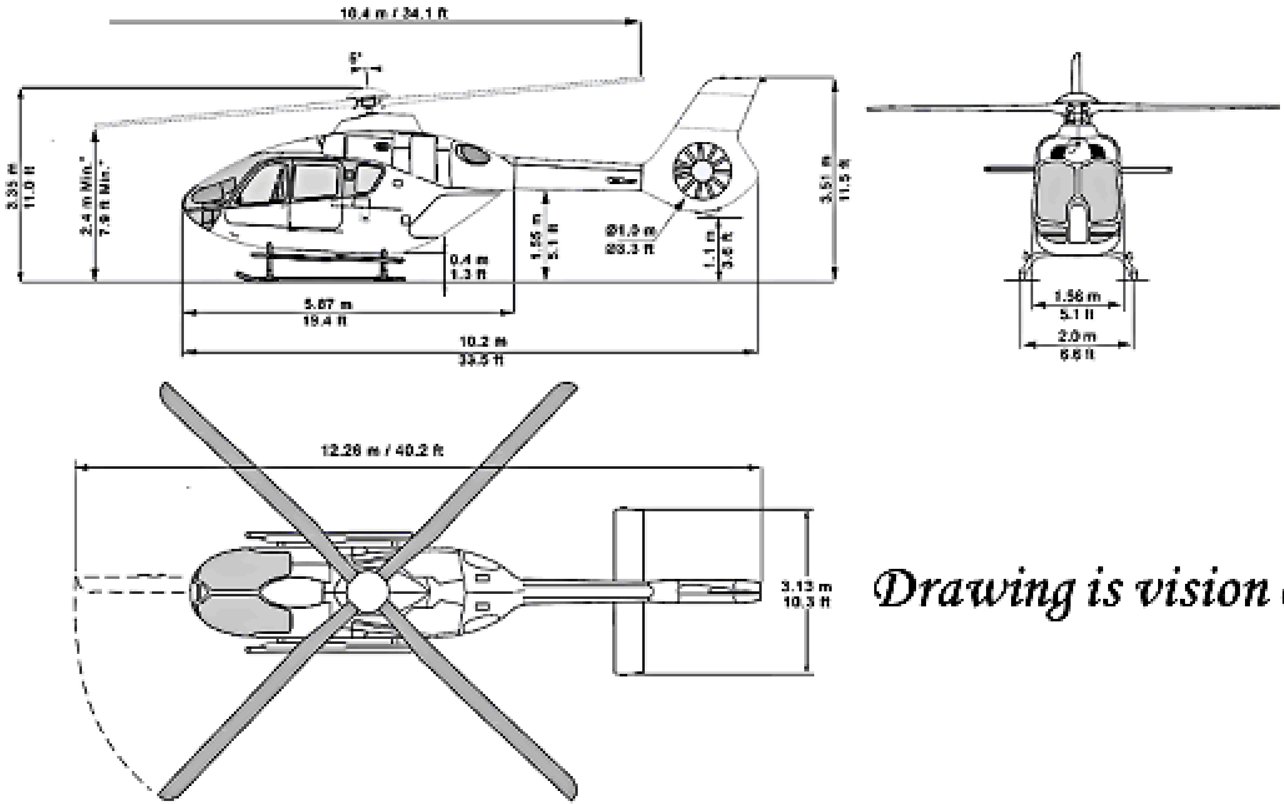
ES 101: Engineering Graphics



Drawing is vision on paper

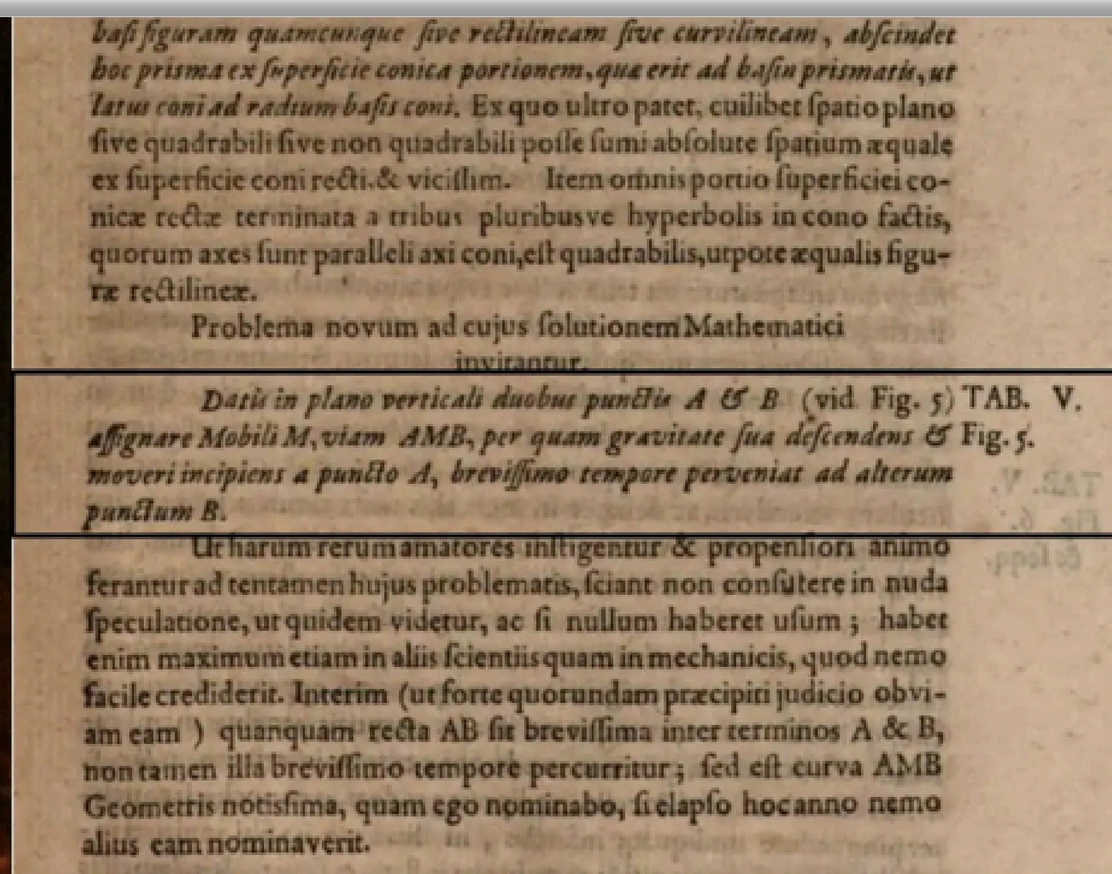
-Andrew Loomis

https://www.aiut-alpin-dolomites.com/english/technical_details.html

Class#4 – 25th September 2024

Sameer Patel
Assistant Professor
Civil Engineering & Chemical Engineering
IIT Gandhinagar

Brachistochrone (least time) problem



baſi figuram quamcunque ſive rectilineam ſive curvilineam, abſcindet hoc priſma ex ſuperficie conica portionem, quæ erit ad baſin priſmatis, ut latus coni ad radium baſis coni. Ex quo ultro patet, cuilibet ſpatio plano ſive quadrabili ſive non quadrabili poſſe ſumi abſolute ſpatium æquale ex ſuperficie coni recti, & viciſſim. Item omnis portio ſuperficiei conicæ rectæ terminata a tribus pluribusve hyperbolis in cono factis, quorum axes ſunt paralleli axi coni, eſt quadrabilis, utpote æqualis figuræ rectilineæ.

Problema novum ad cujus ſolutionem Mathematici invitantur.

