

A Theory of Saving under Risk Preference Dynamics

Ma, Song & Toda (2024) - Reading Group Presentation

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Paper Overview

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Central Question:

- Why do wealthy households save so much?
- How to generate realistic wealth distributions in macro models?

Key Innovation: Time-varying risk aversion

Main Finding:

- Zero asymptotic MPCs arise naturally
- No need for complex return processes

Outline

Motivation

Model Setup

Main Results

Stylized Facts

- Wealthy households have substantially higher saving rates
- Top 1% exhibit markedly lower MPC than median households
- This pattern persists across countries and time periods

Theoretical Challenge

Existing models require restrictive assumptions to yield zero asymptotic MPCs:

- Ma & Toda (2021): Need stringent conditions on return risk
- Benhabib et al. (2015): Require specific capital income risk structure
- Carroll and Shanker (2025): $(\frac{R}{\beta})^{1/\gamma} \geq R$

Overall, we seem to have a poor understanding of why the rich are thrifty.

Why it matters:

- Need a theory of consumption that explains behaviour across agent types.
- Matters for policy design with heterogeneous agents and generating IRFs.

Why it (may) not matter:

- Is the question about MPCs, tails, risk perceptions or frictions?
- We do not really know, but saying asymptotic MPCs cannot be zero in a standard model may not be enough motivation to reject it.

Incorporate stochastic risk aversion in an otherwise vanilla **income fluctuation problem**.

Key Features:

- Risk preferences vary across states and over time
- Captures empirical evidence of preference heterogeneity
- No need for complex return processes/ edge case conditions on returns and discounting

Main Result

Zero asymptotic MPCs arise when agents can become less risk averse in the future.

Agent's Dynamic Problem

$$\begin{aligned} \max_{\{c_t, w_t\}} \quad & \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{i=0}^t \beta_i \right) u(c_t, Z_t) \right] \\ \text{s.t.} \quad & w_{t+1} = R_{t+1}(w_t - c_t) + Y_{t+1} \\ & 0 \leq c_t \leq w_t \end{aligned}$$

Key State Variables:

- Z_t : Markov chain with **preference shocks**
- β_t : Stochastic discount factor
- R_t : Stochastic returns
- Y_t : Non-financial income

Novel feature: Z_t affects risk aversion $\gamma(Z_t)$ directly.

Utility specification

$$u(c, z) = \begin{cases} \frac{c^{1-\gamma(z)}}{1-\gamma(z)} & \text{if } \gamma(z) > 0, \gamma(z) \neq 1 \\ \log c & \text{if } \gamma(z) = 1 \end{cases}$$

Key feature: $\gamma(z)$ varies with state z .

- State decomposition: $Z_t = (\bar{Z}_t, \tilde{Z}_t)$
- Risk aversion driven by $\bar{Z}_t \in \{\bar{Z}_1, \dots, \bar{Z}_N\}$.
- Ordering: $0 < \gamma(\bar{Z}_1) < \dots < \gamma(\bar{Z}_N)$.
- Transition matrix: $\bar{P} = (\bar{p}_{ij})_{1 \leq i, j \leq N}$.

Euler Equation

The optimal consumption function $c^*(w, z)$ satisfies:

$$u'(c^*(w, z), z) = \max \left\{ \mathbb{E}_z[\hat{\beta} \hat{R} u'(c^*(\hat{w}, \hat{Z}), \hat{Z})], u'(w, z) \right\}$$

where $\hat{w} = \hat{R}(w - c^*(w, z)) + \hat{Y}$

Assumption (2.2: Returns and Discounting)

1. For all $z \in Z$:
 - $\mathbb{E}_z[u_c(\hat{Y}, \hat{Z})] < \infty$
 - $\mathbb{E}_z[\hat{\beta} \hat{R} \cdot u_c(\hat{Y}, \hat{Z})] < \infty$
2. *Spectral radius condition*: $r(K(1)) < 1$
where the matrix $K(\theta)$ is defined by:

$$K_{zz'}(\theta) = P(z, z') \int \beta(z, z', \varepsilon) R(z, z', \varepsilon)^\theta \pi(d\varepsilon)$$

Interpretation

$r(K(1)) < 1$ ensures wealth doesn't explode in present value terms
Generalizes the standard condition $\beta R < 1$

Theorem (2.1: Existence and Uniqueness)

If Assumptions 2.1 and 2.2 hold, then:

- 1. The time iteration operator $T : \mathcal{C} \rightarrow \mathcal{C}$ has a unique fixed point c^* .*
- 2. For any $c \in \mathcal{C}$:*

$$\rho(T^k c, c^*) \rightarrow 0 \quad \text{as } k \rightarrow \infty$$

where ρ is a metric based on marginal utilities.

Theorem (3.2: Positive Transition Matrix)

If every entry of \bar{P} is strictly positive and $\Pr_{z,z'}(\hat{\beta}\hat{R} > 0) > 0$ for all $(z, z') \in Z^2$, then for all states with $i \neq 1$:

$$\lim_{w \rightarrow \infty} \frac{c^*(w, z_{ij})}{w} = 0 \quad (1)$$

Key Insight (from more general result)

With fully mixing risk aversion Markov chain, zero asymptotic MPCs arise at all or almost all states.

Why does the possibility of lower risk aversion drive saving?

The Mechanism

1. Agent anticipates possible **decrease** in risk aversion.
2. Lower future risk aversion \Rightarrow Higher future consumption desire.
3. Creates **perpetual precautionary saving motive**.
4. Saving motive persists regardless of wealth level.

Key difference from existing theory:

- Ma & Toda (2021): Zero MPC is knife-edge case.
- **This result:** Zero MPC is generic feature.

Result holds even with:

- Constant returns ($R_t \equiv R$).
- No income risk.
- Constant discount factor.

Theorem (3.4: Strictly Increasing Risk Aversion)

If risk aversion strictly increases next period from state i and R is bounded below by $m > 0$:

$$\lim_{w \rightarrow \infty} \frac{c^*(w, z_{ij})}{w} = 1 \quad \text{for all } j \quad (2)$$

*(Consumption function is **nonconcave**).*

Key Contrast

- Theorem 3.2: Downward transitions $\Rightarrow \bar{c} = 0$ (save everything).
- Theorem 3.4: Upward transitions $\Rightarrow \bar{c} = 1$ (consume everything).

Intuition: When agents expect to become **more** risk averse in the future, they consume aggressively today before their preferences change.

Proposition (3.1: No Downward Transitions Required)

If $\bar{c}(z_{ij}) > 0$ for some i, j , then:

$$\bar{p}_{i1} = \cdots = \bar{p}_{i,i-1} = 0 \quad (3)$$

(Zero probability of moving to lower risk aversion).

Main takeaway:

- Zero MPCs are **generic** when γ can decrease.
- Strictly positive MPCs require **restrictive** conditions disallowing downward transitions.

Why this matters:

- Single, intuitive mechanism, does not rely on growth rates.
- Robust theoretical result.

Why I want to know more:

- Do we need limiting MPCs to be zero? What specific empirical fact are we trying to explain?
- Is this a story about risk or inter-temporal substitution?
- Overall, we are now left with lots of results with ad hoc growth conditions.
- Does risk aversion dynamics pass the "Kath Day-Night Test"?

Question: Why is your MPC limiting to zero?

