INDIVIDUAL TASK 3

APPLICATION OF BAYES' THEOREM IN MEDICAL TESTING

1. Introduction

Probability plays a crucial role in decision-making, especially in medical diagnosis, risk assessment, and artificial intelligence systems. One of the most important concepts in probability theory is Bayes' Theorem, which helps us update probabilities when new information becomes available.

Bayes' Theorem is widely used in real-world applications such as:

Medical testing

Email spam filtering

Fraud detection

Machine learning

Weather forecasting

In this task, we will apply Bayes' Theorem to a medical testing scenario to calculate the probability of a person actually having a disease after receiving a positive test result.

2. Statement of Bayes' Theorem

Bayes' Theorem is mathematically expressed as:

Where:

$P(A|B)$ = Probability of event A occurring given that B has occurred (Posterior Probability)

$P(B|A)$ = Probability of event B occurring given that A is true (Likelihood)

$P(A)$ = Initial probability of event A (Prior Probability)

$P(B)$ = Total probability of event B

In medical testing:

A = Person has the disease

B = Test result is positive

1. 3. Real-World Scenario: Medical Testing

Suppose a certain disease affects 1% of the population. A medical test is developed with the following characteristics:

If a person has the disease, the test correctly identifies it 99% of the time.

(Sensitivity = 99%)

If a person does not have the disease, the test correctly gives a negative result 95% of the time.

(Specificity = 95%)

We want to find:

👉 If a person tests positive, what is the probability that they actually have the disease?

4. Given Data

Let:

P(D) = Probability of having the disease = 0.01

P(Not D) = Probability of not having the disease = 0.99

P(Positive | D) = 0.99

P(Positive | Not D) = 0.05

(Note: Since specificity is 95%, false positive rate = 5% = 0.05)

5. Step-by-Step Calculation

Step 1: Find Total Probability of Testing Positive

Using total probability rule:

Substitute values:

Step 2: Apply Bayes' Theorem

6. Final Answer

This means:

👉 Even if a person tests positive, the probability that they actually have the disease is only 16.7% (approximately 17%).

7. Interpretation and Explanation

This result may seem surprising because the test is 99% accurate. However, since the disease is very rare (only 1% of the population), false positives significantly affect the final probability.

This demonstrates an important concept called the Base Rate Effect — when a disease is rare, even a highly accurate test can produce many false positives.

For example:

If 10,000 people are tested:

100 people actually have the disease.

99 will test positive.

9,900 people do not have the disease.

495 will still test positive (false positives).

Total positive tests = 99 + 495 = 594

Actual disease cases among positives = 99

Probability = 99 / 594 ≈ 16.7%

This confirms our earlier calculation.

8. Importance of Bayes' Theorem in Medical Diagnosis

Bayes' Theorem helps doctors:

Interpret diagnostic test results correctly

Avoid unnecessary panic

Make better treatment decisions

Combine multiple test results for accurate diagnosis

Understand risk levels in population screening

It is especially useful when dealing with rare diseases or mass screening programs.

9. Applications Beyond Medical Testing

Bayes' Theorem is also used in:

Email spam filtering

Fraud detection in banking

Machine learning classification models

Artificial intelligence systems

Weather prediction models

For example, spam filters calculate the probability that an email is spam based on certain keywords and previous patterns using Bayesian probability.

10. Conclusion

Bayes’ Theorem is a powerful statistical tool used to update probabilities based on new evidence. In the medical testing example, we observed that even with a highly accurate test, the probability of actually having the disease after testing positive was only 16.7% due to the low prevalence of the disease.

This example highlights the importance of understanding conditional probability in real-world decision-making. Bayes’ Theorem plays a critical role in healthcare, artificial intelligence, and risk analysis, making it one of the most valuable concepts in probability theory.

If you want, I can also create another Individual Task 1 using Email Spam Filtering with detailed probability calculations.