Datis in plano verticali duobus punctis A & B (vid. Fig. 5) TAB. V.
aſſignare Mobili M, viam AMB, per quam gravitate ſua deſcendens & Fig. 5.
moveri incipiens a puncto A, breviſſimo tempore perveniat ad alterum punctum B.

Ut harum rerum amatores inſtigentur & propenſiori animo ferantur ad tentamen hujus problematis, ſciant non conſiſtere in nuda ſpeculatione, ut quidem videtur, ac ſi nullum haberet uſum; habet enim maximum etiam in aliis ſcientiis quam in mechanicis, quod nemo facile crediderit. Interim (ut forte quorundam præcipiti judicio obviam eam) quanquam recta AB ſit breviſſima inter terminos A & B, non tamen illa breviſſimo tempore percurritur; ſed eſt curva AMB Geometris notiſſima, quam ego nominabo, ſi elapſo hoc anno nemo alius eam nominaverit.

"Given in a vertical plane two points A and B, assign to the moving (body) M, the path AMB, by means of which - descending by its own weight and beginning to be moved (by gravity) from point A - it would arrive at the other point B in the shortest time."

https://www.cantorsparadise.com/the-famous-problem-of-the-brachistochrone-8b955d24bdf7

Brachistochrone (least time) problem

- This problem was posed by Johann Bernoulli in the journal *Acta Eruditorum*
- The problem is of finding the path followed by a particle sliding under the influence of gravity starting at point A and terminating at point B in least time
- Owing to the conservation of energy (frictionless path), the velocity of the particle at any arbitrary height y is given by

$$v = \frac{ds}{dt} = \sqrt{2g(h-y)}$$

Note that the particle starts with zero initial velocity

- The time taken to traverse from point A to point B is

$$T = \int_A^B \frac{ds}{\sqrt{2g(h-y)}}$$

- Considering the trajectory to be a differentiable function $y = f(x)$, from the differential geometry we have

$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{df}{dx}\right)^2} \Rightarrow T = \frac{1}{\sqrt{2g}} \int_{x_0}^{x_1} \sqrt{\frac{1 + (f')^2}{h - f}}\, dx$$

Brachistochrone (least time) problem

- The problem thus comes to the question of finding a function $f(x)$ such that the time T is minimum. Accordingly, for each $f(x)$ passing through the two points A and B, the time T can be evaluated. Such quantities are called functionals, i.e. T is a function of a function $f(x)$
- The functional is quite difficult to evaluate

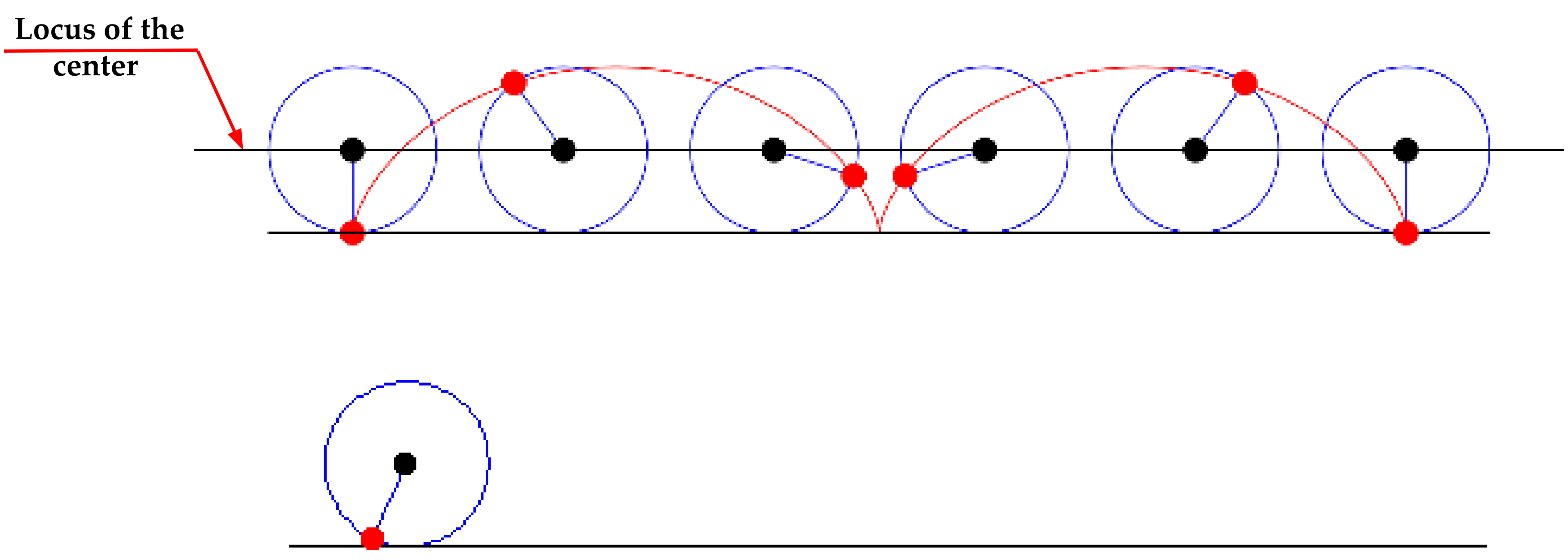
$$T\big(f(x)\big) = \frac{1}{\sqrt{2g}} \int_{x_0}^{x_1} \sqrt{\frac{1+(f')^2}{h-f}}\, dx$$

- The expression $T\big(f(x)\big)$ depends on the whole behavior of the function $f(x)$
- The curve $y = f(x)$ which results in the least possible $T\big(f(x)\big)$ corresponds to *cycloid* (it is the locus of a point on the circumference of a circle that rolls without slipping along a straight line)
- The solution was given by Johann Bernoulli, Jacob Bernoulli, Newton and l'Hospital

https://mathworld.wolfram.com/Cycloid.html https://gfycat.com/biodegradableflowery americankestrel https://en.wikipedia.org/wiki/Brachistochrone_curve

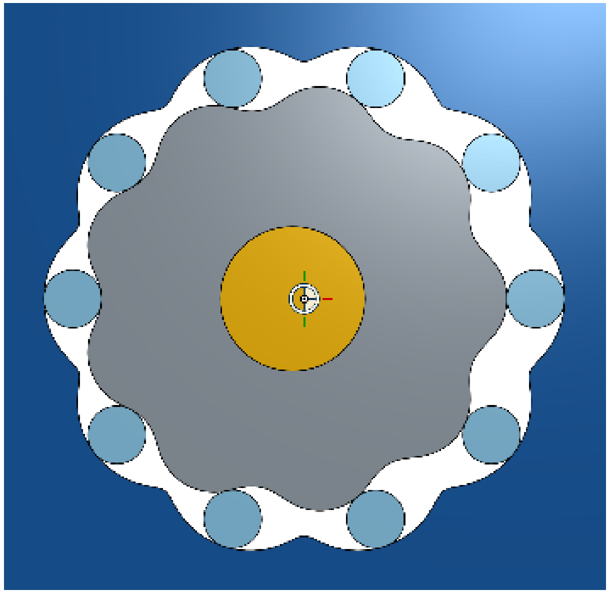
Cycloid

- Cycloid is the curve traced by a point on a circle as it rolls along a straight line without slipping
- The name was coined by Galileo Galilei, who studied it extensively
- The exact solution of the famous Brachistochrone problem is a cycloid and the solution was provided by Johann Bernoulli



