARKutilities import approxUniform, getPercentiles
from HARKparallel import multiThreadCommands
from HARKestimation import minimizeNelderMead
from ConsIndShockModel import IndShockConsumerType
from SetupParamsCSTWnew import init_infinite # dictionary with
```

An Application: Fagereng et al, Table 9

Specify estimation parameters, MPC targets, and prize sizes



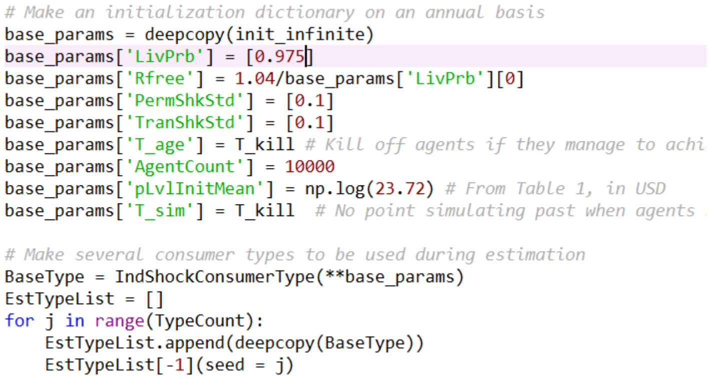
```
TypeCount = 8    # Number of consumer types with heterogeneous discount factors
AdjFactor = 1.0  # Factor by which to scale all of Fagereng's MPCs in Table 9
T_kill = 100     # Don't let agents live past this age
Splurge = 0.0    # Consumers automatically spend this amount of any lottery prize
do_secant = True # If True, calculate MPC by secant, else point MPC
drop_corner = False # If True, ignore upper left corner when calculating distance

# Define the MPC targets from Table 9; element i,j is lottery quartile i, deposit
MPC_target_base = np.array([[1.047, 0.745, 0.720, 0.490],
                            [0.762, 0.640, 0.559, 0.437],
                            [0.663, 0.546, 0.390, 0.386],
                            [0.354, 0.325, 0.242, 0.216]])
MPC_target = AdjFactor*MPC_target_base

# Define the four lottery sizes, in thousands of USD; these are eyeballed centers
lottery_size = np.array([1.625, 3.3741, 7.129, 40.0])
```

An Application: Fagereng et al, Table 9

Modify parameters, make list of consumer types for estimation

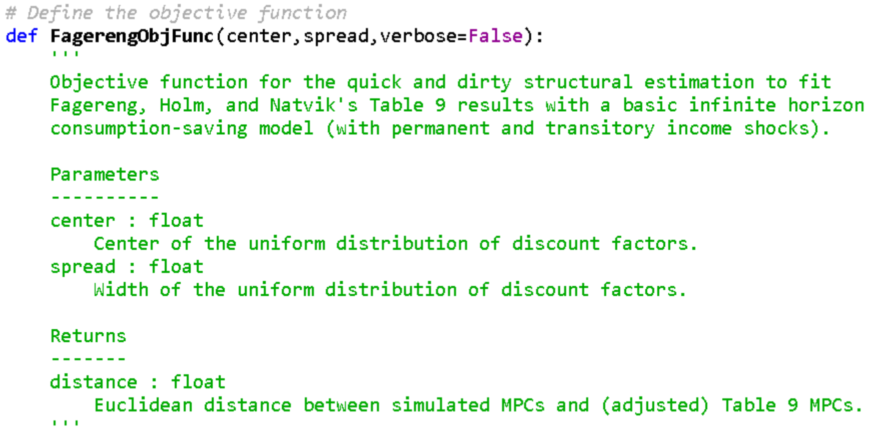


```
# Make an initialization dictionary on an annual basis
base_params = deepcopy(init_infinite)
base_params['LivPrb'] = [0.975]
base_params['Rfree'] = 1.04/base_params['LivPrb'][0]
base_params['PermShkStd'] = [0.1]
base_params['TranShkStd'] = [0.1]
base_params['T_age'] = T_kill # Kill off agents if they manage to achi
base_params['AgentCount'] = 10000
base_params['pLvlInitMean'] = np.log(23.72) # From Table 1, in USD
base_params['T_sim'] = T_kill  # No point simulating past when agents

# Make several consumer types to be used during estimation
BaseType = IndShockConsumerType(**base_params)
EstTypeList = []
for j in range(TypeCount):
    EstTypeList.append(deepcopy(BaseType))
    EstTypeList[-1](seed = j)
```

An Application: Fagereng et al, Table 9

Objective function docstring



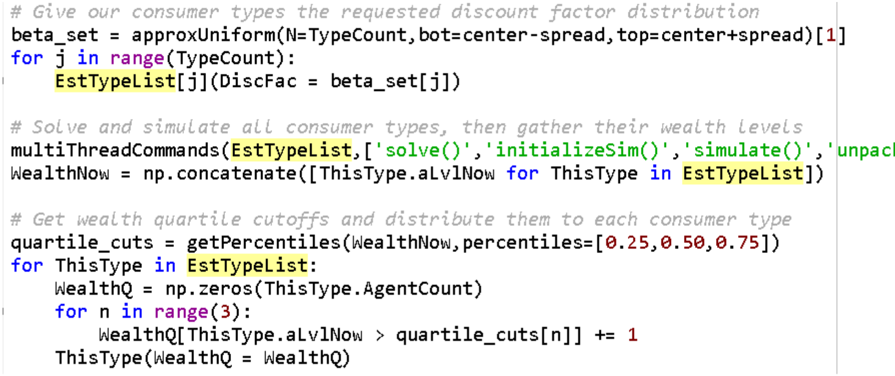
```
# Define the objective function
def FagerengObjFunc(center,spread,verbose=False):
    '''
    Objective function for the quick and dirty structural estimation to fit
    Fagereng, Holm, and Natvik's Table 9 results with a basic infinite horizon
    consumption-saving model (with permanent and transitory income shocks).

