Power and Machines

Study Guides for ENSC3016

Study Guide 1: Review of Electrical Engineering Fundamentals

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Introduction

This study guide provides a revision of the basics of Electrical Engineering. The first section is a review of basic circuit analysis techniques and its constraints. Section two is a review of time-varying signals and phasors. Section three discusses electric power followed by three-phase circuits

Review of Basic Circuit Analysis

Circuit analyses are techniques and axioms that help us to find the variables (voltage and current) in a typical electrical network. An electric network consists of nodes, which are junctions between two or more devices, and loops, which are closed path formed by tracing through an ordered sequence of nodes without passing through any node twice. Kirchhoff's voltage and current axioms state the constraints of network variables in connections. These axioms are expressed as:

Kirchhoff's Current Law - The algebraic sum of currents entering a node is zero at every instant in time.

$$\sum_{Node} i_k = 0 \qquad (1)$$

Kirchhoff's Voltage Law - The algebraic Sum of all voltages around a loop is zero at every instant in time.

$$\sum_{loop} v_k = 0 \qquad (2)$$

Another constraint on the network is originated from the devices, namely the $v - i$ characteristic of devices in each branch, i.e. connected between two nodes. The relation of current in each branch to the voltage of connected nodes are indicated by the characteristics of the elements in that branch. The $v - i$ characteristic of a resistor, capacitor, and inductor are depicted in the table1 below:

Table 1 the v-i characteristics of the lumped component

Ohm's Law	$v = iR$
Capacitor Equation	$i = C\frac{dv}{dt}$
Inductor Equation	$v = L\frac{di}{dt}$

The $v - i$ characteristic of elements could be linear or nonlinear, time-variant or time-invariant. However, the relation of voltage and current should satisfy the constraint.

Time-varying signals:

In most of power systems, we are dealing with sinusoidal variables with fixed frequencies. Most power systems run on 50Hz frequency (60 Hz in the US and Japan). Thus, it sounds reasonable to limit the circuit analysis of power systems to fixed frequency sinusoidal signals. This assumption based on the fact that the frequency will not change across the network even though some elements in the network are nonlinear. The time-varying signals could be expressed as:

$$v(t) = v_{max}\sin(\omega t + \theta_v) \qquad (3)$$

$$i(t) = i_{max}\sin(\omega t + \theta_i) \qquad (4)$$

In these relation, ω is the angular frequencies and θ_v and θ_i are the phase of voltage and current, respectively. It is also possible to represent the time-varying signals in the form of a phasor. A phasor can be represented as a complex number with real and imaginary components such that a phasor of magnitude (or length) R that is at a phase angle of θ with respect to the x-axis (the Real axis). It can be written as $\boldsymbol{R} = x + jy = R.e^{j\theta} = Rcos(\theta) + jRsin(\theta)$. Note that R is the magnitude and θ the phase of the phasor, and $R = \sqrt{x^2 + y^2}$ and $\theta = \arctan({}^{y}/_{x})$. The phasor can also be considered as a vector variable and could be shown graphically as in Fig.1.

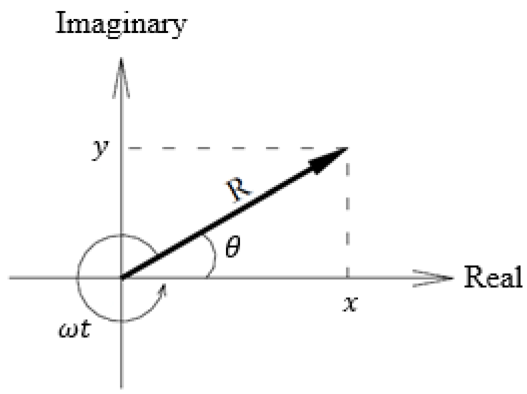


Figure 1 Vector Representation of a Phasor

For a sinusoidal voltage as $v(t) = v_{max}\cos(\omega t + \theta_v)$ the phasor representation is

$$\boldsymbol{V} = Ve^{j\theta_v} \qquad (5)$$

If the phasor is rotating counter-clockwise about the origin at a rate of ω radians per second, then we multiply the phasor by $e^{j\omega t}$ such that $\boldsymbol{V}e^{j\omega t} = Ve^{j\theta_v}e^{j\omega t}$. And consequently we can see that:

$$v(t) = Re\{\boldsymbol{V}e^{j\omega t}\} = V\cos(\omega t + \theta_v) \qquad (6)$$

The bold character is used to represent the phasor variable. The relation between the phasor of voltage and current could be written as:

$$\boldsymbol{V} = \boldsymbol{Z}\boldsymbol{I} \qquad (7)$$

Where $\boldsymbol{Z}$ is the impedance of the branch which is expressed as $\boldsymbol{Z} = R + JX$ where R is the resistance and X is the reactance of the element in the branch. The advantage of using phasors is that the differential equations are replaced by algebraic equations. These relations are shown in table 2. It should be noted that these relations are considering the steady states only.

