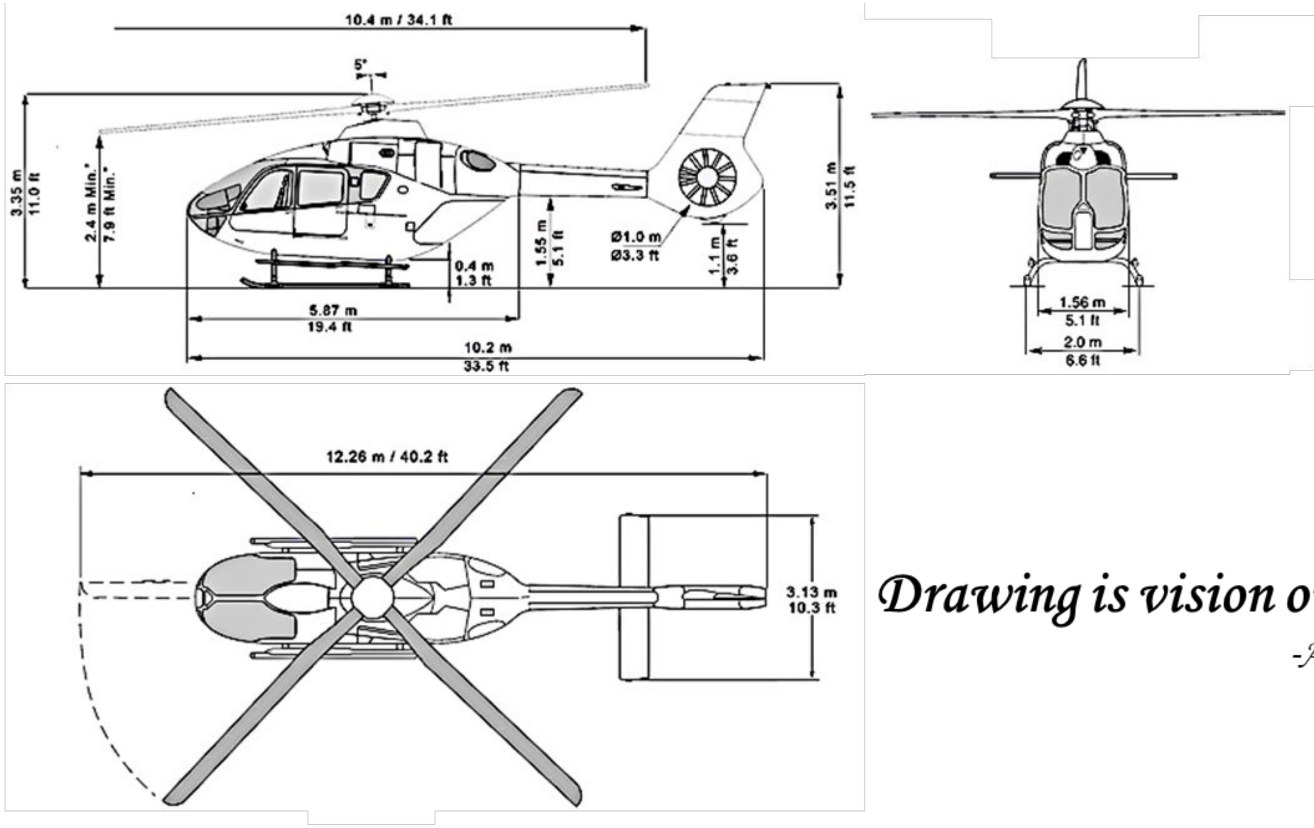
ES 101: Engineering Graphics



Drawing is vision on paper

-Andrew Loomis

https://www.aiut-alpin-dolomites.com/english/technical_details.html

Class#5 – 16th October 2024

