Electrical Machines and Systems

Part 1: Electromagnetism and Transformers

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1. Introduction

These notes are the first part of course notes prepared for the unit Power and Machines in the Department of Electrical and Electronics Engineering at the University of Western Australia.

The aim of the first part of the course note is to develop an understanding of fundamental electromagnetic principles. These concept make the foundation of electric machines which would be discussed in the second part.

In the second chapter the effect of magnetic field on current carrying conductor in from of induced voltage is discussed. It also discusses the developed forced on an iron from magnetic field. The following chapter discusses the magnetic field and representation of magnetic field intensity in form of flux density. Chapter 4 introduces equivalent magnet circuits as a technique to analyse sophisticated magnetic structures. Chapter 5 expands on the concept of magnetic induction and chapter 6 discusses structure and applications of Transformers.

Acknowledgment:

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2. Magnetic Effect:

2.1 Conductor in a magnetic field:

Magnets have the remarkable property of affecting other objects without physical contact. These effects are described in terms of the *magnetic field* of the magnet, measured by the *magnetic flux density* **B**. This is a vector quantity, which has both magnitude and direction.

Suppose that the magnetic field is uniform, so that **B** has a constant magnitude and direction. Consider a straight conductor of length *l* placed in this field, in a direction perpendicular to **B**, as shown in figure 2-1

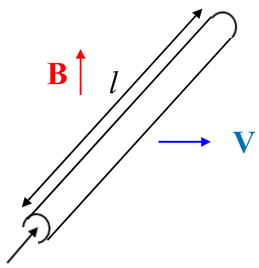


Figure 2-1 Conductor in a magnetic field.

Two effects may be observed:

- If the conductor moves with a velocity $\boldsymbol{v}$ perpendicular to both the conductor and the field, there will be a voltage *e* generated in the conductor, given by:

  $$e = (\boldsymbol{v} \times \boldsymbol{B}) \,.\, \boldsymbol{L} \qquad [V]$$

  For the directions in the figure we get ($\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$,$sin(90°) = 1$):

  $$e = v\,Bl \qquad [V] \qquad (2-1)$$

- If the conductor carries a current *i*, there will be a force **F** exerted on the conductor, perpendicular to both the conductor and the field. The magnitude is given by:

  $$\boldsymbol{F} = i(\boldsymbol{L} \times \boldsymbol{B}) \qquad [N]$$

  For the directions in the figure we get ($i\boldsymbol{L}$ is perpendicular to $\boldsymbol{B}$,$sin(90°) = 1$):

  $$f = Bli[N] \qquad (2-2)$$

With the direction of i shown in figure 2-1, the direction of **F** is the same as the direction of $\boldsymbol{v}$. It should be noted that $\times$ represent the cross product of the two vector and . the scalar product of the two vector.

Both of these effects have wide applications in engineering. Moving-coil devices such as loudspeakers and microphones use these effects in a direct way. Motors and generators are also based on these effects, although most of the force is usually exerted on steel parts rather than the conductors themselves.

2.2 Lorentz Equation:

Equations 2-1 and 2-2 are derived from one of the most fundamental equations of electromagnetism: the Lorentz equation for the force **f** on a moving charge *q*:

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})\ [\mathrm{N}] \qquad (2-3)$$

Where q is the charge in coulombs, **E** is the *electric field strength* in volts/metre, $\boldsymbol{v}$ is the velocity in metres/second, and **B** is the *magnetic flux density* in tesla (T).

The force on a charge thus has an electric component:

$$\boldsymbol{F}_e = q\boldsymbol{E} \qquad (2-4)$$

And a magnetic component:

$$\boldsymbol{F}_m = q\boldsymbol{v} \times \boldsymbol{B} \qquad (2-5)$$

The direction of the magnetic component is given by the right-hand screw rule of the vector product. The force $\boldsymbol{F}_m$ is perpendicular to the plane containing $\boldsymbol{v}$ and **B**, with a clockwise rotation from $\boldsymbol{v}$ to **B** when viewed in the direction of $\boldsymbol{F}_m$.

Both force components can be seen in the deflection of the electron beam in a cathode-ray tube (CRT) [1]. CRTs in oscilloscopes use electrostatic deflection, exploiting $\boldsymbol{F}_e$. CRTs in video monitors and TV sets use magnetic deflection, exploiting $\boldsymbol{F}_m$.

2.2.1 Force on a conductor:

An electric current flowing in a wire represents charge in motion, so the moving charges experience a $\boldsymbol{v} \times \boldsymbol{B}$ force when the wire is placed in a magnetic field. Consider a small length of wire $\boldsymbol{dl}$, carrying a current i, as shown in figure 2-2.

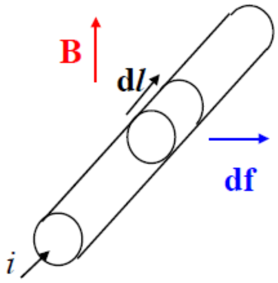


Figure 2-2 Forces in an infinitesimal part of conductor

If dq is the charge in the length $\boldsymbol{dl}$, and $\boldsymbol{v}$ is the velocity, the force acting on the current element is:

$$d\boldsymbol{F} = dq\,(\boldsymbol{v} \times \boldsymbol{B}) \qquad (2-6)$$

The charge will travel a distance $\boldsymbol{dl}$ in a time dt so the velocity is given by:

$$\boldsymbol{v} = \frac{\boldsymbol{dl}}{dt} \qquad (2-7)$$

Since current is the rate of flow of charge, we have:

$$i = \frac{dq}{dt}\ or\ dq = idt \qquad (2-8)$$

Substituting for $\boldsymbol{v}$ and dq in equation 2-6 gives:

$$d\boldsymbol{F} = i\boldsymbol{dl} \times \boldsymbol{B} \qquad (2-9)$$

The force on a conductor of length l may be found by integration. If the field is uniform and perpendicular to the conductor, then the magnitude of the force is given by equation 2-2:

$$f = Bli \qquad [2-2]$$

And the direction is given by equation 2-9.

2.2.2 Voltage induced in a conductor

Consider the conductor shown in figure 2-1. Any free charge in the conductor will experience a force$q\ (\boldsymbol{v} \times \boldsymbol{B})$, and will therefore start to move. Negative electrons will move towards one end, leaving behind a surplus of positive charge. This separation of charge produces an electric field $\boldsymbol{E}$ inside the conductor, with a force $q\boldsymbol{E}$ on the charge, and the process continues until there is equilibrium:

$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B} \qquad (2-10)$$

This is illustrated in figure 2-3

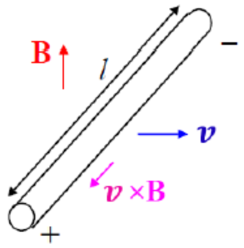


Figure 2-3 Charge Separation

If the flux density $\boldsymbol{B}$ is uniform and perpendicular to the conductor, the voltage between the ends of the conductor is given by:

$$e = \int \boldsymbol{E}\,.\,\boldsymbol{dl} = \int \boldsymbol{uB}\,.\,\boldsymbol{dl} = uB\int dl = Blu \qquad (2-11)$$

This is the formal derivation of equation 2-1.

2.3 Magnetic Forces on iron:

A different kind of magnetic effect is seen in magnets for exerting forces on objects made of iron or steel. Figure 2-4 shows an electromagnet with two coils on a U-shaped steel core attracting a steel object or *armature*. In most applications, as in this case, there is a gap between each of the magnet poles and the armature. Gaps of this kind between magnetic components are termed *airgaps*. Currents flowing in the magnet coils set up a magnetic field in the surrounding space. This field is most intense in the airgaps, resulting in a large force of attraction.

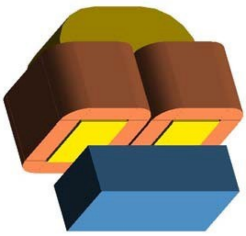


Figure 2-4 U- Core electromagnet

Figure 2-5 shows a computer-generated magnetic field plot, or *flux plot*, for the electromagnet, where the lines give a visual representation of the invisible magnetic field. The direction of the lines gives the direction of the field, and the spacing is an indication of the magnitude: the closer the lines, the greater the magnitude

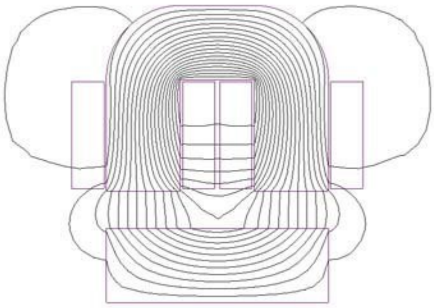


Figure 2-5 : Flux plot for U-core electromagnet

In each airgap, the magnetic field is almost uniform and at right angles to the surface of the armature.

When the magnetic field is uniform and at right angles to an iron surface, the force of attraction per unit area is given by:

$$\frac{f_m}{A} = {}^1\!/_2 \frac{B^2}{\mu_0} \left[\frac{N}{m^2}\right] \qquad (2-12)$$

Where f_m is the magnetic force and A is the area perpendicular to **B**. The quantity μ_0 is the *primary magnetic constant*, with a value of $4\pi \times 10^{-7}$ H/m. A formal derivation of this equation is given in part section 2, and a method of calculating the value of B is given in section 4.5.

Equation 2-12 can be applied to the electromagnet as follows. If B_g is the flux density in the airgap, f_p is the force of attraction on one pole, and A_p is the cross-sectional area of the pole, then equation 2-12 gives:

$$\frac{f_p}{A_p} = {}^1\!/_2 \frac{{B_g}^2}{\mu_0} \qquad (2-13)$$

The total force of attraction is thus:

$$f_t = 2f_p = \frac{{B_g}^2 A_p}{\mu_0} \qquad (2-14)$$

Magnetic forces can be very large. It is possible to design an electromagnet with a flux density of 1.0 T in the airgap, which gives the following force per unit area:

$$\frac{f_m}{A} = {}^1\!/_2 \frac{B^2}{\mu_0} = \frac{(1)^2}{2 \times 4\pi \times 10^{-7}} \approx 4 \times 10^5 \left[\frac{N}{m^2}\right] \qquad (2-15)$$

This represents a force density of about four times the normal atmospheric pressure.

2.3.1 Fringing effect and leakage:

Notice two features of the magnetic field in this electromagnet. First, the magnetic field in the airgap between the poles and the armature is not confined to the pole region, but spreads into the surrounding air; this is termed *fringing*. Secondly, some flux takes a short cut across the space between the poles, instead of crossing the airgap and passing through the armature; this is termed *leakage*.

2.3.2 Applications:

In addition to the obvious applications in lifting magnets and devices like relays, electromagnets with electronic control can be used to eliminate mechanical contact in moving systems [2]. Figure 2-6 shows a magnetic bearing using this principle, and figure 2-7 shows the German Transrapid advanced transport system where magnetic suspension takes the place of wheels.

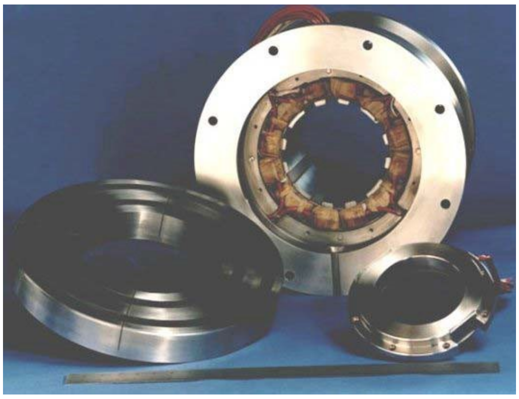


Figure2- 6 Magnetic Bearing (Federal-Mogul Magnetic Bearings)

Figure 2-7 Advanced transport system with

2.4 Permanent Magnet:

Figure 2-8 shows a permanent magnet attracting a steel plate, and figure 2-9 shows the corresponding flux plot. In the electromagnet of figure 2-3, the source of the magnetic field is the current in the coils wound on the poles. In the permanent magnet of figure 2-8, the source of the magnetic field is a block of permanent-magnet material.

Figure 2-8 Permanent Magnet

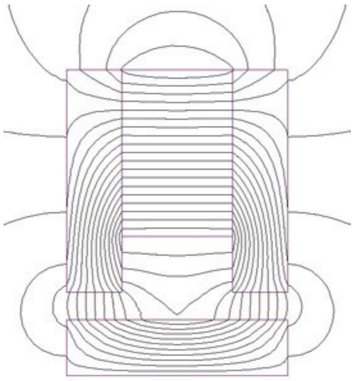


Figure 2-9 Flux plot for permanent magnet.

In many applications, permanent magnets incorporate steel poles, as in figure 2-8, because the performance can be improved significantly. However, before this can be explained, it is necessary to consider the concept of magnetic flux.

2.4.1 Magnetic Flux;

A basic concept in electromagnetism is the *magnetic flux*. If the magnetic flux density **B** is uniform and perpendicular to a flat surface of area A, then the magnetic flux ϕ is defined to be:

$$\phi = B.A \qquad (2-16)$$

Thus, the flux density magnitude B is the flux per unit area. In general, if **B** is not uniform and perpendicular to the surface, the flux is given by a surface integral:

$$\phi = \int \boldsymbol{B}.\boldsymbol{dA} \qquad (2-17)$$

But the simpler form of equation 2-16 will be adequate for this course (based on the assumption that the magnetic field is uniform and always at right angle to the surface).

In a flux plot such as figure 2-9, the curved lines are actually the contours of a flux function that represents flux per unit depth. If $\Delta\phi$ is the change in flux between two successive lines, and Δs is the perpendicular distance between the lines, then from equation 2-16 the flux density is:

$$B \approx \frac{\Delta\phi}{\Delta A} = \frac{\Delta\phi}{\Delta s} \qquad (2-18)$$

There are equal increments of flux $\Delta\phi$ between successive pairs of lines, so widely spaced lines indicate a low flux density, and tightly packed lines indicate a high flux density.

2.4.2. Flux (Gauss Law)

Associated with the concept of flux is the *flux law*, which may be stated as follows:

- For any closed region in the magnetic field, the total flux entering the region is equal to the total flux leaving.

This is analogous to Kirchhoff's current law in electric circuit theory, or to the behaviour of an incompressible fluid, where the total flow into a region is equal to the total flow out. Although nothing is actually flowing in the magnetic field, the flow analogy is useful, and is implied by the name flux.

A useful application of the flux law is to find the relationship between the flux density in the steel and the flux density in the airgap for a device such as the magnet in figure 2-8. The airgap region is shown in more detail in figure 2-10.

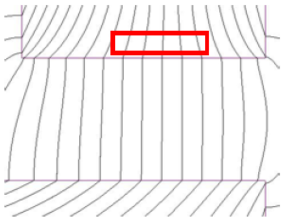


Figure 2-10 Flux plot for the airgap region

Consider a closed region shown by the red rectangle in figure 2-10. The rectangle represents the cross-section of a box that has the same depth as the magnet poles perpendicular to the plane of the diagram. The top face of the box is in the steel pole, and the bottom face is in the airgap. Flux enters the top face of the box in the steel, and leaves the bottom face in the air. The direction of the field is nearly perpendicular to these faces of the box, so there is negligible flux out of the sides of the box. Let B_s be the value of the flux density in the steel, and B_g the value of the flux density in the airgap. Since both faces have the same area A, we have:

$$\phi = B_s A = B_g A \qquad (2-19)$$

It follows that the two flux density values are equal. This is an important result, which is only valid when the field is perpendicular to the surface.

2.4.3 Flux Concentration:

In figure 2-9, there is a good deal of flux leakage between the poles and into the air behind the magnet. If the airgap is made very small, the leakage disappears, and all the flux from the permanent magnet is directed through the poles to the armature, as shown in figure 2-11.

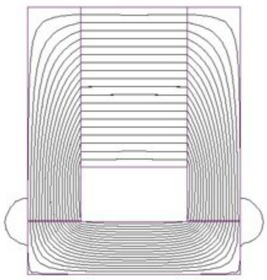


Figure 2-11Flux plot with a small airgap

Let A_m be the cross-sectional area of the magnet, perpendicular to the flux, as shown in figure 2-12, and A_p the corresponding pole area.

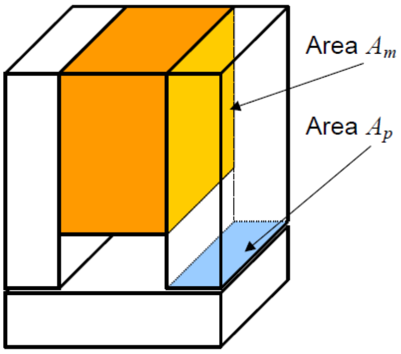


Figure 2-12 Magnet and pole areas.

The flux ϕ leaving the magnet through the area A_m enters the pole, and then leaves the pole through the area A_p. From the flux law:

$$\phi = B_m A_m = B_s A_p \qquad (2-20)$$

Where B_s is value of the flux density in the steel at the bottom of the pole. From equation 2-18, the flux density in the airgap is $B_g = B_s$. Thus:

$$B_g = B_s = \frac{A_m}{A_p} B_m \qquad (2-21)$$

If the pole area A_p is made smaller than the magnet area A_m, equation 2-20 shows that the flux density in the airgap will be greater than the flux density in the magnet. This is the *flux concentration* effect of steel poles.

It is shown in section 4.6 that the magnet flux density B_m will remain constant under these conditions, so that reducing the area A_p will increase the airgap flux density B_g. The effect is to increase the force of attraction, for the following reason. From equation 2-14, the force is:

$$f_t = \frac{{B_g}^2 A_p}{\mu_0} \qquad [2-14]$$

Suppose that the area A_p is halved by reducing the pole thickness. From equation 2-20, the value of B_g will be doubled, so the value of ${B_g}^2$ will be quadrupled. Therefore, the total force will double.

This method of increasing the force is used in the magnetic door catch shown in figure 2-13, which has narrow poles to concentrate the flux from an inexpensive ceramic ferrite permanent magnet.

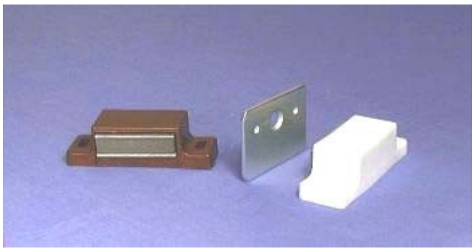


Figure2- 13 Magnetic door catch

2.4.4 Properties of permanent magnets:

The relative advantages of permanent magnets and electromagnets can be summarised as follows:

Electromagnets	**Permanent magnets**
Require a current supply	Do not require a supply
Heat generated in coil	No heat generated
Field is controllable	Field is fixed*
Can be turned off	Cannot be turned off*

The field can only be varied or the magnet turned off by changing the mechanical configuration

2.4.5. Applications

Permanent magnets are widely used for providing the working flux in moving-coil devices such as loudspeakers, and in small DC and AC motors (section 3 and 4 part 2). They are also used in magnetic holding devices and lifting magnets, in which case they may incorporate coils to counteract the effect of the permanent magnet so that the load can be released. Other applications include magnetic filtration and separation, mechanical drives and couplings, low-friction bearings, MRI analysers and scanners, and magnetrons for radar and microwave ovens. See reference [3] for further details.