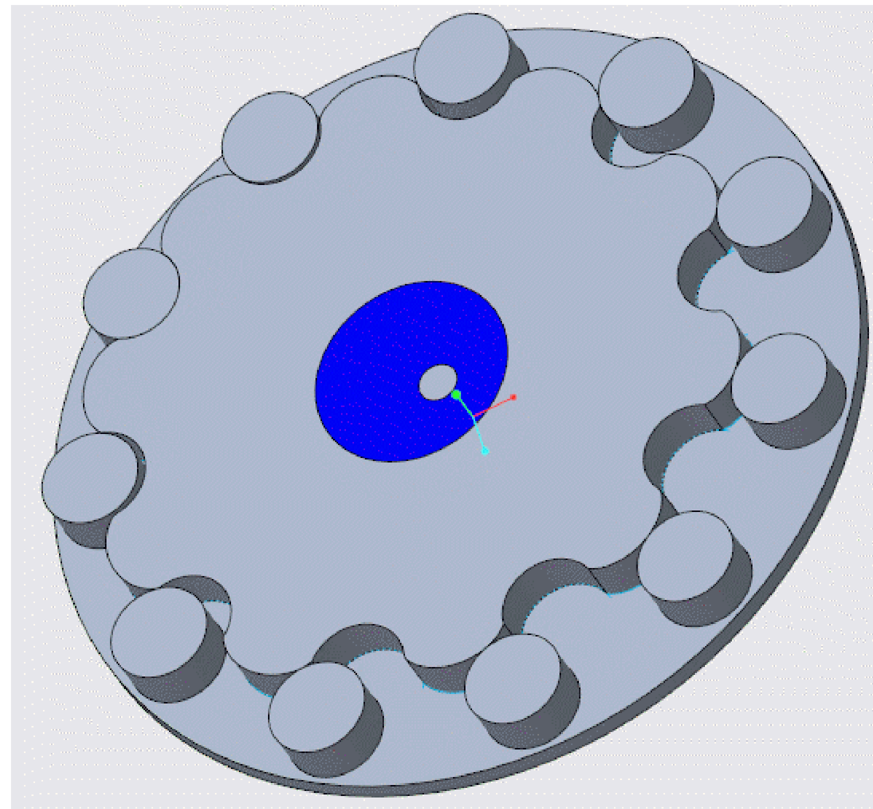
https://mathworld.wolfram.com/Cycloid.html

Cycloid



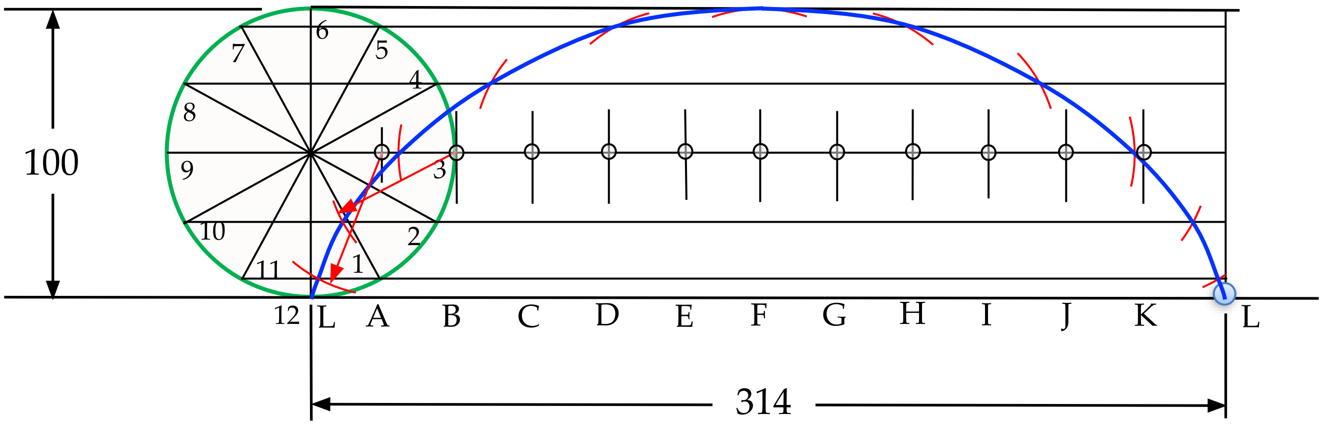
Kimbell Art Museum, Texas

https://en.wikipedia.org/wiki/Cycloid
http://creoparametricarmenia.blogspot.com/2015/04/cycloidal-gear-101-kinematic.html
https://forum.onshape.com/discussion/7054/cycloidal-gear-animation

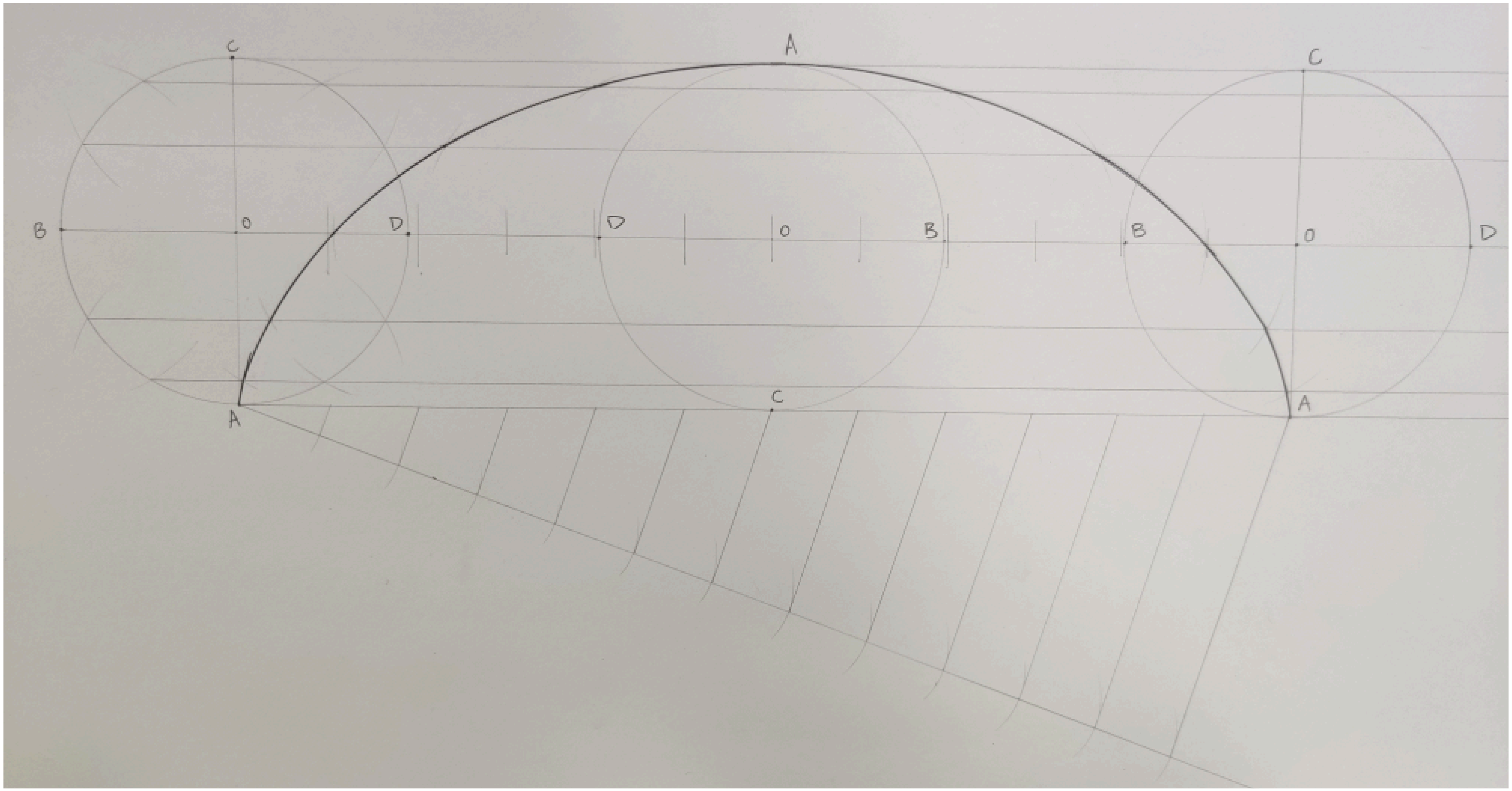


https://en.wikipedia.org/wiki/Cycloidal_drive
https://community.ptc.com/t5/PTC-University-Training/Designing-a-cycloidal-gear-help/td-p/682379