Table 2 relation of phasor of voltage to the phasor of current for lumped elements

Ohm's Law	$\boldsymbol{V} = R\boldsymbol{I}$	
Capacitor Equation	$\boldsymbol{V} = -jX_C\boldsymbol{I}$	$X_C = \frac{1}{\omega c}$
Inductor Equation	$\boldsymbol{V} = jX_L\boldsymbol{I}$	$X_L = \omega L$

In the relation of the phasor, it is normal to use the root mean square (RMS) values instead of the peak value. The RMS value is defined based on the relation below:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T [V_{max}\cos(\omega t + \theta_v)]^2 dt} = \frac{V_{max}}{\sqrt{2}} \qquad (8)$$

Electric Power

The instantaneous power is the product of instantaneous current and voltage:

$$p(t) = v(t)\, i(t) = V_{max}\cos(\omega t + \theta_v) \,.\, I_{max}\cos(\omega t + \theta_i)$$

$$= 2V_{rms}\, I_{rms}\cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \qquad (9)$$

Using the relation $\cos(a) \,.\, \cos(b) = \frac{1}{2}(\cos(a+b) + \cos(a-b))$, we get:

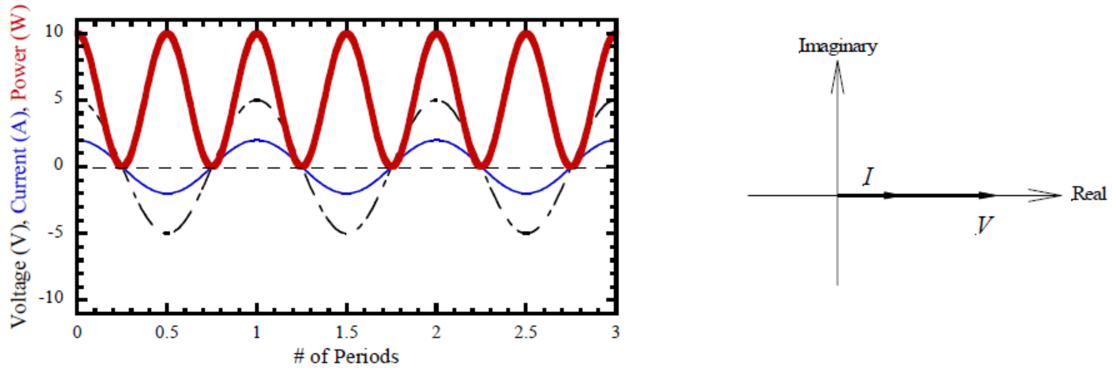
$$p(t) = V_{rms}\, I_{rms}[\cos(2\omega t + \theta_v + \theta_i) + \cos(\theta_v - \theta_i)] \qquad (10)$$

Looking at the relation of instantaneous power, it has an oscillating part and a time-independent part. If we take the average of power by integrating over a period, the average of the oscillating part would be zero. Thus the average power over a period would be:

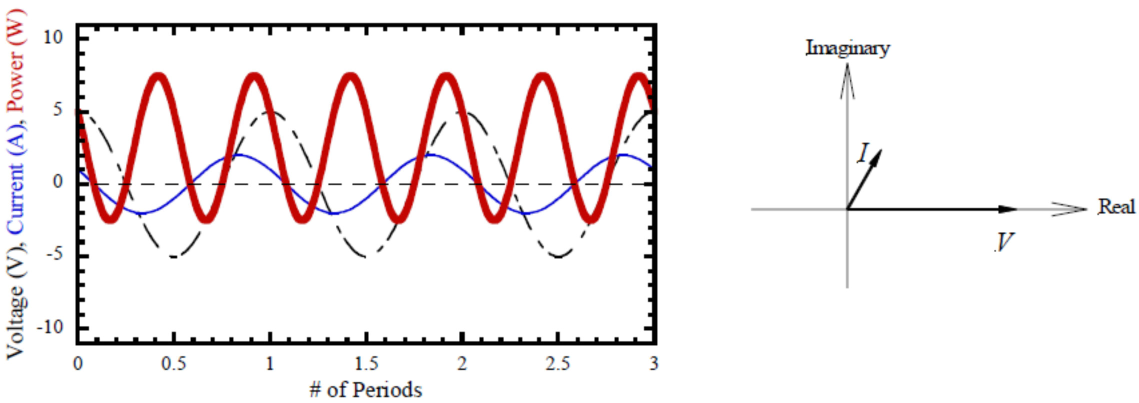
$$P = V_{rms}I_{rms}\cos(\theta_v - \theta_i) \qquad (11)$$