3 Magnetic Field

3.1 Magnetic Field of a Conductor

Current flowing in a conductor sets up a magnetic field in the surrounding space. The magnetic flux density $\boldsymbol{dB}$ that results from a current i in a small length of wire $\mathbf{d}\boldsymbol{l}$ is given by a formula attributed to Ampère [1], also known as the Biot-Savart formula. The field resulting from a conductor of any shape may be determined by integration, and there is a particularly simple result for the special case of a long straight wire. The magnitude of the flux density at a distance r from the axis of the wire is given by

$$B = \frac{\mu_0 i}{2\pi r} \quad [T] \qquad (3-1)$$

where μ_0 is the *primary magnetic constant* with a value given by

$$\mu_0 = 4\pi \times 10^{-7} \approx 1.257 \times 10^{-6} \frac{H}{m} \qquad (3-2)$$

The direction of the field is tangential to a circle of radius r, so the field can be represented by circles centred on the axis as shown in the flux plot of figure 3-1. A right-hand screw rule applies to the direction of the field, which is clockwise round the circle when viewed in the direction of the current

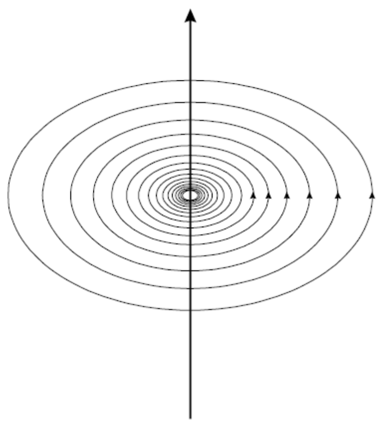


Figure 3-1Magnetic field of a single wire.

For a wire of finite length, as shown in figure 3-2, the magnitude of the flux density at a distance r from the wire is [1]:

$$B = \frac{\mu_0 i}{2\pi r} (\cos\alpha_2 - \cos\alpha_1) \qquad (3-3)$$

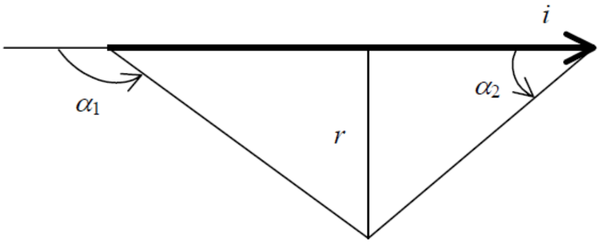


Figure 3-2 Wire of finite length

This result can be used to calculate the field of currents flowing in wire loops of various shapes, as the following example shows.

- Example: A wire of length 1.2 m is bent into the shape of a regular hexagon, so the length of each side is 200 mm. If the wire carries a current of 1.0 A determine the magnitude of the flux density at the center of the hexagon.
- Solution:
  If lines are drawn from the two ends of one side to the center of the hexagon, the three lines form an equilateral triangle. The angles are therefore $\alpha_1 = 120^o$, $\alpha_2 = 60^o$. The distance from the center to the mid-point of the side is:

  $$r = 100 \tan 60^{\circ} = 100 \times 1.732 = 173\ mm$$

  The contribution of one side to the flux density is:

  $$B = \frac{\mu_0 i}{2\pi r}\,(\cos\alpha_2 - \cos\alpha_1) = \frac{4\pi \times 10^{-7}}{4\pi \times 173 \times 10^{-3}}(\cos 60 - cos 120) = 5.78 \times 10^{-7}\ T$$

  The total flux density is therefore:

  $$B_t = 6B_s = 6 \times 5.7810^{-7}\ T$$

3.2 Forces on parallel currents:

Consider two long parallel current-carrying conductors as shown in figure 3-3. The first conductor creates a magnetic field in the space around it. At the axis of the second conductor, this field is represented by the flux density vector $\mathbf{B}_1$. Similarly, the second conductor creates a flux density $\mathbf{B}_2$ at the axis of the first conductor. The resulting forces will be $\mathbf{f}_1$ and $\mathbf{f}_2$ as shown in figure 3-3.

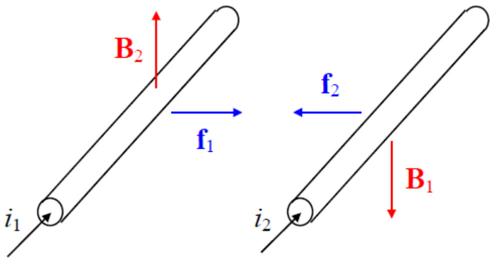


Figure 3-3 Parallel currents.

From equation 2-2, the magnitude of the force on a length *l* of the first conductor is:

$$f_1 = B_2 l i_1 \quad (3\text{-}4)$$

And the magnitude of the force on a length *l* of the second conductor is:

$$f_2 = B_1 l i_2 \quad \text{(3-5)}$$

Substituting for *B* in terms of the current from equation 3-1 gives the result:

$$f_1 = f_2 = \frac{\mu_0 i_1 i_2}{2\pi r} \qquad (3-6)$$

3.3 Magnetic Field of parallel currents:

To calculate the force on parallel conductors, we have used partial fields: only the field due to the first conductor has been used to calculate the force on the second, and *vice versa*. The total magnetic field of two currents is given by the vector sum of the fields that each would produce on its own. This is an instance of the *principle of superposition*, which is often used in electromagnetism. Note, however, that it is only valid for a linear system. It will be shown in section 4 that the principle is not valid when magnetic materials are present.

Figure 3-4 shows the flux plot when the conductors carry equal currents in the same direction. Figure 3-5 shows the corresponding plot for currents in opposite directions.

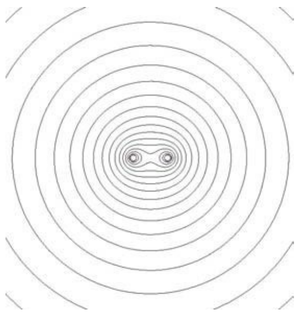


Figure 3-4 Currents in the same direction.

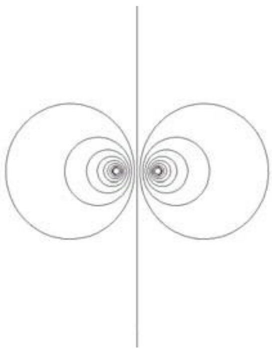


Figure 3-5 Currents in opposite directions

With currents in the same direction, the flux lines become more like circles with increasing distance from the conductors. When the distance is large compared with the conductor separation, the field is similar to that of a single conductor carrying twice the current. In between the conductors, the fields tend to cancel, and there is a null point mid-way between the conductors where the flux density is zero.

When the currents are in opposite directions, the resultant field is completely different. Between the conductors, the fields combine. Away from the conductors, the fields tend to cancel, as can be shown by adding the fields vectorially. If d is the distance between the axes of the conductors, and r is the distance from a point mid-way between the conductors, the magnitude of the flux density is given by:

$$B \approx \frac{\mu_0 i d}{2\pi r^2} \qquad (3-7)$$

Provided that $r >> d$.

Equation 3-7 has an important consequence for magnetic fields in the environment. Power cables usually contain pairs of conductors carrying equal and opposite currents, so the flux density decays rapidly with distance. Its magnitude can be reduced by making the separation d very small. The external magnetic field of a normal domestic or industrial power cable is negligibly small.

The field of a single conductor, given by equation 3-1, decays as $1/r$ instead of $1/r^2$, and there is no factor corresponding to d. The external magnetic field is therefore much greater. This property of a single conductor is exploited by *induction loops* for hearing aids. An induction loop is a long wire taken round the perimeter of a room, supplied with current from a power amplifier in the public address system. The alternating magnetic field set up by the loop induces a small voltage in a special coil in the hearing aid, which can be amplified in the same way as the signal from a microphone

3.4 Ampere's Circuital Law:

3.4.1 Magnetic Intensity:

When magnetic materials are considered (see section 3-8), it is necessary to introduce a new magnetic quantity H to describe the properties of the materials. In the absence of magnetic materials, H is defined as follows:

$$B = \mu_0 H \qquad (3-8)$$

Where H is the *magnetic intensity,* with units of amperes/metre (A/m). In terms of H, equation 3-1 for the field of a long straight wire becomes:

$$H = \frac{B}{\mu_0} = \frac{i}{2\pi r} \quad \left[\frac{A}{m}\right] \qquad (3-9)$$

Equation 3-9 can be re-arranged to express i in terms of H:

$$i = H.2\pi r = H.l \quad [A] \qquad (3-10)$$

Where $l = 2\pi r$ is the circumference of a circle of radius r. Thus the current magnitude i is equal to H times the length of the path encircling the current.

3.4.2 Ampere's circuital law:

Equation 3-10 is a particular case of a general law that applies to any closed path in the magnetic field. Figure 3-6 shows a closed path or *contour* of arbitrary shape encircling a conductor carrying a current i.

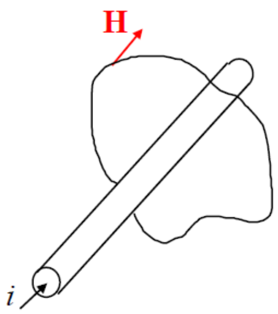


Figure 3-6Closed path encircling a current.

In general, the magnetic intensity *H* will vary from point to point along the contour, and it will not be parallel to the path at all points. Consider a short length **d*l*** of the path, and take the scalar product of **H** and **d*l***:

$$\boldsymbol{H}\,.\,\boldsymbol{dl} = H\,dl\,cos\theta \qquad (3-11)$$

Where θ is the angle between **H** and **dl**. The quantity H cosθ is the component of **H** parallel to **dl**, so equation 3-11 represents the component multiplied by the length dl. If the complete contour is divided up into small pieces, an expression like equation 3-11 applies to each piece. The sum of these terms may be written as an integral, and Ampère's circuital law states that this integral is equal to the current in the conductor:

$$\oint \boldsymbol{H}\,.\,\boldsymbol{dl} = i \qquad (3-12)$$

If there are several conductors, each carrying current, passing through the contour, then the general form of equation 3-12 is:

$$\oint \boldsymbol{H}\,.\,\boldsymbol{dl} = \sum i \qquad (3-13)$$

Where the summation on the right-hand side represents the algebraic sum of all the currents passing through the contour. It does not matter where the currents are placed in relation to the contour. Equation 3-13 can be used to calculate the unknown field *H* in many situations, as will be shown in sections 3.5 and 3.6, and it forms the basis of the magnetic circuit method introduced in section 4.

In most applications of equation 3-13, the contour is chosen to make the integration as simple as possible. If the problem has any kind of symmetry, this can be used to simplify the integration

3.5 Field of cylindrical conductors

3.5.1 Field of a long straight wire:

We already know that the field of a long straight wire has circular symmetry, as shown in figure 3-1. To exploit this fact in the application of Ampère's circuital law, we choose a circular contour as shown in figure 3-7

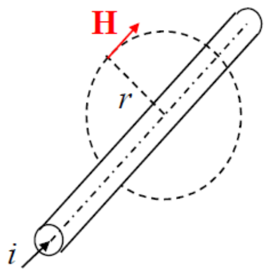


Figure 3-7 Contour for a long straight wire

By symmetry, the direction of **H** is tangential to the circle, and its magnitude is the same at all points on the circle. Equation 3-12 becomes:

$$i = \oint \boldsymbol{H} \,.\, \boldsymbol{dl} = \oint H \,.\, dl \cos\theta = \oint H \,.\, dl = H \oint dl = H.2\pi r \qquad (3-14)$$

Thus, the value of H is given by.

$$H = \frac{i}{2\pi r}\left[\frac{A}{m}\right] \qquad (3-15)$$

This is the same expression as in equation 3-9.

3.5.2 Field of a solid Cylinder:

Consider a long solid cylinder of radius a, carrying a current I, as shown in figure 3-8. The problem is to find the magnitude of H at a distance r from the axis.

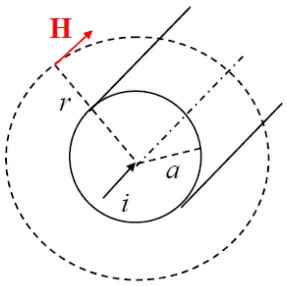


Figure 3-8 Contour for a solid cylinder

Since this structure also has circular symmetry, we expect the field lines to be circles centred on the axis of the cylinder, but we do not yet know the magnitude of **H**. We choose a circular contour of radius $r > a$, and follow the same argument as before

$$i = \oint \boldsymbol{H} \,.\, \boldsymbol{dl} = \oint H \,.\, dl \cos\theta = \oint H \,.\, dl = H \oint dl = H.2\pi r \qquad (3-16)$$

Therefore, the value of H is again given by:

$$H = \frac{i}{2\pi r}\left[\frac{A}{m}\right] \qquad (3-17)$$

3.5.3 Field of a hollow cylinder:

Figure 3-9 shows a hollow cylinder of inner radius a_i and outer radius a_o, carrying a current i. It is required to find the magnitude of H two cases:

- Outside the cylinder, at a radius $r_o > a_o$,
- Inside the hole, at a radius $r_i < a_i$.

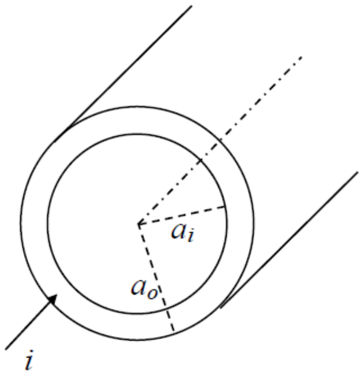


Figure 3-9 Hollow cylinder

Figure 3-10 shows a circular contour for calculating the field H_o outside the cylinder at a radius r_o.

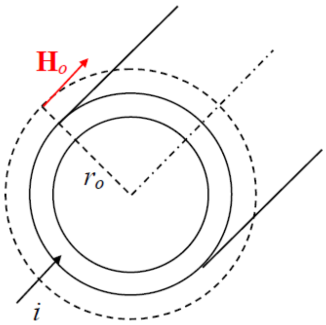


Figure 3-10Contour for external field.

Applying Ampère's circuital law gives:

$$i = \oint \boldsymbol{H} \, . \, \boldsymbol{dl} = H_0 . 2\pi r_0 \qquad (3-18)$$

$$H_0 = \frac{i}{2\pi r_0} \qquad (3-19)$$

Figure 3-11 shows a circular contour for calculating the field H_i inside the hole at a radius r_i

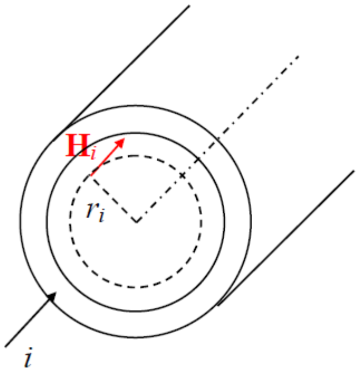


Figure 3-11 Contour for internal field.

There is no current passing through this contour, so Ampère's circuital law gives:

$$0 = \oint \boldsymbol{H} \,.\, \boldsymbol{dl} = H_i . 2\pi r_i \qquad (3-20)$$

$$H_i = 0 \qquad (3-21)$$

This interesting result, that there is no magnetic field in the hole, depends on perfect circular symmetry. If the hole is offset, there will be a field in the hole

3.5.4 Field of a coaxial cable:

The two conductors of a coaxial cable can be represented by a solid cylinder inside a hollow cylinder. If the conductors carry equal and opposite currents, Ampère's circuital law shows that there will be no magnetic field outside a coaxial cable provided the axes of the cylinders are coincident.

3.6 Field of a toroidal coil

Figure 3-12 shows a coil wound on a ring or toroid. The ring is the *core* of the coil, and for the present, it is assumed non-magnetic.

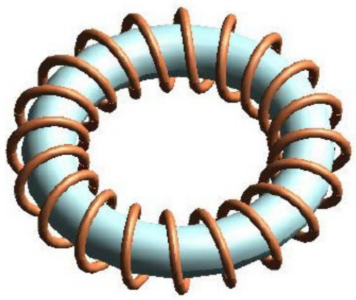


Figure 3-12Toroidal coil

Figure 3-13 shows the cross-section of a toroidal coil, where crosses indicate current flowing inwards and dots indicate current flowing outwards. It is required to find the magnetic field in three regions:

- inside the toroidal core, at a radius r,
- inside the hole, at a radius r_i.
- outside the toroid, at a radius r_o,

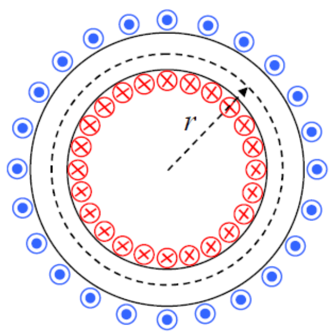


Figure 3-13 Contour for the field in the core

3.6.1 Field in the core:

Consider a circular contour of radius r in the core, as shown in figure 3-13. If the turns of the coil are closely spaced and uniformly distributed around the core, we may assume:

- the field direction is tangential to the circle,
- the field magnitude is constant at all points.

If the coil has N turns, and carries a current i, then the total current passing through the contour is Ni. Ampère's circuital law (equation 3-13) gives:

$$\sum i = Ni = \oint \boldsymbol{H} \, . \boldsymbol{dl} = H . 2\pi r \qquad (3-22)$$

Therefore, the value of H in the core is given by:

$$H = \frac{Ni}{2\pi r} \quad \left[\frac{A}{m}\right] \qquad (3-23)$$

Note that the value of H will vary over the cross- section of the core, since r is not the same at all points, but this variation will be ignored. If $D = 2r$ is the mean diameter of the toroid, then the mean value of H is:

$$H = \frac{Ni}{\pi D} \quad \left[\frac{A}{m}\right] \qquad (3-24)$$

And the corresponding flux density is:

$$B = \mu_0 H = \frac{\mu_0 Ni}{\pi D} \quad [T] \qquad (3-25)$$

3.6.2 Field in the hole:

Figure 3-14 shows the integration contour for finding the field in the hole.

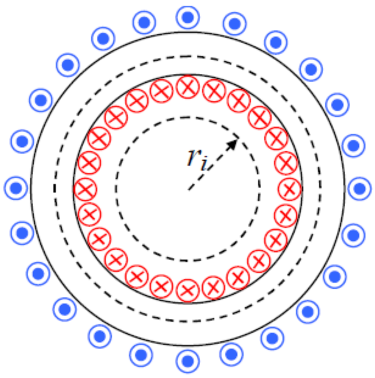


Figure 3-14 Contour for the field in the hole.

As with the hollow cylinder, there is no current passing through this contour, so we have:

$$0 = \oint \boldsymbol{H} \, . \boldsymbol{dl} = H_i . 2\pi r_i \qquad (3-26)$$

$$H_i = 0 \qquad (3-27)$$

3.6.3 Field outside the toroid

Figure 3-15 shows the integration contour for finding the field outside the toroid.

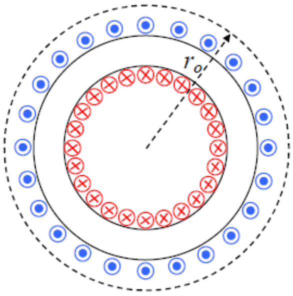


Figure 3-15 Contour for the exterior field.

Current from each turn of the coil passes through this contour twice: once on the inside of the core, and once on the outside. Since the currents are in opposite directions, they cancel out. Equation 3-13 gives:

$$\sum i = 0 = \oint \boldsymbol{H} \,.\, \boldsymbol{dl} = H_0 . 2\pi r_0 \qquad (3-28)$$

Therefore, the value of H_o outside the toroid is zero.

3.6.4 Applications:

A perfect toroidal coil has the remarkable property that the magnetic field is confined entirely to the core: there is no external magnetic field. Practical coils will depart from this ideal because the turns are not perfectly distributed, but the external field is usually very small. For this reason, toroids are widely used in transformers and inductors. Figure 3-16 shows a small toroidal inductor of this kind.