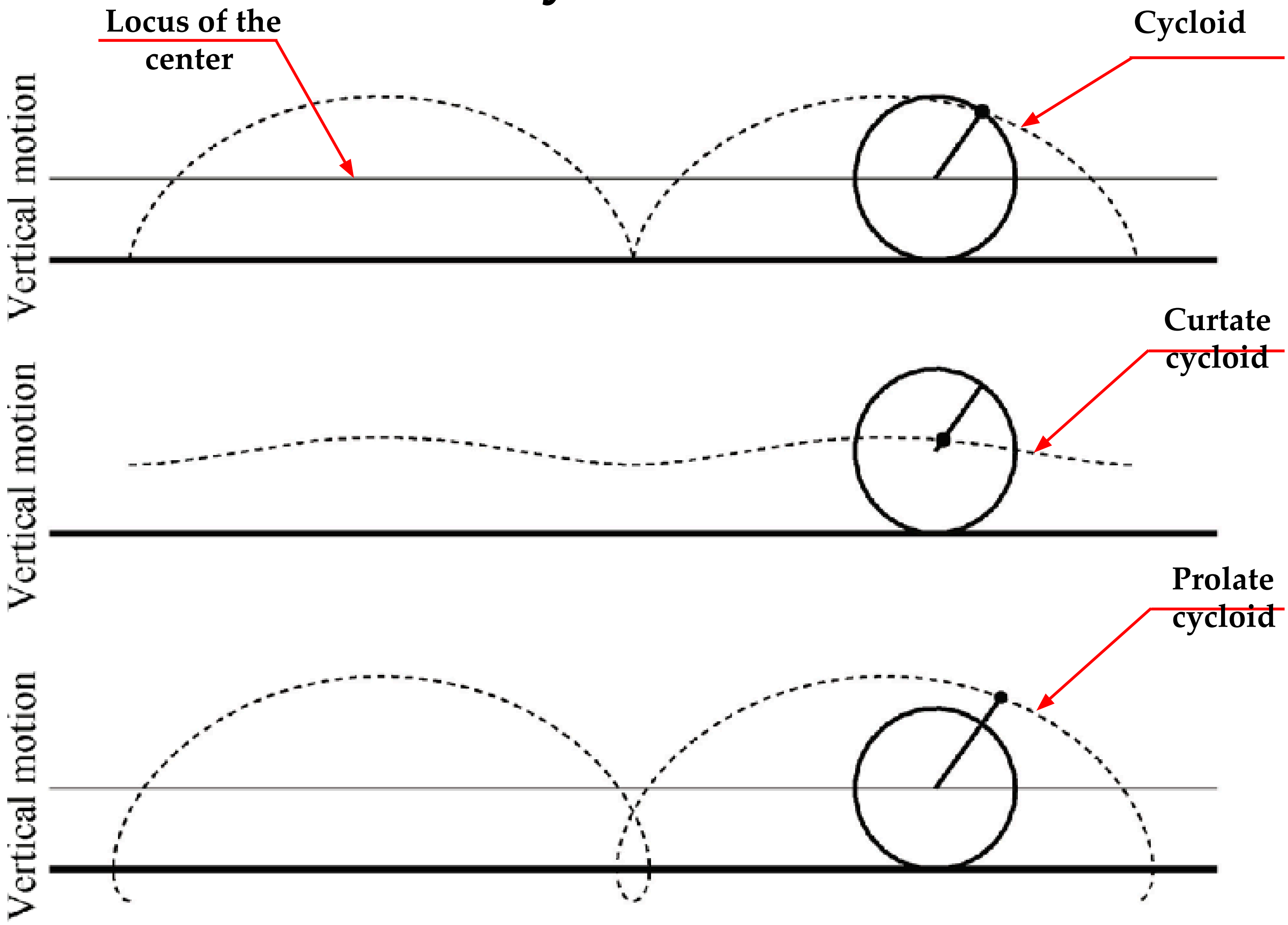
Cycloid



Cycloid



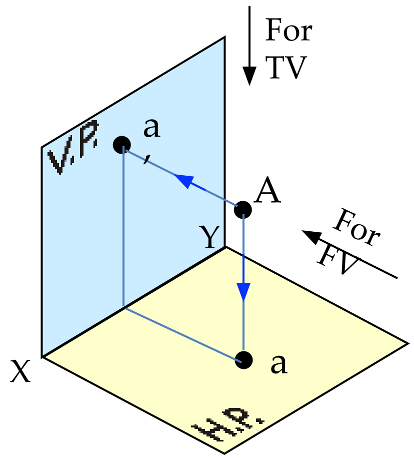
Cycloid



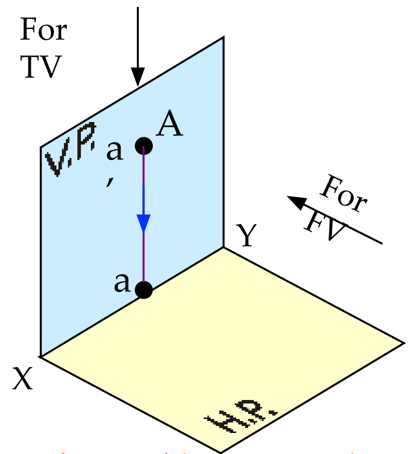
J. Carpentier, M. Benallegue, J-P. Laumond, On the center of Mass Motion in Human Walking, *International Journal of Automation and Computing*, DOI: 10.1007/s11633-017-1088-5

Projections of point

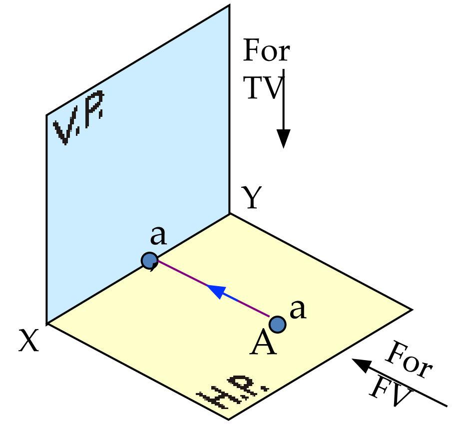
Point A above HP & in front of VP



Point A above HP & on VP

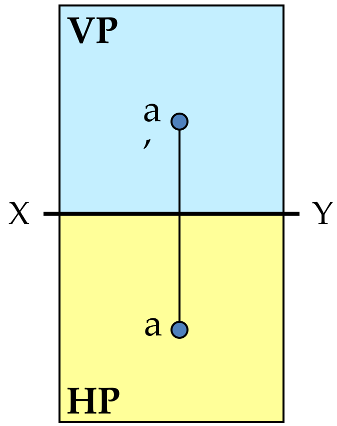


Point A on HP & in front of VP

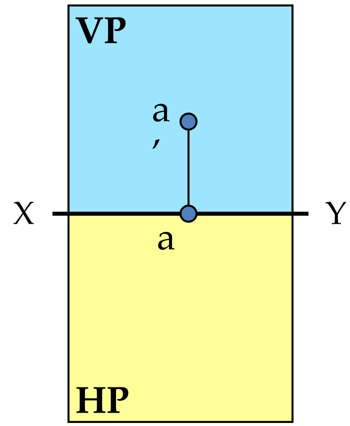


Orthographic presentations of all above cases.