    Parameters
    ----------
    center : float
        Center of the uniform distribution of discount factors.
    spread : float
        Width of the uniform distribution of discount factors.

    Returns
    -------
    distance : float
        Euclidean distance between simulated MPCs and (adjusted) Table 9 MPCs.
    '''
```

An Application: Fagereng et al, Table 9

Distribute β to consumers; solve and simulate; mark quartiles



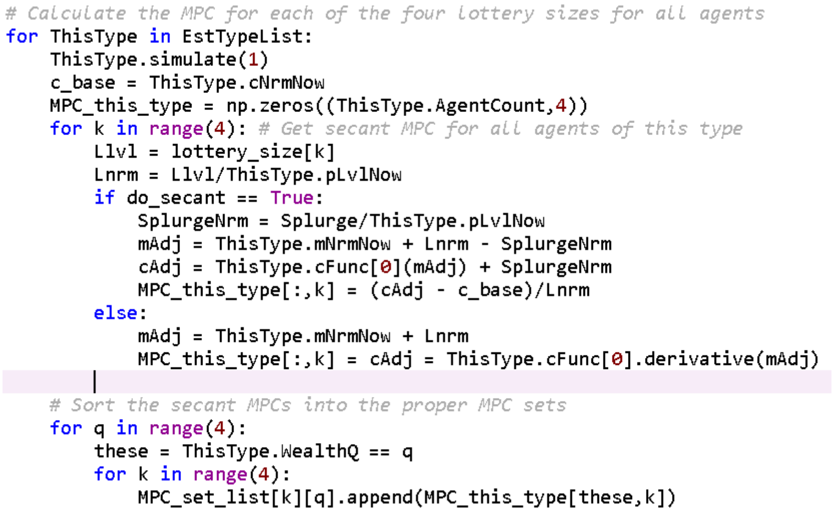
```
# Give our consumer types the requested discount factor distribution
beta_set = approxUniform(N=TypeCount,bot=center-spread,top=center+spread)[1]
for j in range(TypeCount):
    EstTypeList[j](DiscFac = beta_set[j])

# Solve and simulate all consumer types, then gather their wealth levels
multiThreadCommands(EstTypeList,['solve()','initializeSim()','simulate()','unpac
WealthNow = np.concatenate([ThisType.aLvlNow for ThisType in EstTypeList])

# Get wealth quartile cutoffs and distribute them to each consumer type
quartile_cuts = getPercentiles(WealthNow,percentiles=[0.25,0.50,0.75])
for ThisType in EstTypeList:
    WealthQ = np.zeros(ThisType.AgentCount)
    for n in range(3):
        WealthQ[ThisType.aLvlNow > quartile_cuts[n]] += 1
    ThisType(WealthQ = WealthQ)
```

An Application: Fagereng et al, Table 9

Make nested list of MPCs by prize size and deposit quartiles



```
# Calculate the MPC for each of the four lottery sizes for all agents
for ThisType in EstTypeList:
    ThisType.simulate(1)
    c_base = ThisType.cNrmNow
    MPC_this_type = np.zeros((ThisType.AgentCount,4))
    for k in range(4): # Get secant MPC for all agents of this type
        Llvl = lottery_size[k]
        Lnrm = Llvl/ThisType.pLvlNow
        if do_secant == True:
            SplurgeNrm = Splurge/ThisType.pLvlNow
            mAdj = ThisType.mNrmNow + Lnrm - SplurgeNrm
            cAdj = ThisType.cFunc[0](mAdj) + SplurgeNrm
            MPC_this_type[:,k] = (cAdj - c_base)/Lnrm
        else:
            mAdj = ThisType.mNrmNow + Lnrm
            MPC_this_type[:,k] = cAdj = ThisType.cFunc[0].derivative(mAdj)

    # Sort the secant MPCs into the proper MPC sets