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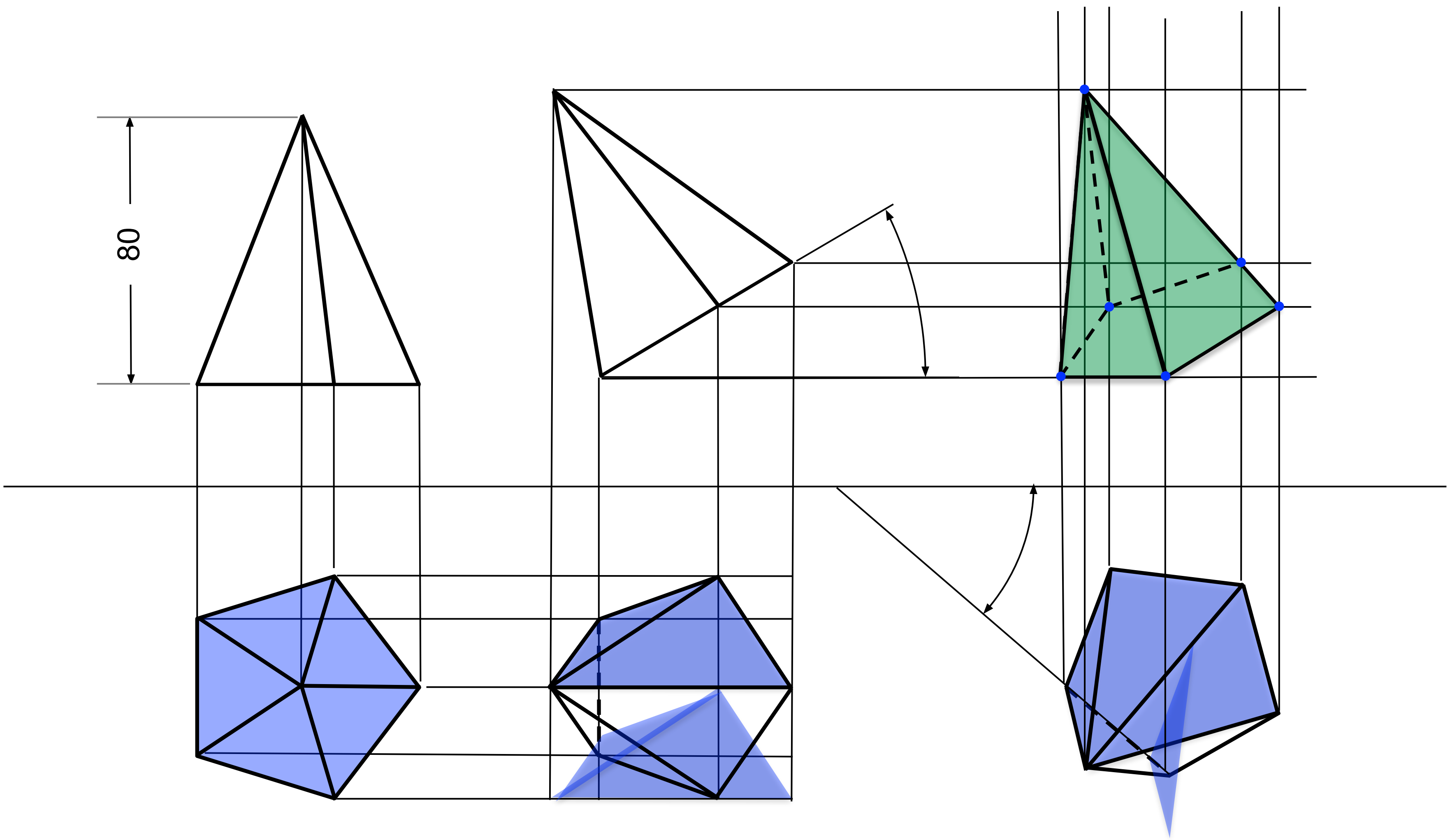
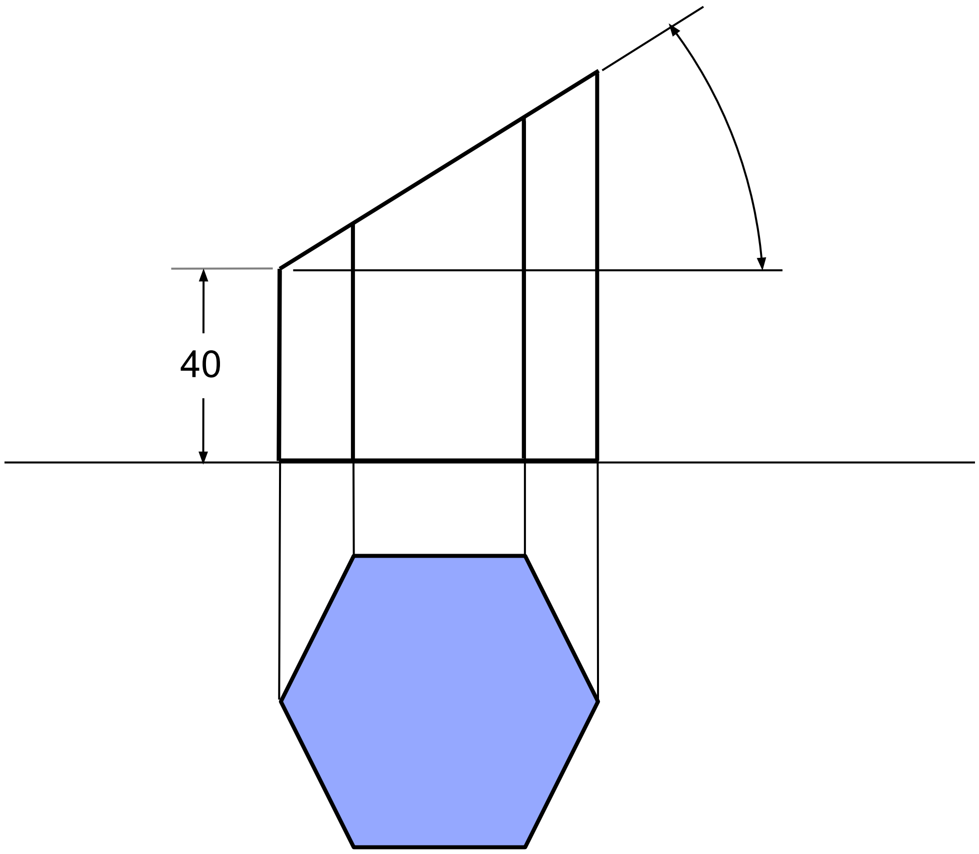
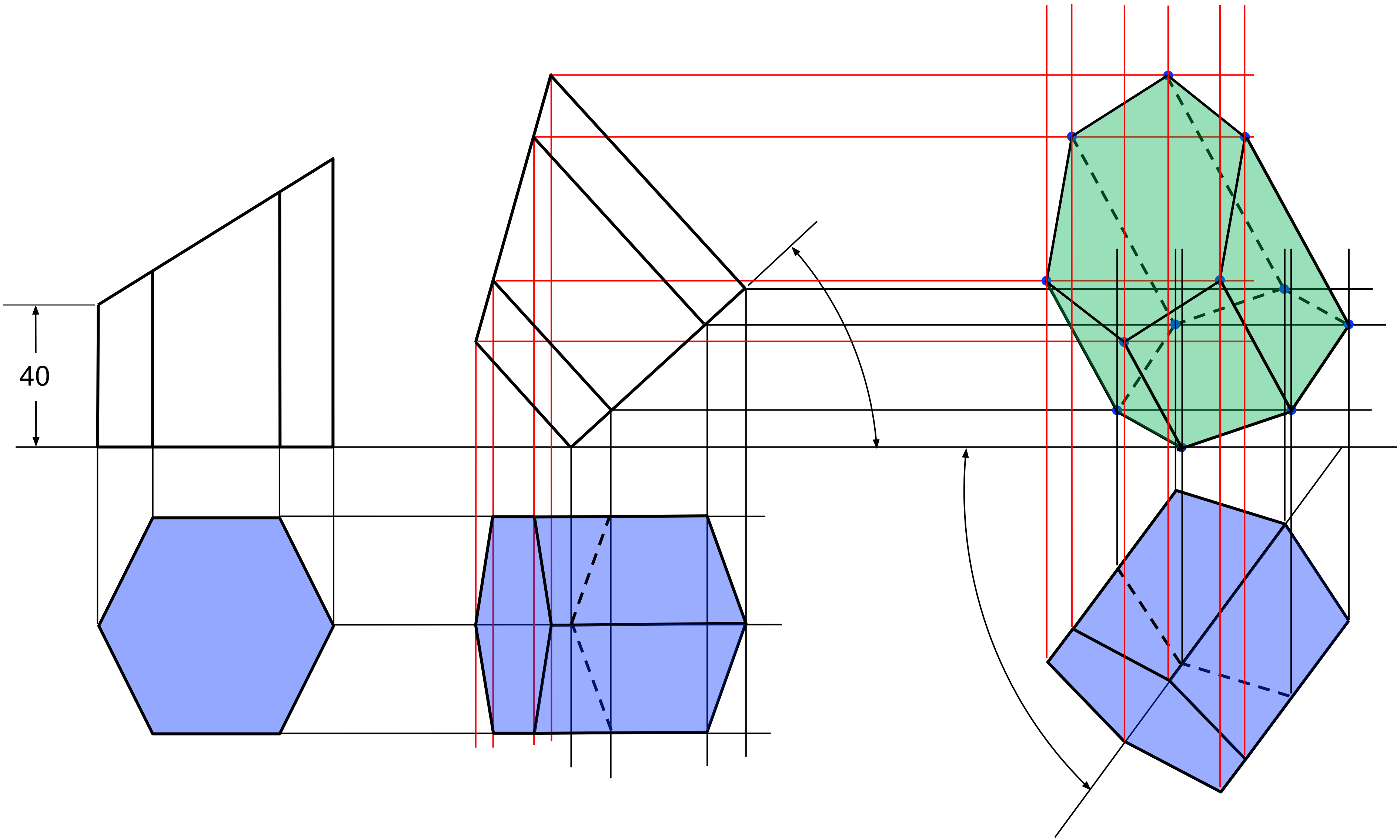
Projection of Solids