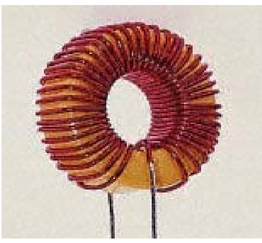


Figure 3-16 Toroidal inductor.

3.7 Magnetic Flux and Inductance:

3.7.1 Flux linkage

Magnetic flux was defined in equation 2-16:

$$\phi = BA \qquad [2-16]$$

For the toroidal coil, B is the flux density in the core, and A is the cross-sectional area of the core. Another quantity known as the *flux linkage* is important for coils. It is defined as the sum of the fluxes for each turn of the coil. If the coil has N turns, and each turn has the same flux ϕ, then the flux linkage is:

$$\lambda = N\phi = NBA \quad [Wb] \qquad (3-29)$$

In many cases, the flux is not the same for each turn, so the general definition of flux linkage is:

$$\lambda = \sum \phi \quad [Wb] \quad (3-30)$$

Where the summation is over the number of turns in the coil, and the individual flux values are given by equation 2-16. Equation 3-29 will be used in this course.

3.7.2 Self-inductance and stored Energy:

Suppose that a coil carries a current *i*. In the absence of magnetic materials, the magnetic flux density *B* at any point will be proportional to *i*. It follows that the flux and the flux linkage are also proportional to *i*, so we may put:

$$\lambda = N\phi = Li \quad (3-31)$$

Where L is a constant known as the self-inductance of the coil. It will be shown in section 5 that this is consistent with the self-inductance of circuit theory. Re-arranging equation 3-31 gives:

$$L = \frac{\lambda}{i} \quad [henry, H] \quad (3-32)$$

From circuit theory, the magnetic energy stored in the coil is given by:

$$W_m = {}^1/_2\, Li^2 \quad [joule\ , j] \quad (3-33)$$

Substituting for *Li* from equation 3-31 gives:

$$W_m = {}^1/_2\, Li^2 = {}^1/_2\, Li.i = {}^1/_2\, \lambda i \quad (3-34)$$

Compare this with the electric energy stored in a capacitor charged to a voltage *v*:

$$W_e = {}^1/_2\, Cv^2 = {}^1/_2\, qv \quad (3-35)$$

Where *C* is the capacitance and *q* is the charge.

3.7.3Inductance of a toroidal Coil:

For the toroidal coil considered in section 3.6, we have:

$$\lambda = N\phi = NBA = NA.\frac{\mu_0 Ni}{\pi D} \quad (3-36)$$

The self-inductance is therefore given by:

$$L = \frac{\lambda}{i} = \frac{\mu_0 N^2 i}{\pi D} \quad [H] \quad (3-37)$$

Note that *L* depends on the coil geometry and the square of the number of turns. This expression for the inductance is valid only for a non-magnetic core.

3.7.3 Unit of μ_0:

Re-arranging equation 3-37 gives:

$$\mu_0 = \frac{\pi D}{N^2 A} \quad \left[\frac{m.H}{m^2} = \frac{H}{m}\right] \quad (3-38)$$

Thus the units of μ_0 are H/m.

3.8 Magnetic Material

3.8.1 Introduction:

Suppose that the toroidal core is wound on a core of magnetic material such as iron or steel. Experimentally, the inductance is much larger, and it varies with the current in the coil. Thus, we may put:

$$L' = \mu_r L \qquad (3-39)$$

Where μ_r is a dimensionless property of the material known as the *relative permeability.* Since the inductance is still defined by equation 3-32, it follows that the flux linkage λ, and therefore the flux density *B*, has increased by the same factor μ_r. We choose to retain the definition of *H* in terms of the current through Ampère's circuital law (equation 3-13). Since *B* has changed, it follows that equation 3-8 is no longer valid:

$$B \neq \mu_0 H \qquad (3-40)$$

Instead, we must have:

$$B = \mu_r \mu_0 H \qquad (3-41)$$

Where $\mu = \mu_r\mu_0$ is the *permeability*. From equation 3-41, the relative permeability is:

$$\mu_r = \frac{B}{\mu_0 H} \qquad (3-42)$$

The effect of the magnetic material is to make the flux density *B* much larger than it would have been with the coil alone. Physically, this happens because the original field from the coil has magnetised the material of the core. The magnetised material sets up its own field, adding to the original field of the coil.

When considering magnetic materials it is important to preserve the distinction between **B** and **H**. Ideas of cause and effect can be helpful:

- **H** is the cause, related to currents through Ampère's circuital law.
- **B** is the effect, related to observable quantities such as force through the Lorentz equation.

Note that the unit of **H** is the ampere/metre (A/m), whereas the unit of **B** is the tesla (T).

3.8.2 Engineering Materials:

The relative permeability μ_r is very close to 1 except for two groups of materials:

- Ferromagnetic: iron and iron alloys (steels), cobalt and nickel. These are electrical conductors.
- Ferromagnetic: ferrites – based on oxides of iron. These are electrical insulators.

For these groups, μ_r is very large and variable. Everything else is effectively non-magnetic for engineering purposes. Magnetic materials are further classified as *soft* and *hard*.

- Soft magnetic materials have very little residual magnetization when the external source of **H** is removed.
- Hard magnetic materials retain considerable magnetisation when the external source of **H** is removed.

Hard magnetic materials are permanent magnets, which will be considered in section 4.3. Only soft magnetic materials are considered in the present section.

3.8.3 Measurement of B and H:

The relationship between *B* and *H* can be measured with two coils wound on a ring sample of the material:

- The primary coil carries a current *i*, giving a known value of *H* in the ring (equation 3-24).
- The corresponding value of *B* is determined from the voltage induced in the secondary coil when the primary current changes.

From Faraday's law (see section 5, equation 5-2) the voltage is:

$$e = \frac{d\lambda}{dt} = NA\frac{dB}{dt} \qquad (3-43)$$

The voltage is applied to the input of an electronic integrator, giving an output proportional to *B*:

$$v_0 = k\int edt = kNA\int \frac{dB}{dt}dt = kNAB \qquad (3-44)$$

3.8.4. Relationship between B and H:

Figure 3-17 shows the relationship between *B* and *H* from measurements on a ring sample of a soft magnetic material.

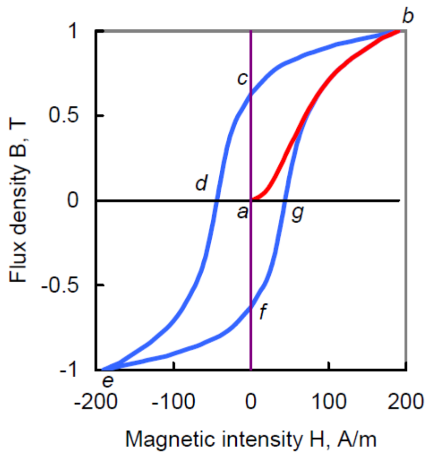


Figure 3-17 B/H relationship for soft material

If the material is initially demagnetised, and the value of *H* is progressively increased by raising the current *i* in the primary coil, the resulting *B* follows a curve such as the red curve *ab* in figure 3-17.

When the value of H is reduced to zero, B does not fall to zero. Instead, the blue curve *bc* is traversed, and H must be reversed to bring the value of B to zero at point *d*. Further increase of negative H takes the curve to point *e*. If H is gradually changed from negative to positive, the curve *efgb* is traversed, and the cycle repeats.

The curve *ab* is known as the *initial magnetisation curve,* and the loop *bcdefg* is the *hysteresis loop.* It may be shown that the area of this loop represents energy lost per unit volume in taking the material round one cycle of magnetisation.

3.4.8 Magnetisation Characteristic:

For many purposes the difference between the two sided of the hysteresis loop in figure 3-17 can be ignored. An average value is reasonably representative of the material properties, and the resulting curve is known as the *magnetisation characteristic.* Figure 3-18 shows part of the magnetisation characteristic for a typical silicon steel of the kind used in transformers and motors. Further graphs are given in section 7.1

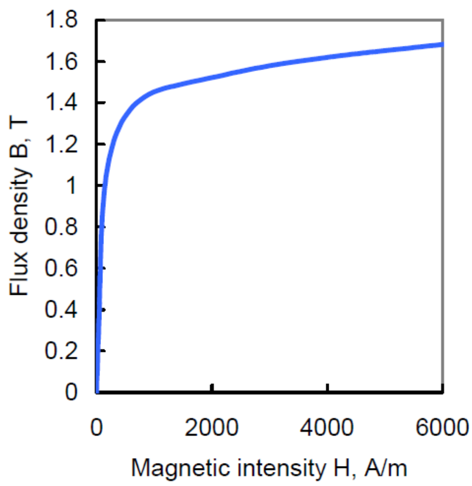


Figure 3-18 Characteristic for silicon steel

The shape of the curve in figure 3-18 is typical of soft magnetic materials. It has two distinct regions, joined by a 'knee':

- A steeply rising initial part, where a small increase in H gives a large increase in B.
- A saturated region, above about 1.5 T, where any further increase in B requires a large increase in H.

Equation 3-42 gives the relative permeability
μ_r in terms of B and H. Figure 3-19 shows the resulting variation of μ_r with B, calculated for the same silicon steel.

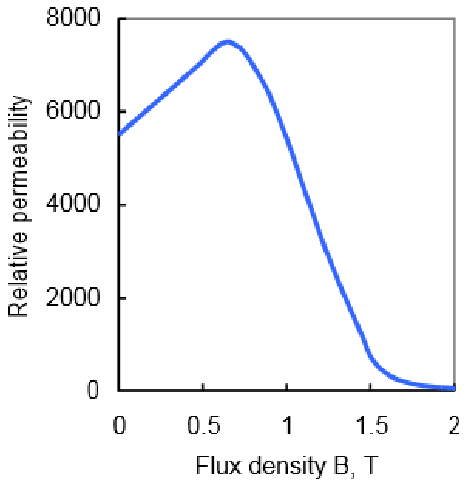


Figure 3-19 Relative permeability.

4 Magnetic Circuits:

4.1 The magnetic Circuit Concept:

The soft magnetic materials, considered in section 3.8, have a remarkable property. If the relative permeability is high enough, they act as ‘conductors’ of magnetic flux. To demonstrate this, figure 4-1 shows a ring core, with a concentrated coil instead of the uniformly wound coil of section 3.6

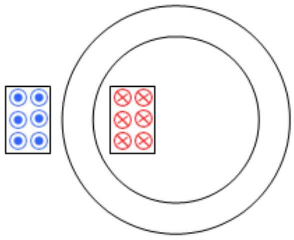


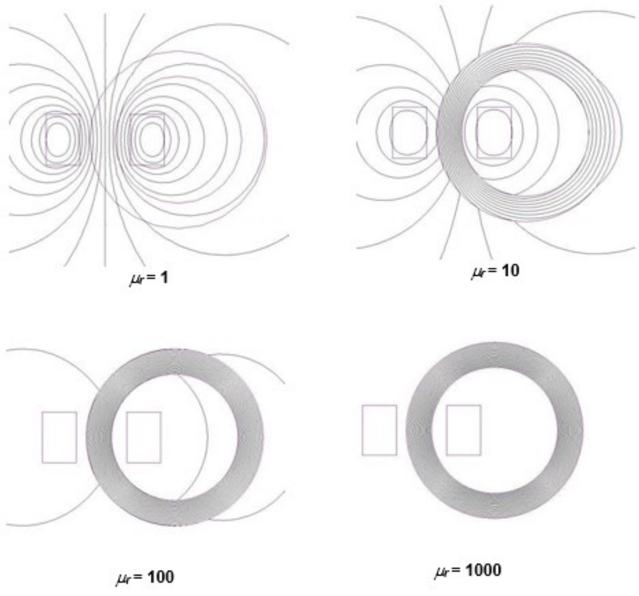
Figure 4-1 Ring core with concentrated coil

Suppose that cores of different materials are used, with relative permeability values ranging from 1 to 1000. The resulting flux plots are shown in figure 4-2. A value of 1 means that the material is non-magnetic, so this flux plot is the same as the field of the coil without a core.

With increasing values of relative permeability, more of the coil flux is diverted from the air to the core. When μ_r = 1000, the flux is virtually confined to the core, giving the same distribution of flux density as a uniformly wound toroidal coil. The exact shape and location of the coil have little effect on the flux distribution in the core. The core that guides the flux in a circular path is a simple example of a *magnetic circuit*. From the graph of relative permeability for silicon steel in figure 3-19, the value of μ_r will exceed 1000 if the value of *B* is less than 1.5 T. The material is *unsaturated*, and it acts as an effective flux guide. However, if the flux density is significantly greater than 1.5 T, the material is *saturated* and the value of μ_r decreases rapidly. It then ceases to be an effective flux guide.

Practical magnetic circuits normally exploit the unsaturated properties of magnetic materials. Designers need to ensure that the material is not saturated. Users need to be aware of the possibility of saturation if currents exceed the design values, which may alter the behavior of the device.

Figure 4-2 Effect of core permeability.



Suppose that an airgap is cut in the core, as shown in figure 4-3. The resulting flux plot is shown in figure 4-4.

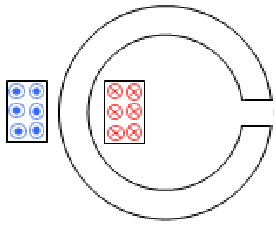


Figure 4-3Ring core with an airgap

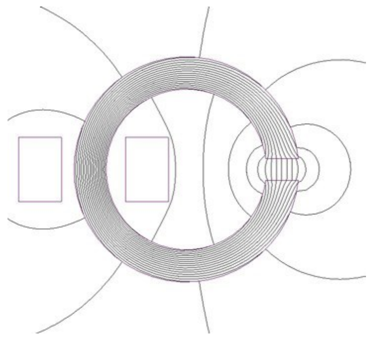


Figure 4-4 Flux plot for a ring with an airgap.

Although the relative permeability of the core is still 1000, the flux is no longer confined to the core. As with the magnets considered in section 2, there is some fringing of the field at the edges of the airgap, and some leakage of flux from one side of the ring to the other, across the air space.

Although the field is significantly different from that of a closed ring, the ring with an airgap is another example of a magnetic circuit.

The formal concept of the magnetic circuit depends on the flux law, which was introduced in section 2.4:

- For any closed region in the magnetic field, the total flux entering the region is equal to the total Flux leaving.

This statement is exactly true if the flux in the air is included in addition to the flux in the core. To a first approximation, we may ignore the effects of leakage and fringing, and suppose that the flux in the core is the same for all cross-sections. With this approximation, the magnetic structure is analogous to a series electric circuit, where the current is the same at all points. It is analysed in a similar way, with Ampère's circuital law taking the place of Kirchhoff's voltage law.

The magnetic circuit concept gives a quick and simple way of getting an approximate answer.

Where an accurate solution is required, powerful computer modelling methods are available. Even when computer methods are used, it is valuable to check the result by a magnetic circuit calculation

4.2 Magnetic Circuit without Air-gap:

Figure 4-5 shows a simple magnetic circuit without an airgap, where the dotted circular contour of length l is the mean circumference of the ring. The coil has N turns and carries a current i.

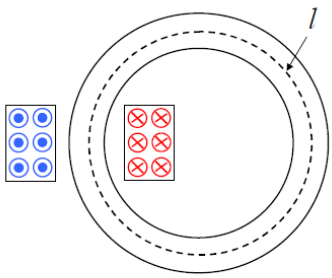


Figure 4-5Magnetic circuit without an airgap

The core is assumed to have a constant cross- sectional area. It follows from the flux law that the flux density *B* is constant, so *H* is also constant. Applying Ampère's circuital law gives:

$$\sum i = Ni = \oint \boldsymbol{H} \,.\, \boldsymbol{dl} = Hl \qquad (4-1)$$

The value of *H* in the ring is therefore given by:

$$H = \frac{Ni}{l} \quad \left[\frac{A}{m}\right] \qquad (4-2)$$

The corresponding value of *B* is determined from the curve for the material. There are two cases:

1. The coil current *i* is specified, and it is required to find the corresponding value of *B*.
2. The value of *B* is specified, and it is required to find the corresponding coil current *i*.

For case 1, the value of *H* is determined from equation 4-2, and the value of *B* found from the material curve. For case 2, the process is reversed: the value of *H* is found from the curve, and then substituted in equation 4-2 to get *i*.

- Example 4-1: A ring of silicon steel has a mean diameter of

80 mm, and there is a coil of 1200 turns wound on the ring. Determine the current in the coil that will give a flux density of 1.1 T in the ring.

- Solution: From the curve in figure 12-2, *H* = 200 A/m. Equation 4-2 gives:

$$i = \frac{Hl}{N} = \frac{200 \times \pi \times 80 \times 10^{-3}}{1200} = 41.9\, mA$$

4.2.1 Non-Linearity:

An important consequence of the non-linear relationship between *B* and *H* is that *B* is not proportional to the current in the coil. If the calculation of worked example 4-1 is repeated for a flux density of 1.4 T instead of 1.1 T, the value of *H* increases from 200 to 700 A/m, and the coil current increases from 41.9 mA to 147 mA. Thus, a 27% increase in *B* requires a 250% increase in current. It follows that the inductance given by equation 3-32 is not constant but varies over a wide range.

A further consequence of non-linearity is that the principle of superposition no longer holds for the magnetic flux density. Suppose that the ring in worked example 4-1 is wound with two coils, each of 1200 turns. A current of 41.9 mA in either coil will give a flux density of 1.1 T in the ring, but the same current in both coils will not give a flux density of 2.2 T. The total value of *H* is the sum of the contributions from each coil, so H increases from 200 to 400 A/m, and the flux density increases from 1.1 to 1.3 T.

4.3 Magnetic Circuit with airgap:

Figure 4-6 shows a magnetic circuit with an airgap. The effective length of the steel ring, in the direction of the magnetic flux, is the dotted path of length ls, and the length of the airgap is lg. The coil has N turns and carries a current *i.*

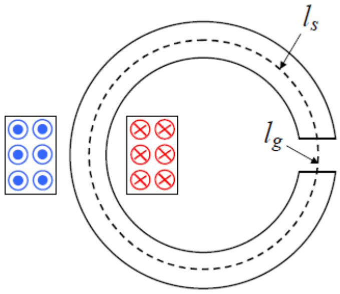


Figure 4-6 Ring core with an airgap

The first step is to establish the relationship between the values of B and H in the airgap and the values in the steel. Let ϕ_s be the flux in the steel, and ϕ_g the flux in the airgap. From the flux law, $\phi_s \approx \phi_g$. If A_s is the cross-sectional area of the steel, and A_g is the cross-sectional area of the airgap, then:

$$B_s A_s \approx B_g A_g \qquad (4-3)$$

Often, as in figure 4-6, the two areas are equal. In this case, we have the result:

$$B_s \approx B_g \qquad (4-4)$$

The closeness of the approximation depends on the amount of fringing and leakage: see section 4.5. For simplicity, these effects will be ignored, and it will be assumed that $B_s = B_g$. However, the values of H cannot be the same. In the airgap, equation 3-8 gives:

$$H = \frac{B}{\mu_0} \qquad (4-5)$$

In the steel, H is obtained from the magnetisation curve for the given value of B, and this will be different from the value given by equation 4-5.