This is known as *real* power. The maximum real power is obtained when $\theta_v = \theta_i$ i.e., there is no phase difference between voltage and current. If the phase difference between the voltage and current is π/2 rad, the real power would be zero. However, the current is not zero. In order to better visualize the instantaneous and real power, the instantaneous power is plotted below based on different phases of voltage and current.

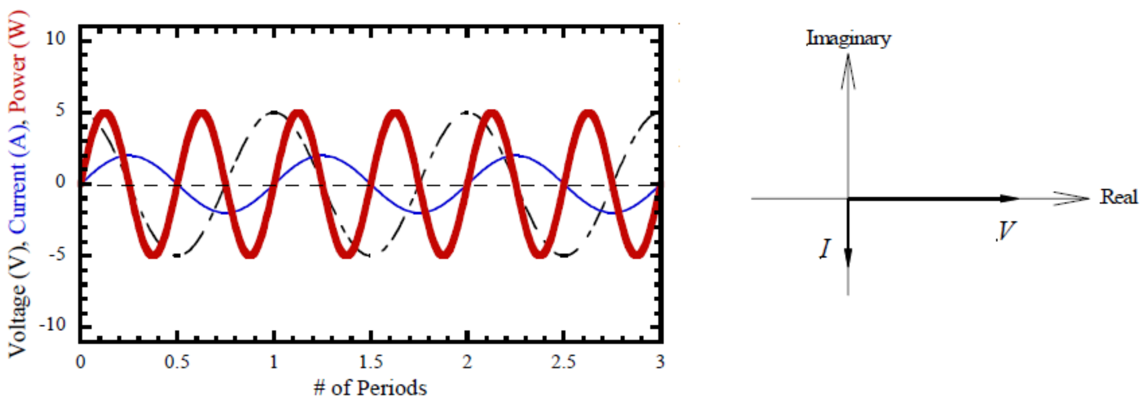
a) $\theta_v = \theta_i = 0$, in this case, the network is completely resistive and the average power is maximum. As seen in the picture, the instantaneous power is not negative over the whole cycle.



b) $\theta_v = 0\,, \theta_i = \frac{\pi}{3}$, in this case, a chunk of instantaneous power is negative in each cycle, the real power is decreased by the ratio of $\cos(\frac{\pi}{3})$



C) $\theta_v = 0\,, \theta_i = -\frac{\pi}{2}$. In this case, the instantaneous power curve is symmetrical over the horizontal axis, and consequently, the real power is zero.



Since the instantaneous power relation is cumbersome to deal with, the power relation is defined is a form of vectors as follow:

$$\boldsymbol{S} = \boldsymbol{P} + j\boldsymbol{Q} \qquad (12)$$

In the relation above, the vector $\boldsymbol{S}$ is known as apparent power and its unit is Volt Ampere (VA), $\boldsymbol{P}$ is the real or active power, and its unit is Watt (W), and $\boldsymbol{Q}$ is known as reactive power and its unit is Volt-Amp Reactive (VAR). The relation of the amplitude of apparent and reactive power are:

$$S = V_{rms} I_{rms} \quad VA \qquad (13)$$

$$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i) \quad VAR \qquad (14)$$

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \quad W \qquad (15)$$

These three victors create the triangle of power as shown in Fig.2.

