Q 1 **Logarithmic *p*-series**

a. Show that (Part (a) was already done last time.)

$$\int_2^{\infty} \frac{dx}{x(\ln x)^p} \quad (p \text{ a positive constant})$$

converges if and only if $p > 1$.

b. What implications does the fact in part (a) have for the convergence of the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p} \; ?$$

Give reasons for your answer.

Q 2 Which of the following series converge and which ones diverge?

a) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 1}$ b) $\sum_{n=1}^{\infty} \tan \frac{1}{n}$

Q3 Which of the series converge, and which diverge?

a) $\sum_{n=1}^{\infty} n^2 e^{-n}$ b) $a_1 = 1, \quad a_{n+1} = \frac{1 + \tan^{-1} n}{n} a_n$

Q 4 Which of the series converge, and which diverge?

a) $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{(n^2)}}$ b) $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^{(n/2)}}$

PART II

Q5. Discuss convergence / divergence of

① $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ ② $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ ③ $\sum_{n=1}^{\infty} \left(\frac{1}{n+1}\right)^n$

Q6. Do the following series converge?

① $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n^3)}$ ② $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$

③ $\sum_{n=1}^{\infty} (-1)^n \operatorname{sech}(n)$.

Q7. Show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} - \sum_{n=1}^{\infty} \frac{1}{(2n)^2}$.