My research covers two areas of macroeconomic theory. Firstly, my research seeks to develop computational and mathematical foundations for heterogeneous agent macroeconomic models. Secondly, my research seeks to understand the relationship between technological change, income inequality, the environment and growth. I am also interested in applying machine learning techniques to help solve heterogeneous agent models on the computer and understanding the impact of artificial intelligence technologies on growth.

Modelling Heterogeneity in Macroeconomics

The rational expectations revolution in the 1970s firmly placed the micro-economic foundations of consumer behaviour at the centre of macroeconomic analysis. The key assumption that allowed the micro-foundations of macroeconomic models to be mathematically tractable was the representative agent, a single agent whose decisions reflected macroeconomic variables.

However, macroeconomists now recognise that traditional representative agent models cannot adequately account for the vast heterogeneity of economic life — heterogeneity in wealth, income, skills and preferences. This criticism, along with the availability of greater computing power, has led to macroeconomists computing models with many heterogeneous agents facing incomplete markets (the seminal papers are Bewley (1977), Aiyagari (1994), Huggett (1993) and Krusell and Smith (1998)).

Not only are representative agent models unrealistic, but heterogeneity has been shown to be important for questions about steady state levels of macroeconomic aggregates such as capital (Aiyagari, 1995), how macroeconomic variables such as consumption and investment respond to monetary policy (Auclert, 2017; Kaplan et al., 2016), to the timing of fiscal policy (Heathcote, 2005) and to aggregate shocks (Krueger et al., 2016).

Heterogeneity is also key to understanding the impact of policy on welfare (Dávila et al., 2012; Park, 2017); for example, policies discouraging saving have a burden on the wealth poor since they depress wage rates, but may in fact benefit the wealthy since they increase the return on capital.

Despite the popularity of heterogeneous agent models, because we no longer have the mathematical simplicity of a representative agent, there remain large gaps in the theoretical and computational foundations of these models. My research

agenda focuses on helping develop this foundation. In particular, the following questions form my immediate research agenda:

1. When do recursive rational expectations equilibria in heterogeneous agent models exist? When does a computational algorithm converge to the equilibrium (see Cao (2016) for a technical summary of the issues)?

2. Under what circumstances can we represent people's decision rules and aggregate relationships as functions of simple statistics, such as the mean, of aggregate distributions? When do we require each person's decision rule to depend on the entire high dimensional distribution of all other people (see discussion by Heathcote et al. (2009), p32 and appendix of Krueger et al. (2016))?

3. How can we accurately and efficiently represent high dimensional distributions on the computer? What are the best ways to overcome the curse of dimensionality in computer simulations of heterogeneous agent models? How can we apply techniques emerging from the machine learning literature, such as reinforcement learning, to compute equilibrium models?

These foundations can help us improve our understanding of how heterogeneity affects aggregate macroeconomic relationships, allow us to model more realistic forms of aggregate uncertainty and individual diversity and help create more efficient and faster algorithms to perform policy analysis.

To date, my PhD thesis has focused on foundations of optimal policy in heterogeneous agent models. I have proven an existence theorem for social planner problems in a standard heterogeneous agent model with production (Shanker, 2017). The proof overcomes the mathematical challenges brought on by the infinite dimensional structure of the policy problem by developing a new maximum theorem for non-compact topological spaces.

The existence proof implied optimal policies have a recursive structure, which allowed me to develop an algorithm guaranteed to converge to the true optima, based on infinite dimensional value function iteration (paper in progress). To speed up the computational procedure, I use a novel application of adaptive sparse grids with reinforcement learning (Pfluger, 2010; Brumm and Scheidegger, 2017; Garcke and Klompmaker, 2014). The procedure reduces grid points for the high dimensional state-space and avoids having to perform infinite dimensional optimisation at each iteration. To the best of my knowledge this is the the first time

value function iteration and reinforcement has been used in a model where the state is infinite dimensional.