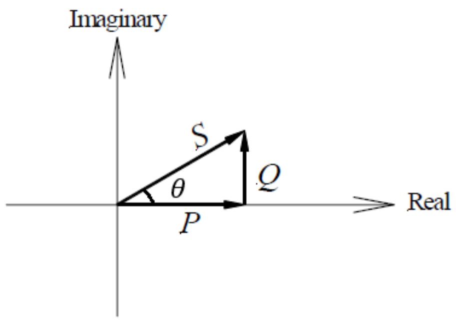


Figure 2 the power triangle

As can be seen in Fig.2. that the angle of P is always zero and the angle of Q is always $\pm\frac{\pi}{2}$. It is evident from that:

$$S = \sqrt{P^2 + Q^2} \qquad (16)$$

$$Q = S\sin(\theta) \qquad (17)$$

$$P = S\cos(\theta) \qquad (18)$$

Where, $\theta = \theta_v - \theta_i$ is the phase angle between voltage and current, which is also the angle of the impedance of the load ($\boldsymbol{Z} = \boldsymbol{z}\angle\theta$). If $\boldsymbol{Z}$ is inductive, this angle is positive and if it is capacitive this angle is negative. For inductive load the phase of current is smaller than voltage or in other words, the current lags the voltage. For capacitive loads, the current will lead the voltage. Sometimes instead of inductive and capacitive, the terms lagging and leading are used respectively. From the power relation, it could be inferred that the reactive power is related to the inductive or capacitive part of the impedance and the active power is related to the resistive part of the impedance and the apparent power. This the relation of power could be rewritten in terms of current and impedance.

$$P = I^2\, Z\cos(\theta) = I^2\, R \qquad (19)$$

$$Q = I^2\, Z\sin(\theta) = I^2 X \qquad (20)$$

$$S = I^2 Z \qquad (21)$$

Also, the apparent power could be expressed based on the voltage and current of the source.

$$\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* = V\angle\theta_v . I\angle{-\theta_i} \qquad (22)$$

The term $\boldsymbol{I}^*$ is the conjugate phasor of the current to produce the negative of the phase angle.

It should be noted that the reactive power which is created by the component of the current that is in 90 degrees out of phase with the source voltage is not desirable. In fact, this represents the power that is continually bounced back and forth between the source and the reactive part of the load in every cycle. In other words, this power is transferred from source to the load and is stored in capacitor or inductor (in the form of the electric field for capacitor and magnetic field in inductor). The sored power is then transferred from load to the source and consequently, the average power fetched from source would be zero. Normally electric loads like electric motors have inductive characteristics and render the current to be lagging. To compensate this lagging current, the power network operators usually install a capacitor in a certain location of the grid to compensate this inductive load and lagging current.

Three Phase systems

Most of the electrical generation and transmission systems are in the form of three-phase AC systems. The three-phase AC systems have to major advantages over single-phase systems. (1) they deliver more power compared to a single-phase machine with the same amount of metal used in machine and lines. (2) the delivered power of the three-phase system is constant at all times compared to pulsing power in single-phase systems.

A three-phase voltage is generated using three single-phase sources with the same amplitude and 120° phase difference. Thus, the voltage relation of the three phases would be:

$$v_A(t) = v_{max}\sin(\omega t) \qquad (23)$$

$$v_B(t) = v_{max}\sin(\omega t - 120) \qquad (24)$$

$$v_C(t) = v_{max}\sin(\omega t - 240) \qquad (25)$$

And in phasor form:

$$\boldsymbol{V_A} = V\angle 0 \qquad (26)$$

$$\boldsymbol{V_B} = V\angle - 120 \qquad (27)$$

$$\boldsymbol{V_C} = V\angle - 240 \qquad (28)$$

The same relations could be written for currents in a three-phase system. The vector representation of three-phase voltages is shown in Fig.3.

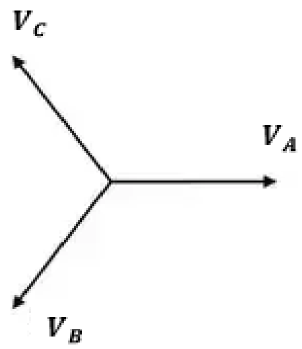


Figure 3 Phasor diagram of three phases, abc phase sequence

It is evident that due to the 120° phase difference, the summation of three voltages would be zero. So we can write: $\boldsymbol{V_A + V_B + V_C = 0}$

In the three-phase system shown in Fig.3, the phase sequence is *abc*. The phase sequence indicates the order at which the three-phase will have their peak. Another form of phase sequence is acb. The phasor diagram of such a system is shown in Fig.4.A three-phase generator has three windings which have 120° physical displacement. Thus, we will have three terminals (6 wires) coming out of a three-phase generators. There are two possible connections of theses terminals which are discussed in the next section.