If the value of B is specified, then the values of H can be determined in the steel and in the airgap. The required current in the coil can then be determined from Ampère's circuital law as follows:

$$\sum i = Ni = \oint \boldsymbol{H} \,.\, \boldsymbol{dl} = H_s l_s + H_g l_g \qquad (4-6)$$

This gives the result:

$$i = \frac{H_s l_s + H_g l_g}{N} \qquad (4-7)$$

Unlike the previous problem, it is not straightforward to find t4he value of B that results from a given coil current i, because of the non- linear relationship between H_s and B_s.

In many practical magnetic circuits, the value of $H_g l_g$ greatly exceeds the value of $H_s l_s$, so the airgap has a dominant effect.

- Example 4-2: A ring of silicon steel has a mean diameter of 80 mm, and there is a coil of 1200 turns wound on the ring. An airgap of length 3 mm is cut in the ring. Determine the current in the coil that will give a flux density of 1.1 T in the ring.
- Solution: We have $B_s = B_g = 1.1$ T. From the curve in figure 12-2, $H_s = 200$ A/m. From equation 4-5, H_g is given by:

$$H_g = \frac{B_g}{\mu_0} = \frac{1.1}{4\pi \times 10^{-7}} = 875 \quad kA/m$$

Thus:

$$H_s l_s = 200 \times (8\pi - 3) \times 10^{-3} = 49.7\ A$$

$$H_g l_g = 875 \times 10^{-3} \times 3.0 \times 10^{-3} = 2.63\ kA$$

Equation 4-7 gives:

$$i = \frac{H_s l_s + H_g l_g}{N} = \frac{49.7 + 2.63 \times 10^3}{1200}\ A = 2.23\ A$$

4.3.1 Non-Linearity:

A useful effect of the airgap in a magnetic circuit is to reduce the non-linearity. If the calculation of worked example 4-2 is repeated for a flux density of 1.4 T instead of 1.1 T, the value of H_s increases from 200 to 700 A/m. However, $H_s l_s$ is still much smaller than $H_g l_g$, and the coil current increases from 2.23 A to 2.93 A. In this case, a 27% increase in B requires a 31% increase in current. Compare this with the previous case of the ring without an airgap, where the same increase in B required a 250% increase in the current. The non-linearity is greatly reduced, and the inductance is almost constant.

4.3.2 Variable cross-sectional area:

The air and iron parts of the magnetic circuit may have different cross-sectional areas, as shown in figure 4-7, where the air cross-sectional area A_g is larger than the steel cross-sectional area A_s.

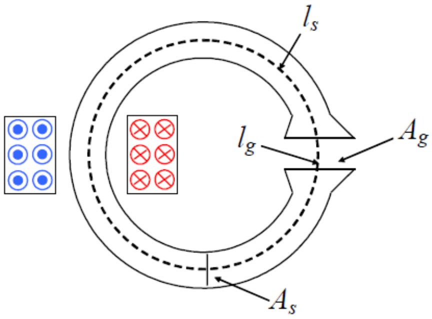


Figure 4-7 Ring core with an airgap

If the flux values are similar, it follows from equation 4-3 that the flux density in the steel will be larger than the flux density in the airgap:

$$B_s A_s \approx B_g A_g \qquad (4-8)$$

4.4 Electric Circuit Analogy:

The quantity *Ni* is the driving force in a magnetic circuit, with a similar function to the electromotive force or voltage in an electric circuit. It is therefore termed the *magnetomotive force,* with the symbol $\mathcal{F}$. Corresponding to the current in an electric circuit, we have the flux ϕ for a magnetic circuit. Equation 4-6 can be re-cast in terms of flux if we express *H* in terms of *B* through equation 3-41:

$$B = \mu_r \mu_0 H = \mu H \qquad [3-41]$$

This gives:

$$\mathcal{F} = Ni = H_s l_s + H_g l_g = \frac{B_s}{\mu_s} l_s + \frac{B_g}{\mu_g} l_g = \phi\left(\frac{l_s}{\mu_s A_s} + \frac{l_g}{\mu_g A_g}\right) = \phi(\mathcal{R}_s + \mathcal{R}_g) \qquad (4-9)$$

In equation 4-9, A_s is the cross-sectional area of the steel, and A_g is the cross-sectional area of the airgap (see figure 4-7). The quantities R_s and R_g are termed the *reluctances* of the two parts of the circuit. They are analogous to resistances in an electric circuit, and are given by:

$$\mathcal{R}_s = \frac{l_s}{\mu_s A_s}, \quad \mathcal{R}_g = \frac{l_g}{\mu_0 A_g} \qquad (4-1-)$$

Equation 4-9 is identical in form to the voltage equation of the electric circuit in figure 4-8:

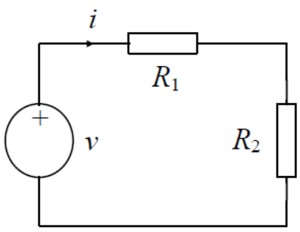
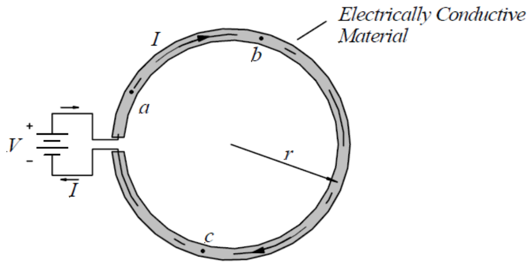
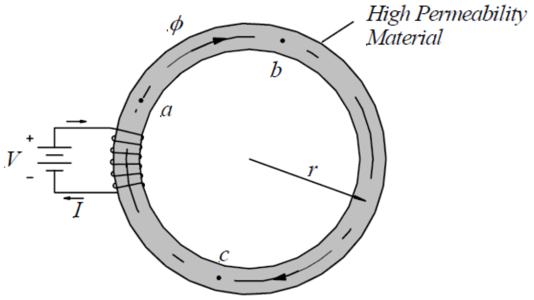
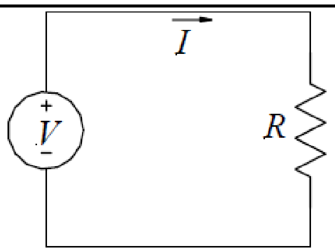
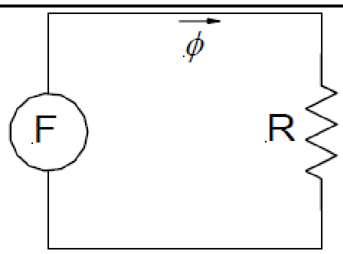


Figure 4-8 Analogous electric circuit

Table 4-1 on the next page compares electric and magnetic circuits. The analogy is useful because the familiar ideas of electric circuits can suggest ways of handling the less familiar concepts of magnetic circuits. In problems that are more complex, where the magnetic circuit contains parallel paths, this approach can be very helpful

Table 1 Circuit analogues

Electric Circuit	Magnetic Circuit
	
Driving Force	
applied battery voltage = V	applied ampere-turns = F
Response	
$current = \dfrac{driving\ force}{\text{electric resistance}}$ Or $I = \dfrac{V}{\mathrm{R}}$	$Flux = \dfrac{driving\ force}{\text{Magnetic Reluctance}}$ Or $\phi = \dfrac{F}{\mathcal{R}}$
Impedance Impedance is used to indicate the impediment to the driving force in establishing a response.	
$R = \dfrac{l}{\sigma \mathrm{A}}$	$\mathcal{R} = \dfrac{l}{\mu \mathrm{A}}$
Equivalent Circuit	
 $V = IR$	 $\mathcal{F} = \phi \mathcal{R}$
Fields	
Electric Field Intensity $E = \dfrac{V}{l} = \dfrac{V}{2\pi r}\ \ (V/m)$ or $\int E.dl = V$	Magnetic Field Intensity $H = \dfrac{\mathcal{F}}{l} = \dfrac{V}{2\pi r}\ \ (A-t/m)$ or $\int H.dl = \mathcal{F}$

Potential	
Electric Circuit	**Magnetic Circuit**
Electric Potential Difference $V_{ab} = \int_a^b E.dl = \frac{V}{l}\int_a^b dl = \frac{IR}{l} l_{ab} = \frac{I}{l}\frac{l}{\sigma A} l_{ab} = IR_{ab}$	Magnetic Potential Difference $\mathcal{F}_{ab} = \int_a^b H.dl = \frac{\mathcal{F}}{l}\int_a^b dl = \frac{\phi R}{l} l_{ab} = \frac{\phi}{l}\frac{l}{\mu A} l_{ab} = \phi\mathcal{R}_{ab}$
Flow Densities	
Current Density $J \equiv \frac{I}{A} = \frac{V}{AR} = \frac{El}{A\left({}^{l}/_{\sigma A}\right)} = \sigma E$	Flux Density $B \equiv \frac{\phi}{A} = \frac{\mathcal{F}}{A\mathcal{R}} = \frac{Hl}{A\left({}^{l}/_{\mu A}\right)} = \mu H$

4.5 Magnetic Circuit Problems:

When there are airgaps in the magnetic circuit, the assumption that the flux is constant around the circuit is an oversimplification. Consider the ring core of worked example 4-2. The ring has a constant cross-sectional area, so the flux density should be constant if the flux is constant. Figure 4-9 shows a computer-generated flux plot together with a shaded plot of the flux density magnitude, where values are represented by colours ranging from blue for minimum to red for maximum. In this case, the fringing and leakage fluxes have a significant effect, and the flux density is far from constant.

The computer results show that the flux density varies from 1.05 T in the middle of the airgap to 1.65 T on the opposite side of the core. In the worked example, a flux density of 1.1 T was assumed for all parts of the core and the airgap. The simple magnetic circuit method has therefore under-estimated the value of *H* in most of the core.

In this case, the error has had little effect on the result. With a current of 2.23 A, the true value of the airgap flux density is 1.05 T instead of 1.1 T. However, if a higher airgap flux density were required, the error would be much greater. With a coil current of 2.93 A, the magnetic circuit method gives an airgap flux density of 1.4 T, but the true value is only 1.11 T. The flux density on the opposite side of the core is then 1.77 T, which is well into the saturation region of the magnetisation characteristic for the material. To prevent saturation, it would be necessary to increase the cross-sectional area of the core in this region.

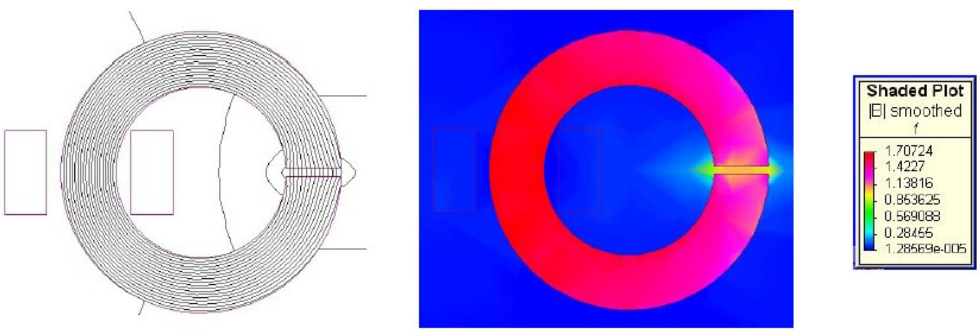


Figure 4-9 Flux plot and shaded plot of |B|.

4.6 Electromagnet:

The magnetic circuit concept can be used to calculate the flux density in electromagnets of the kind considered in section 2.3. Figure 4-10 shows the cross-section of a simple electromagnet. Coils are wound on the steel poles, which are linked by a steel yoke to form the magnet core.

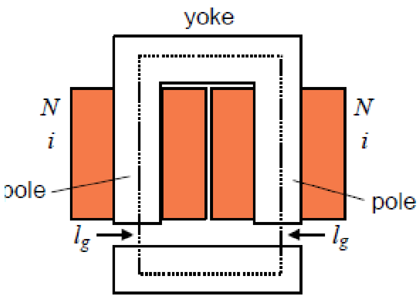


Figure 4-10 Simple electromagnet

Applying Ampère's circuital law to the dotted contour in figure 4-10 gives:

$$\sum i = 2Ni = \oint \boldsymbol{H} \cdot \boldsymbol{dl} = H_s l_s + H_g l_g \qquad (4-12)$$

Where H_s is the value of H in the steel parts, which is assumed to be constant, and l_s is the total length of the steel path. At the boundary between the steel pole and the airgap, we have from equation 4-4:

$$B_s = B_g \qquad [4-4]$$

Equation 4-4 can be expressed in terms of H:

$$\mu_r \mu_0 H_s = \mu_0 H_g \qquad (4-13)$$

Thus:

$$H_s = \frac{H_g}{\mu_r} \qquad (4-14)$$

If the steel is unsaturated, so that μ_r is very large, equation 4-14 shows that H_s will be very much smaller than H_g. provided the airgap length is not too small, it follows that:

$$H_s l_s \ll 2H_g l_g \qquad (4-15)$$

Equation 4-12 therefore becomes:

$$2Ni \approx 2H_g l_g \qquad (4-16)$$

The flux density in the airgap is thus:

$$B_g = \mu_0 H_g = \frac{\mu_0 Ni}{l_g} \qquad (4-17)$$

The approximation in equation 4-16 is equivalent to neglecting the reluctance of the steel parts in comparison with the reluctance of the airgap, as the following consideration shows. From equation 4-9, with $2N$ in place of N, we have:

$$\mathcal{F} = 2Ni = \phi(\mathcal{R}_s + \mathcal{R}_g) \qquad (4-18)$$

The reluctances are:

$$\mathcal{R}_s = \frac{l_s}{\mu_0 \mu_r A_s} \qquad (4-19)$$

$$\mathcal{R}_g = \frac{2l_g}{\mu_0 A_g} \qquad (4-20)$$

If the steel relative permeability μ_r is large, then $R_s << R_g$, so equation 4-18 becomes:

$$2Ni \approx \phi\mathcal{R}_g = B_g A_g \cdot \frac{2B_g}{\mu_0 A_g} = \frac{2B_g l_g}{\mu_0} \qquad (4-21)$$

This gives the same result for B_g as equation 4-17.

Since the steel parts no longer feature in the equation for B_g, it follows that the exact distribution of H – and therefore B – in the steel is unimportant. In fact, there will be considerable variation, as the flux plot in figure 4-10 shows, and the designer may need to increase the thickness of part of the core to prevent saturation.

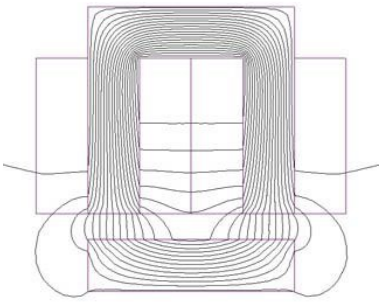


Figure 4-11 Flux plot for simple electromagnet

The total force is given by equation 2-14:

$$f_t = \frac{{B_g}^2 A_p}{\mu_0} \qquad [2-14]$$

If the flux density is calculated from equation 4-17, the error in the force given by equation 2-14 is generally less than 10% in simple electromagnets if the steel is unsaturated.

Accurate calculation of the airgap flux density and the force requires electromagnetic simulation software, particularly if parts of the magnetic circuit are saturated. Nevertheless, equations 4-17 and 2-14 are useful preliminary design tools, and they give a valuable check on the results of computer simulation

4.6.1 Alternative Coil position:

Although it is common to put coils on the two poles, as in figure 4-10, an alternative is a single coil on the yoke, as shown in figure 4-12. Because of the flux-guiding properties of the steel core, the position makes very little difference to the magnet performance. The flux plot in figure 4-13 is very similar to the one in figure 4-11, and the force of attraction is virtually unchanged.

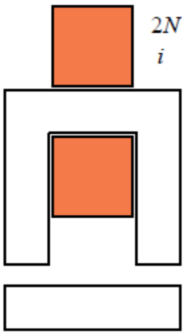


Figure 4-12Electromagnet with a single coil

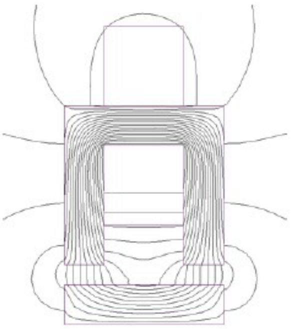


Figure 4-13Flux plot for electromagnet

4.7 Permanent Magnets:

Ampère's circuital law applies to permanent magnets as well as electromagnets. Consider the dotted contour in figure 4-14.

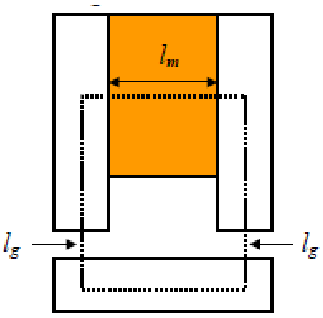


Figure 4-14 Contour for Ampère's circuital law.

There is no current passing through this contour, so we have:

$$0 = \oint \boldsymbol{H} \,.\, \boldsymbol{dl} = H_m l_m + H_s l_s + 2H_g l_g \qquad (4-22)$$

As with the electromagnet, H_s is the value of H in the steel parts, which is assumed to be constant, and l_s is the total length of the steel path. By a similar argument, if the steel poles are unsaturated, $H_s l_s$ will be small in comparison with the other terms in equation 4-22, so we have:

$$H_m l_m \approx 2H_g l_g \qquad (4-23)$$

If the contour is drawn in the direction of the magnetic flux, H_g is positive. Equation 4-23 shows that H_m must be negative, which is in the opposite direction to B_m in the permanent magnet. In the airgap we have $B_g = \mu_0 H_g$. If leakage is neglected, we also have the relationship between B_m and B_g from the flux law:

$$B_g = \frac{A_m}{A_P} B_m \qquad [2-19]$$

With these relationships, equation 4-23 can be re- cast in terms of H_m and B_m:

$$H_m \approx -2\frac{B_m}{\mu_0} \cdot \frac{A_m}{A_P} \cdot \frac{l_g}{l_m} \qquad (4-24)$$

Equation 4-24 is the equation of a straight line through the origin. No matter what may be the relationship between B and H for the permanent- magnet material, the magnetic circuit imposes the condition that the working values H_m and B_m must lie on this line.

4.7.1 Permanent-magnet Material

Figure 4-15 shows the hysteresis loop for a permanent-magnet material.

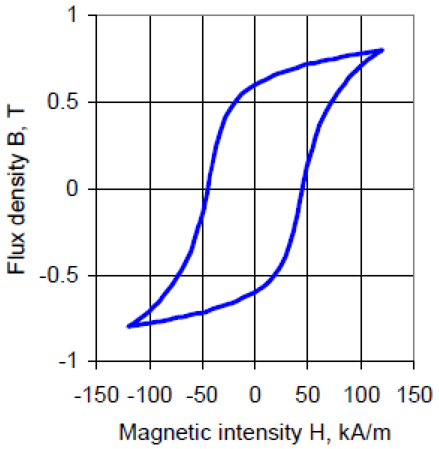


Figure 1 Material B/H characteristic

It follows from equation 4-24 that the working part of the material curve for a permanent magnet is the second quadrant, where H is positive and B is negative, as shown in figure 4-16. Because H is acting in a direction to demagnetise the material, this is known as the *demagnetisation characteristic*

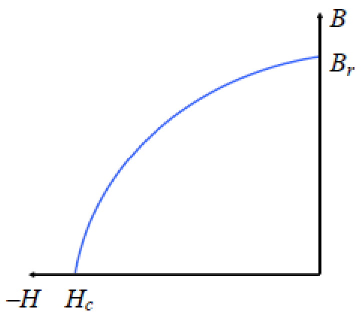


Figure 2 Demagnetization characteristic

Two important properties of a permanent magnet material are the points where the curve intersects the axes.

