Lecture 1: Magnetic fields and magnetic force

Learning outcomes

Assessable

- Calculate the force on moving charges in a magnetic field via the cross product.
- Calculate the direction of resulting vectors from the cross product or, where appropriate, apply basic rules to determine the direction.
- Calculate potential difference in a moving conductor.

Understanding

- Visualize and conceptually explain the motion of charged particles in magnetic fields.

1. Summary

1.1. Purpose of PHYS2003

The purpose of PHYS2003 is to provide you with the fundamental physics basis of phenomena and devices that you will draw on as you advance in your studies. In particular, units such as ELEC3016 (Power and Machines), ELEC3014 (Electronic Materials and Devices) and ELEC4407 (Engineering Electromagnetics) will draw heavily on the understanding of electromagnetism and quantum mechanics that you develop in PHYS2003.

1.2. Content of PHYS2003

PHYS2003 comprises two modules, the first on magnetism and optical properties (lecturer: Dr David Gozzard) and the second on electronic material and devices (lecturer: A/Prof Vince Wallace).

Module 1 will cover topics including:

- Magnetic fields and magnetic force
- Ampere's law
- Electromagnetic induction and Faraday's law
- AC circuits (RC, RL, RLC circuit), complex impedance, phasors, and resonance
- Magnetic moment and magnetic dipole, magnetic susceptibility, diamagnetism, paramagnetism, Curie temperature
- Magnetic domains, B-H hysteresis, ferromagnetism, rare earth magnetic materials, and
- Basic optical properties of materials

2. Assumed prior knowledge

This unit assumes knowledge from MATH1011 (differentiation, integration and vector fields), MATH1012 (Fourier series and differential equations), and PHYS1001 (waves and optics, and electricity).

3. Magnetic fields and magnetic force

Understanding of magnetic fields and magnetic force is critical for applications such as electric motors, solenoids, speaker coils, actuators, etc. This will also lead on to understanding electromagnetic (EM) waves.

3.1. Producing a magnetic field

Electric field is represented by $\vec{E}$ and is produced by an electric charge. Magnetic fields, represented by $\vec{B}$ are produced in two ways.

3.1.1. Electromagnet

Moving charged particles create a magnetic field. We can use the current (movement of electrons) in a wire to create an **electromagnet**. This will be discussed in detail in lecture 2.

3.1.2. Permanent magnet

Elementary particles such as electrons have an *intrinsic* magnetic field that is a basic property of the particle, just as charge and mass are basic properties of particles. In certain materials (crystal structures) the magnetic fields of the electrons add to give a net magnetic field around the material, i.e. a **permanent magnet**. (In most materials, the magnetic fields of the electrons cancel each other out, leaving no net magnetic field.) This will be discussed further in lecture 12.

4. Defining $\vec{B}$

The electric field $\vec{E}$ at a given point in space is defined by putting a test particle of charge q at rest at that point and measuring the electric force $\overrightarrow{F_E}$ acting on the particle, giving $\vec{E}$ as

$$\vec{E} = \frac{\overrightarrow{F_E}}{q} \tag{1.1}$$

In contrast, we define $\vec{B}$ in terms of the magnetic force $\overrightarrow{F_B}$ exerted on a moving electrically charged particle. Charged particles with different velocity vectors $\vec{v}$ will experience different resulting forces. Particles moving along one particular axis will experience zero force. For all other particle velocities, we would find that the force $\overrightarrow{F_B}$ is always perpendicular to the direction of $\vec{v}$ (you should recognize this as meaning a cross product is involved), and that the magnitude of $\overrightarrow{F_B}$ is proportional to $v \sin\phi$ where ϕ is the angle between the zero-force axis and the direction of $\vec{v}$. The **magnetic field** $\vec{B}$ is defined to be a vector quantity directed along the zero-force axis.

The definition of $\vec{B}$ is summarized with the vector equation:

$$\overrightarrow{F_B} = q\vec{v} \times \vec{B} \tag{1.2}$$

That is, the force on the particle is given by the charge times the cross product of the velocity of the particle with the magnetic field.