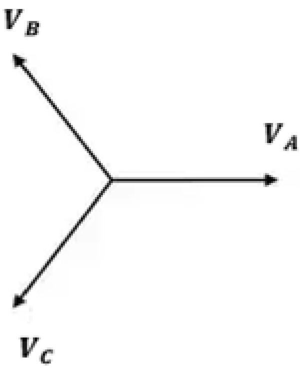


Figure 4 phasor diagram of three phases, acb phase sequence

Y and Δ Connection:

Fig.5 shows the two different ways of connecting the three single-phase sources to form a three-phase system. In Fig.5a. the Start or wye connection is shown where the neutral terminals of each source are connected, and the other terminal is connected to each line of the three-phase system. Fig.5b. shows the delta connection where the neutral of each source is connected to the phase of another source, and the lines of the three-phase system are connected to these connection points.

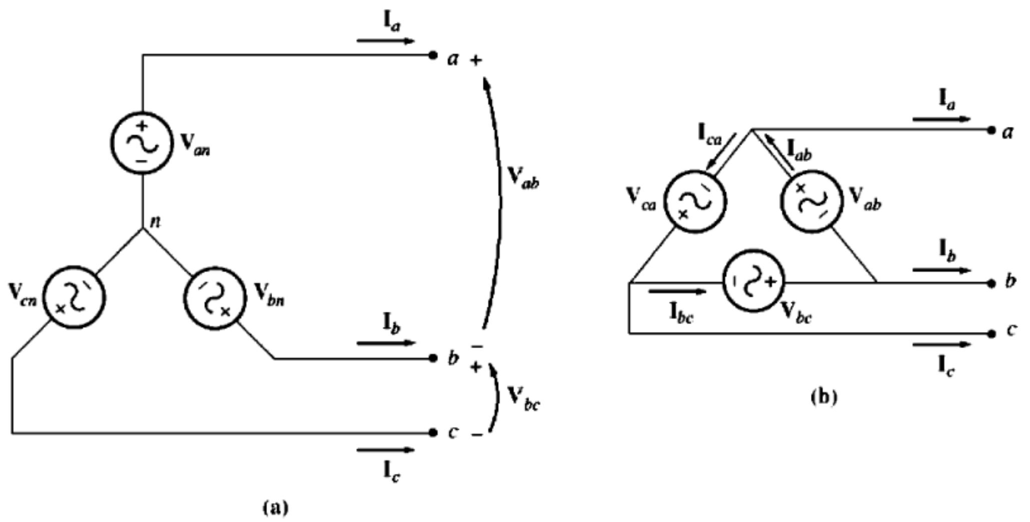


Figure 5 a) Y connection b) Δ connection

As could be seen in Fig.5. in Y connection a neutral point (n) exists. By looking at the output terminals of a three-phase system, it is possible to have two different value for voltage. If the voltage between each line and the neutral point is considered ($\boldsymbol{V_{an}}, \boldsymbol{V_{bn}} \boldsymbol{V_{cn}}$), the voltage is called phase voltage and is indicated by $\boldsymbol{V_{\phi}}$. If the voltage between the two lines is considered ($\boldsymbol{V_{ab}}, \boldsymbol{V_{bc}} \boldsymbol{V_{ca}}$), the voltage is called line voltage and is indicated by $\boldsymbol{V_L}$. Since there is no neutral point in delta connection it is not possible to have the phase voltage. The same notation could be used for currents in phases and lines as well. The lines and phases are shown in Fig. 6. where a Y connected source is connected to a resistive load.

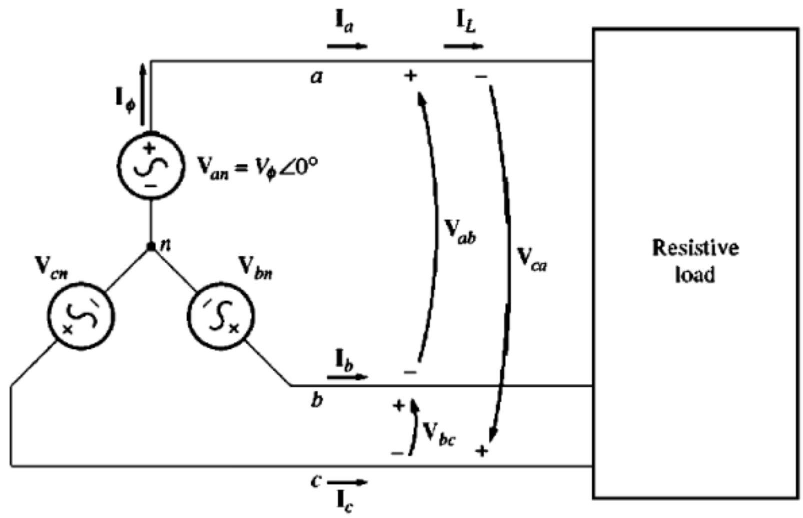


Figure 6 Y connected generator with resistive load

Looking at Fig.6. it is possible to find the relation of phase and line quantities. It is evident that:

$$\boldsymbol{I_L} = \boldsymbol{I_\phi} \quad for\ Y - cnencted \quad (29)$$

Also, we can write:

$$\boldsymbol{V_{ab}} = \boldsymbol{V_a} - \boldsymbol{V_b} = V_\phi\angle 0° - V_\phi\angle 120° = \sqrt{3}V_\phi\angle 30° \quad (30)$$

This we can conclude that:

$$V_L = \sqrt{3}V_\phi \quad for\ Y - cnencted \quad (31)$$

It is notable that in Eq.31. the relation is for the amplitude of line and phase voltage and not the vectors. From Eq.30. it is clear that the line voltages are at 30° phase different with phase voltages. These phasor diagrams of these voltages are shown in Fig.7.

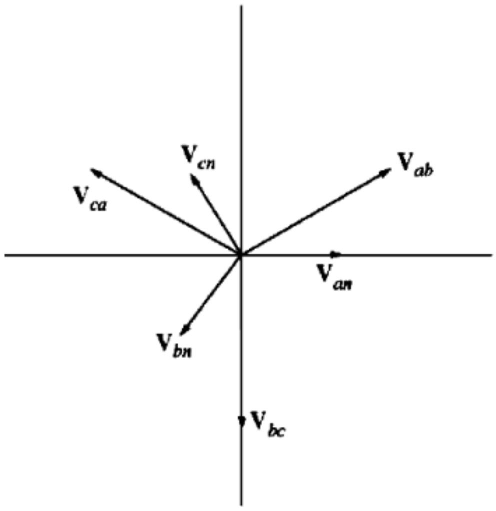


Figure 7 Line and phase voltages for Y connected sources

Same procedures could be done for delta connected and the following relation could be derived.

$$\boldsymbol{V_L} = \boldsymbol{V_\phi} \quad for\ \Delta - cnencted \quad (32)$$

$$I_L = \sqrt{3}I_\phi \quad for\ \Delta - cnencted \quad (33)$$

It is worth noting that the load connection could also be one of the two Y or Δ connection, as shown in Fig.8.

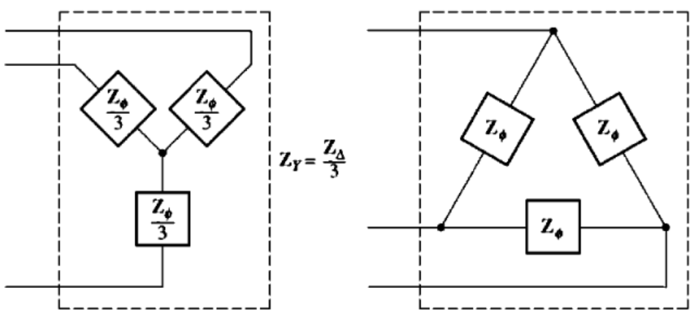


Figure 8 Y and Δ connected load

As could be seen in Fig.4. it is possible to transform a delta connected load to a star connected by scaling the load impedance by the relation: $\boldsymbol{Z_Y} = {}^{\boldsymbol{Z_\Delta}}/_{\boldsymbol{3}}$.

Now consider a balanced Y-connected three-phase load as in Fig.9.

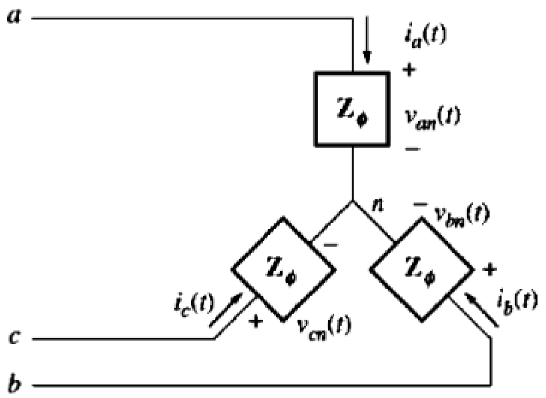


Figure 9 A balanced Y-connected Load

The word balanced means the load and the parameters of the lines are exactly the same for all the phases. The instantaneous power supplied to one phase is given by:

$$p(t) = v(t)\, i(t)$$

So for each phase, we can write:

$$P_a(t) = V_{an}(t) i_a(t) \sin(\omega t) \sin(\omega t - \theta) \qquad (34)$$

Where θ is the phase difference of voltage and current of each phase. Using trigonometric relations, Eq.34 could be sorted to:

$$P_a(t) = VI[cos\theta - cos(2\omega t - \theta)] \qquad (35)$$

And knowing the $120°$ phase different for other phases we can write:

$$P_b(t) = VI[cos\theta - cos(2\omega t - 240° - \theta)] \qquad (36)$$

$$P_c(t) = VI[cos\theta - cos(2\omega t - 480° - \theta)] \qquad (37)$$

The power in each phase has two parts. A constant part and the pulsing part. The total power would be the sum up of the power in each phase, so that:

$$P_{total}(t) = P_a(t) + P_b(t) + P_c(t) \qquad (38)$$

The pulsing component of the three instantaneous powers will cancel each other's out. This is due to $120°$ phase different of these oscillating component. Thus the total power supplied to balanced three-phase loads is always constant as shown in Fig.10. It is evident from Fig.10 that the summation of all three oscillating powers would be a constant over all time.

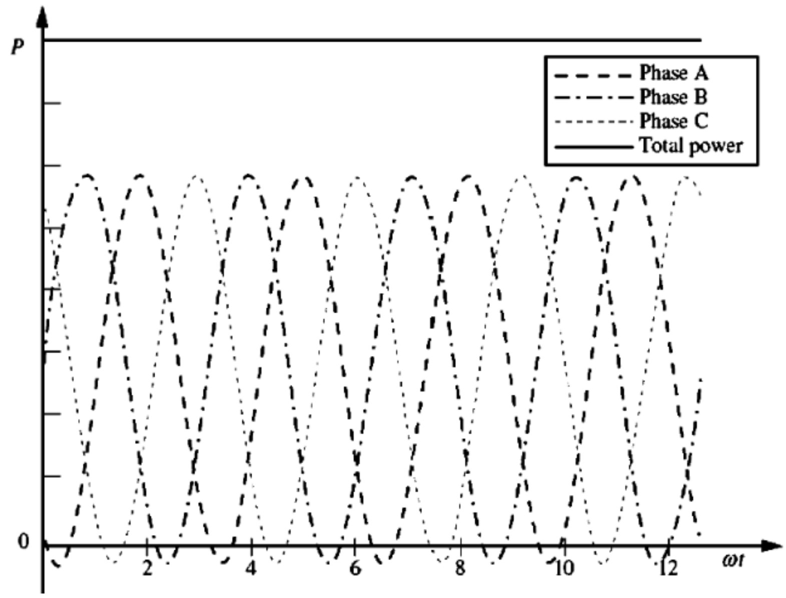


Figure 10 instantaneous power in three phases and the total power

From Eq.35, we can find the active power of each phase as:

$$P_a = V_\phi I_\phi cos\theta \qquad (39)$$

So the total active power of the three systems would be:

$$P = 3V_\phi I_\phi cos\theta \qquad (40)$$

Similarly, the relation of the reactive and apparent power of the three-phase system would be:

$$Q = 3V_\phi I_\phi sin\theta \qquad (41)$$

$$S = 3V_\phi I_\phi \qquad (42)$$

Knowing that for Y- connected load $V_L = \sqrt{3}V_\phi$ we can write the relation of powers based on line voltage and current:

$$P = \sqrt{3}V_L I_L cos\theta \qquad (43)$$

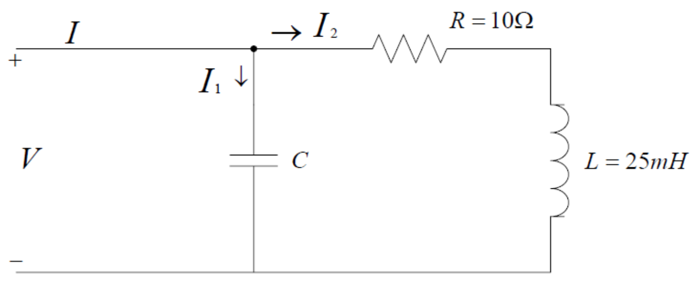
And also:

$$Q = \sqrt{3}V_L I_L sin\theta \qquad (44)$$

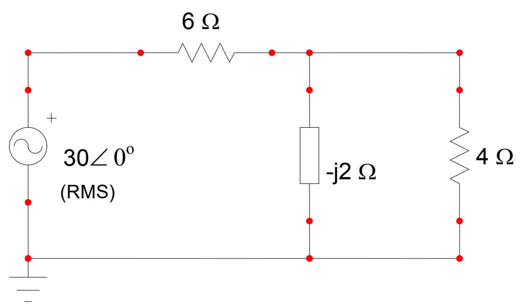
$$S = \sqrt{3}V_L I_L \qquad (45)$$

Exercises:

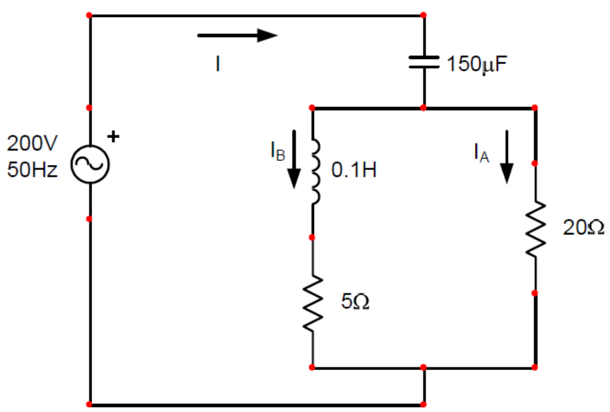
1-1 The circuit of figure below is excited by a 240-V, 60-Hz sinusoidal source. If C=10μF, determine: (240-V is RMS value)
(a) The magnitude of current ***I***
(b) The average value of power dissipated by resistor ***R***



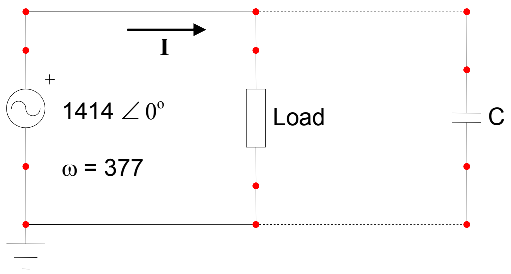
2-1 Calculate the power factor of the entire circuit of the figure below as seen by the source. What is the average power supplied by the source?



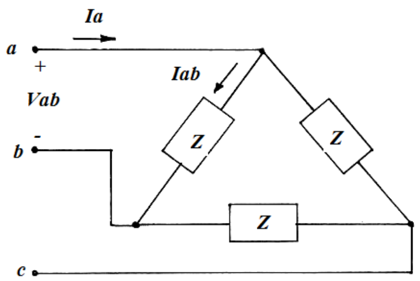
3-1. A network is arranged as indicated in the figure below, the values being as shown. Calculate the value of the current in each branch and its phase relative to the supply voltage. Draw the complete phasor diagram.



4-1 Consider the situation shown in the figure below. A 1000-V rms source delivers power to a load. The load consumes 100 kW with a power factor of 25% lagging.
(a) Find the phasor **I** assuming that the capacitor is not connected to the circuit.
(b)Find the value of the capacitance that must be connected in parallel with the load to achieve a power factor of 100 percent. Usually, power-systems engineers rate capacitances used for power-factor correction in terms of their reactive power rating. What is the rating of this capacitance in kVAR? Assuming that this capacitance is connected, find the new value of the phasor **I**.
(c)Suppose that the source is connected to the load by a long distance. What are the potential advantages and disadvantages of connecting the capacitance across the load?



5-1. A set of balanced three-phase voltages is impressed on a balanced, three-phase, delta-connected load. If $V_{ab} = 7200\angle 0°$ V and $\boldsymbol{I}_a = 12\angle 0°$ A. Determine:
(a) The phase current $\boldsymbol{I}_{ab}$
(b) The total average power supplied to the load.



6-1 Figure below shows a three-phase power system with two loads. The **Δ**-connected generator is producing a line voltage of 480V, and the line impedance is $0.09 + j0.16\ \Omega$. Load1 is Y-connected, with a phase impedance of $2.5\angle 36.87°\ \Omega$ and load 2 is **Δ**-connecte, with a phase impedance of $5\angle -20°\ \Omega$.

(a) What is the voltage of the two loads?
(b) What is the voltage drop on the transmission line?
(c) Find the real and reactive powers supplied to each load
(d) Find real and reactive power losses in the transmission line.
(e) Find the real power, reactive power, and power factor supplied by the generator.