- The *remanence* B_r is the value of B that remains in the material when H is reduced to zero.

- The *coercivity* H_c is the value of H that must be applied to the material to bring B to zero.

Typical values of B_r and H_c are given in table 4-2 for some representative permanent-magnet materials [3]. The higher the value of H_c, the more difficult it is to demagnetise the material.

Table 2 Permanent-Magnet materials

Material	B_r (T)	$-H_c$ (kA/m)
1% carbon steel	0.9	4
35% cobalt steel	0.9	20
Alnico 4	1.0	120
Ceramic ferrite	0.4	290
Samarium cobalt	1.0	480
Neodymium iron boron	1.2	910

4.7.2 Working point:

The working values of B and H in the permanent- magnet material is given by the intersection of the line of equation 4-24 and the material curve, as shown in figure 4-17. By analogy with electric circuits, the line is often called a *load line*.

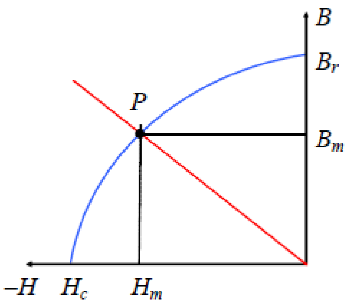


Figure 3 Material curve and load line

From equation 4-24, the slope of the line is inversely proportional to the length of the airgap, so the effect of increasing the airgap is to move the operating point P further down the curve. If the airgap is very small, the load line is nearly vertical. The value of H_m is then very small, and the value of B_m is close to the remanence B_r. This condition is unaffected by the value of B_s in the steel poles, provided that the poles remain unsaturated, so that $H_s l_s$ is still small in comparison with $H_m l_m$ in equation 4-22.

4.7.3 Demagnetization:

Suppose that the airgap in the magnetic circuit is increased, so that the working point moves from P_1 to P_2 in figure 4-18.

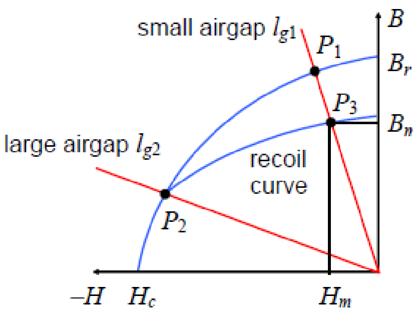


Figure 4Recoil curve

If the airgap is restored to its original value, the original curve may not be followed. Instead, a new curve known as the *recoil curve* is traversed, as shown in figure 4-18, giving a new working point P_3 with a lower value of B_m. The material has become partly demagnetized. This is a particular problem with the Alnico materials, which are still widely used. With materials such as ceramic ferrite or neodymium iron boron, the recoil curve is indistinguishable from the main curve unless a strong demagnetizing field is applied by passing current through a coil.

5 Electromagnetic Induction:

5.1 Introduction:

Electromagnetic induction is an astonishing effect, which describes the generation of a voltage in a conductor or a circuit. It can happen in two entirely different ways:

- Motional induction, where a conductor moves in a magnetic field.
- Transformer induction, where the circuit is stationary and the field varies with time.

Faraday's law of electromagnetic induction, which relates the generated voltage to the change of flux, can take account of both effects. Electromagnetic induction is widely misunderstood, and this misunderstanding has resulted in numerous engineering failures. One example is a patent for a whole range of 'improved' DC motors and generators, none of which will work [4]. Motional induction has already been considered in section 2, where it was shown that the voltage induced in a moving conductor is given by:

$$e = Blu \qquad [2-1]$$

Although it is the basis of many practical devices, this equation has a deceptive simplicity. Some of the problems that arise from its application are discussed in section 5.3.

5.2 Transformer Induction

5.2.1 Faraday's Law:

Figure 5-1 represents a coil of N turns and cross- sectional area A placed in a magnetic field.

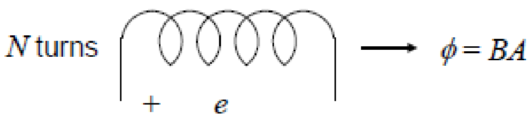


Figure 5-1 Coil in a magnetic field

Initially the field will be assumed uniform, so that the flux linkage is given by:

$$\lambda = N\phi = NBA \qquad (5-1)$$

It is a remarkable fact that a voltage is induced in the coil whenever the flux changes, regardless of what causes it to change. *Faraday's law* of electromagnetic induction states that the voltage e is given by:

$$e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} \quad [V] \qquad (5-2)$$

Some textbooks put a negative sign in equation 5-4, out of deference to Lenz's law, which states that induced currents act in a direction to oppose the change. However, a positive sign is required for consistency with the normal conventions of circuit theory: see the discussion of self-inductance below.

- Example 5-1: Find the maximum value of the voltage induced in a coil of 150 turns by a magnetic flux $\phi = \Phi_m \sin \omega t$ if $\Phi_m = 400$ μWb and the frequency is 60 Hz.

- Solution:

The flux linkage is:

$$\lambda = N\phi = N\phi_m \sin \omega t$$

So the induced voltage is:

$$e = \frac{d\lambda}{dt} = \omega N\phi_m \cos \omega t = E_m \cos \omega t$$

The maximum value of e is:

$$E_m = \omega N\phi_m = 2\pi \times 60 \times 150 \times 400 \times 10^{-6} = 22.6 \ V$$

5.2.2 Self-Inductance:

Suppose that the magnetic flux in a coil is the result of a current i in the same coil, as shown in figure 5-2. It will be assumed that the coil does not have a magnetic core, or that the magnetic circuit has a large airgap, so that the flux is proportional to the current.

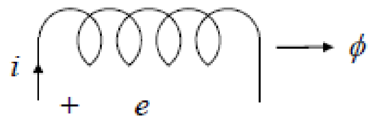


Figure 5-2 Self-inductance

From equation 3-31 the flux linkage is:

$$\lambda = N\phi = Li$$

Faraday's law, equation 5-2, gives the induced voltage as:

$$e = \frac{d\lambda}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt} + i\frac{dL}{dt} \qquad (5-3)$$

If L is constant, this reduces to the defining equation for inductance in circuit theory:

$$e = L\frac{di}{dt} \qquad (5-4)$$

If the current i is held constant, but a displacement of part of the system changes the value of L, then equation 5-3 shows that there will be a generated voltage given by:

$$e = i\frac{dL}{dt} = i\frac{dL}{dx}\frac{dx}{dt} = iu\frac{dL}{dx} \qquad (5-5)$$

Where u is the velocity corresponding to the rate of change of the displacement x. This is an important property of *variable reluctance* devices, which are considered in part 2, section 7.

5.2.3 Mutual Inductance:

Figure 5-3 shows a pair of adjacent coils, where some flux links the second coil when there is a current i_1 in the first coil. The coils are said to be *coupled* magnetically.

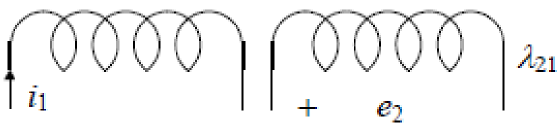


Figure 5-3 Mutual inductance-1

Let the flux linkage with the second coil beλ_{21}. As with a single coil, it will be assumed that the coils are wound on non-magnetic cores, or the airgap in the magnetic circuit is large, so that we may put:

$$\lambda_{21} = M_{21} i_1 \qquad (5-6)$$

Where M_{21} is a constant. In figure 5-4, the roles are reversed: the second coil carries a current i_2, and we consider the flux linkage with the first coil.

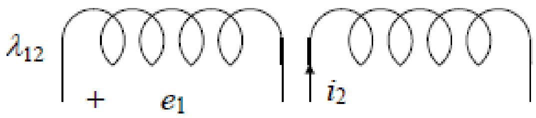


Figure 5-4 Mutual inductance – 2

Let the flux linkage with the first coil be λ_{12}. With the same assumptions as before, we have:

$$\lambda_{12} = M_{12} i_2 \qquad (5-7)$$

where M_{12} is a constant. It may be shown that:

$$M_{12} = M_{21} = M \qquad (5-8)$$

where M is termed the *mutual inductance* of the two coils. The reciprocal property of mutual inductance expressed in equation 5-10 does not depend on any similarity of the coils – they can have different shapes, sizes and numbers of turns. Combining equations 5-6, 5-7 and 5-8 gives:

$$M = \frac{\lambda_{12}}{i_2} = \frac{\lambda_{21}}{i_1} \qquad (5-9)$$

Equation 5-9 gives two different ways of determining the mutual inductance, and one way is often easier than the other. An application of this principle is given below for coaxial cables. If the current i_1 in figure 5-3 changes with time, then the voltage e_2 induced in the second coil is given by:

$$e_2 = \frac{d}{dt}(M i_1) = M \frac{di_1}{dt} \qquad (5-10)$$

Similarly, if the current i_2 in figure 5-4 changes with time, then the voltage e_1 induced in the first coil is given by:

$$e_1 = \frac{d}{dt}(M i_2) = M \frac{di_2}{dt} \qquad (5-11)$$

It is assumed that the coils do not move, so M is a constant.

5.2.4 Circuit aspects of coupled coils:

Figure 5-5 shows a voltage source connected to each coil, so that currents flow in both coils.

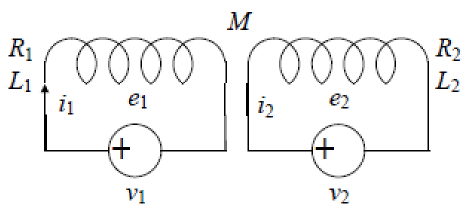


Figure 5-5 Coupled coils

Kirchhoff's voltage law applied to each coil gives the circuit equations for coupled coils:

$$v_1 = R_1 i_1 + e_1 = R_1 i_1 + \frac{d}{dt}(L_1 i_1 + M i_2) = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad (5-12)$$

$$v_2 = R_2 i_2 + e_2 = R_2 i_2 + \frac{d}{dt}(L_2 i_2 + M i_2) = R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_2}{dt} \qquad (5-13)$$

It may be shown that the total stored magnetic energy in the system is given by:

$$W_m = {}^1\!/_2\, L_1 i_1^2 + {}^1\!/_2\, L_2 i_2^2 + M i_1 i_2 \qquad (5-14)$$

5.2.5 Coaxial cable:

Coaxial cables are widely used for signal connections because they reject interference from external magnetic fields. To show that they have this property, suppose that the source of the magnetic field is a circuit carrying a current i_1. The circuit. To calculate *M,* let the cable carry a current i_2, so that $M = \lambda_{12} / i_2$. Since a perfect coaxial cable produces no external magnetic field (see section 3.5), it follows that $\lambda_{12} = 0$, so $M = 0$. In practice, the conductors may not be perfect coaxial cylinders, so there may be some residual induced voltage.

5.3 Further aspects of induction:

5.3.1 Motional Induction revisited:

Equation 2-1 is sometimes known as the *motional induction formula:*

$$e = Blu \qquad [2-1]$$

Its simplicity makes it a trap for the unwary, and it is important to be aware of the restrictions:

- B is the local field at the conductor.
- B is the field of a single source, or the sources of B are not in relative motion.
- u is the velocity relative to the source of B.

The following example shows how the incorrect use of equation 2-1 can give completely wrong answers. Figure 5-6 shows a single-turn coil moving with velocity **u** in a magnetic field **B**. One side of the coil is enclosed in a steel tube, which screens it from the magnetic field.

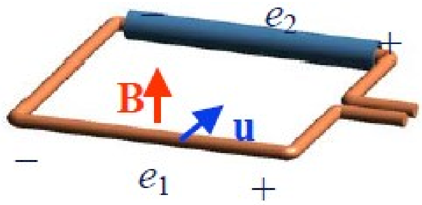


Figure 5-6 partly screened coil

If the external field **B** is uniform in the absence of the steel tube, the effect of the tube is shown by the flux plot in figure 5-7. As an example of the screening effect, if the material of the tube has a relative permeability of 1000, and the inside diameter is 90% of the outside diameter, the flux density inside the tube is about 1% of the value for the uniform field.

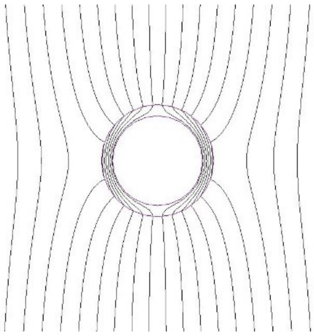


Figure 5-7 Screening effect of a steel tube

The two coil sides at right angles to the direction of motion are of length l. There is a voltage e_1 induced in unscreened side, given by:

$$e_1 = B_1 lu = Blu \qquad (5-15)$$

However, the flux density B_2 in the vicinity of the other coil side is very small, so the direct application of equation 2-1 gives:

$$e_1 = B_2 lu \approx 0 \qquad (5-16)$$

The voltage at the terminals of the coil therefore appears to be:

$$e = e_1 - e_2 \approx Blu - 0 = Blu \qquad (5-17)$$

If the coil has N turns instead of a single turn, it appears to be possible to generate a voltage given by:

$$e \approx NBlu \qquad (5-18)$$

This would provide a neat way of generating electricity by moving a coil of wire through the Earth's magnetic field. However, it will not work for the following reason. The field $\mathbf{B}_2$ is the small difference between two fields: the external field $\mathbf{B}$, and the field $\mathbf{B}_s$ created by the steel tube, which is itself magnetised by the external field. Thus:

$$\boldsymbol{B}_2 = \boldsymbol{B}_e - \boldsymbol{B}_s \qquad (5-19)$$

The conductor is in motion with respect to the source of $\mathbf{B}$, but it is stationary with respect to the source of $\mathbf{B}_s$, since this is the cylinder that is moving with the conductor. Therefore, it is only the component $\mathbf{B}$ that gives rise to a generated voltage, so the voltage e_2 is equal in magnitude to e_1, and the voltage at the coil terminals is zero. Similar considerations apply to the non- working inventions mentioned in reference [4].

5.3.2 Transformer induction revisited:

In Faraday's law (equation 5-2), the value of the induced voltage depends only on the rate of change of flux linkage.

- The voltage does not depend on the nature of the source of the flux density B.
- The voltage does not depend on the local value of B at the conductor.
- It is possible to have an induced voltage in a circuit even if $B = 0$ in the conductor.

Consider the toroidal coil of section 3.6. Outside the coil, the value of *B* is very small. If the coil is energised from an AC source, as it is in a toroidal transformer, there will be an alternating magnetic flux in the core. Suppose that a loop of wire encircles the core. The flux linkage with the loop is equal to the flux in the core, so there will be an induced voltage in the loop although the magnetic field is negligible in the wire. The explanation of this result is that the magnetic field is only half the story. In textbooks on electromagnetic theory [5], it is shown that, whenever a magnetic field is changing with time, there is an associated electric field. Figure 5-8 shows the lines of the electric field **E** around a toroidal coil when there is an alternating magnetic flux in the core.

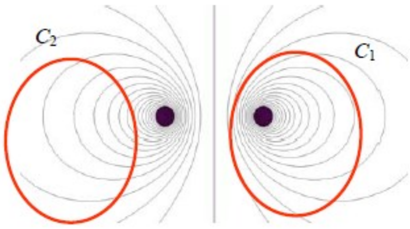


Figure 5 Electric field of a toroidal coil

If a wire completely encircles the core, such as the loop C_1 in figure 5-8, there will be an induced voltage given by:

$$e = \oint_{C_1} \boldsymbol{E}.\boldsymbol{dl} = \frac{d\lambda}{dt} \qquad (5-20)$$

However, if the wire does not encircle the core, such as the loop C_2 in figure 5-8, then the induced voltage is zero:

$$e = \oint_{C_2} \boldsymbol{E}.\boldsymbol{dl} = 0 \qquad (5-21)$$

5.3.3 Practical application:

The voltage given by equation 5-20 has an important practical consequence. If a toroidal transformer is fastened by a bolt passing through the hole, it is essential that the two ends of the bolt are not connected together by metal parts of the equipment. If they are connected, a conducting loop will be formed around the core, and the induced voltage will circulate a large current around the loop.

5.3.4 General use of Faraday's Law:

Faraday's law (equation 5-2) holds for any closed circuit, and takes account of both transformer and motional effects.

$$e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} \qquad (5-2)$$

This gives the correct value for the induced voltage, no matter what causes the flux ϕ to change. In a circuit with a fixed boundary, if ϕ does not change, there can be no induced voltage. This is often the quickest way of establishing whether a claimed invention for a new electromechanical device will work. For example, in the coil shown in figure 5-6, there is no change of flux through the coil when it moves in a uniform field, and therefore no voltage will appear at the terminals.

Circuits where the boundary shapes change with time, or those that involve sliding contacts, are notoriously difficult, and require careful analysis to predict the induced voltage correctly [6]

6 Transformer

6.1 Introduction:

A transformer is a practical application of magnetically coupled coils. Usually the purpose is to transfer energy from one coil to the other, as in figure 6-1 where energy is transferred from the AC source to the lamp through the space between the coils.

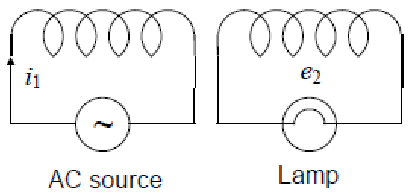


Figure 6-1 Transformer principle

Transformers are essentially AC devices, because there has to be a change of flux to give an induced voltage in a coil. There are two main reasons for transferring energy in this way:

- To provide electrical isolation between the source and the load.
- To change the voltage and current levels.

The coils are usually placed on a common magnetic core to improve the coupling. Figure 6-2 shows the flux plots for two coupled coils (a) without a core, (b) with an open core, (c) with a closed core

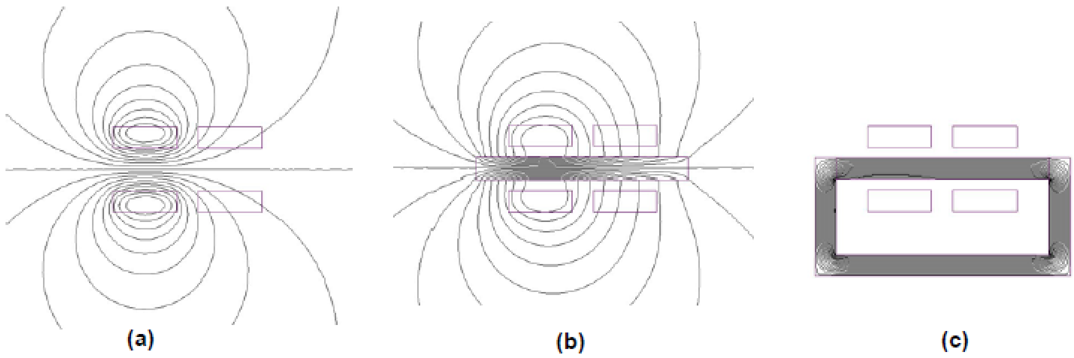


Figure 6-2 Effect of a magnetic core:

6.1.2 Practical aspects:

The cores for high-frequency transformers are often made from magnetically soft ferrites, which are electrical insulators. For power frequencies, the cores are made from an iron alloy such as silicon steel. These materials have better magnetic properties than ferrites, but they are also electrical conductors. If the core were made of solid steel, currents – known as *eddy currents* – would be induced in the core by the alternating magnetic flux. Steel transformer cores are therefore made from thin laminations, insulated from one another, to minimise eddy currents [1, 6].

Small power transformers are commonly made with the shell-type construction shown in figure 6-3, where the coils are wound on the central limb of the core, and the two outer limbs provide symmetrical flux return paths. The centre limb is twice the width of the outer limbs because it carries twice the flux, as shown by the flux plot in figure 6-4.