Remembering the definition of the cross-product, the magnitude of $\overrightarrow{F_B}$ is

$$F_B = |q|vB\sin\phi \tag{1.3}$$

where B is the magnitude of the magnetic field and can be represented by

$$B = \frac{F_B}{|q|v} \tag{1.4}$$

The force $\overrightarrow{F_B}$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

Note that $\overrightarrow{F_B}$, being perpendicular to $\vec{v}$ *never* has a component parallel to $\vec{v}$ and so cannot change the particle's speed (and thus kinetic energy), only its direction of travel.

The SI unit for $\vec{B}$ is called the **tesla** (T).

$$1\ tesla = 1\frac{newton}{(coulomb)(meter/second)} \tag{1.5}$$

Recall that a coulomb per second is an *ampere*, this becomes

$$1\ T = 1\frac{newton}{(coulomb/second)(meter)} = 1\frac{N}{A{\cdot}m} \tag{1.6}$$

Another common (non-SI) unit for magnetic field is the **gauss** (G). 1 T = 10^4 gauss. The Earth's magnetic field at the surface is around 10^{-4} T (1 gauss), while a large electromagnet, such as used for picking up cars at scrap yards, is on the order of 1 T.

4.1. Finding the magnetic force on a particle

A simple way to remember the direction of the force on a moving particle due to a magnetic field is the "left-hand rule" shown in Figure 1 below. Note that this is for a **positively charged** particle (current is the direction in which positive charges flow) and the that direction of the force is reversed for a negative particle.

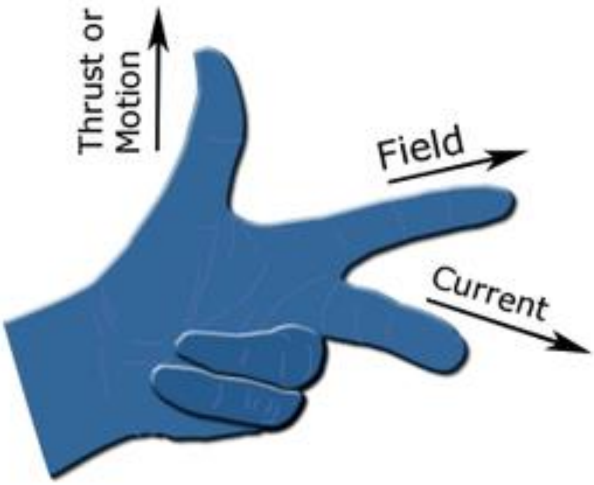

Figure 1: "Left-hand rule" for finding the direction of force due to a magnetic field. In this image, the middle finger is the velocity of the particle (current), the index finger is the

magnetic field, and the thumb gives the direction of the resulting force for *positively charged* particle. For a negatively charged particle, the "current" is in the opposite direction to the motion of the particle. (Image from Wikipedia.)

Note that, a particle moving parallel to the magnetic field will experience no force, and that a particle that is stationary relative to the magnetic field will also experience no force.

4.2. Magnetic field lines

Conventionally, we represent magnetic fields with field lines. The lines emerge from the north pole of the magnet and enter the magnet at the south pole. The direction tangent to a magnetic field line gives the direction of $\vec{B}$ at that point. The spacing of the lines represents the magnitude of $\vec{B}$ in that region. (Closer lines signify a stronger field.) Because a magnet has two poles it is called a **magnetic dipole**.

