MA 103: Quiz 1 (2024)

(1) **(2 points each)** Pick the correct answer out of the choices given for each of the questions below. **No justification required.**

(a) Which of the following is NOT possible for the linear system below, irrespective of the value of k?

$$\begin{aligned} x + 2y &= 4 \\ 7x + 14y &= k. \end{aligned}$$

(i) Infinitely many solutions
(ii) Unique solution
(iii) No solution.

(b) Let

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 1 \\ 1 & 0 & 4 \end{bmatrix}.$$

Then the reduced row echelon form of A is

$$\text{(i)} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(ii)} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & -21/2 \end{bmatrix} \quad \text{(iii)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) If A and B are two invertible $n \times n$ matrices, which of the following is/are true?

(i) $A - B$ is invertible.
(ii) $A + B^{-1}$ is invertible.
(iii) $A^T B$ is invertible.
(iv) None of these.

(d) There does not exist a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ such that $T(0, 0) = (0, 1)$.

(i) True
(ii) False.

(e) If A is an $n \times n$ matrix that is not invertible, then the linear system $A\mathbf{x} = 0$ has infinitely many solutions.

(i) True
(ii) False.

(2) **(10 points)** Solve the following linear system of equations using Cramer's rule.

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

(3) **(10 points)** Using elementary row operations, find the inverse of the matrix below, if it exists.

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

(4) Let $A\mathbf{x} = 0$ be a homogeneous system of n linear equations in n unknowns, and let Q be an invertible $n \times n$ matrix. Prove that $A\mathbf{x} = 0$ has only the trivial solution if and only if $(QA)\mathbf{x} = 0$ has only the trivial solution.

(**Note:** You have to show **both** the implications, that is, if $A\mathbf{x} = 0$ has only the trivial solution, then $(QA)\mathbf{x} = 0$ has only the trivial solution, and vice-versa.)