Projection of Solids



Projection of Solids



Brachistochrone (least time) problem

baſi figuram quamcunque ſive rectilineam ſive curvilineam, abſcindet hoc prisma ex ſuperficie conica portionem, quæ erit ad baſin prismatis, ut latus coni ad radium baſis coni. Ex quo ultro patet, cuilibet ſpatio plano ſive quadrabili ſive non quadrabili poſſe ſumi abſolute ſpatium æquale ex ſuperficie coni recti, & viciſſim. Item omnis portio ſuperficiei conicæ rectæ terminata a tribus pluribusve hyperbolis in cono factis, quorum axes ſunt paralleli axi coni, eſt quadrabilis, utpote æqualis figuræ rectilineæ.

Problema novum ad cujus ſolutionem Mathematici invitantur.

Datis in plano verticali duobus punctis A & B (vid. Fig. 5) *aſſignare Mobili M, viam AMB, per quam gravitate ſua deſcendens & moveri incipiens a puncto A, breviſſimo tempore perveniat ad alterum punctum B.* TAB. V. Fig. 5.

Ut harum rerum amatores inſtigentur & propenſiori animo ferantur ad tentamen hujus problematis, ſciant non conſiſtere in nuda ſpeculatione, ut quidem videtur, ac ſi nullum haberet uſum; habet enim maximum etiam in aliis ſcientiis quam in mechanicis, quod nemo facile crediderit. Interim (ut forte quorundam præcipiti judicio obviam eam) quanquam recta AB ſit breviſſima inter terminos A & B, non tamen illa breviſſimo tempore percurritur; ſed eſt curva AMB Geometris notiſſima, quam ego nominabo, ſi elapſo hoc anno nemo alius eam nominaverit.

"Given in a vertical plane two points A and B, assign to the moving (body) M, the path AMB, by means of which - descending by its own weight and beginning to be moved (by gravity) from point A - it would arrive at the other point B in the shortest time."

https://www.cantorsparadise.com/the-famous-problem-of-the-brachistochrone-8b955d24bdf7

Brachistochrone (least time) problem

- This problem was posed by Johann Bernoulli in the journal *Acta Eruditorum*
- The problem is of finding the path followed by a particle sliding under the influence of gravity starting at point A and terminating at point B in least time
- Owing to the conservation of energy (frictionless path), the velocity of the particle at any arbitrary height y is given by

$$v = \frac{ds}{dt} = \sqrt{2g(h-y)}$$

Note that the particle starts with zero initial velocity

- The time taken to traverse from point A to point B is

$$T = \int_A^B \frac{ds}{\sqrt{2g(h-y)}}$$

- Considering the trajectory to be a differentiable function $y = f(x)$, from the differential geometry we have

$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{df}{dx}\right)^2} \Rightarrow T = \frac{1}{\sqrt{2g}} \int_{x_0}^{x_1} \sqrt{\frac{1 + (f')^2}{h - f}}\, dx$$