    for q in range(4):
        these = ThisType.WealthQ == q
        for k in range(4):
            MPC_set_list[k][q].append(MPC_this_type[these,k])
```

An Application: Fagereng et al, Table 9

Make simulated MPC table, calculate distance from targets

```
# Calculate average within each MPC set
simulated_MPC_means = np.zeros((4,4))
for k in range(4):
    for q in range(4):
        MPC_array = np.concatenate(MPC_set_list[k][q])
        simulated_MPC_means[k,q] = np.mean(MPC_array)

# Calculate Euclidean distance between simulated MPC averages and
diff = simulated_MPC_means - MPC_target
if drop_corner:
    diff[0,0] = 0.0
distance = np.sqrt(np.sum((diff)**2))
if verbose:
    print(simulated_MPC_means)
else:
    print (center, spread, distance)
return distance
```

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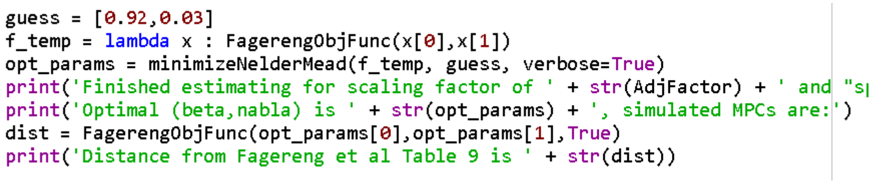
Estimation and reporting



```
guess = [0.92,0.03]
f_temp = lambda x : FagerengObjFunc(x[0],x[1])
opt_params = minimizeNelderMead(f_temp, guess, verbose=True)
print('Finished estimating for scaling factor of ' + str(AdjFactor) + ' and "s
print('Optimal (beta,nabla) is ' + str(opt_params) + ', simulated MPCs are:')
dist = FagerengObjFunc(opt_params[0],opt_params[1],True)
print('Distance from Fagereng et al Table 9 is ' + str(dist))
```

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- Some small demonstration exercises
- Carroll, Slacalek, Tokuoka, and White (2017) estimations

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- Some small demonstration exercises
- Carroll, Slacalek, Tokuoka, and White (2017) estimations
- Much room for improvement: endogenous labor supply (e.g.)

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And even more to come...

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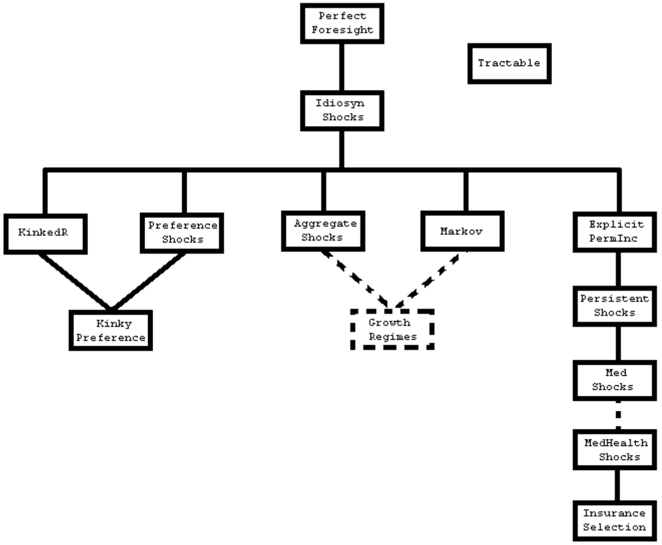
- MedHealthShock:* Medical shocks plus discrete health states
- DynInsSel:* ...plus choice over medical insurance contracts

And even more to come...

Consumption-Saving Model Tree



Topics for Further Discussion

Time is short, but I could talk about...