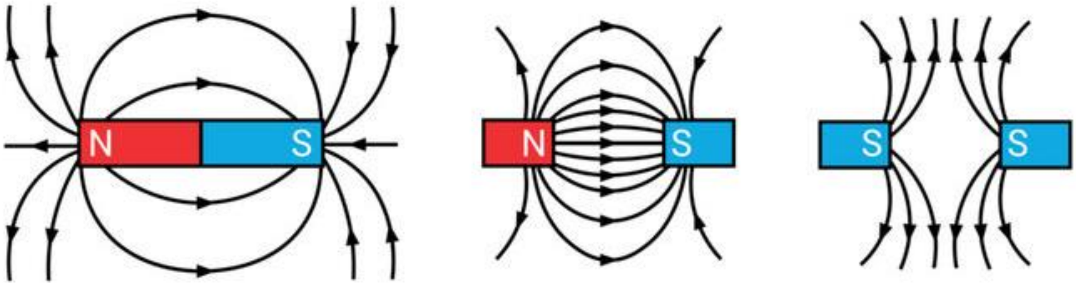

Figure 2: Magnetic field lines representing the magnetic field of bar magnets.

4.3. Example – magnetic force on a moving charged particle

In an experiment in the laboratory a magnetic field with magnitude 1.2 mT is directed vertically up (floor-to-ceiling) through a test chamber. A proton is fired horizontally through the chamber with a kinetic energy of 5.3 MeV. (The mass of the proton is 1.67×10^{-27} kg and an MeV = 1.60×10^{-13} J.)

(a) What is the magnitude of the force acting on the proton?

We get v from the kinetic energy $K = \frac{1}{2}mv^2$.

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3\ MeV)(1.60\times10^{-13}\ J/MeV)}{1.67\times10^{-27}\ kg}} = 3.2\times10^{7}\ m/s$$

$$F_B = |q|vB\sin\phi = (1.60\times10^{-19}\ C)\left(3.2\times10^{7}\ \frac{m}{s}\right)(1.2\times10^{-3}\ T)\sin 90$$
$$= 6.1\times10^{-15}\ N$$

(b) What is the acceleration of the proton?

$$a = \frac{F_B}{m} = \frac{6.1\times10^{-15}\ N}{1.67\times10^{-27}\ kg} = 3.7\times10^{12}\ m/s^2$$

(c) In which direction will the proton be deflected? (Assume you are standing at the side of the chamber the proton is launched from, and the direction is from your perspective.)

Answer: to the right from your perspective. (i.e. If the proton was fired south to north through the chamber, it will be deflected towards the east.)

5. Moving charged particles in a magnetic field

5.1. Cathode ray tubes and the discovery of the electron

Both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ can produce a force on a charged particle. Around 1897, J. J. Thomson used these effects in an experiment that resulted in the discovery of the electron.

Thomson create a *cathode ray tube* in which charged particles (now known to be electrons) were fired towards a phosphor screen. Before the screen, the electrons passed through electric and magnetic field that were set perpendicular to each other (so that both fields deflected the electrons in same plane).

Thomson would set $\vec{E}$ and $\vec{B}$ to zero and measure the position of the spot on the screen due to the charged particles hitting it. He would then turn on $\vec{E}$, deflecting the electrons in one direction (e.g. up). He would then turn on $\vec{B}$and adjust the strength of $\vec{B}$ to counteract the deflection caused by $\vec{E}$ (e.g. $\vec{B}$ was set to deflect the electrons back down).

From eqtn 1.1, the force on a charged particle due to an electric field is:

$$F_E = |q|E$$

When the $\vec{E}$ and $\vec{B}$ fields are adjusted to counteract each other, the relation is:

$$|q|E = |q|vB$$

which we can rearrange to

$$v = \frac{E}{B}$$

Thus, the speed of the particles could be measured. By measuring the distance of the deflection of the particles on the screen (and so calculating their acceleration) Thomson could determine the ratio of their mass to their charge. He knew that they were negatively charged, and found that, per unit of charge, they were lighter than hydrogen (the lightest atom) by a factor of more than 1,000. This is considered to be the discovery of the electron.

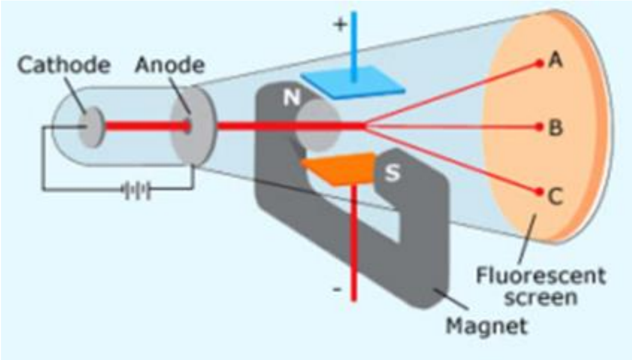

Figure 3: Simplified diagram of the J. J. Thomson experiment that discovered the electron.