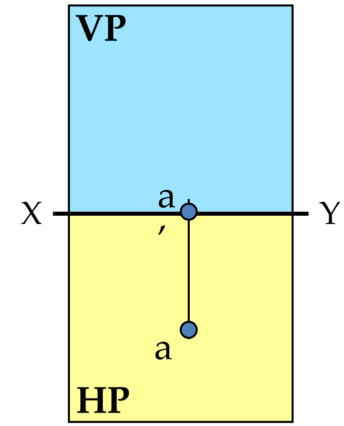
FV above XY, TV below XY



FV above XY, TV on XY



FV on XY, TV below XY

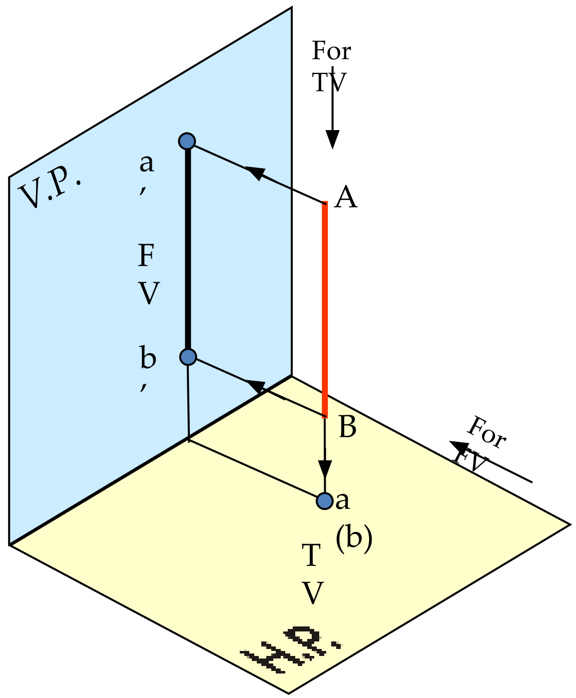


Projection of straight lines

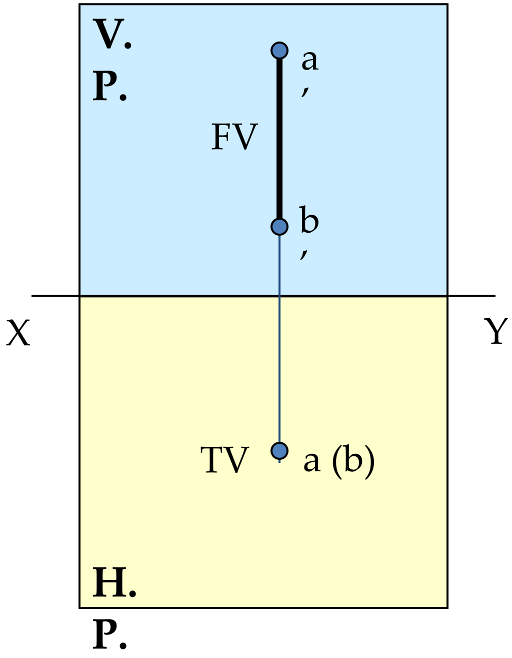
- Information needed:
  - Length
  - Position of its ends with respect to HP and VP
  - Inclination with respect to HP and VP
- Aim – draw its projections (FV and TV)

Projection of straight lines

1. Line perpendicular to HP and parallel to VP



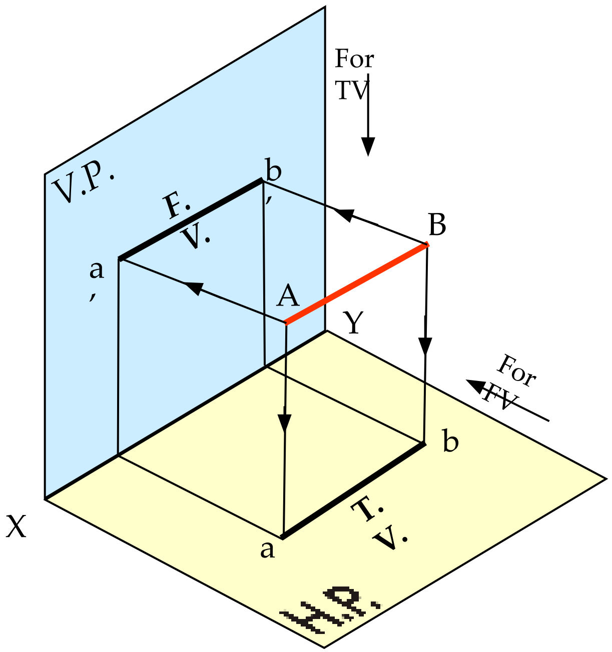
Orthographic projection



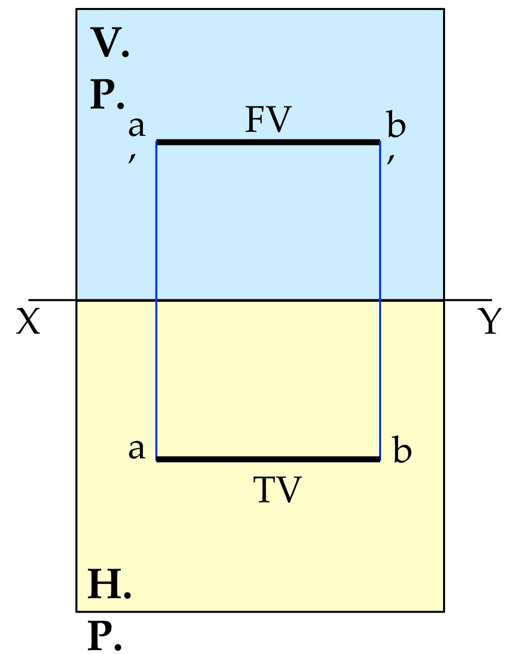
- FV is perpendicular to XY
- TV is a point