- Class inheritance / model recombination Link
- “Macroeconomic” framework and models Link
- To do: endogenous labor supply models Link
- To do: durable goods models Link
- To do: various bits, large and small Link

The Future of HARK: Incorporating Labor (1/4)

Model of labor supply on intensive margin:

$$u(c,\ell) = ((1-\ell)^{\alpha} c)^{1-\rho}/(1-\rho),$$

$$v_t(b_t,\theta_t) = \max_{c_t,\ell_t} u(c_t,\ell_t) + \beta \cancel{D}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(b_{t+1},\theta_{t+1}) \right] \text{ s.t.}$$

$$y_t = \ell_t \theta_t, \qquad \ell_t \in [0,1],$$

$$a_t = m_t + y_t - c_t, \quad a_t \geq \underline{a},$$

$$b_{t+1} = R/(\Gamma_t \psi_{t+1}) a_t,$$

$$\psi_{t+1} \sim F_{\psi t+1}(\psi), \quad \theta_{t+1} \sim F_{\theta t+1}(\theta), \quad \mathbb{E}[\psi_{t+1}] = 1.$$

The Future of HARK: Incorporating Labor (2/4)

Model of labor supply on extensive margin:

$$u(c,\ell) = c^{1-\rho}/(1-\rho) - \alpha\ell,$$

$$v_t(b_t,\theta_t,\ell_{t-1}) = \max_{c_t,\ell_t} u(c_t,\ell_t) + \beta\cancel{D}_t\mathbb{E}_t\left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(b_{t+1},\theta_{t+1},\ell_t)\right]$$

$$y_t = \ell_t\theta_t, \qquad \ell_t \in \{0,\ell_{t-1}\},$$

$$a_t = m_t + y_t - c_t, \qquad a_t \geq \underline{a},$$

$$b_{t+1} = R/(\Gamma_t\psi_{t+1})a_t,$$

$$\psi_{t+1} \sim F_{\psi t+1}(\psi), \qquad \theta_{t+1} \sim F_{\theta t+1}(\theta), \quad \mathbb{E}[\psi_{t+1}] = 1.$$

The Future of HARK: Incorporating Labor (3/4)

Model of endogenous employment search:

$$u(c,s) = ((1-s)^{\alpha} c)^{1-\rho}/(1-\rho),$$

$$v_t(m_t, e_t) = \max_{c_t, s_t} u(c_t, s_t) + \beta \cancel{D}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(m_{t+1}, e_{t+1}) \right] \text{ s.t.}$$

$$a_t = m_t - c_t, \quad a_t \geq \underline{a}, \quad s_t \in [0,1],$$

$$m_{t+1} = R/(\Gamma_t^e \psi_{t+1}) a_t + \theta_t e_{t+1} + \underline{b}(1 - e_{t+1}),$$

$$\text{Prob}(e_{t+1} = 1 | e_t = 0) = s_t, \qquad \text{Prob}(e_{t+1} = 0 | e_t = 1) = \mho,$$

$$\psi_{t+1} \sim F^e_{\psi t+1}(\psi), \quad \theta_{t+1} \sim F_{\theta t+1}(\theta), \quad \mathbb{E}[\psi_t] = 1.$$

The Future of HARK: Incorporating Labor (4/4)

Applications of `Market` for labor models:

- Non-trivial calculation of $L_t = \int_0^1 \ell_{it} p_{it} \theta_{it} di$ for Cobb-Douglas
- Disutility of employment search and probability of job loss depend on labor market slackness
- Can look at behavior in response to change in SS, etc

Back

The Future of HARK: Durable Goods (1/3)

General durable goods model:

$$u(c,d) = (c^{\alpha}, d^{1-\alpha})^{1-\rho}/(1-\rho).$$

$$v_t(m_t, d_t) = \max_{c_t, i_t} u(c_t, d_t) + \beta \cancel{D}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(m_{t+1}, d_{t+1}) \right] \text{ s.t.}$$

$$a_t = m_t - c_t, \quad a_t \geq \underline{a},$$

$$D_t = d_t + g(i_t), \quad d_{t+1} = (1-\delta_{t+1})D_t, \quad \delta_{t+1} \sim F_{\delta}(\delta),$$

$$m_{t+1} = R/(\Gamma_t \psi_{t+1}) a_t + \theta_{t+1},$$

$$\psi_{t+1} \sim F_{\psi t+1}(\psi), \quad \theta_{t+1} \sim F_{\theta t+1}(\theta), \quad \mathbb{E}[\psi_t] = 1.$$

The Future of HARK: Durable Goods (2/3)

Variations of durable goods model require different solvers:

- Easiest case: $g(i_t)$ is concave, $i_t \in \mathbb{R}$. Every end-of-period state (a_t, D_t) associated with *some* beginning of period state.
- Slightly harder: $i_t \geq 0$, must handle constraint

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- Easiest case: $g(i_t)$ is concave, $i_t \in \mathbb{R}$. Every end-of-period state (a_t, D_t) associated with *some* beginning of period state.
- Slightly harder: $i_t \geq 0$, must handle constraint
- Somewhat harder: $g(i_t) = \pi i_t$. One locus in (a_t, D_t) space is optimal; each point on optimal (a_t, D_t) locus associated with locus in (m_t, d_t) space.

The Future of HARK: Durable Goods (2/3)

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- Just ugh: $g(i_t) = \pi i_t + K\mathbf{1}(i_t \neq 0)$, $i_t \geq 0$.

The Future of HARK: Durable Goods (3/3)

Applications for `Market` with durable goods:

- Endogenous pricing of durable good: housing market
- Dynamics of demand for durables after an aggregate shock
- Some specifications overlap with health models

Back

The Future of HARK: Small To-Do Items

Contributions that would get your feet wet in HARK:

- Extend perfect foresight to handle borrowing constraint
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- Fix/improve aggregate shocks model: handle life cycle models
- Fix/generalize `ExplicitPermInc` models: `PermGroFunc`
- Portfolio allocation models; eventually: asset pricing
- Advanced features on more solvers: cubic spline interpolation
- Various numeric methods detached from particular models

The Future of HARK: Heavy Lifting

If you're feeling ambitious or are comfortable with HARK:

- Incorporate `opencl4py` with basic consumption-saving model. "Repack" model inputs into memory buffers, pass to OpenCL solver. OpenCL simulator: easier, big gains for some models.