5.2. The Hall Effect

In a conductor, when there is no current through it, the electrons move randomly with no net or average motion in any direction. When a current is passed through the conductor, the electrons continue to move randomly, but with an average **drift** in the opposite direction to the electric field causing the current. The drift speed v_d is very smaller compared with the speed of the electrons in their individual random motion. In a copper wire at room temperature, the electrons are randomly moving at around 10^6 m/s, while the drift speed is only around 10^{-4} m/s (i.e. on the order of centimetres per hour).

In 1879, Edwin H. Hall showed that the drifting conduction electrons in a copper wire can be deflected by an outside magnetic field. This is the **Hall Effect** and is used in a wide range of sensor technology.

Imagine a copper strip with current flowing down the page. The electrons then drift up the page. If we apply a magnetic field pointing into the page, the force due to the magnetic field will deflect the electrons to the right-hand edge of the strip.

This caused a greater density of electrons at the right-hand edge of the strip, and so a separation of charges that produces an electric field within the strip. The force on the electrons due to the electric field (equivalently, their own mutual repulsion) will eventually cancel the force due to the magnetic field, resulting in a maximum value of the electric field.

The electric field results in a potential difference across the strip, which can be measured with a voltmeter.

In addition to allowing us to determine things such as whether the moving charges within a conductor are positive or negative, and measure the drift speed of electrons (if we move the strip through the magnetic field with same speed as the electron drift, but in the opposite direction, the Hall Effect voltage will be zero), we can create a device where the output voltage is proportional to the strength of the magnetic field. Such a **Hall effect sensor** (or Hall sensor) has many applications.

Hall effect sensors can detect both the magnitude and orientation of a magnetic field, and they can work with static (unchanging) magnetic fields. They have no moving parts, so are not prone to wear, and can operate at much higher speeds than mechanical switches. Hall effect sensors have a wide range of applications in proximity sensing, positioning, speed detection, and current sensing. They are widely used in speed controllers for brushless DC motors because they are used to detect the position of magnets in the rotor.

5.3. Example – potential difference across a moving conductor

Imagine a metal cube in an x-y-z cartesian coordinate system. The cube has an edge length of d = 1.5 cm and is moving in the positive y-direction. A magnetic field of magnitude 0.050 T is oriented along the positive z-axis.

(a) Which faces of the cube are at the lower and higher electrical potential?

Applying eqtn 1.2 and/or the left-hand-rule, we find that the electrons experience a force in the negative-x direction. The free conduction electrons will move towards the negative-x side of the cube, making the negative-x face negatively charged and the positive-x face positively charged. The positive-x face is at a higher electric potential and the negative-x face at a lower electric potential.

(b) What is the potential difference between the faces of the cube?

The potential difference V due to an electric field of magnitude E across a strip of length d is:

$$V = Ed$$

So we need to find the magnitude of the electric field induced by the motion of the cube.

Once the cube is moving steadily and the electrons have settled to their new positions (i.e. the system has reached a steady state) the electrons are prevented from bunching further together by their mutual electrostatic repulsion, which is given by $|q|E$. At steady state, this force will balance out the force on the electrons due to the magnetic field, so we have:

$$|q|E = |q|vB \sin 90 = |q|vB$$

Which cancels out to:

$$E = vB$$

Thus, we get:

$$V = vBd = (4.0)(0.050)(0.015) = 0.0030 = 3.0\,mV$$

6. Summary of key formulae from lecture 1

Magnetic field

$$\overrightarrow{F_B} = q\vec{v} \times \vec{B} \quad (1.2)$$

$$F_B = |q|vB \sin\phi \quad (1.3)$$