Using the Sequence Space Jacobian to Solve and Estimate Heterogeneous-Agent Models

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Econometrica, 2021 (September)

Sydney Macro reading group presentation by: Akshay Shanker
Sydney, Australia

March 20, 2023

Introductory discussion

Computing models with heterogeneity and the macro-economy

How can we efficiently and accurately solve macroeconomic models where a rich distribution of agents interacts in rational expectations equilibrium through time?

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- However, the policy functions can be sufficiently log-linearised in finite moments of the distribution in the special case of KS (1998)
- IMPORTANT: KS $\nRightarrow$ representative agent
- IMPORTANT: $\neg$representative agent $\nRightarrow$ $\neg$heterogeneity

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- Mathematical 'complexity' a challenge for proofs: complexity + abstraction = hard
- Dimensionality a challenge for practical algorithms

While theory lags.....efficient computation methods have burgeoned

- KS style algorithms (approximate, non-linear)
- Linearised methods (Reiter (2009, 2010), Boppart, Krusell and Mittman (2018))
- This paper in the spirit of Boppart, Krusell and Mittman (2018): MIT shocks

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- Even while the formal theory lags, we need to explore the space with computation
- Start with experiments, formalise later if appropriate (Jonathan Borwein)

Main idea of contribution

Based on linearising on the sequence space of an aggregate path

Main contribution to present simple results to quickly compute Jacobian of response of a shock

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Focus on computation method rather than IRFs and estimation

The K-S-B-A-H Model

The canonical heterogeneous agent model

We start with a model with no uncertain aggregate shocks

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In the lineage of to Aiyagari (1993) also related to Huggett (1992) and Bewley (1972)

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Let S, where $S := A \times E$, denote the agents' state space

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At time zero, each agent $i \in I$ draws asset level x_0^i, with x_0^i taking values in A

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All shocks defined on common probability space $(\bar{\Omega}, \Sigma, \bar{\mathbb{P}})$

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We have $\{k_t^i, e_t^i\} \sim D_t$ for each i, where $(D_t)_{t=0}^{\infty}$ are the aggregate (distributions) of the economy

Firms

Standard price taking firms with neoclassical production function F, consider sequence of determistic shocks (Z_t)

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Interest and wage rates in the economy will be:

$$r_t := Z_t F_1(\mathcal{K}_t, L) - \delta, \quad w_t : \; = Z_t F_2(\mathcal{K}_t, L)$$

where:

$$\mathcal{K}_t = \int \int k D_t(de, dk) \tag{2}$$

Utility

Let $u \colon \mathbb{R}_+ \to \bar{D}$ be each consumer's utility function, where $\bar{D} = \mathbb{R}_+$ or $\bar{D} = \mathbb{R} \cup \{-\infty\}$

Time t utility for agent i will be $u(c_t^i)$

Value function

Each period, agents' policies satisfy:

$$V_t(e, k') = \max_{c,k'} u(c) + \beta \mathbb{E}_t V_{t+1}(k', e') \tag{3}$$

s.t. $c + k' = (1 + r_t)k + w_t e,\; k' \geq 0$

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thus we have $h_t(k, e) = \arg\max_{c,k'} u(c) + \beta \mathbb{E}_t V_{t+1}(k', e')(1)$

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Sequential equilibria exist, see Cao (2020), and the proof is correct

The Sequence Space Jacobian Method

ABRS H-function

Let $\mathbf{K} = \{K_0, \ldots\} \in \mathbb{R}^N$ and $\mathbf{Z} = \{Z_0, \ldots\} \in \mathbb{R}^N$ be sequence of transitory ‘shocks’

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Capture Rational Expectations:

$$\mathbf{H}_t(\mathbf{K}, \mathbf{Z}) = \mathcal{K}_t \left(\{F_1(K_s, Z_s) - \delta, F_2(K_s, Z_s)\}_{s \geq 0} \right) - K_{t+1} \tag{4}$$

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The Galaxy (well, at-least ABRS) Know(s) as much as Hari Seldon Condition:

$$\mathbf{H}_t(\mathbf{K}, \mathbf{Z}) = \mathbf{0} \tag{5}$$

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We are perturbing the system around the steady state along the sequence

In sequence space, no need to worry about differentiating w.r.t to distributions c.f. Reiter (2009, 2019)

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$$[\mathbf{H}_K]_{t,s} = \underbrace{\frac{\partial \mathcal{K}_t}{\partial r_{s+1}}}_{\text{numerical}} \overbrace{\frac{\partial r_{t+1}}{\partial K_t}}^{\text{analytical}} + \underbrace{\frac{\partial \mathcal{K}_t}{\partial w_{s+1}}}_{\text{numerical}} \overbrace{\frac{\partial w_{t+1}}{\partial K_t}}^{\text{analytical}} \tag{7}$$

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Contribution of paper is to show how to efficiently calculate $\frac{\partial \mathcal{K}_t}{\partial r_{s+1}}$ and $\frac{\partial \mathcal{K}_t}{\partial w_{s+1}}$

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The term $\mathcal{F}_{t,s}$ is the difference between the response of the aggregate capital today to a shock $s+1$ periods in the future and the the response of the aggregate capital yesterday to a shock $s+1$ periods in the future from $t-1$

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$$\mathcal{F}_{t,s} = \mathcal{E}'_{t-1} d\mathbf{D}_1^s \tag{9}$$

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Response to distribution to s period ahead 'news'

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Similarly for derivative w.r.t to w_{s+1} (or use matrix notation)

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6. Evaluate $\frac{\partial \mathcal{K}_t}{\partial r_{s+1}} = \sum_{k=0}^{\min\{s+1,t\}} \mathcal{F}_{t-k,s-k}$ to arrive at the Jacobian (similarly for w_t)

Timings

TABLE II

DIRECT AND FAKE NEWS ALGORITHMS TO COMPUTE 300 × 300 JACOBIANS $\mathcal{J}$

	Krusell–Smith	HD Krusell–Smith	One-Asset HANK	Two-Asset HANK
Direct	**21 s**	**2102 s**	**156 s**	**956 s**
step 1 (backward)	13 s	1302 s	132 s	846 s
step 2 (forward)	8 s	800 s	24 s	111 s
Fake news	**0.086 s**	**10.467 s**	**0.317 s**	**3.498 s**
step 1	0.060 s	8.654 s	0.236 s	3.159 s
step 2	0.011 s	1.061 s	0.022 s	0.119 s
step 3	0.011 s	0.758 s	0.045 s	0.201 s
step 4	0.003 s	0.003 s	0.014 s	0.018 s
Grid points n_g	3500	250,000	3500	10,500
Inputs n_x	2	2	4	5
Outputs n_y	2	2	4	4
Jacobians $n_x \times n_y$	4	4	16	20

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Stochastic process thus admits a moving average representation using the calculated Jacobian (see paper)

Concluding remarks

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- Non-convexities?
- Overall, linearisation the question is open

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- Model has limitations where future states (distributions) affect policy that cannot be summarised by aggregates (money search, OLG models with bequests etc.)
- To solve a non-linear system, far easier to go back to recursive policies
- Then we are back to the question: what features of the distribution matter?

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- When aggregate dynamics can be linearised?

- Do aggregate dynamics depend on the current state only?

- What features of the state matter for describing aggregate dynamics?

We do not have efficient enough global solution methods to compare approximate solutions to!

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We do not know whether recursive rational expectations equilibria exist