Growth and Technological Change

Alongside the rational expectations revolution in macroeconomics, growth theory also sought to explain long-run growth using micro-founded, profit driven entrepreneur behaviour — endogenous growth theory due to Romer (1990) and Aghion and Howitt (1992). Initially endogenous growth theory featured symmetric technologies, again allowing us to represent the aggregate macro-economy using simple, deterministic balanced growth relationships using a half dozen or fewer equations.

It was not till the work of Acemoglu (2002) that the focus of growth theory shifted to understanding the direction of technical change, how technical change augments different factors of production. Directed technical change theory has been used to explain rising wage inequality (Acemoglu, 2002), the stylized fact of a constant capital-output ratio (Acemoglu, 2003) and how policy can lead to an environmentally sustainable growth path (Acemoglu et al., 2012), among other questions.

My immediate research in this area seeks to understand how new forms of technologies that can substitute for labour, namely artificial intelligence (AI), affect income inequality. Acemoglu and Restrepo (2017) show how AI can exasperate the income gap between high and low skilled workers. My immediate agenda is to extend this framework[1] to understand how AI affects the *shape* of the distribution — for example, is it that the top of the wealth distribution gets further and further away from the middle? Or is it that the middle and top half of the distribution get richer leaving behind a bottom?

To this point in my PhD thesis, I have used the theory of directed technical change to understand the relationship between energy and growth (Shanker and Stern, 2017). For most of the twentieth and twenty first centuries, the energy intensity of Gross Domestic Product (GDP) across the world has declined, while energy prices have stayed stationary. Much of the energy and climate policy modelling literature treated this decline as an exogenous autonomous trend (the AEEI). In (Shanker and Stern, 2017), we show how energy intensity can decline as a result of

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[1]Building on ideas in such as Acemoglu and Cao (2015) and Jones and Kim (2017), where the growth process is linked to Pareto distributions in wealth.

profit driven energy augmenting technical change, which firms undertake even if prices do not rise. However, whether or not this decline in energy intensity continues depends on how much current research increases the productivity of future research. The autonomous decline in energy intensity may in fact stop if current research has a high impact on the productivity of future research, implying the assumption of a continued AEEI made by climate policy models may be overly optimistic.

I conclude with two longer term areas of research. First, in the spirit of Schumpeter (1942), who envisioned growth through entrepreneurial creative destruction as being inherently unstable, how can we link the key themes raised in this research statement — growth, technical change and business cycles in a general equilibrium heterogeneous agent framework?[2] In particular, as AI dominates technical change, should we expect economies to be more or less volatile?

Secondly, is sustained growth possible within environmental limits? Is some sort of automatic decoupling inevitable? Can a sustainable development path be cost-less or even improve welfare (the Porter-Hypothesis)? Both Acemoglu et al. (2012) and Shanker and Stern (2017) seem to suggest that without government intervention, growth eventually leads to environmental disaster.[3] How do our answers change with broader assumptions on technical change, such as in the presence of rapid AI driven technical change?

References

Acemoglu, D., 2002. Directed Technical Change. The Review of Economic Studies 69 (4), 781–809.

Acemoglu, D., 2003. Labour and Capital Augmenting Technical Change. Journal of the European Economic Association 1 (1), 1–37.

Acemoglu, D., Aghion, P., Bursztyn, L., Hemous, D., feb 2012. The Environment and Directed Technical Change. American Economic Review 102 (1), 131–166. URL `http://pubs.aeaweb.org/doi/abs/10.1257/aer.102.1.131`

Acemoglu, D., Cao, D., 2015. Innovation by entrants and incumbents. Journal of

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[2]Some progress in this regard has been made by the agent-based modelling literature, see for example, Dosi et al. (2010).

[3]See also Hassler et al. (2016) who show a slow down in growth from resource scarcity

Economic Theory 157, 255–294.
URL http://dx.doi.org/10.1016/j.jet.2015.01.001

Acemoglu, D., Restrepo, P., 2017. The Race between Machine and Man: Implications of Technology for Growth, Factor Shares and Employment.
URL http://www.nber.org/papers/w22252

Aghion, P., Howitt, P., 1992. A Model of Growth Through Creative Destruction. Econometrica 60 (2).