Back

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Back

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- Models of firm creation / bankruptcy / investment / hiring

Back

Example Model: Basic Consumption-Saving

Consumption-saving model with idiosyncratic permanent and transitory shocks to income (normalized format):

$$u(c) = c^{1-\rho}/(1-\rho).$$

$$v_t(m_t) = \max_{c_t} u(c_t) + \beta \cancel{D}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_{t+1})^{1-\rho} v_{t+1}(m_{t+1}) \right] \text{ s.t.}$$

$$a_t = m_t - c_t, \quad a_t \geq \underline{a},$$

$$m_{t+1} = R/(\Gamma_{t+1}\psi_{t+1}) a_t + \theta_{t+1},$$

$$\psi_{t+1} \sim F_{\psi t+1}(\psi), \quad \theta_{t+1} \sim F_{\theta t+1}(\theta), \;\; \mathbb{E}[\psi_t] = 1.$$

Example Model: Basic Consumption-Saving

Model solution in two lines:

$$\text{FOC: } u'(c_t) = R\beta\cancel{D}\mathbb{E}_t\left[(\psi_{t+1}\Gamma_{t+1})^{-\rho}v'_{t+1}(m_{t+1})\right],$$

$$\text{EC: } v'_t(m_t) = u'(c_t).$$

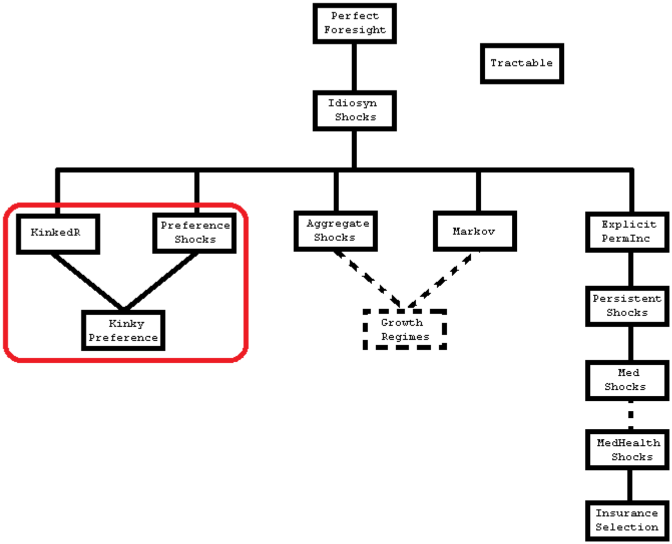
Will use endogenous grid method:

$$\mathfrak{v}'_t(a_t) \equiv R\beta\mathbb{E}_t\left[(\psi_{t+1}\Gamma_{t+1})^{-\rho}v'_{t+1}(m_{t+1})|a_t\right],$$

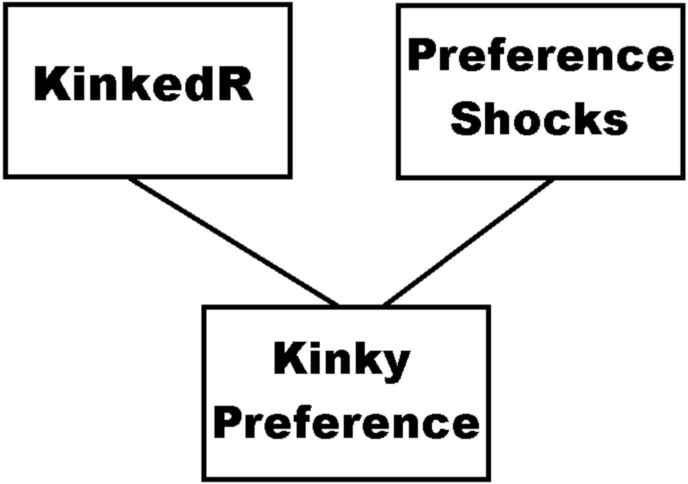
$$c_t = \mathfrak{v}'_t(a_t)^{-\rho}, \qquad m_t = a_t + c_t \text{ (for exogenous set of } \{a_t\}).$$

Back

Consumption-Saving Model Tree



Consumption-Saving Model Tree



Kinked R: Costly Borrowing (1/3)

Make one small adjustment to idiosyncratic income shocks model: interest rate on borrowing is higher than rate on saving.