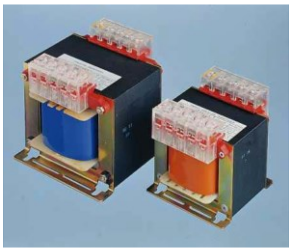


Figure 6-3 Shell-type transformers.

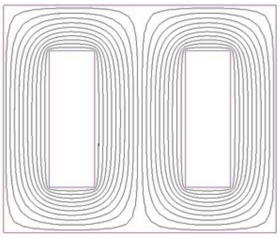


Figure 6-4 Flux plot: shell-type transformer

Toroidal transformers exploit the remarkable properties of toroidal coils described in section 3.6. Although they are more expensive than shell-type transformers, the performance is better. They are used in high-quality electronic equipment and for instrument transformers (see section 6.3) where measurement accuracy is important. Typical toroidal transformers are shown in figure 6-5.

Figure 6-5 Toroidal transformers.

6.2 Transformer Principle:

The action of a transformer is most easily understood if the two coils are wound on opposite sides of a magnetic core, as shown in the model of figure 6-6. This form is used for some low-cost transformers, but the magnetic coupling is not as good as with the shell-type construction.

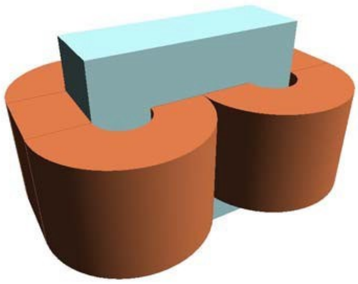


Figure 6-6 Core-type transformer

Figure 6-7 is a schematic representation of the transformer. It will be assumed that the coupling is perfect: the same magnetic flux ϕ passes through each turn of each coil. The coil connected to the source is termed the *primary,* and the coil connected to the load is termed the *secondary*. It is usual to refer to the coils as *windings.*

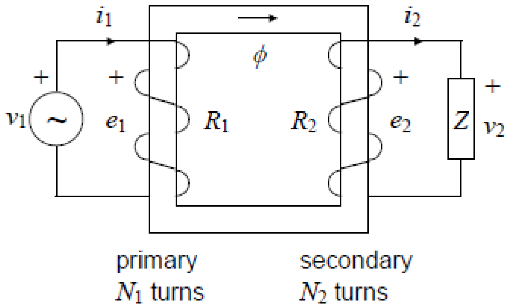


Figure 6-7 Transformer with source and load

6.2.1 Voltage Relationship;

Faraday's law (equation 5-2) applied to the two windings gives the following expressions for the voltages induced in the windings:

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi}{dt} \qquad (6-1)$$

$$e_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\phi}{dt} \qquad (6-2)$$

Kirchhoff's voltage law applied to the two windings gives:

$$v_1 = R_1 i_1 + e_1 = R_1 i_1 + N_1 \frac{d\phi}{dt} \qquad (6-3)$$

$$v_2 = R_2 i_2 + N_2 \frac{d\phi}{dt} \qquad (6-4)$$

The sign difference arises from the reference directions for current in the two windings.

If the resistances R_1 and R_2 are negligible, then equations 6-3 and 6-4 become:

$$v_1 \approx N_1 \frac{d\phi}{dt} \qquad (6-5)$$

$$v_2 \approx N_2 \frac{d\phi}{dt} \qquad (6-6)$$

Dividing these equations gives the important result:

$$\frac{v_1}{v_2} \approx \frac{N_1}{N_2} \qquad (6-7)$$

Thus, the secondary voltage can be made larger or smaller than the primary voltage by changing the ratio of the numbers of turns on the two windings. Voltage transformation is one of the most common uses of transformers, on a large scale in electrical power transmission and distribution, and on a small scale in the power supplies for electronic equipment.

6.2.2 Sinusoidal operation:

If the voltage source is sinusoidal, then the core flux will also be sinusoidal, so we may put:

$$\phi = \phi_m \sin(\omega t) \qquad (6-8)$$

Substituting this expression in equation 6-5 gives:

$$v_1 = N_1 \frac{d\phi}{dt} = N\omega\phi_m \cos(\omega t) \qquad (6-9)$$

Thus the maximum primary voltage is:

$$V_{1m} = N_1\omega\phi_m = 2\pi f N_1 A B_m \qquad (6-10)$$

where A is the cross-sectional area of the core and B_m is the maximum flux density in the core. We can deduce some of the limits of transformer operation from equation 6-10. It is normal practice to design a transformer with B_m close to the saturation value for the core material when V_1 and f have their normal rated values. A typical value for B_m is 1.4 T for the silicon steel characteristic in figure 6-8.

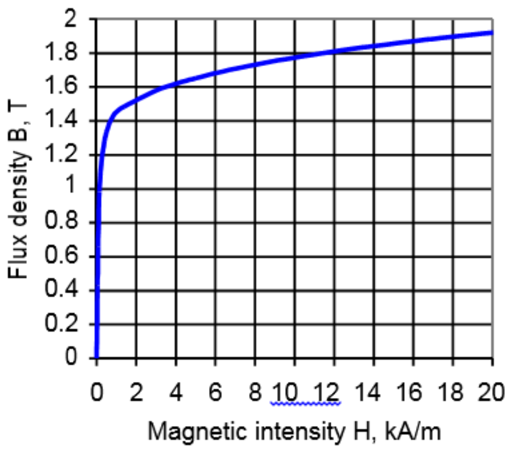


Figure 6-8 Silicon transformer steel

From equation 6-10, if f is increased or V_1 is reduced, then the value of B_m will fall. The transformer will be under-run, but this will not cause any problems. However, if f is reduced below the design value, or V_1 is increased above the design value, then B_m will be required to increase. This will take the core into saturation. The magnetisation curve in figure 6-8 shows that there will be a very large increase in the value of H, which in turn will require a large current in the primary. The resulting I^2R power loss in the primary winding will cause the temperature to rise, which may damage or destroy the transformer

6.2.3 Current Relationship:

The relationship between the primary and secondary currents can be found by considering the magnetic circuit of the transformer. From the basic magnetic circuit equation, we have:

$$\mathcal{F} = N_1 i_1 - N_2 i_2 = \mathcal{R}\phi \qquad (6-11)$$

The negative sign arises from the reversed reference direction of the secondary current. $\mathcal{R}$ is the reluctance of the core, given by:

$$\mathcal{R} = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_0 A} \qquad (6-12)$$

If the relative permeability μ_r is large, $\mathcal{R}\phi$ will be small in comparison with $N_1 i_1 - N_2 i_2$, so so equation 6-11 becomes:

$$N_1 i_1 - N_2 i_2 \approx 0 \qquad (6-15)$$

Thus:

$$N_1 i_1 \approx N_2 i_2 \qquad (6-16)$$

Equation 6-14 may be expressed in a similar form to the voltage relationship in equation 6-7:

$$\frac{i_1}{i_2} \approx \frac{N_2}{N_1} \qquad (6-17)$$

Observe that the current is related to the turns ratio in the opposite way to the voltage. If the secondary voltage is smaller than the primary voltage, then the current is larger, and *vice versa*. If losses are ignored, then the power input to the primary will be equal to the power output from the secondary.

If sinusoidal voltages and currents are represented by phasors, the corresponding forms for the basic voltage and current equations are:

$$\frac{\boldsymbol{V_1}}{\boldsymbol{V_2}} = \frac{N_1}{N_2} \qquad (6-18)$$

$$\frac{\boldsymbol{I_1}}{\boldsymbol{I_2}} = \frac{N_2}{N_1} \qquad (6-19)$$

6.2.4 Transformer rating:

The maximum voltage at the primary terminals of a transformer is determined by equation 2-8, and is independent of the current. The maximum primary current is determined by the I^2R power loss in the resistance of the transformer windings, which generates heat in the transformer. This power loss is independent of the applied voltage. Consequently, for a given design of transformer, there is a maximum value for the product V_1I_1 at the primary terminals. To a first approximation, this is also equal to the product V_2I_2 at the secondary terminals. This maximum value does not depend on the phase angle between the voltage and the current. Transformer ratings therefore specify the apparent power VI (volt-amperes, VA) rather than the real power $VI\cos\phi$(watts, W).

6.3 Type of power transformers

In addition to the ordinary single-phase power transformer, two other types are in common use: auto-wound transformers, and 3-phase transformers.

6.3.1 Auto-wound transformer:

A transformer can have a single coil with an output taken from a portion of the coil, as shown in figure 6-9. This is known as an *auto-wound* transformer or auto-transformer

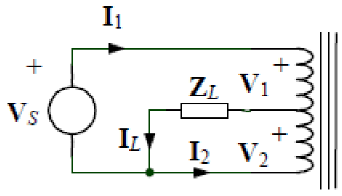


Figure 6-9 Auto-wound transformer

Unlike the normal transformer with two windings, known as a *double-wound* transformer, the auto-wound transformer does not provide electrical isolation between the primary and the secondary. However, an auto-wound transformer can have a much larger apparent power rating than a double-wound transformer of the same physical size. Let N_1 be the number of turns on the upper part of the winding in figure 6-9 , and N_2 the number of turns on the lower part. The conventional transformer equations 6-18 and 6-19 apply to these parts of the winding, since they are equivalent to two separate windings with a common connection.

Applying Kirchhoff's law to this circuit gives:

$$\boldsymbol{V_s = V_1 + V_2} \quad (6-20)$$

$$\boldsymbol{I_L = I_1 + I_2} \quad (6-21)$$

As an example, suppose that $N_1 = N_2$. If the transformer is regarded as ideal (see section 2.3), then $\mathbf{I}_1 = \mathbf{I}_2$ and $\mathbf{V}_1 = \mathbf{V}_2$. Equations 6-20 and 6-21 give:

$$\boldsymbol{V_s = V_1 + V_2 = 2V_2 = 2V_L} \quad (6-22)$$

$$\boldsymbol{I_L = I_1 + I_2 = 2I_1 = 2I_L} \quad (6-23)$$

Where $\mathbf{V}_L$ is the voltage across the load and $\mathbf{I}_S$ is the current supplied by the source. This auto-wound transformer behaves as a step-down transformer with a ratio of 2:1, and the current in each winding is equal to half of the load current.

An elegant application of the auto-wound transformer principle is the variable transformer, which has a single-layer coil wound on a toroidal core. The output is taken from a carbon brush that makes contact with the surface of the coil; the brush can be moved smoothly from one end of the coil to the other, thus varying the output voltage. Examples of variable transformers are shown in figure 6-10.

Figure 6-10 Variable transformers.

6.3.2 3 Phase Transformer:

In 3-phase systems, it is common practice to use sets of three single-phase transformers. It is also possible, however, to make 3-phase transformers with three sets of windings on three limbs of a core, as shown in figure 6-11.

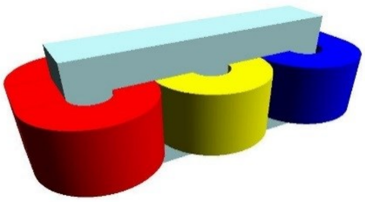


Figure 6-11 3-phase transformer model

The corresponding fluxes are shown in figure 6-12

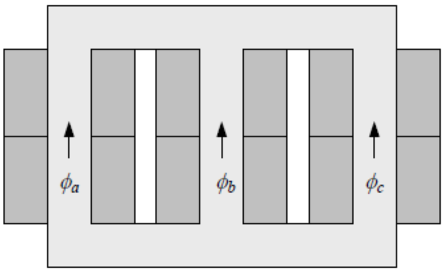


Figure6-12 3-phase transformer flux

In a balanced system with sinusoidal phase voltages, the fluxes will be given by:

$$\phi_a = \phi_m cos\omega t$$

$$\phi_b = \phi_m \cos(\omega t - 120^\circ) \qquad (6-24)$$

$$\phi_c = \phi_m \cos(\omega t - 240^\circ)$$

Figure 6-13 shows flux plots for the transformer at the instants when ω*t* = 0, 120º and 240º.

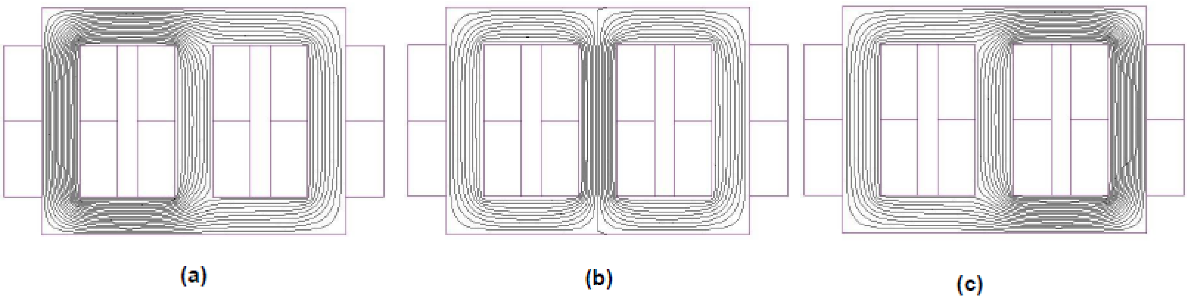


Figure 6-13 3-phase transformer flux plots: (a) 0º, (b) 120º, (c) 240º

There is no requirement for another limb to form a flux return path, because the fluxes ϕ_a, ϕ_b and ϕ_c sum to zero in a balanced 3-phase system. The proof is as follows. From equation 6-24, the sum is given by:

$$\frac{\phi_a + \phi_b + \phi_c}{\phi_m} = cos\omega t + \cos(\omega t - 120^\circ) + \cos(\omega t - 240^\circ) = 0 \qquad (6-25)$$

Because the fluxes in the three limbs sum to zero at all instants of time, there is no leakage of flux from the core, as the flux plots in figure 6-13 demonstrate.

6.4 Ideal Transformer Properties:

If the primary and secondary windings have zero resistance, and the magnetic core has zero reluctance, then the approximate equalities in equations 6-18 and 6-19 become exact equalities. This leads to the concept of an *ideal transformer element,* to accompany the other ideal elements of circuit theory. Figure 6-14 shows a circuit symbol for the ideal transformer element.

The voltage and current relationships in the time and frequency domains are given in table 6-1.

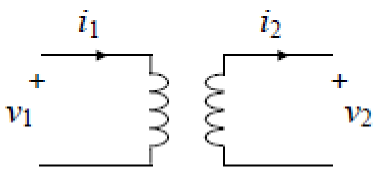


Figure 6-14 Ideal transformer element

Table 3 Ideal Transformer relationship

Time Domain	Frequency Domain
$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n$	$\frac{\boldsymbol{V_2}}{\boldsymbol{V_1}} = \frac{N_2}{N_1} = n$
$\frac{i_1}{i_2} = \frac{N_2}{N_1} = n$	$\frac{\boldsymbol{I_1}}{\boldsymbol{I_2}} = \frac{N_2}{N_1} = n$

The following properties of the ideal transformer may be deduced from equations 6-22 and 6-23:

- The voltage transformation is independent of the current, and *vice versa*.
- If the secondary is short-circuited, so that $v_2 = 0$, the primary terminals appear to be short-circuited since $v_1 = 0$.
- If the secondary is open-circuited, so that $i_2 = 0$, the primary terminals appear to be open-circuited since $i_1 = 0$.
- The output power is equal to the input power, so there is no power loss in the element.

6.4.1 Impedance transformation:

The ideal transformer has the important property of transforming impedance values in a circuit. Consider an ideal transformer with an impedance $\mathbf{Z}_L$ connected to its secondary terminals, as shown in figure 6-15.

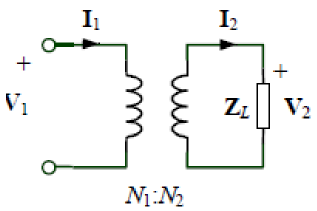


Figure 6-15: Ideal transformer with a load

The secondary impedance is given by:

$$\boldsymbol{Z_L} = \frac{\boldsymbol{V_2}}{\boldsymbol{I_2}} \qquad (6-26)$$

At the primary terminals, the circuit presents an impedance given by:

$$\boldsymbol{Z_{in}} = \frac{\boldsymbol{V_1}}{\boldsymbol{I_1}} = \frac{\frac{N_1\boldsymbol{V_2}}{N_2}}{\frac{N_2\boldsymbol{I_2}}{N_1}} = \left(\frac{N_1}{N_2}\right)^2 \frac{\boldsymbol{V_2}}{\boldsymbol{I_2}} = \left(\frac{N_1}{N_2}\right)^2 \boldsymbol{Z_L} = \frac{\boldsymbol{Z_L}}{n^2} \qquad (6-30)$$

Thus the combination of an ideal transformer of ratio n and an impedance $\mathbf{Z}_L$ can be replaced by an equivalent impedance $\mathbf{Z}_L / n^2$

6.4.2 Referred impedances:

Figure 6-16(a) shows an ideal transformer with a load impedance $\mathbf{Z}_L$ connected to the secondary. Another impedance $\mathbf{Z}_2$ is in series with $\mathbf{Z}_L$. The input impedance of this circuit is:

$$\boldsymbol{Z_{in}} = \frac{\boldsymbol{Z_2} + \boldsymbol{Z_L}}{n^2} \qquad (6-31)$$

The input impedance of the circuit in figure 6-16(b) is:

$$\boldsymbol{Z_{in}} = \boldsymbol{Z_2}' + \frac{\boldsymbol{Z_L}}{n^2} \qquad (6-32)$$

The two expressions for $\mathbf{Z}_{in}$ will be identical if:

$$\boldsymbol{Z_2}' = \frac{\boldsymbol{Z_2}}{n^2}\, n \qquad (6-33)$$

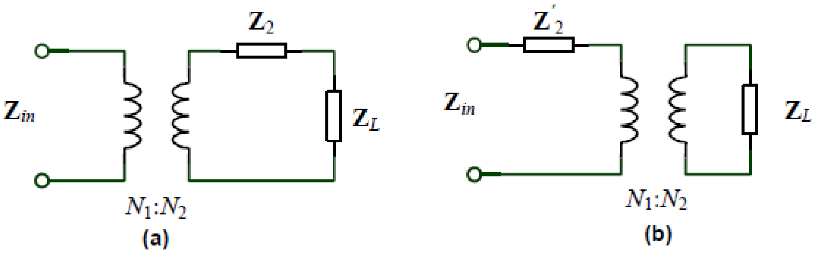


Figure 6-16 Referred impedance – 1.

The impedance $\mathbf{Z}'_2$ is termed the secondary impedance $\mathbf{Z}_2$ referred to the primary. In a similar way, a primary impedance $\mathbf{Z}_1$ can be referred to the secondary, as shown in figure 6-17. In this case, the referred impedance is given by:

$$\boldsymbol{Z_1}'' = n^2\boldsymbol{Z_1} \qquad (6-34)$$

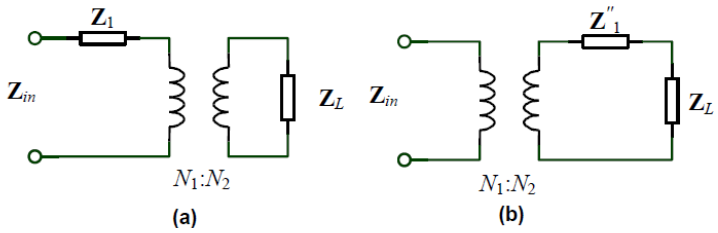


Figure 6-17 Referred impedance – 2.