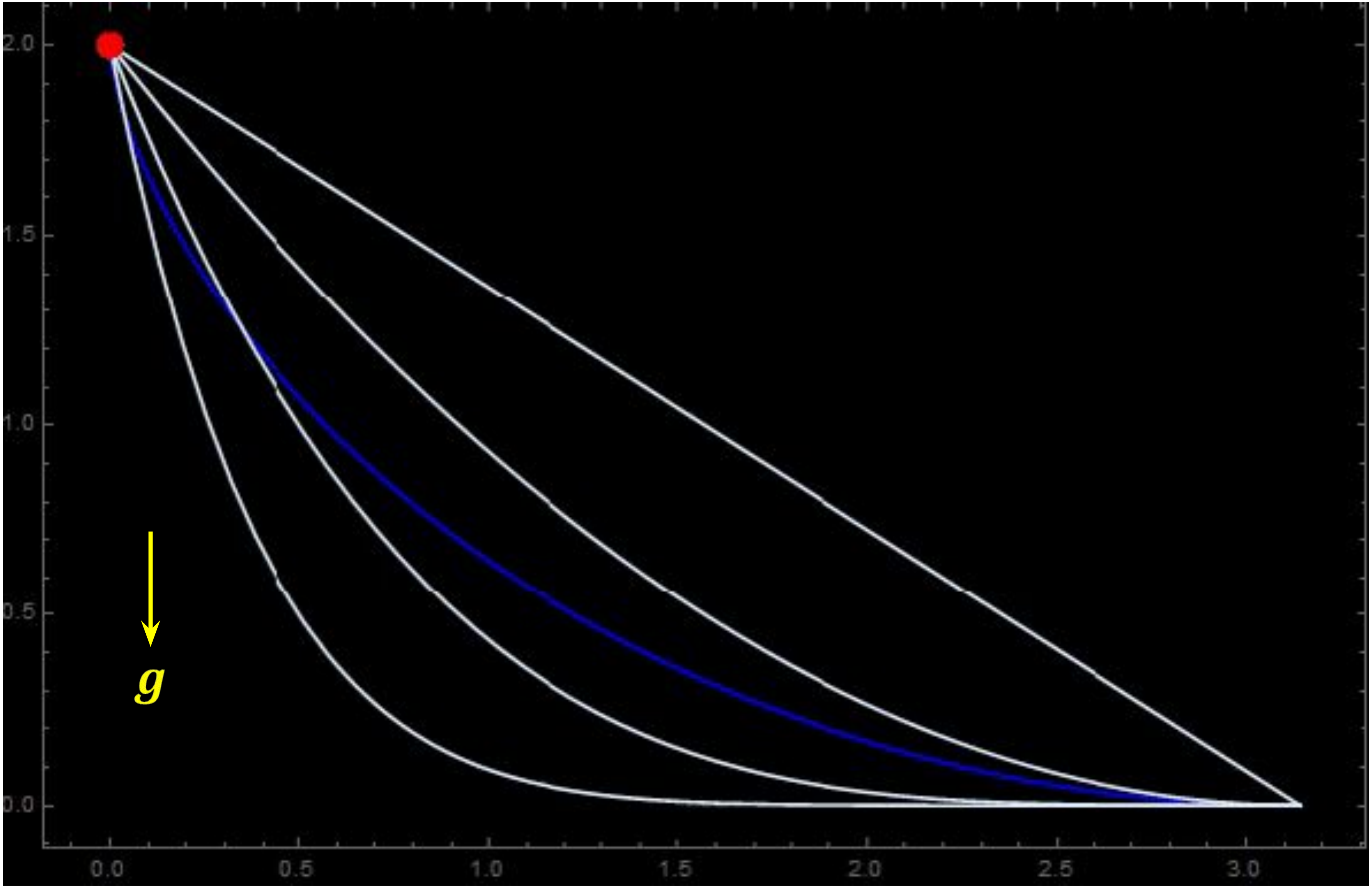
Brachistochrone (least time) problem

- The problem thus comes to the question of finding a function $f(x)$ such that the time T is minimum. Accordingly, for each $f(x)$ passing through the two points A and B, the time T can be evaluated. Such quantities are called functionals, i.e. T is a function of a function $f(x)$
- The functional is quite difficult to evaluate

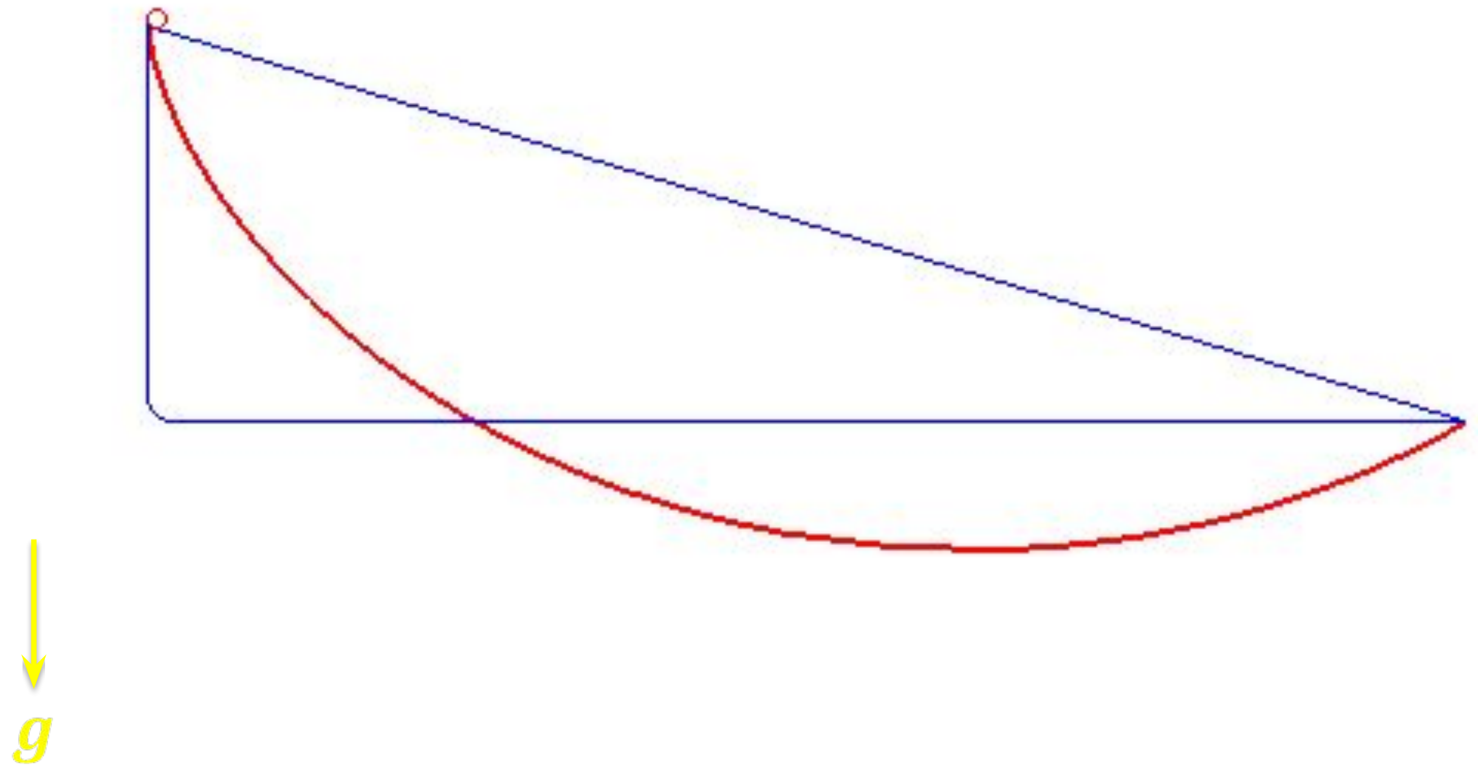
$$T\big(f(x)\big) = \frac{1}{\sqrt{2g}} \int_{x_0}^{x_1} \sqrt{\frac{1+(f')^2}{h-f}}\, dx$$