$$
\begin{aligned}
u(c) &= \frac{c^{1-\rho}}{1-\rho}, \\
v(m_t) &= \max_{c_t} u(c_t) + \beta\cancel{D}_{t+1}\mathbb{E}[v_{t+1}(m_{t+1})], \\
a_t &= m_t - c_t, \qquad a_t \geq \underline{a}, \\
m_{t+1} &= R/(\Gamma_{t+1}\psi_{t+1})a_t + \theta_{t+1}, \\
\theta_{t+1} \sim F_{\theta t+1}, & \qquad \psi_{t+1} \sim F_{\psi t+1}, \quad \mathbb{E}[\psi_{t+1}] = 1, \\
R &= \begin{cases} R_{boro} & \text{if } a_t < 0 \\ R_{save} & \text{if } a_t > 0 \end{cases}, \qquad R_{boro} \geq R_{save}.
\end{aligned}
$$

Kinked R: Costly Borrowing (2/3)

`ConsKinkedRsolver` inherits from `ConsIndShockSolver`

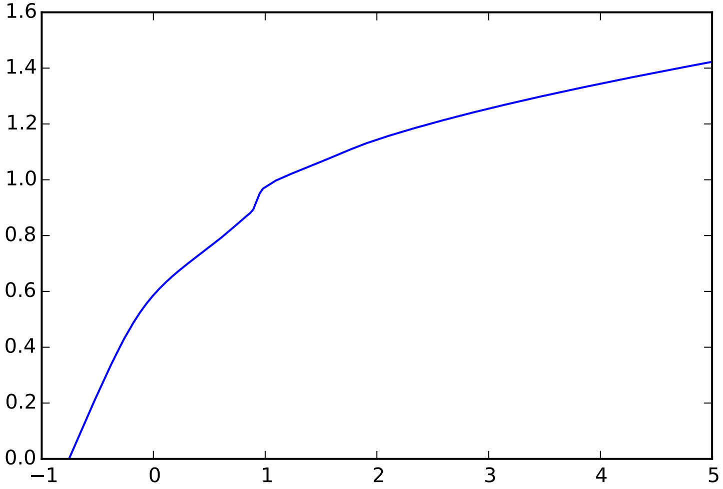
Additions to `__init__` method:

- Store new attributes `Rboro` and `Rsave`

Additions to `prepareToCalcEndOfPrdvP`:

- Four lines to use correct value of R for each value of a_t
- One line to apply that change to calculation of m_{t+1}
- Three lines to recalculate minimum MPC and human wealth

Kinked R: Costly Borrowing (3/3)



Marginal Utility Shocks (1/4)

Consider another small modification to `IndShockModel`:

- Multiplicative (idiosyncratic) shocks to utility each period.
- Consumption "more valuable" in some periods than others.

$$
\begin{aligned}
u(c;\eta) &= \eta\frac{c^{1-\rho}}{1-\rho}, \qquad \eta_t \sim F_\eta,\\
v(m_t,\eta_t) &= \max_{c_t} u(c_t;\eta_t) + \beta \cancel{D}_{t+1}\mathbb{E}[v_{t+1}(m_{t+1})],\\
a_t &= m_t - c_t, \qquad a_t \geq \underline{a},\\
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$$

Marginal Utility Shocks (2/4)

New input `PrefShkDstn` is constructed:

- `PrefShkStd`: Standard deviation of (log) pref shocks
- `PrefShkCount`: Number of discrete shocks in “body”
- `PrefShkTailCount`: Discrete shocks in “augmented tail”

Marginal Utility Shocks (3/4)

`ConsPrefShockSolver` inherits from `ConsIndShockSolver`

Additions to `__init__` method:

- 2 lines: Store preference shock distribution `PrefShkDstn`

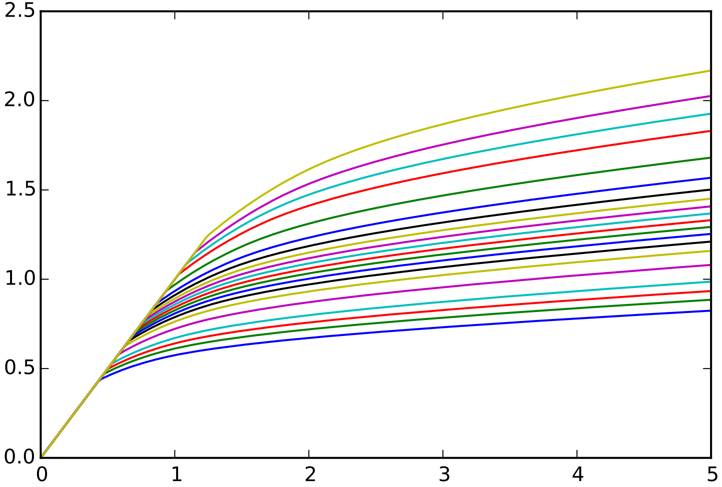
Replace `getPointsForInterpolation`

- 8 lines: Values of c_t and m_t for each η_t in `PrefShkDstn`

Replace `usePointsForInterpolation`

- 6 lines: Construct `cFunc` as a `LinearInterpOnInterp1D`
- 6 lines: Make `vPfunc` by integrating marginal utility across η_t

Marginal Utility Shocks (4/4)



Combination Inheritance: “Kinky Preferences” (1/4)

Combine those two extensions to `IndShockModel`:

- Borrowing has higher interest rate than saving...