The concept of referred impedance is often a useful device for simplifying circuits containing transformers, as will be shown in the next section. It is conventional to use a single prime (′) to denote quantities referred to the primary side, and a double prime (″) to denote quantities referred to the secondary side.

6.5 Circuit Model of a transformer:

In a practical transformer, the winding resistances and the core reluctance are not zero. In addition, there will be some power loss in the core because of eddy currents and hysteresis in the magnetic material. All of these effects can be represented by the equivalent circuit [3, 4] shown in figure 6-18.

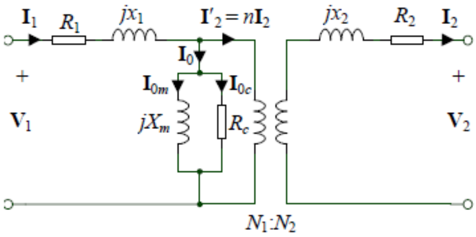


Figure 6-18 Transformer equivalent circuit.

This circuit is based on the ideal transformer element, with additional circuit elements to represent the imperfections. The resistances R_1 and R_2 represent the physical resistances of the windings, and R_c represents the power lost in the core. The reactance X_m, known as the *magnetising reactance,* allows for the current required to magnetise the core when the reluctance is not zero. Reactances x_1 and x_2, known as *leakage reactances,* represent the leakage flux that exists when the magnetic coupling between the primary and the secondary is not perfect. Figure 6-19 shows the leakage flux when the core has an artificially low relative permeability of 10, and one winding at a time is energised. In practice, the leakage is much less than this, so the leakage reactances are normally very much smaller than the magnetising reactance X_m.

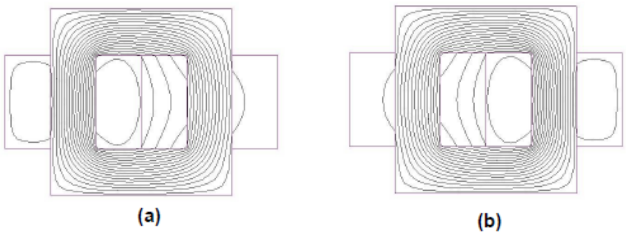


Figure 6-19 Transformer leakage flux:

In the circuit of figure 6-18, the current $\mathbf{I}'_2$ is the effective value of the secondary current as seen from the primary side of the transformer. It is also known as the secondary current *referred* to the primary. The current $\mathbf{I}_0$ is the *no-load current*, which is the current taken by the primary when there is no load connected to the secondary. It has a component $\mathbf{I}_{0m}$, known as the *magnetising current*, which represents the current required to set up the magnetic flux in the core. The current $\mathbf{I}_{0c}$ is the core loss component of the no-load current.

6.5.1 Approximate equivalent circuit.

For power transformers with ratings above 100 VA, the values of the series elements R_1 and x_1 are generally much smaller than the shunt elements R_c and X_m. Under normal working conditions, the voltage drop in $R_1 + jx_1$ is much smaller than the applied voltage V_1. Similarly, the no-load current I_0 is much smaller than the load current I_1. It follows that the shunt elements can be moved to the input terminals, as shown in figure 6-20(a), with very little loss of accuracy. The secondary elements R_2 and jx_2 can be replaced by equivalent elements $R'_2 = R_2 / n^2$ and $x'_2 = x_2 / n^2$ on the primary side (see section 2.3), giving the circuit shown in figure 6-20(b). Finally, the series elements can be combined to give an effective resistance R_e and leakage reactance x_e, as shown in figure 6-20(c), where the values are:

$$R_e = R_1 + \frac{R_2}{n^2} \quad , \qquad x_e = x_1 + \frac{x_2}{n^2} \qquad (6-34)$$

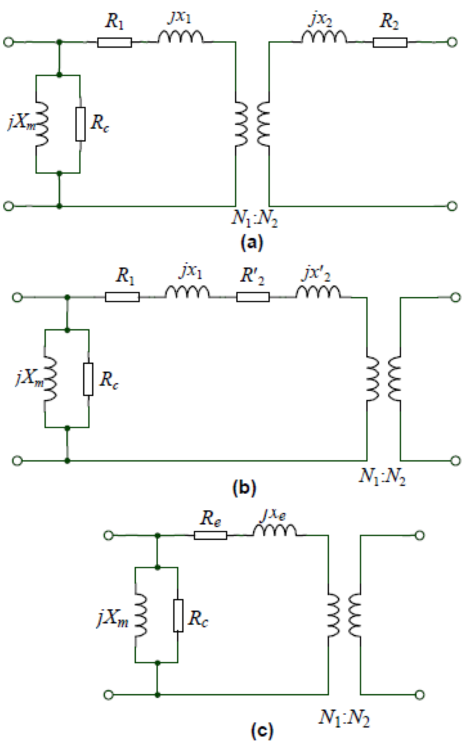


Figure 6-20: Approximate equivalent circuit

6.6 Parameters Determination:

The parameters of the approximate equivalent circuit (figure 2-14) can be determined experimentally from two tests:

- An open-circuit test, where the secondary is left unconnected and the normal rated voltage is applied to the primary.
- A short-circuit test, where the secondary terminals are short-circuited and a low voltage is applied to the primary, sufficient to circulate the normal full-load current.

6.6.1 Open-circuit Test:

With the secondary unconnected, $\mathbf{I}_2 = 0$, so the equivalent circuit reduces to the form shown in figure 6-21.

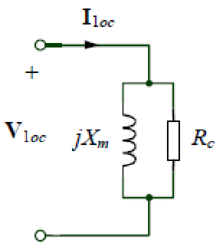


Figure 6-21 Open-circuit test

To a very close approximation, the current I_{1oc} supplied to the primary is equal to the no-load current $\mathbf{I}_0$ in figure 2-18. The values of the elements R_c and X_m can be determined from measurements of the input voltage V_{1oc}, current I_{1oc} and power P_{1oc} as follows. The input power is entirely dissipated in the resistance R_c, giving:

$$R_c = \frac{V_{1oc}^2}{P_{1oc}} \qquad (6-35)$$

The input impedance of the circuit is given by:

$$\boldsymbol{Z}_{1oc} = \frac{\boldsymbol{V}_{1oc}}{\boldsymbol{I}_{1oc}} = \frac{1}{\frac{1}{R_c} + \frac{1}{jX_m}} \qquad (6-36)$$

In terms of magnitudes, equation 6-36 becomes:

$$Z_{1oc} = \frac{V_{1oc}}{P_{1oc}} = \frac{1}{\sqrt{\frac{1}{R_c{}^2} + \frac{1}{X_m{}^2}}} \qquad (6-37)$$

Re-arranging equation 6-37 gives the value of X_m:

$$X_m = \frac{1}{\sqrt{\left(\frac{I_{1oc}}{V_{1oc}}\right)^2 - \frac{1}{R_c{}^2}}} \qquad (6-38)$$

From figure 6-20(c), it follows that the turns ratio is given by:

$$n = \frac{N_2}{N_1} = \frac{V_{2oc}}{V_{1oc}} \qquad (6-39)$$

6.6.2 Short Circuit test:

If the secondary terminals are short-circuited, the ideal transformer in figure 6-20 can be replaced by a short circuit, so the equivalent circuit takes the form shown in figure 6-22(a). In a typical power transformer, the shunt elements R_c and X_m are at least 100 times larger than the series elements R_e and x_e. Consequently, the shunt elements can be neglected, and the circuit reduces to the form shown in figure 6-22(b).

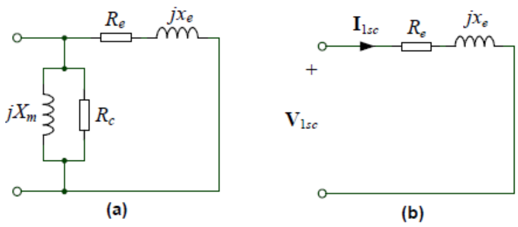


Figure 6-22 Short-circuit test

The values of the elements R_e and x_e can be determined from measurements of the input voltage V_{1sc}, current I_{1sc} and power P_{1sc} as follows. The input power is entirely dissipated in the resistance R_e, giving:

$$R_e = \frac{P_{1sc}}{I_{1sc}^2} \qquad (6-40)$$

The input impedance of the circuit is given by:

$$\boldsymbol{Z}_{1sc} = \frac{\boldsymbol{V}_{1sc}}{\boldsymbol{I}_{1sc}} = R_e + jx_e \qquad (6-41)$$

In terms of magnitudes, equation 2-35 becomes:

$$Z_{1sc} = \frac{V_{1sc}}{I_{1sc}} = \sqrt{R_e{}^2 + x_e{}^2} \qquad (6-42)$$

Re-arranging equation 2-36 gives the value of x_e

$$X_e = \sqrt{\left(\frac{V_{1sc}}{I_{1sc}}\right)^2 - {R_e}^2} \qquad (6-43)$$

In practice, the open-circuit test is usually made on the low-voltage side of the transformer to minimise the value of V_{oc}, and the short-circuit test is made on the high-voltage side to minimise the value of I_{sc}. The resulting parameter values are then referred to the primary side of the transformer.

6.7 Transformer Performance:

Consider a transformer with an impedance ZL connected to the secondary. With the approximate equivalent circuit, this may be represented by the circuit diagram of figure 6-23.

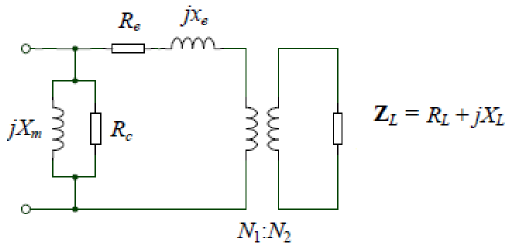


Figure 6-23: Transformer with a load

The load impedance can be referred to the primary side of the ideal transformer element, giving the circuit shown in figure 6-24.

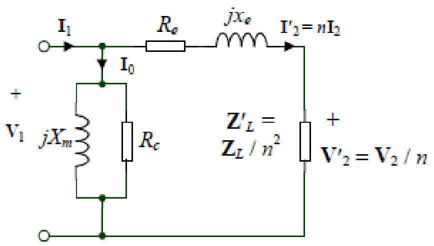


Figure 6-24: Circuit with referred impedance

This circuit is easily solved for the currents $\mathbf{I}_0$ and $\mathbf{I}'_2$. The referred secondary voltage is given by $\mathbf{V}'_2 = \mathbf{Z}_L\mathbf{I}'_2$, and the secondary terminal quantities are given by $\mathbf{V}_2 = n\mathbf{V}'_2$, $\mathbf{I}_2 = \mathbf{I}'_2 / n$.

6.7.1 Voltage regulation and efficiency:

When a load is connected to the secondary of a transformer, there will be a voltage drop in the series elements R_e and x_e, so the secondary terminal voltage will change. The *voltage regulation* is defined as:

$$\varepsilon = \frac{V_{2nl} - V_{2fl}}{V_{2nl}} \qquad (6-44)$$

where V_{2nl} is the magnitude of the no-load secondary terminal voltage, and V_{2fl} is the corresponding full-load voltage. Power is lost as heat in the windings and core of the transformer, represented by the resistance elements R_e and R_c in the equivalent circuit. The *efficiency* is defined in the usual way as:

$$\eta = \frac{P_{out}}{P_{in}} \qquad (6-45)$$

where P_{in} is the power input to the primary and P_{out} is the power output from the secondary. The power lost as heat in the transformer is:

$$P_{loss} = P_{in} - P_{out} \qquad (6-46)$$

so we have the following alternative forms of equation 6-45:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{in} - P_{loss}}{P_{in}} = 1 - \frac{P_{loss}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = 1 - \frac{P_{loss}}{P_{out} + P_{loss}} \qquad (6-47)$$

Large transformers are very efficient. Even a 2 kVA transformer can have an efficiency of about 95%. Above 25 kVA, the efficiency usually exceeds 99%. It is very difficult to make an accurate measurement of efficiency by direct measurement of P_{out} and P_{in}, since this would require an accuracy of measurement of the order of 0.01%. Instead, the normal practice is to determine the losses from measurements, and calculate the efficiency from one of the alternative expressions in equation 6-47. The losses can be calculated with high accuracy from the equivalent-circuit parameters determined from tests on the transformer.

6.7.2 Maximum efficiency:

The power loss in a transformer has two components: the core loss, given by V^2 / R, and the I^2R loss, given by I'^2R_e. The core loss will be constant if the primary voltage V_1 is constant, but the I^2R loss will vary with the secondary current. When the current is low, the output power will be low, but the core loss remains at the normal value. Consequently, the efficiency of the transformer will be low under these conditions. It may be shown that the efficiency is a maximum when the secondary current is such that the variable I^2R loss is equal to the fixed core loss. This result also applies to other devices where the losses have fixed and variable components. Transformers are usually designed to have maximum efficiency at the normal operating value of secondary current, which may be less than the maximum rated current.

6.7.3 Power Relationship:

When calculating the transformer performance, the following power relationships can be useful. The complex power **S** is given by [2]:

$$\boldsymbol{S} = P + jQ = \boldsymbol{VI}^* \qquad (6-48)$$

where **V** is the voltage phasor, **I*** is the complex conjugate of the current, P is the real power, and Q is the reactive power. If ϕ is the phase angle, then:

$$P = S\,cos\phi = VI\,cos\phi = Re(\boldsymbol{VI}^*) \qquad (6-49)$$

$$Q = S\,sin\phi = VI\,sin\phi = Im(\boldsymbol{VI}^*) \qquad (6-50)$$

If the voltage phasor **V** is chosen as the reference quantity, and defined to be purely real ($\mathbf{V} = V + j0$), then the power relationships take a simple form:

$$P = VI\,cos\phi = VRe(\boldsymbol{I}) \qquad (6-51)$$

$$Q = VI\,sin\phi = VIm(\boldsymbol{I}) \qquad (6-52)$$

6.8 Current Transformers

Instrument transformers are special transformers for extending the range of measuring instruments. There are two basic types: voltage transformers for measuring high voltages, and current transformers for measuring high currents. Using transformers for voltage measurement is similar in principle to the ordinary use of transformers to change voltage levels, so it will not be considered further. Current transformers, on the other hand, need special consideration. These are usually toroidal transformers with high-quality core material.

Figure 6-25 shows a load connected to a source. The primary of a current transformer is in series with the load, and the secondary is connected to a meter

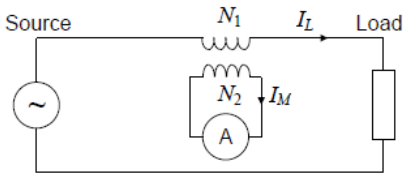


Figure 6-25 Use of a current transformer

Equation 6-19 gives:

$$I_M \approx \frac{N_1}{N_2} I_L \quad or \ I_L \approx \frac{N_1}{N_2} I_M \qquad (6-53)$$

Thus, the unknown current I_L in the load can be determined from the measured current I_M. With a well-designed current transformer, the error in the current measurement from equation 6-18 is typically less than 1%. In many applications, the current-transformer primary is permanently connected in the main circuit, where the current may be very large. The secondary current is usually low – typically, no more than 5 A – and the secondary voltage is also low because the meter resistance is small. It is normal practice to connect one terminal of the meter to earth, so that it is safe to handle the terminals.

- Example:

A current transformer has 10 turns on the primary and 200 turns on the secondary. The secondary is connected to a meter with a resistance of 0.5 Ω, and the secondary current is 5 A. Find the current in the primary and the voltage across the primary terminals. Assume that the transformer is ideal. If the transformer has a toroidal core with a cross-section of 20 mm × 20 mm, and the frequency is 50 Hz, determine the maximum value of the flux density in the core.

- *Solution:*

The secondary terminal voltage is:

$$V_2 = RI_2 = 0.5 \times 5.0 = 2.5 \text{ V}$$

The primary current is:

$$I_1 = \frac{N_2}{N_1} I_L = \frac{200}{10} \times 5.0 = 100\ A$$

The primary voltage is:

$$V_1 = \frac{N_1}{N_2} V_2 = \frac{10}{200} \times 2.5\ V = 125\ mV$$

From Equation 6-10:

$$B_m = \frac{V_{1m}}{2\pi f N_1 A} = \frac{\sqrt{2} \times 125 \times 10^{-3}}{2\pi \times 50 \times 10 \times (25 \times 10^{-3})^2} = 0.141\ T$$

6.8.1 Open-Circuit secondary

Problems arise if it is required to disconnect the meter while current continues to flow in the main circuit. Consider the circuit of figure 6-26, where the load is purely resistive and a switch has been substituted for the meter.

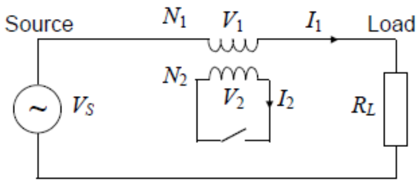


Figure 6-26 Open-Circuit Secondary

With the switch closed, $V_2 = 0$, so $V_1 \approx 0$, and the primary current is determined by the source voltage and the load resistance:

$$I_1 = \frac{V_S}{R_L} \qquad (6-54)$$

The secondary current is thus:

$$I_2 \approx \frac{N_1}{N_2} I_1 \qquad (6-55)$$

With the switch open, the situation is complex. Suppose initially that the transformer can be regarded as ideal. With the switch open, $I_2 = 0$, and therefore $I_1 = 0$. With no current in the load, the transformer primary voltage V_1 must be equal to the source voltage V_S.

In practice, this cannot happen. In worked example 6-1, the primary voltage V_1 is only 125 mV in normal operation, and the core flux density is then 0.141 T. If the source voltage V_s is 400 V, it would require a core flux density of 451 T to make V_1 equal to V_s. Since the core material saturates at about 1.6 T, this is impossible. Opening the secondary switch will therefore not make any significant difference to the primary current, which is still determined by the load resistance R_L. However, the conditions in the transformer are very far from normal. Consider the magnetic circuit equations. With the switch closed,

$$N_1 i_1 - N_2 i_2 \approx 0 \qquad (6-56)$$

With the switch open, there is no secondary MMF to balance the primary MMF N_1I_1, so the core material is subjected to a large alternating H field given by equation 4-2:

$$H = \frac{N_1 i_1}{l} = \frac{N_1 I_{1m} sin\omega t}{l} \qquad (6-57)$$

Where l is the mean length of the magnetic core. The core material is taken well into saturation, so the flux waveform is almost a square wave. This can result in dangerously high voltage pulses being induced in the secondary, which can damage the transformer insulation and present a hazard to the operator.

6.8.2 Safe operation:

If it is necessary to disconnect the secondary load without interrupting the primary current, this can be done safely by including a switch in parallel with the load as shown in figure 6-27. The switch must be closed before disconnecting the load, so that the secondary current I_2 continues to flow through the switch.