- The expression $T\big(f(x)\big)$ depends on the whole behavior of the function $f(x)$
- The curve $y = f(x)$ which results in the least possible $T\big(f(x)\big)$ corresponds to *cycloid* (it is the locus of a point on the circumference of a circle that rolls without slipping along a straight line)
- The solution was given by Johann Bernoulli, Jacob Bernoulli, Newton and l'Hospital

https://mathworld.wolfram.com/Cycloid.html



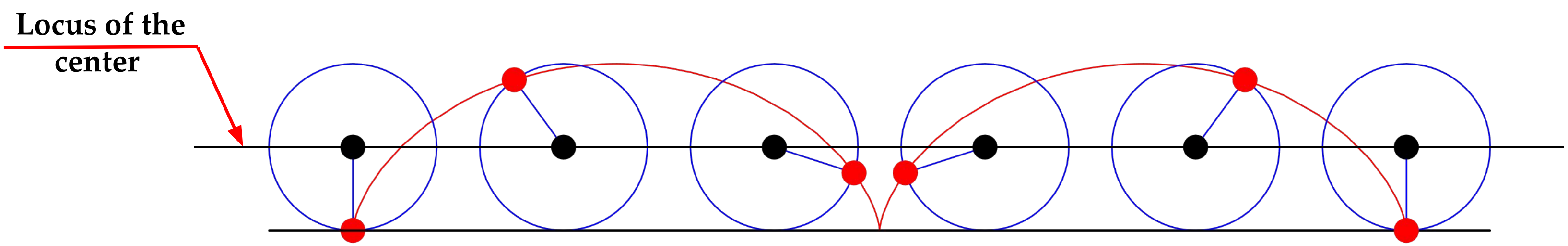
https://gfycat.com/biodegradableflowery americankestrel



https://en.wikipedia.org/wiki/Brachistochrone_curve

Cycloid

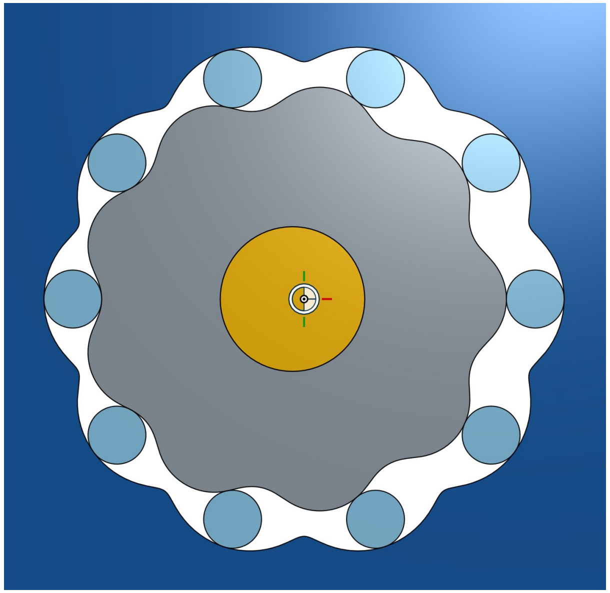
- Cycloid is the curve traced by a point on a circle as it rolls along a straight line without slipping
- The name was coined by Galileo Galilei, who studied it extensively
- The exact solution of the famous Brachistochrone problem is a cycloid and the solution was provided by Johann Bernoulli

