

Big O Notation

A mathematical notation which describes the upper limit of the function depicting the relationship between the time and input size.

For example, for

$$O(N^3)$$

The time complexity cannot exceed $O(n^3)$. It can be $O(n^2)$, $O(n)$, $O(\log n)$, but it can't be greater than $O(n^3)$.

Let's dive into the mathematical part

If

$$f(n) = O(g(n))$$

then,

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} < \infty$$

$$\begin{array}{ccc} f(n) & & O(g(n)) \\ \swarrow & & \nwarrow \\ 6(n^3) + 3n + 5 = O(n^3) \end{array}$$

$$\lim_{n \rightarrow \infty} \frac{6(n^3) + 3n + 5}{n^3}$$

$$= \lim_{n \rightarrow \infty} 6 + \frac{3}{n^2} + \frac{5}{n^3}$$

Substituting $n = \text{infinity}$

$$= 6 + \frac{3}{\infty} + \frac{5}{\infty}$$

$$= 6 + 0 + 0$$

$$= 6$$

$$< \infty$$

And that's why we ignore constants as well as less dominating terms!