Projection of straight lines

2. Line parallel to both HP and VP



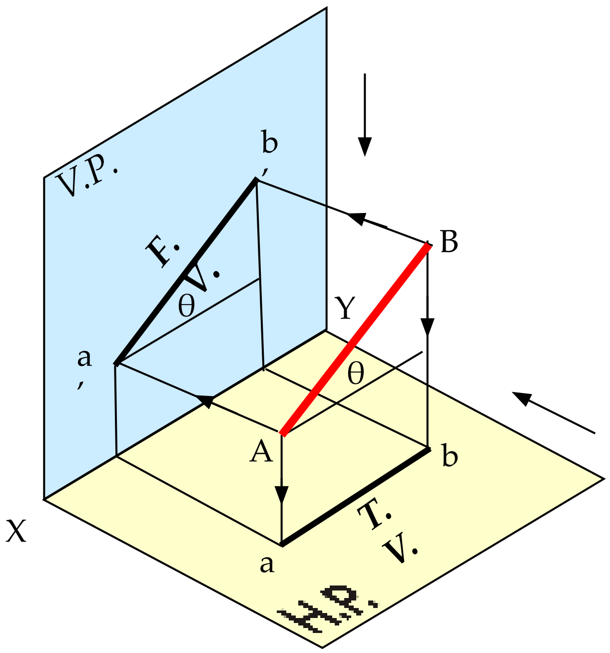
Orthographic projection

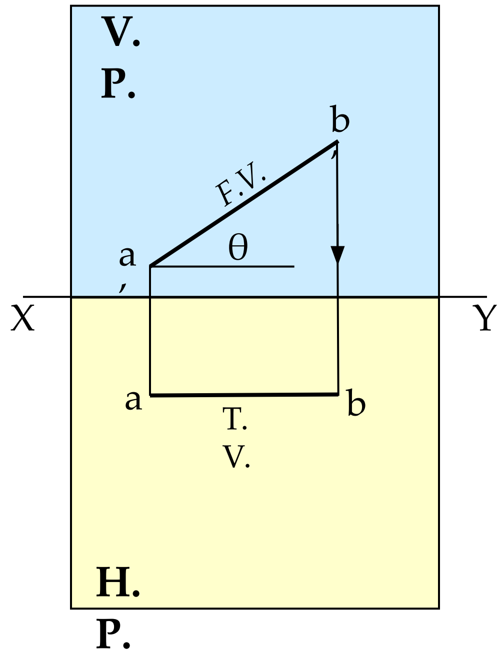


- FV and TV are parallel to XY
- FV and TV show true length

Projection of straight lines

3. Line inclined to HP and parallel to VP

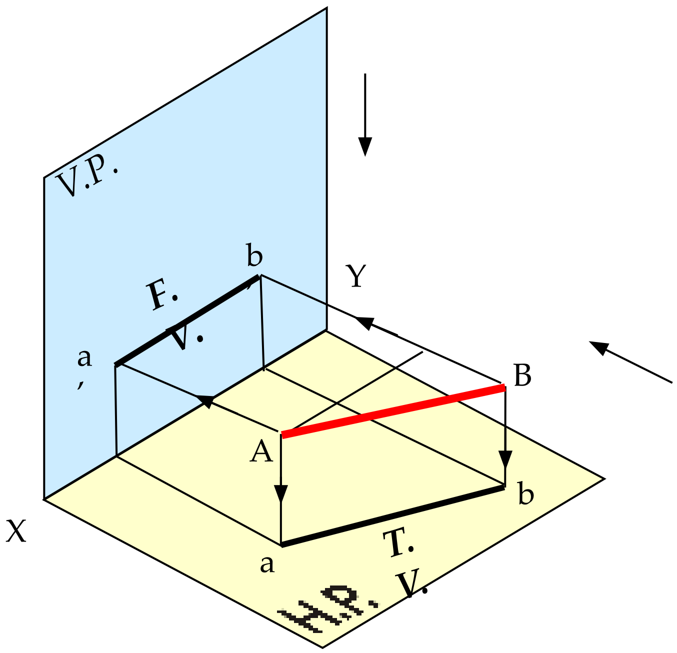


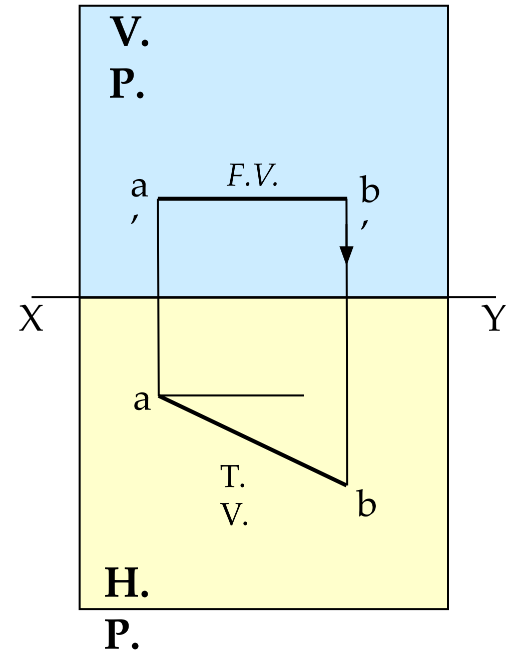


- FV is inclined to XY
- TV shows apparent length
- If TV is parallel to XY, then the FV shows true length and true inclination

Projection of straight lines

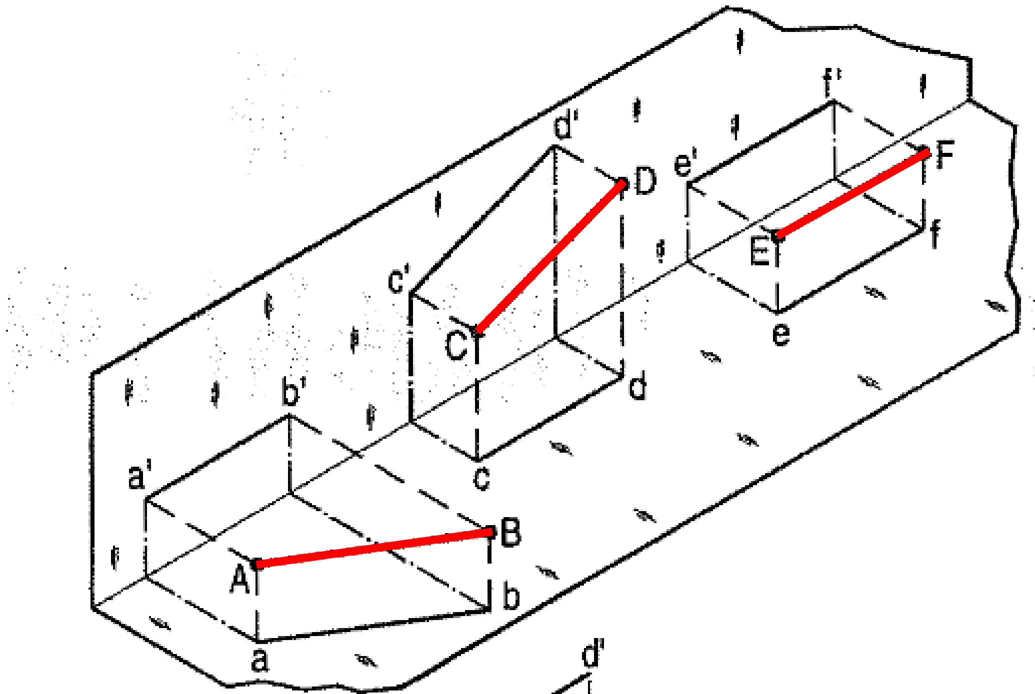
4. Line inclined to VP and parallel to HP

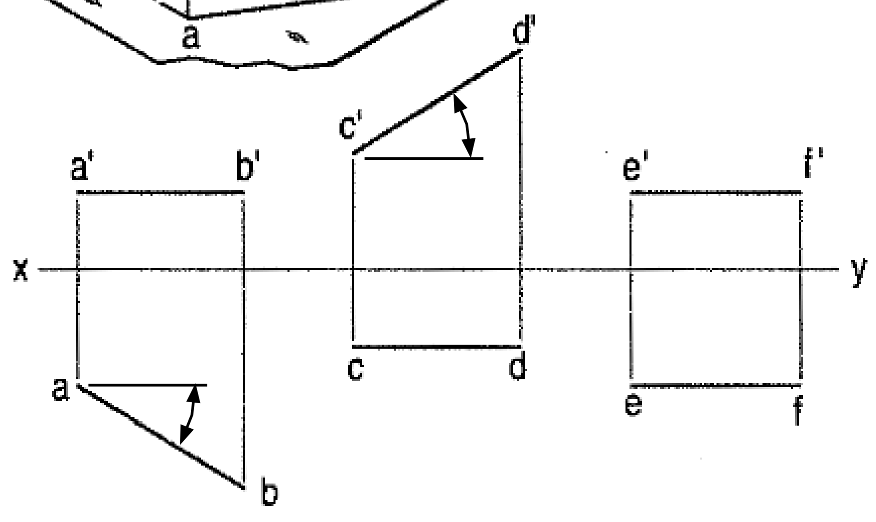




- TV is inclined to XY
- FV shows apparent length
- If FV is parallel to XY, then the TV shows true length and true inclination