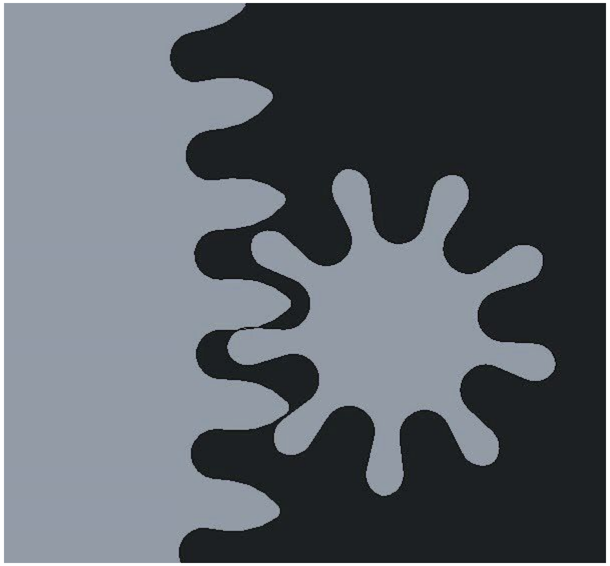


https://mathworld.wolfram.com/Cycloid.html

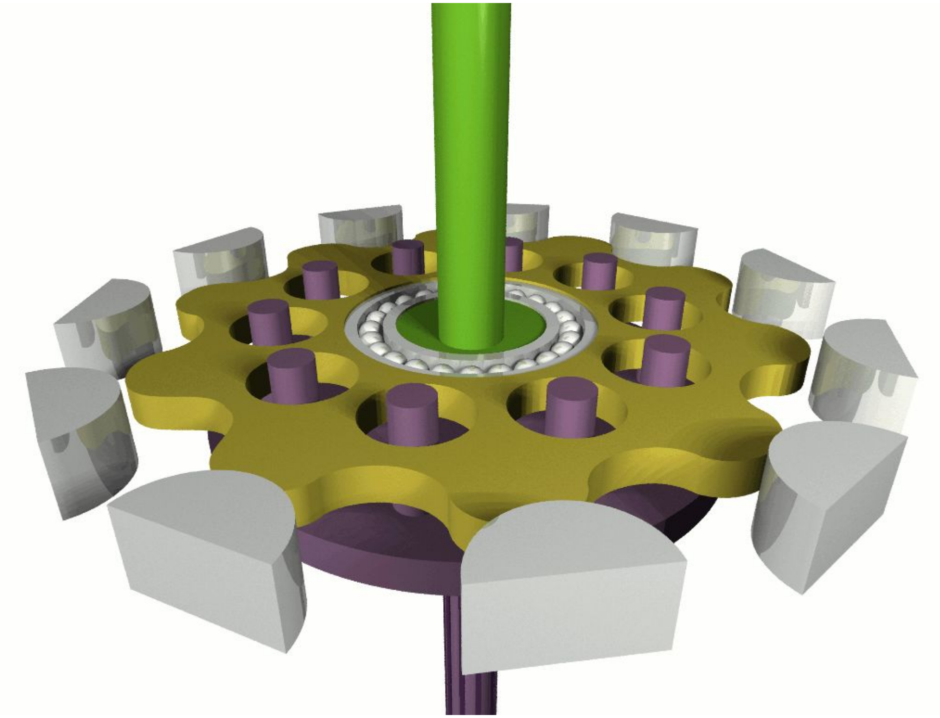
Cycloid

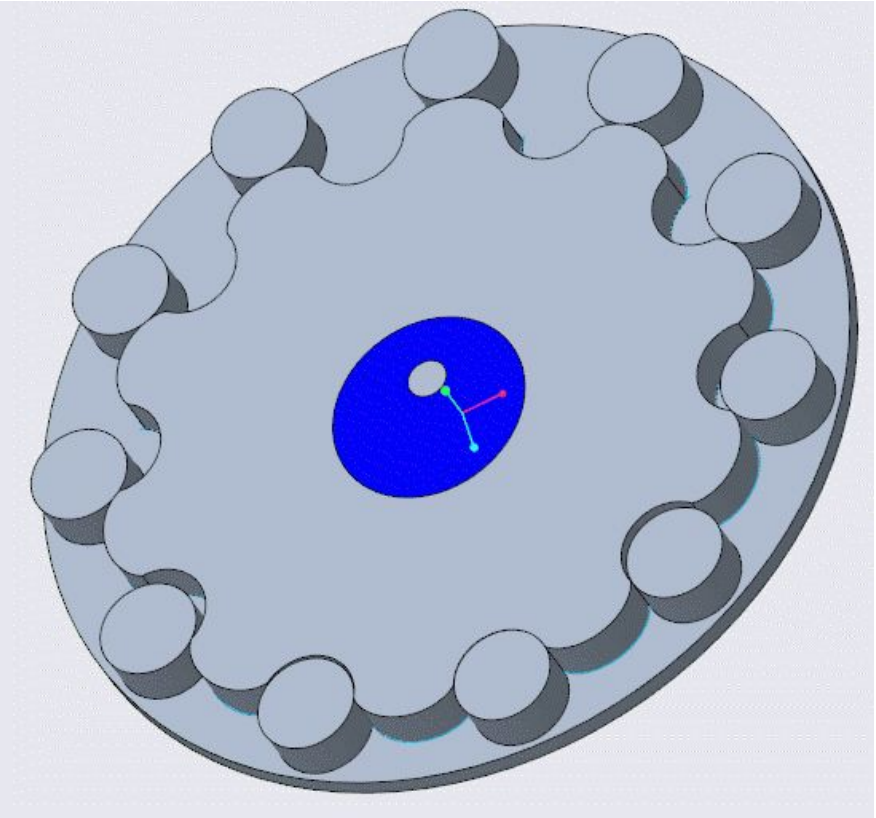
Kimbell Art Museum, Texas





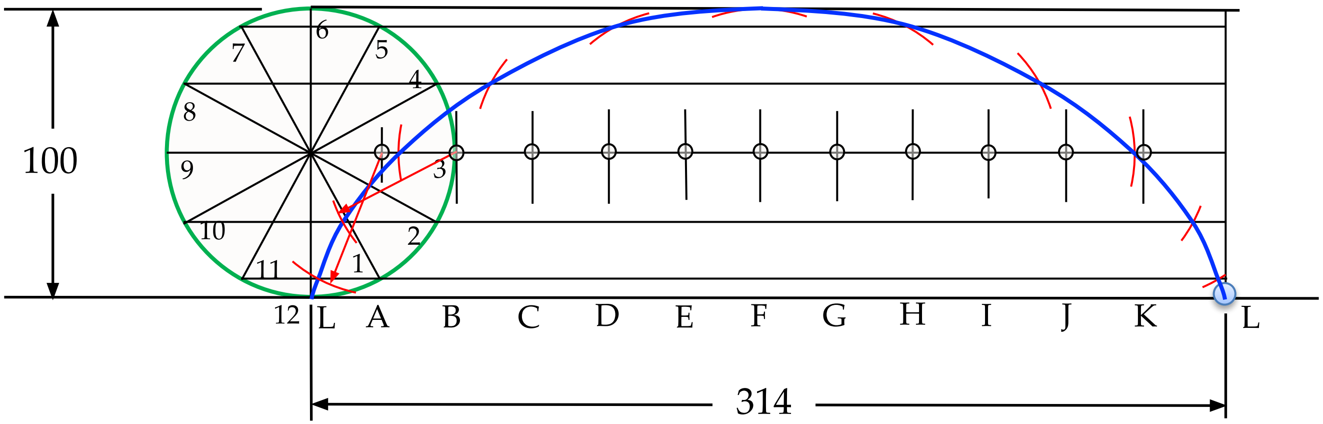
https://en.wikipedia.org/wiki/Cycloid
http://creoparametricarmenia.blogspot.com/2015/04/cycloidal-gear-101-kinematic.html
https://forum.onshape.com/discussion/7054/cycloidal-gear-animation



