# MA 103: Quiz 2 (2024)

(1) **(2 points each)** Pick the correct answer out of the choices given for each of the questions below, or say whether True or False. **No justification required. No partial marking.**

(i) The tangent to the *Witch of Agnesi* curve whose parametrized form is $y = 2\tan(\theta), x = 2\cos^2(\theta)$ at the point $(2, 1)$ is given by

(a) $y - 1 = 1 - x/2$.
(b) $y - 1 = 2 - x$.
(c) $y - 1 = \sqrt{2} - x/\sqrt{2}$.

(ii) Choose the correct statement for a twice-differentiable function $f$ whose domain is the whole real line.

(a) If $f''(c)$ does not exist, then $(c, f(c))$ cannot be an inflection point of $f$.
(b) If $f''(c) = 0$, then $(c, f(c))$ is an inflection point of $f$.
(c) If $f''(x) < 0$ for $x < c$ and $f''(x) > 0$ for $x > c$, then $(c, f(c))$ is an inflection point of $f$.
(d) None of these.

(iii) Let $f(x) = 1 - x^{2/3}$. Then $f(1) = f(-1) = 0$, and yet it is true that $f'(x)$ is never zero in the interval $[-1, 1]$. Why does this not contradict Rolle's theorem?

(a) because $f$ is not continuous on $[-1, 1]$.
(b) because $f$ is not differentiable on $(-1, 1)$.
(c) because $f'$ is not continuous on $(-1, 1)$.
(d) because $f'$ is not differentiable on $(-1, 1)$.

(iv) If $f$ is a continuous function on the interval $[0, 3]$ with $f(0) > 0$ and $f(3) < 27$, then there exists a number $c$ in $(0, 3)$ such that $f(c) = c^3$.

(a) True
(b) False.

(v) Let $n$ be an integer greater than or equal to 2, and suppose $f$ is a polynomial of degree $n$. The maximum number of inflection points that $f$ can have is

(a) $n - 1$
(b) $n$
(c) $n - 2$
(d) None, since $f$ is a polynomial.

(2) (i) **(5 points)** Evaluate the limit **without using L'Hospital's rule or infinite series expansions:**

$$\lim_{x \to 0} \frac{\sqrt{9-x}-3}{3-\sqrt{9+x}}.$$

(ii) **(5 points)** Find the equation of the normal to the curve $x^3y^3 + y^2 = x + y$ at the point $(1, -1)$.

(3) **(10 points)** Prove that there is no value of $k$ such that the equation $x^3 - 3x + k = 0$ has two distinct roots in $[0, 1]$.

(4) **(10 points)** Sketch the graph of the function $f(x) = x^3 - 3x^2 + 3x$ using the following:

(a) Identify where the extrema of $f$ occur.
(b) Find the intervals on which $f$ is increasing and the intervals on which $f$ is decreasing.
(c) Find where the graph of $f$ is concave up and where it is concave down.
(d) Sketch the general shape for $f$.