Modeling Development Priorities for the Heterogeneous Agents Resources and toolKit

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- NOT structural changes or object representation
- UNLESS closely related to a methodological issue

The Big Methodological Issues

The same methodological/numeric issues come up in a wide array of models; details differ on case-by-case basis

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6. Dimensionality "mismatch" between states and controls

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- White: Got that to work in 3D as well
- Still no way to get exact location of discontinuity in 2D+... and "approximate" discont might not be that precise!

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- I got nothin'

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- Always need to compute limiting solution: what do policy and (marginal) value functions asymptote to as state goes to ∞?
- Carroll: Solution asymptotes from below to PF problem
- Argument is complicated by non-concave value
- Does limiting solution exist with discrete choice?

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- Requires computing expectations at a non-optimal candidate– what EGM tries to avoid?
- Examples: Portfolio choice, NEGM, choosing house size
- Sometimes requires using value function itself
- Maybe use implicit function theorem to get a "good guess"?

Major Modeling Areas

1. Human capital acquisition / education
2. Endogenous labor supply (and demand?)
3. Housing and durable goods
4. Health and insurance

Human Capital and Education (1/2)

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- Limits difficulties from discrete choice

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- Seneviratne (2013): Discrete choice, discrete state
- HARK version: Convexify state space, but not choice space

Labor Supply Models (1/5)

1. Intensive margin (working version by T. Magne)
2. Extensive margin (versions by MNW and P. Mogensen)
3. Job search intensity
4. Incorporating into aggregate framework

Labor Supply Models (2/5)

Model of labor supply on intensive margin:

$$u(c, \ell) = ((1 - \ell)^{\alpha} c)^{1-\rho}/(1 - \rho),$$

$$v_t(b_t, \theta_t) = \max_{c_t, \ell_t} u(c_t, \ell_t) + \beta \not{D}_t \mathbb{E}_t \left[(\psi_{t+1} \Gamma_t)^{1-\rho} v_{t+1}(b_{t+1}, \theta_{t+1}) \right] \text{ s.t.}$$

$$y_t = \ell_t \theta_t, \qquad \ell_t \in [0, 1],$$

$$a_t = m_t + y_t - c_t, \qquad a_t \geq \underline{a},$$

$$b_{t+1} = R/(\Gamma_t \psi_{t+1}) a_t,$$

$$\psi_{t+1} \sim F_{\psi t+1}(\psi), \qquad \theta_{t+1} \sim F_{\theta t+1}(\theta), \quad \mathbb{E}[\psi_{t+1}] = 1.$$

Labor Supply Models (3/5)

Model of labor supply on extensive margin:

$$u(c, \ell) = c^{1-\rho}/(1-\rho) - \alpha\ell,$$

$$v_t(b_t, \theta_t, \ell_{t-1}) = \max_{c_t, \ell_t} u(c_t, \ell_t) + \beta \cancel{\mathbb{D}}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(b_{t+1}, \theta_{t+1}, \ell_t) \right]$$

$$y_t = \ell_t \theta_t, \qquad \ell_t \in \{0, \ell_{t-1}\},$$

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Labor Supply Models (4/5)

Model of endogenous employment search:

$$u(c,s) = ((1-s)^{\alpha}c)^{1-\rho}/(1-\rho),$$

$$v_t(m_t,e_t) = \max_{c_t,s_t} u(c_t,s_t) + \beta \not{D}_t \mathbb{E}_t\left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(m_{t+1},e_{t+1})\right] \text{ s.t.}$$

$$a_t = m_t - c_t, \qquad a_t \geq \underline{a}, \qquad s_t \in [0,1],$$

$$m_{t+1} = R/(\Gamma_t^e \psi_{t+1})a_t + \theta_t e_{t+1} + \underline{b}(1-e_{t+1}),$$

$$\text{Prob}(e_{t+1}=1|e_t=0) = s_t, \qquad \text{Prob}(e_{t+1}=0|e_t=1) = \mho,$$

$$\psi_{t+1} \sim F^e_{\psi t+1}(\psi), \qquad \theta_{t+1} \sim F_{\theta t+1}(\theta), \quad \mathbb{E}[\psi_t] = 1.$$

Labor Supply Models (5/5)

Applications of `Market` for labor models:

- Non-trivial calculation of $L_t = \int_0^1 \ell_{it} P_{it} \theta_{it} di$ for Cobb-Douglas
- Disutility of employment search and probability of job loss depend on labor market slackness
- Can look at behavior in response to change in Social Security

Durable Goods: Housing and Health (1/4)

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- Changing level of d_t might involve fixed cost
- Generates "region of inaction" / inactivity constraint
- Grossman (1972): Health is a capital stock, can't disinvest
- Health produces longevity, reduces disutility of work

Durable Goods: Housing and Health (2/4)

General (housing) durable goods model:

$$u(c,d) = (c^{\alpha}, d^{1-\alpha})^{1-\rho}/(1-\rho).$$

$$v_t(m_t,d_t) = \max_{c_t,i_t} u(c_t,d_t) + \beta \not{D}_t \mathbb{E}_t \left[(\psi_{t+1}\Gamma_t)^{1-\rho} v_{t+1}(m_{t+1},d_{t+1})\right] \text{ s.t.}$$

$$a_t = m_t - c_t - i_t, \qquad a_t \geq \underline{a},$$

$$D_t = d_t + g(i_t), \qquad d_{t+1} = (1-\delta_{t+1})D_t, \qquad \delta_{t+1} \sim F_{\delta}(\delta),$$

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Durable Goods: Housing and Health (3/4)

Variations of durable goods model require different solvers:

- Easiest case: $g(i_t)$ is convex, $i_t \in \mathbb{R}$. Every end-of-period state (a_t, D_t) associated with *some* beginning-of-period state.
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- Somewhat harder: $g(i_t) = i_t/\pi_t$. One locus in (a_t, D_t) space is optimal; each point on optimal (a_t, D_t) locus associated with locus in (m_t, d_t) space.

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- Just ugh: $g(i_t) = i_t/\pi_t + K\mathbf{1}(i_t \neq 0)$, $i_t \geq 0$.

Durable Goods: Housing and Health (4/4)

Applications for `Market` with housing durable goods:

- Endogenous pricing of durable good: housing market
- Dynamics of demand for durables after an aggregate shock

Next steps for health models:

- White (2018) does health investment model with exogenous income (retired people only)
- Can add income process, interact with labor supply decision
- Health insurance and endogenous health status

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- Take my Topics in Dynamic Modeling course at UDel!