Aiyagari, S. R., 1994. Uninsured Idiosyncratic Risk and Aggregate Saving. The Quarterly Journal of Economics 109 (3), 659–684.

Aiyagari, S. R., 1995. Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints and Constant Discounting. Journal of Political Economy 103 (6), 1158–1175.
URL http://www.jstor.org/stable/2138707

Bewley, T., 1977. The Permanent Income Hypothesis: A Theoretical Formulation. Journal of Economic Theory 16, 252–292.

Brumm, J., Scheidegger, S., 2017. Using Adaptive Sparse Grids to Solve High-Dimensional Dynamic Models. Econometrica 85 (5), 1575–1612.
URL https://www.econometricsociety.org/doi/10.3982/ECTA12216

Cao, D., 2016. Existence of Generalized Recursive Equilibrium in Krusell and Smith (1998).
URL http://faculty.georgetown.edu/dc448/KSExistenceV17.pdf

Dávila, J., Hong, J. H., Krusell, P., Ríos-Rull, J.-V., 2012. Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks. Econometrica 80 (6), 2431–2467.

Dosi, G., Fagiolo, G., Roventini, A., sep 2010. Schumpeter meeting Keynes: A policy-friendly model of endogenous growth and business cycles. Journal of Economic Dynamics and Control 34 (9), 1748–1767.
URL http://linkinghub.elsevier.com/retrieve/pii/S016518891000148X

Garcke, J., Klompmaker, I., 2014. Adaptive sparse grids in reinforcement learning. Lecture Notes in Computational Science and Engineering 102, 179–194.

Hassler, J., Krusell, P., Olovsson, C., 2016. Directed Technical Change as a Response to Natural-Resource Scarcity.
URL hassler-j.iies.su.se/PAPERS/estc.pdf%0A

Heathcote, J., 2005. Fiscal Policy with Heterogeneous Agents and Incomplete Markets. Review of Economic Studies 72 (1), 161–188.

Heathcote, J., Storesletten, K., Violante, G. L., 2009. Quantitative Macroeconomics with Heterogeneous Households.
URL http://www.nber.org/papers/w14768

Huggett, M., 1993. The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies. Journal of Economic Dynamics and Control 17, 953–969.

Jones, C. I., Kim, J., 2017. A Schumpeterian Model of Top Income Inequality. Journal of Political Economy (Forthcoming).
URL http://www.nber.org/papers/w20637

Krueger, D., Mitman, K., Perri, F., 2016. Macroeconomics and Household Heterogeneity. In: Taylor, J. B., Uhlig, H. (Eds.), Handbook of Macroeconomics. Vol. 2. Elsevier, pp. 843–921.

Krusell, P., Smith, A. A., 1998. Income and Wealth Heterogeneity in the Macroeconomy. Journal of Political Economy 106 (5), 867–896.

Park, Y., 2017. Constrained Efficiency in a Risky Human Capital Model. American Economic Journal: Macroeconomics (Forthcoming).

Pfluger, D., 2010. Spatially Adaptive Sparse Grids for High-Dimensional Problems. Verlag Dr. Hut, Munchen.
URL http://www5.in.tum.de/pub/pflueger10spatially.pdf

Romer, P., 1990. Endogenous Technological Change. The Journal of Political Economy 98 (5), S71–S102.

Schumpeter, J. A., 1942. Capitalism, Socialism and Democracy. Harper, New York.

Shanker, A., 2017. Existence of Recursive Constrained Optima in the Heterogenous Agent Neoclassical Growth Model.
URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3011662

Shanker, A., Stern, D. I., 2017. Energy and the Direction of Growth: Will Autonomous Energy Efficiency Improvements Continue?
URL github.com/akshayshanker/energy