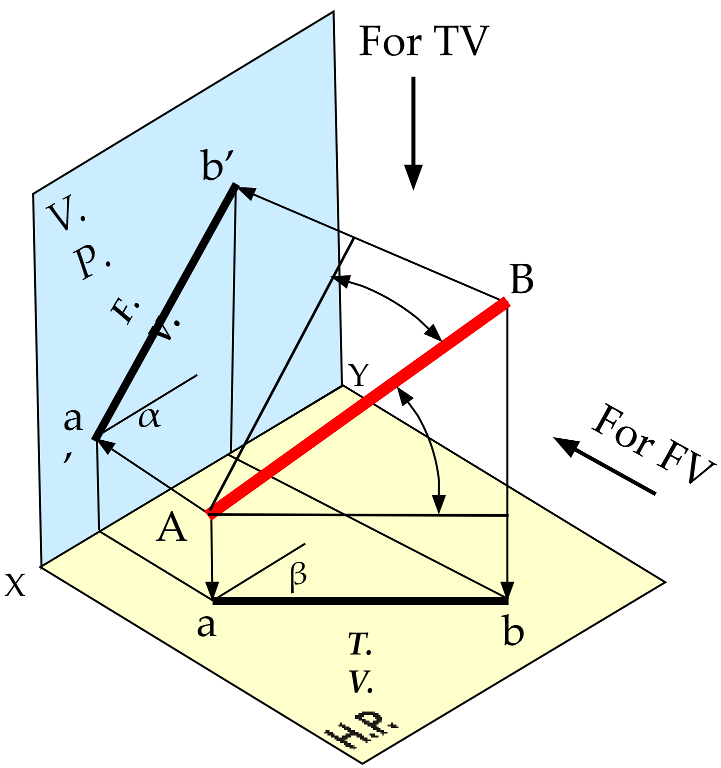
Projection of straight lines

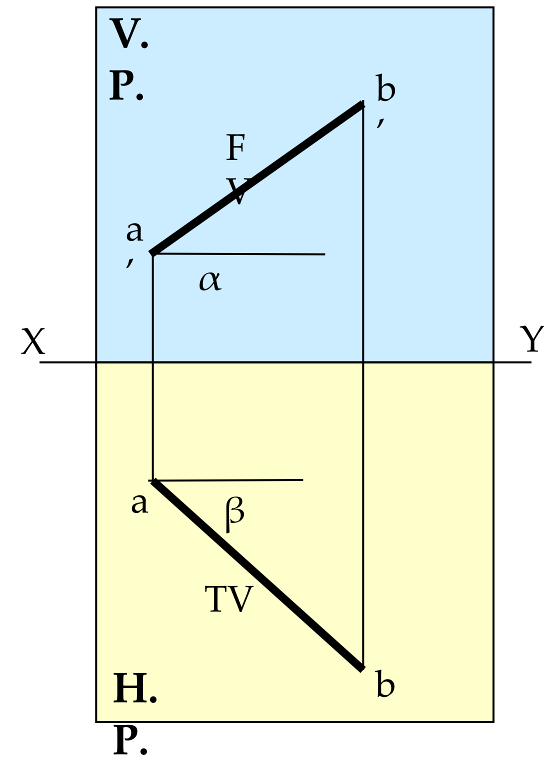




Projection of straight lines

5. Line inclined to both HP and VP



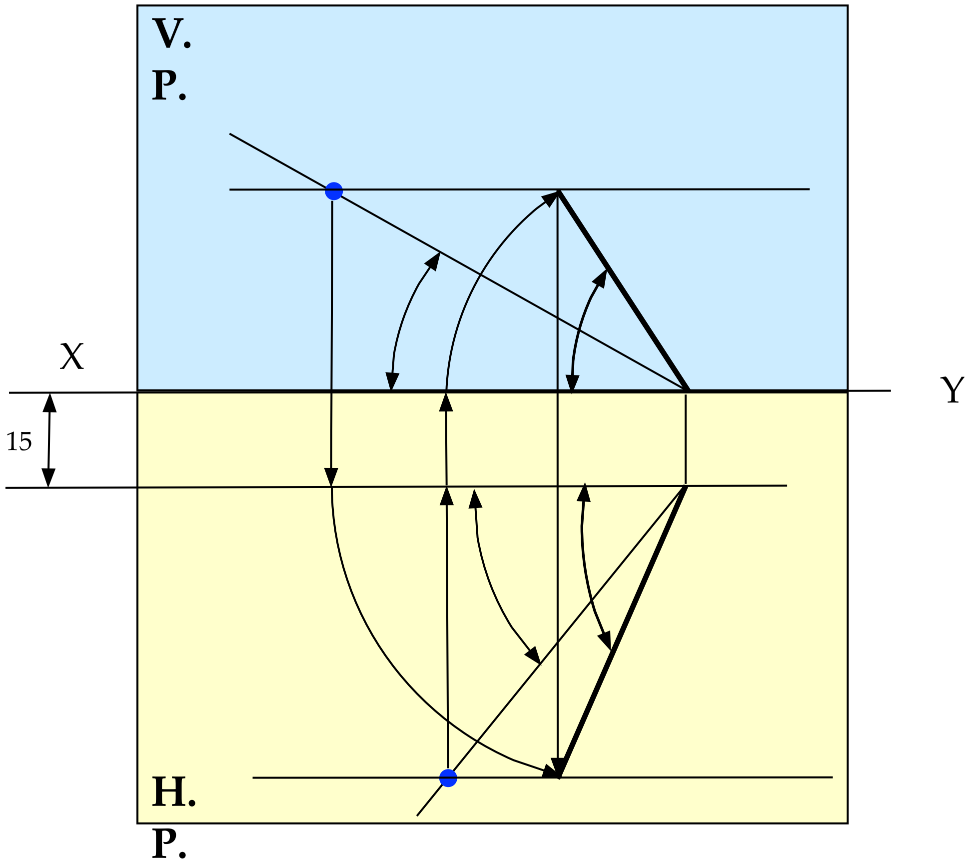


- Both FV and TV inclined to XY
- Both views show apparent length and inclination