- ...and there are shocks to marginal utility
- HARK makes this pretty easy

Combination Inheritance: “Kinky Preferences” (2/4)

$$
\begin{aligned}
u(c,\eta) &= \eta\frac{c^{1-\rho}}{1-\rho}, \qquad \eta_t \sim F_\eta, \\
v(m_t,\eta_t) &= \max_{c_t} u(c_t) + \beta\cancel{D}_{t+1}\mathbb{E}[v_{t+1}(m_{t+1})], \\
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$$

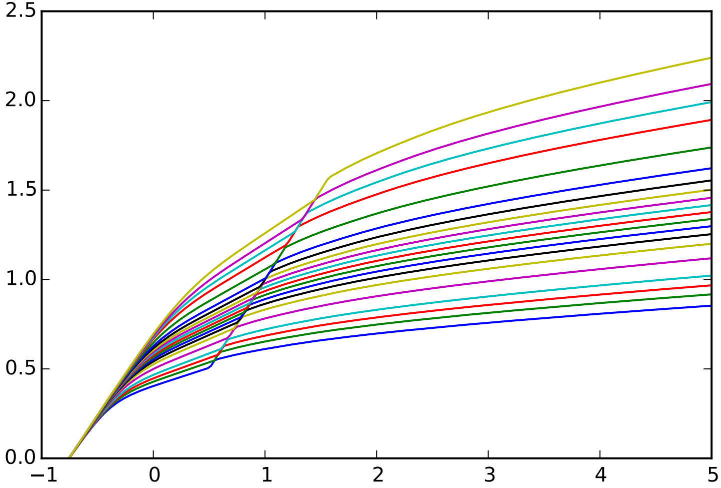
Combination Inheritance: “Kinky Preferences” (3/4)

`ConsKinkyPrefSolver` inherits from two parent classes. Entirety of the code for the solver:

```
class ConsKinkyPrefSolver(ConsPrefShockSolver,ConsKinkedRsolver):
   def __init__(self,solution_next,...):
       ConsKinkedRsolver.__init__(self,solution_next,...)
       self.PrefShkPrbs = PrefShkDstn[0]
       self.PrefShkVals = PrefShkDstn[1]
```

Combination Inheritance: "Kinky Preferences" (4/4)

Back



Macroeconomics in HARK (1/5)

- Some inputs to micro models are exogenous to each agent...
- ...But endogenous to collective whole of agents
- Might be static quantities or dynamic processes

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- Fixed point in the space of beliefs

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- Equilibrium: consistency between what agents *believe* the endogenous objects are and what values / dynamic processes *actually occur* when agents act on those beliefs
- Fixed point in the space of beliefs
- Need a representation of beliefs about endogenous objects
- And a rule for how agents form beliefs from observing history

Macroeconomics in HARK (2/5)

“The computational algorithm has two key features. First, it is based on bounded rationality in the sense that we endow agents with boundedly rational perceptions of how the aggregate state evolves... Second, we use solution by simulation, which works as follows: (i) given the boundedly rational perceptions, we solve the individuals’ problems using standard dynamic programming methods; (ii) we draw individual and aggregate shocks over time for a large number of individuals; (iii) ...we generate a time series for all aggregates; and finally (iv) we compare the perceptions about the aggregates to those in the actual simulations, and these perceptions are then updated. We think this approach... can be productive for other applications.”

–Per Krusell and Tony Smith (2006)

Macroeconomics in HARK (3/5)

HARK operationalizes K-S method with a farming metaphor:

1. Solve agents' microeconomic problem for some beliefs
2. Simulate many agents for many periods by looping on:
    - `sow`: Distribute current aggregate variables to agents
    - `cultivate`: Agents act according to their micro solution
    - `reap`: Collect some individual variables from the agents
    - `mill`: Generate aggregate variables from individual vars
    - `store`: Record some information in a "history"
3. Use history to update beliefs about endogenous objects

`Market`'s method `solve` loops on this process until convergence.

Macroeconomics in HARK (4/5)

Attributes of a `Market` instance (or subclass):

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- `sow_vars`: Names of aggregate variables
- `reap_vars`: Individual variables that form aggregates
- `track_vars`: Aggregates that need to be recorded in history
- `millRule`: Function that transforms ind $\longrightarrow$ agg variables
- `calcDynamics`: Function that transforms history into beliefs

Macroeconomics in HARK (5/5)

Extra methods of a `Market`-compatible `AgentType`:

- `marketAction`: What agents *do* to generate `reap_vars`. Often just simulate one period with `simOnePeriod`
- `reset`: How to initialize for a new history: reset states

Trivial to add more *ex ante* heterogeneity: just add more `AgentType` instances to `agents`!