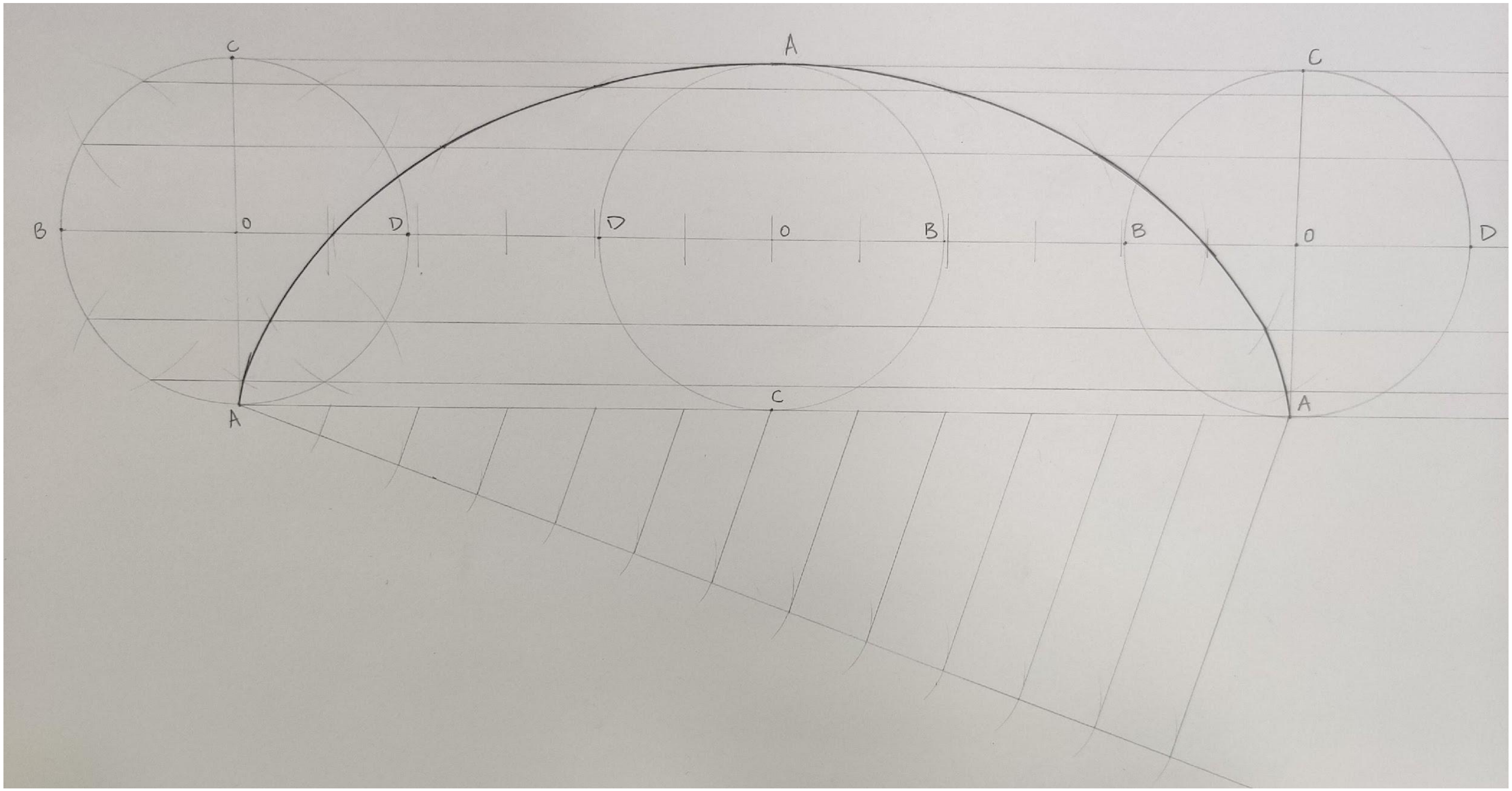


https://en.wikipedia.org/wiki/Cycloidal_drive
https://community.ptc.com/t5/PTC-University-Training/Designing-a-cycloidal-gear-help/td-p/682379