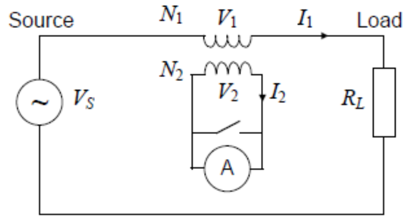


Figure 6-27 Safe Operation

6.8.3. Numerical Example:

Suppose that the current transformer has a toroidal core with a cross-section of 20 mm × 20 mm and a mean diameter of 100 mm. There are 10 turns on the primary, 200 turns on the secondary, the RMS current in the primary is 89 A, and the frequency is 50 Hz. The magnetic material is silicon steel, with the magnetisation data given in section 12-1. From equation 6-21, the maximum value of H is:

$$H_m = \frac{N_1 I_{1m}}{\pi D} = \frac{10 \times \sqrt{2} \times 89}{\pi \times 100 \times 10^{-3}}$$

The period of the waveform is:

$$T = \frac{1}{f} = \frac{1}{50} = 20\ ms$$

Figure 6-14(a) shows the waveform of H, which is proportional to the sinusoidal primary current. This is applied to the magnetisation curve in figure 6-14(b). It takes the magnetic material well into saturation, so the resulting waveform of B, shown in figure 6-14(c), is almost a square wave. The red dots on these graphs represent successive instants of time during the first quarter cycle of the current waveform. The voltage induced in the secondary is:

$$e_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\phi}{dt} = N_2 A \frac{dB}{dt}$$

where A is the cross-sectional area of the core. For most of the time, B is almost constant, so dB / dt is small. However, when the sinusoidal current changes from negative to positive, B suddenly changes from negative saturation to positive saturation, and the resulting dB / dt is very large. There is a similar effect at the negative-going transition. Figure 6-28(d) shows a graph of dB / dt, obtained by numerical differentiation of the graph of B. From this graph, the maximum value of dB / dt is 14.4 T/ms. The maximum induced voltage in the secondary is therefore:

$$e_{2max} = N_2 A \frac{dB}{dt} = 200 \times (20 \times 10^{-3})^2 \times 14.4\ \times 10^3 = 1150\ V$$

6.8.4 Current Transformer errors:

In a well-designed current transformer, the core flux density is low and the core is made from a high-quality magnetic material. Under normal operating conditions, the core loss will be negligibly small, so the core loss resistance R_c can be omitted from the equivalent circuit. A circuit model for a current transformer connected to a load therefore takes the form shown in figure 6-29.

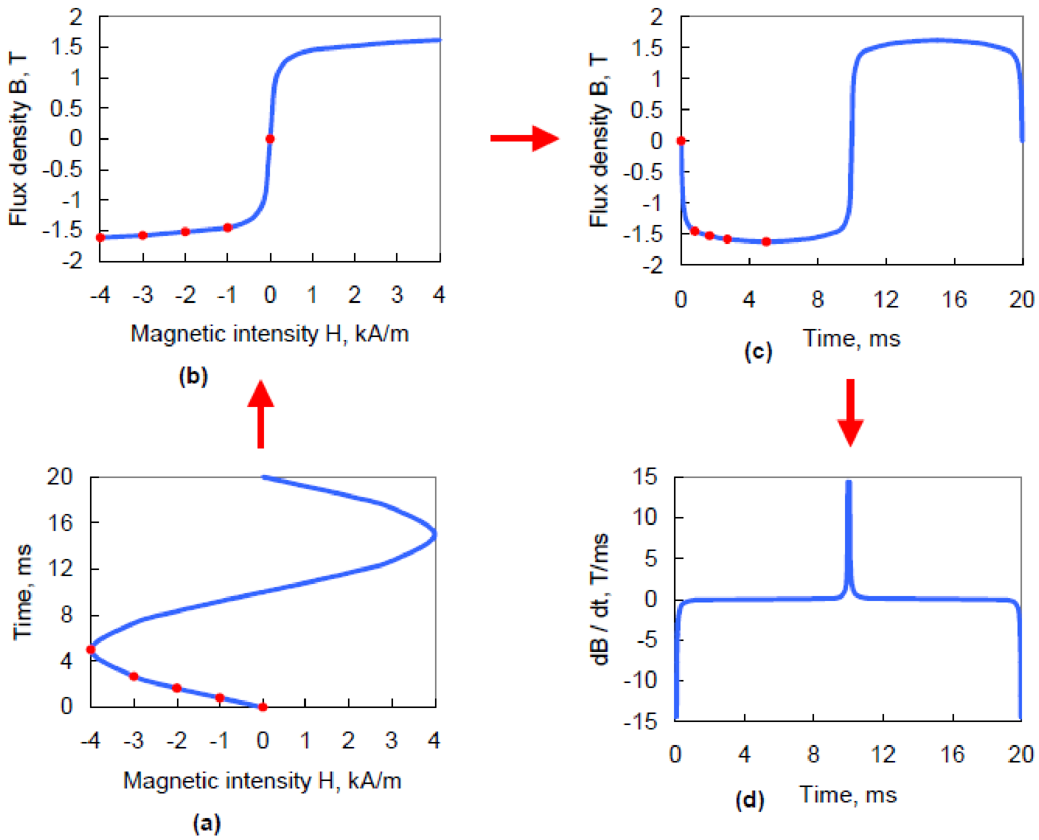


Figure 6-28 : Current transformer with open-circuit secondary. (a) H waveform, (b) magnetisation curve, (c) B waveform, (d) dB / dt waveform.

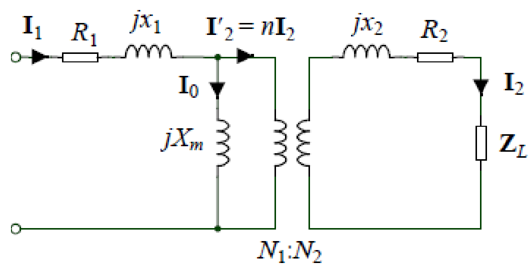


Figure 6-29 Current Transformer Circuit Model

Only the relationship between currents is of interest, so the primary impedance ($R_1 + jx_1$) can be disregarded. It is now convenient to refer the primary quantities to the secondary side, giving the circuit model shown in figure 6-30. The circuit acts as a current divider, where the current in the secondary branch is given by:

$$\mathbf{I_2} = \frac{jX''_m \mathbf{I''_1}}{\mathbf{Z_2} + \mathbf{Z_L} + jX''_m} \qquad (6-58)$$

where $\mathbf{I}''_1$ is the primary current referred to the secondary, and $\mathbf{Z}_2 = R_2 + jx_2$. In a well-designed transformer, the secondary impedance Z_2 is very small in comparison with the referred magnetising reactance X''_m, so this term introduces very little error. Equation 2-48 shows that it is desirable to keep the load impedance $\mathbf{Z}_L$ as small as possible if the error is to be minimised.

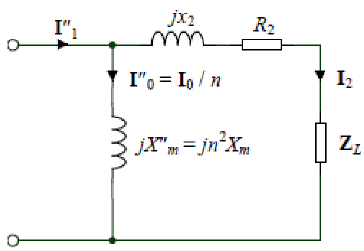


Figure 6-30 Modified Circuit Model

In practice, current transformers are designed for a specified maximum secondary voltage at the rated secondary current. This defines a maximum apparent power for the secondary load, or *burden*. Typically, a small current transformer will have a rated secondary burden of 5 VA. With the usual secondary current rating of 5 A, this implies that the maximum secondary voltage is 1 V, and the maximum impedance magnitude is 0.2 Ω.

6.8 Transformer Design:

The majority of single-phase transformers use the shell type of construction shown in figure 6-3. Normally the core laminations are made in two parts, termed E and I laminations, as shown in figure 6-31.

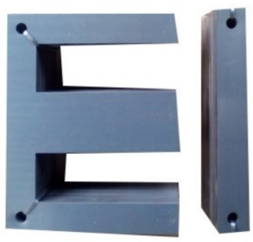


Figure 6-31 E I laminations

The center limb is twice the width of the outer limbs because it carries twice the flux, as shown by the flux plot in figure 6-32.

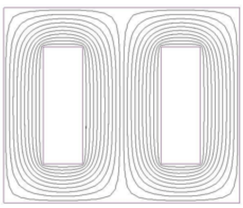


Figure 6-32 Flux Plot: Shell type transformer

The coils are wound on a bobbin that fits the center limb of the core, and the core is assembled by inserting E laminations alternately from each side and adding matching I laminations. Dimensions are chosen so that two E and two I laminations can be punched from a rectangular steel sheet without any waste. Current flowing in the resistance of the transformer windings will produce heat, which must escape through the surface of the windings. In addition, there will be power loss in the core, which also appears as heat.

The power output from a given size of transformer is governed by the rate at which heat can be removed. Large transformers are usually cooled by circulating oil, but small transformers rely on natural convection cooling in air. A simple design approach for small transformers is given below.

6.8.1 Thermal Model

The rate of cooling depends on the exposed surface area of the transformer and the temperature rise above ambient. An exact calculation is complex, since it needs to take account of temperature gradients within the transformer as well as the cooling conditions on different surfaces.

A simple thermal model ignores temperature gradients, and the power loss in the core. It just considers the I^2R loss in the windings, and assumes that this heat escapes through the exposed surfaces of the windings. It is assumed that the temperature rise is proportional to the power loss per unit area:

$$\Delta T = \frac{k_c P_l}{A_s} \qquad (6-59)$$

Where ΔT is the temperature rise above ambient, P_l is the total power loss in the windings, A_s is the exposed surface area, and k_c is a cooling coefficient with a typical value of 0.04 Km^2/W. Figure 6-33 shows the side view and top view of a shell-type transformer.

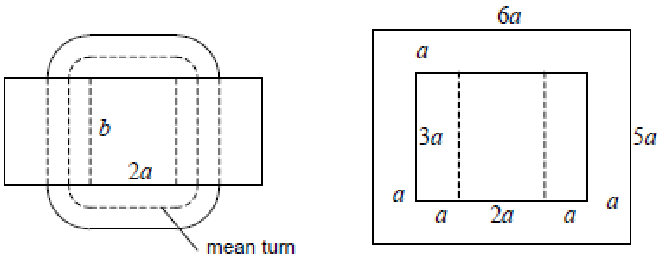


Figure 6-33 Shell-type transformer dimensions

It is assumed that the core is made from laminations punched as shown in figure 6-32. If a is the width of each outer limb of the core, then the width of the centre limb is $2a$, and the other dimensions are as shown in figure 6-33.

6.8.2 Winding resistance

The total cross-sectional area of the two windings is the *window area* of height $3a$ and width a. Each winding occupies half of this area, so the conductor cross-sectional area for each winding is:

$$A_c = 1.5 k_s a^2 \qquad (6-60)$$

where k_s is the conductor space factor, which allows for insulation and space between the turns.

For simplicity, it will be assumed that the primary and secondary windings are placed side- by-side on the core, and that they have the same number of turns N. From figure 2-25, the mean turn length of each winding is:

$$l_m = \pi a + 4a + 2b = (\pi + 4)a + 2b \qquad (6-61)$$

If the winding has N turns, then the total length of wire is Nl_m, and the cross-sectional area of the wire is A_c / N. The winding resistance is therefore:

$$R = \frac{\rho l}{A} = \frac{\rho N l_m}{A_c/N} = \frac{\rho N^2\{(\pi + 4)a + 2b\}}{1.5 k_s a^2} \qquad (6-62)$$

6.8.3 *Temperature Rise*

If the RMS current in one winding is *I*, the power loss is I^2R. The cooling surface area of the winding, from figure 6-33, is:

$$A_s = 1.5a(4a + 2\pi a) + 4a^2 + \pi a^2 = a^2(10 + 4\pi) \quad (6-63)$$

Substituting in equation 6-59 gives:

$$\Delta T = \frac{k_c P_l}{A_s} = \frac{k_c I^2 R}{A_s} = \frac{k_c \rho N^2 I^2 \{(\pi + 4)a + 2b\}}{3k_s a^4 (5 + 2\pi)} \quad (6-64)$$

Thus, the current is given by:

$$I^2 = \frac{k_s a^4 (5 + 2\pi)\Delta T}{k_c \rho N^2 \{(\pi + 4)a + 2b\}} \quad (6-65)$$

From equation 6-15:

$$N = \frac{V_{max}}{2\pi f A B_{max}} = \frac{\sqrt{2}V}{4\pi f a b B_{max}} \quad (6-66)$$

- Example

A transformer has a primary wound for 230 V, and the core measures 60 × 50 × 30 mm. The maximum winding temperature is 90ºC, the ambient temperature is 30ºC, the winding space factor is 0.4, the cooling coefficient is 0.04 Km^2/W, and the resistivity of copper at 90ºC is 21.9 nΩm. If the maximum flux density in the core is 1.4 T and the frequency is 50 Hz, determine (a) the number of turns on the primary, (b) the maximum current in the primary.

- *Solution*

From figure 2-25, *a* = 10 mm and *b* = 30 mm. From equation 6-66, we have:

$$N \frac{\sqrt{2}V}{4\pi f a b B_{max}} = \frac{\sqrt{2} \times 230}{4\pi \times 50 \times 10 \times 10^{-3} \times 1.4} = 1230$$

Equation 6-64 gives:

$$I^2 = \frac{k_s a^4 (5 + 2\pi)\Delta T}{k_c \rho N^2 \{(\pi + 4)a + 2b\}}$$

$$I = \sqrt{\frac{k_s a^4 (5 + 2\pi)\Delta T}{k_c \rho N^2 \{(\pi + 4)a + 2b\}}}$$

$$I = \sqrt{\frac{3 \times 0.4 \times (0 \times 10^{-3})^4 (5 + 2\pi)(90 - 30)}{0.04 \times 21.9 \times 10^{-9} \times (1230)^2 \times \{(\pi + 4) \times 10 \times 10^{-3} + 2 \times 30 \times 10^{-3}\}}} = 0.216A$$

6.8.4 *Rating and size:*

A relationship between the apparent power rating of the transformer and the dimensions can be obtained by substituting for *N* from equation 6-66 in equation 6-65:

$$VI = \frac{2\pi B_{max} b a^3 \sqrt{3k_s(5 + 2\pi)\Delta T}}{\sqrt{k_c \rho \{(\pi + 4)a + 2b\}}} \qquad (6-67)$$

If it is assumed that the core depth b is proportional to the dimension a, and other quantities remain constant, then equation 6-67 gives the following relationship:

$$VI \propto a^{3.5} \qquad (6-68)$$

From equation 6-68, if the dimensions of the transformer are doubled, the rating will increase by a factor of 11.3. A similar result is obtained for the increase in the power output of a DC machine when the dimensions are doubled.

References:

[1] Smith, R.J. and Dorf, R.C. *Circuits, Devices and Systems* (5th Edition, Wiley 1992)

[2] Jayawant, B.V.: *Electromagnetic Levitation and Suspension Techniques* (Edward Arnold, 1981).

[3] McCaig, M. and Clegg, A.G.: *Permanent Magnets in Theory and Practice* (2nd edition, Pentech Press, 1987).

[4] Edwards, J.D.: *Electrical Machines and Drives* (Macmillan, 1991).

[5] Kraus, J.D.: *Electromagnetics* (5th edition, McGraw-Hill, 1999).

[6] Carter, G.W.: *The Electromagnetic Field in its Engineering Aspects* (2nd edition, Longmans, 1967), chapter 8.

7. Appendices

7.1 Silicon steel characteristics

The magnetization characteristics in figures 7-1 and 7-2, are extended and expanded versions of figure 3-18. They are derived from the characteristics for Carpenter silicon steel from the material library of the electromagnetic simulation software MagNet (Mentor Corporation).
Numerical *B* and *H* values are as follows.

H (kA/m)	*B* (T)
0	0
0.05	0.431
0.07	0.66
0.1	0.845
0.14	0.984
0.2	1.101
0.3	1.214
0.4	1.286
0.5	1.335
0.7	1.4
1	1.453
1.4	1.485
2	1.522
3	1.58
4	1.62
5	1.652
7	1.708
10	1.773
14	1.842
20	1.921
30	2.017
40	2.089

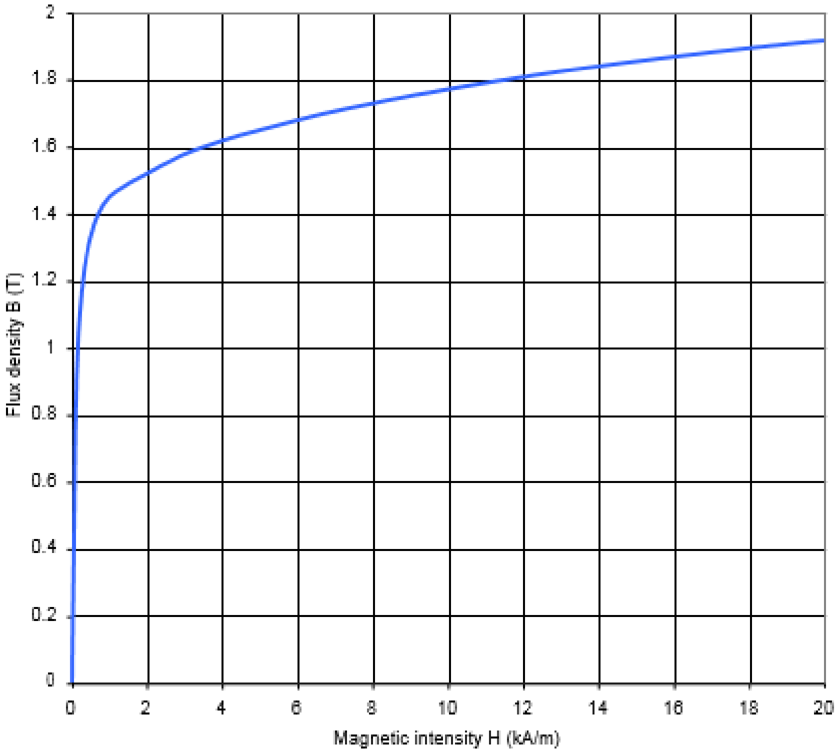

Flux density B (T)
Magnetic intensity H (kA/m)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
2
4
6
8
10
12
14
16
18
20

Figure 7-1: Carpenter silicon steel: extended characteristic

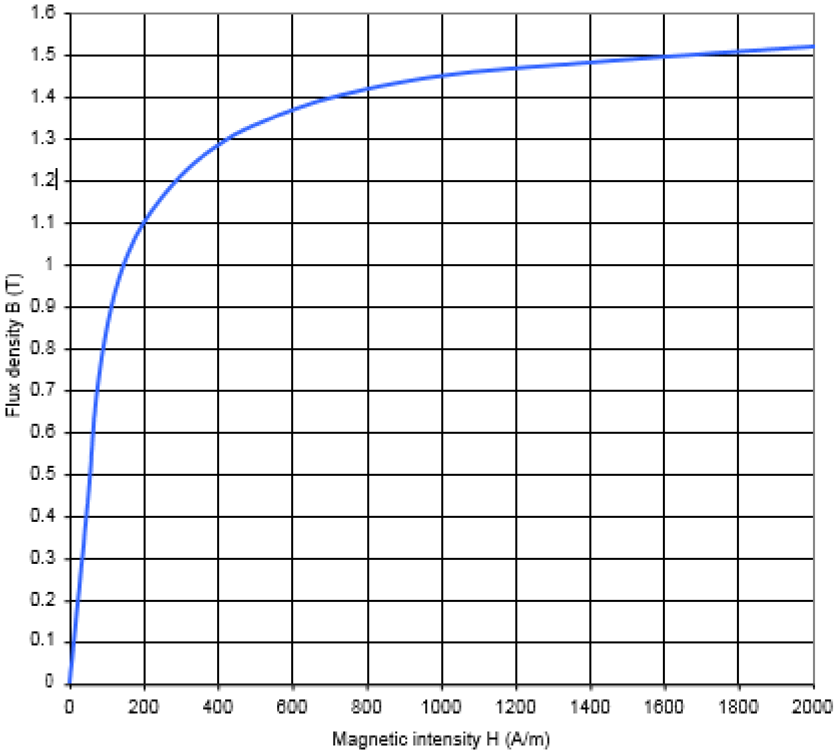


Figure 7-2: Carpenter silicon steel: expanded characteristic