Consumption-Saving with Aggregate Productivity Shocks

$$
\begin{aligned}
v_t(m_t, M_t) &= \max_{c_t} u(c_t) + \beta \cancel{D}_{t+1} \mathbb{E}[v_{t+1}(m_{t+1}, M_{t+1})], \\
a_t &= m_t - c_t, \qquad a_t \geq 0, \\
m_{t+1} &= \frac{R_{t+1} a_t}{\Gamma_{t+1} \psi_{t+1} \Psi_{t+1}} + W_{t+1} \theta_{t+1} \Theta_{t+1} \ell, \\
A_t = \mathbf{A}(M_t), & \qquad k_{t+1} = (1-\delta) A_t / (\Psi_{t+1} \ell), \\
R_{t+1} = \mathbf{R}(k_{t+1}/\Theta_{t+1}), & \qquad W_{t+1} = \mathbf{W}(k_{t+1}/\Theta_{t+1}), \\
M_{t+1} &= R_{t+1} k_{t+1} + W_{t+1} \Theta_{t+1} \ell \\
\theta_{t+1} \sim F_{\theta t+1}, & \qquad \psi_{t+1} \sim F_{\psi t+1}, \quad \mathbb{E}[\psi_{t+1}] = 1, \\
\Theta_{t+1} \sim F_{\Theta}, & \qquad \Psi_{t+1} \sim F_{\Psi}, \quad \mathbb{E}[\Psi_{t+1}] = \mathbb{E}[\Theta_{t+1}] = 1.
\end{aligned}
$$

Consumption-Saving with Aggregate Productivity Shocks

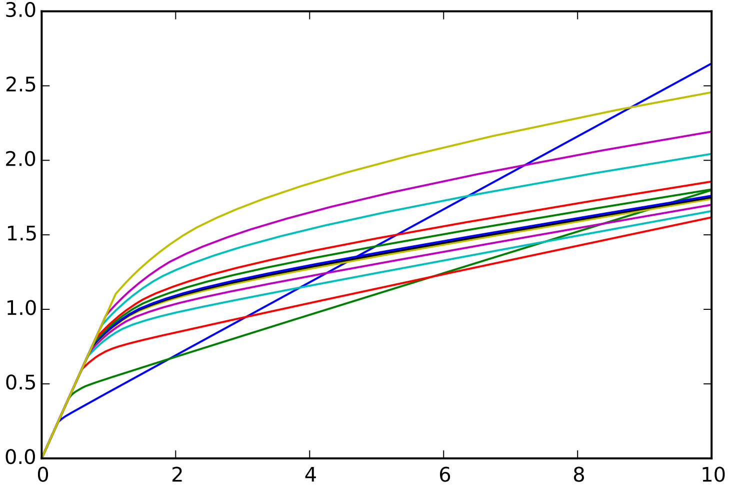
Some totally new inputs for an `AggShockConsumerType`:

- `Rfunc` and `wFunc`: Factor payments as function of k_t
- `Mgrid`: Grid of M_t state values (sort of constructed)
- `Afunc`: $\mathbb{E}[A_t | M_t] = \mathbf{A}(M_t)$

`IncomeDstn` combines idiosyncratic and aggregate shocks:

- Discrete approximation to aggregate shock distribution constructed like idiosyncratic shocks: `PermShkAggStd`, `TranShkAggStd`, `PermShkAggCount`, `TranShkAggCount`
- `IncomeDstn` has five elements: probs, idio shocks, agg shocks

Consumption-Saving with Aggregate Productivity Shocks



Cobb-Douglas Economy in the `Market` Framework

`CobbDouglasEconomy` is a subclass of `Market`:

- `sow`: Distribute $(M_t, R_t, W_t, \Theta_t, \Psi_t)$ to consumers
- `cultivate`: Consumers draw (θ_t, ψ_t), choose c_t
- `reap`: Collect assets a_t and productivity P_t from consumers
- `mill`: Calc A_{t+1}, draw $(\Theta_{t+1}, \Psi_{t+1})$, calc k_{t+1}, M_{t+1}, get (R_{t+1}, W_{t+1})
- `store`: Record M_t and A_t in their histories

Loop that process for (say) 1000 periods

- `calcDynamics`: Regress $log(A_t)$ on $log(M_t)$, make new **A**
- Distribute new **A** to consumers as `Afunc`

Other Applications of `Market`

Krusell and Smith were right: method is applicable to other topics

- Premiums of medical insurance contracts: actuarial constraint maps *who* buys each contract to break even premium, subject to informational constraints (sex, age, health, etc)

Back

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Back

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- Papageorge et al risky sex framework: probability of contracting HIV from risky sex act depends on HIV infection rate and risky sex choices of the population
- Agent-to-agent interaction: could `sow` a permutation of what is `reaped`: imperfect knowledge, contagion of information, moves closer to "agent-based modeling"

Back