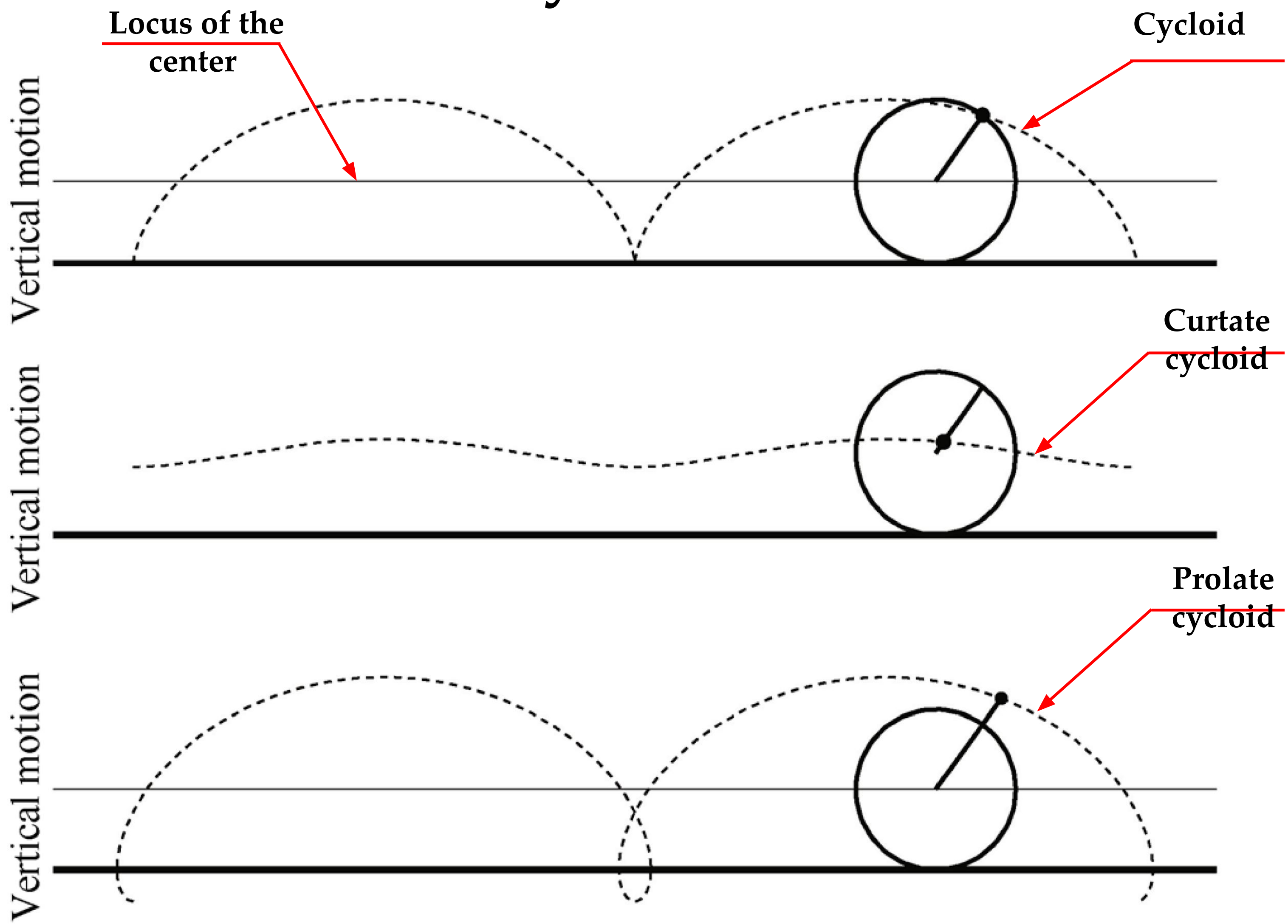
Cycloid



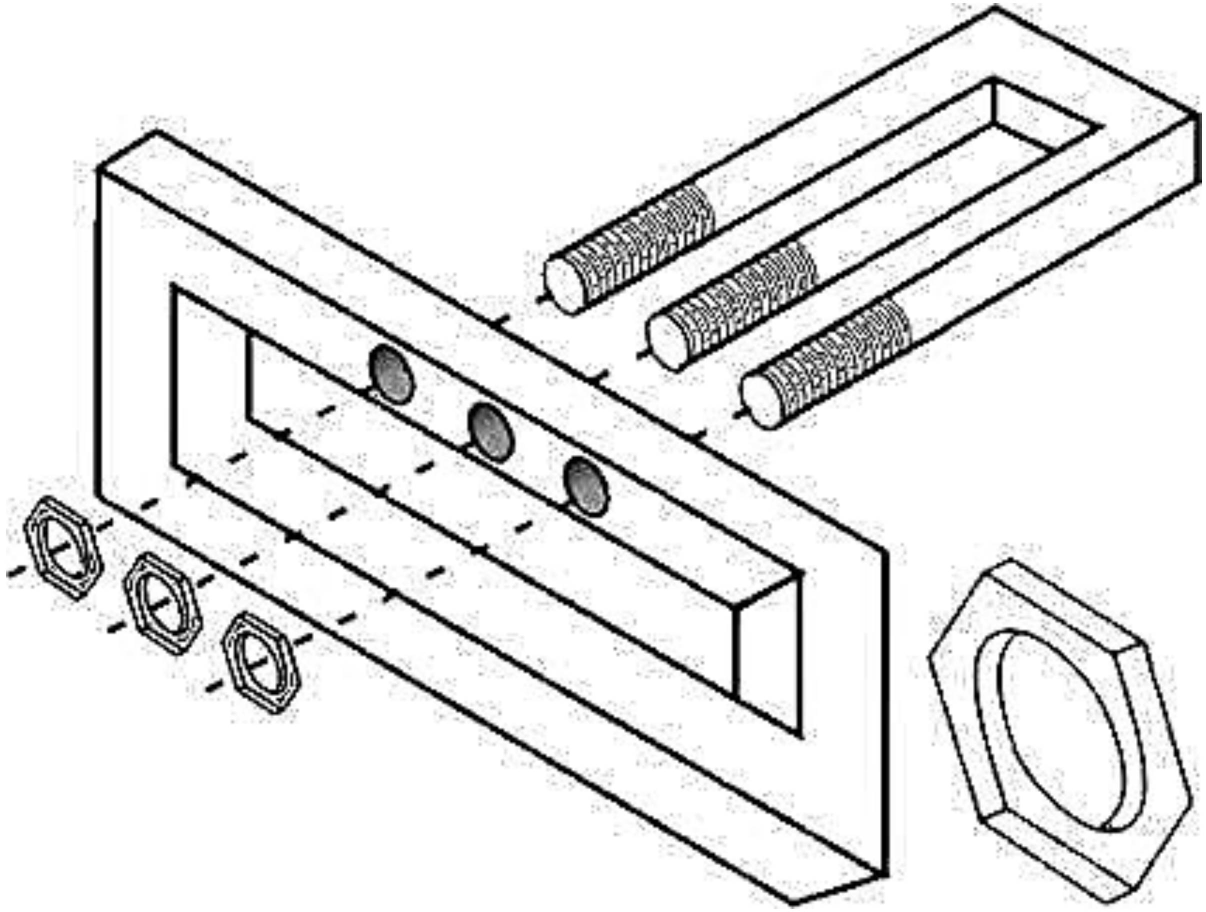
Cycloid



Cycloid



J. Carpentier, M. Benallegue, J-P. Laumond, On the center of Mass Motion in Human Walking, *International Journal of Automation and Computing*, DOI: 10.1007/s11633-017-1088-5



Thank you

https://www.goillusions.com/