Projection of straight lines

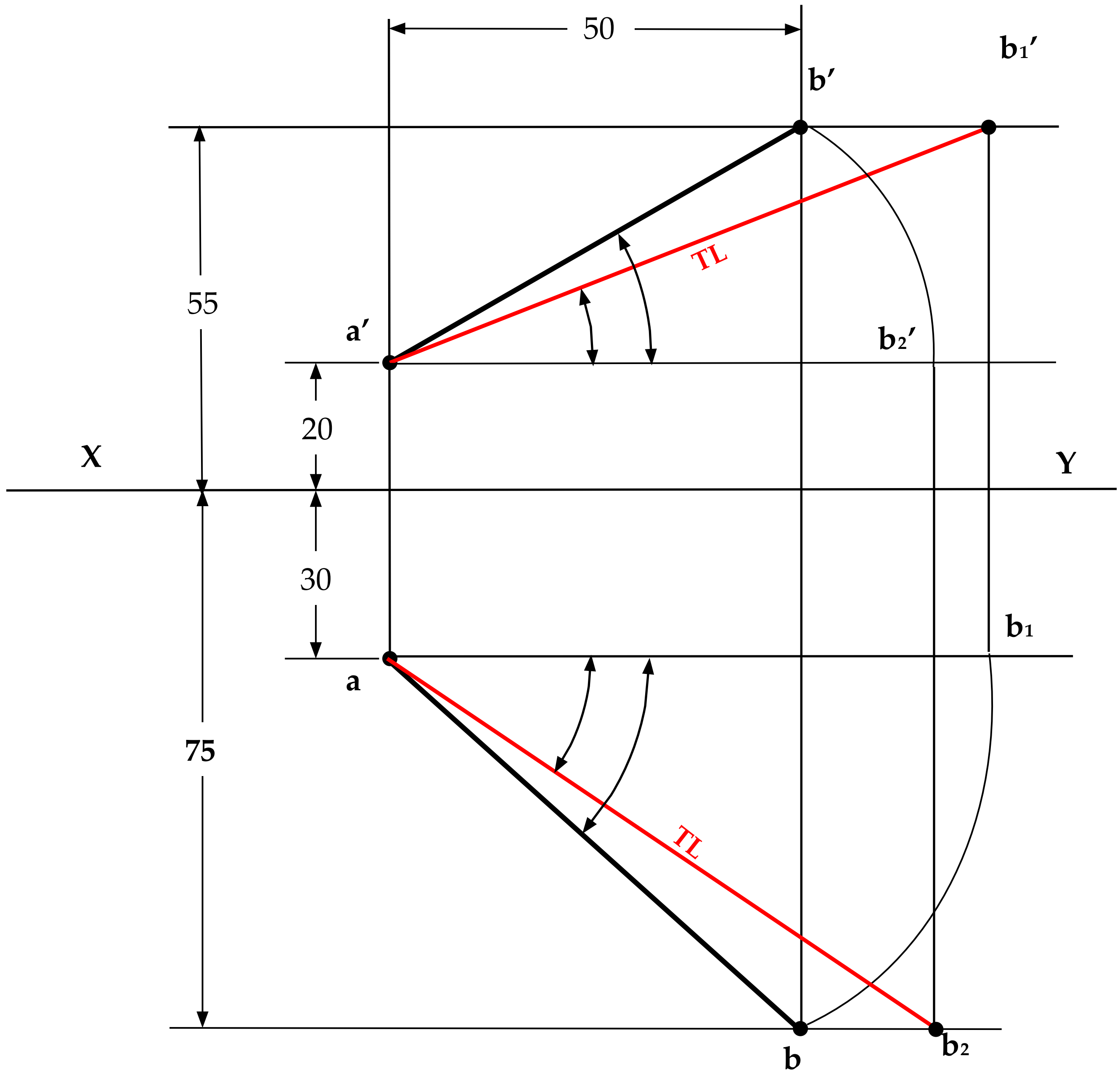
A line AB, 65 mm long, has its end B in the H.P. and 15 mm in front of the V.P. The end A is in the third quadrant. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.

Projection of straight lines



Projection of straight lines

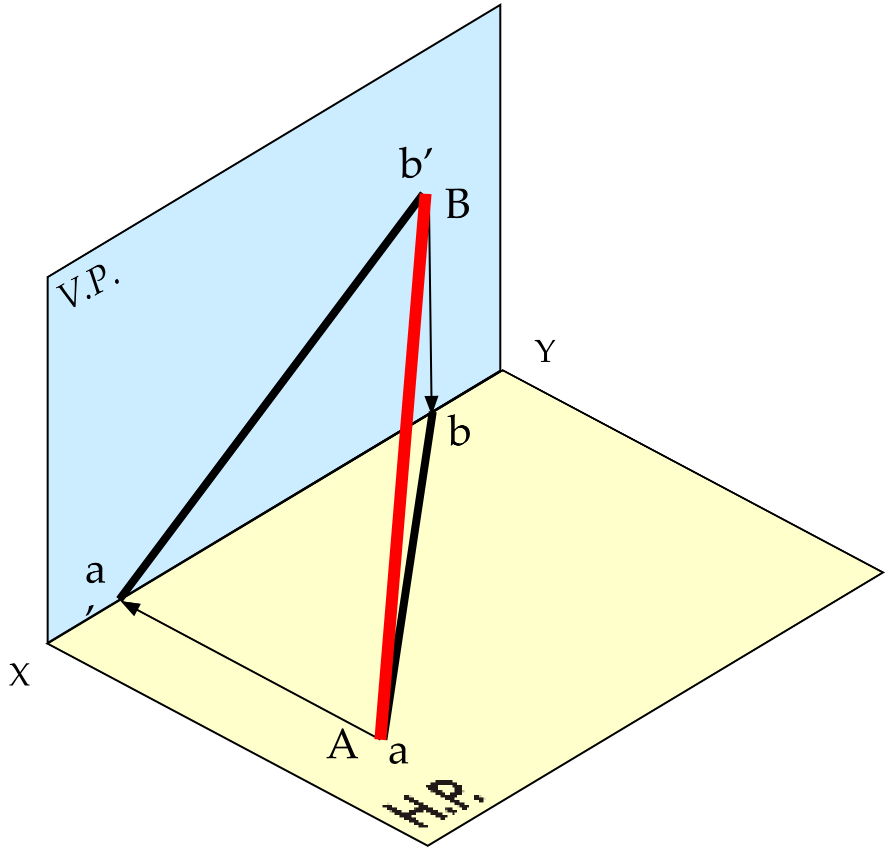
End A of a line AB is 20mm above HP and 30mm in front of VP while end B is 55mm above HP and 75mm in front of VP. The distance between end projectors is 50mm. Find true length and true inclination of the line

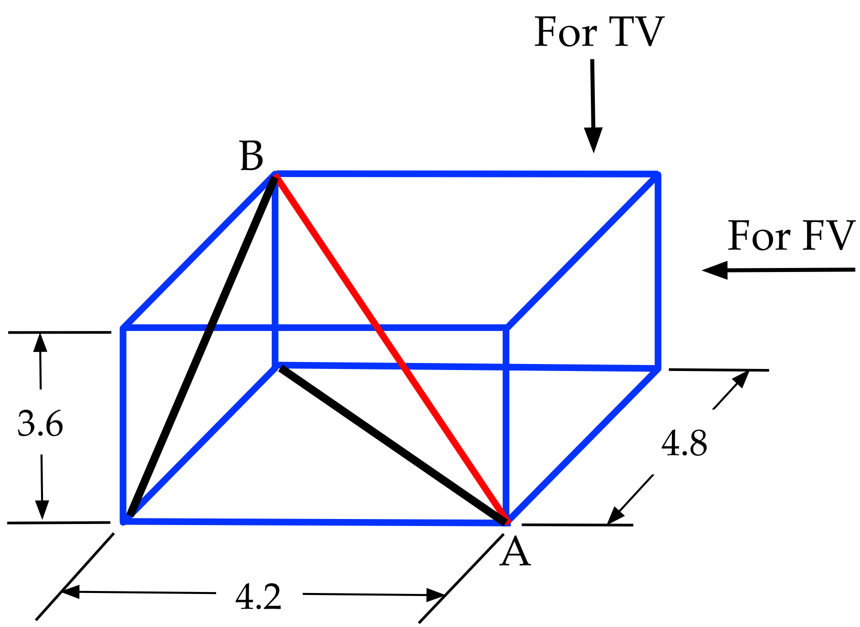


50
b₁′
b′
55
TL
a′
b₂′
20
X
Y
30
b₁
a
75
TL
b
b₂

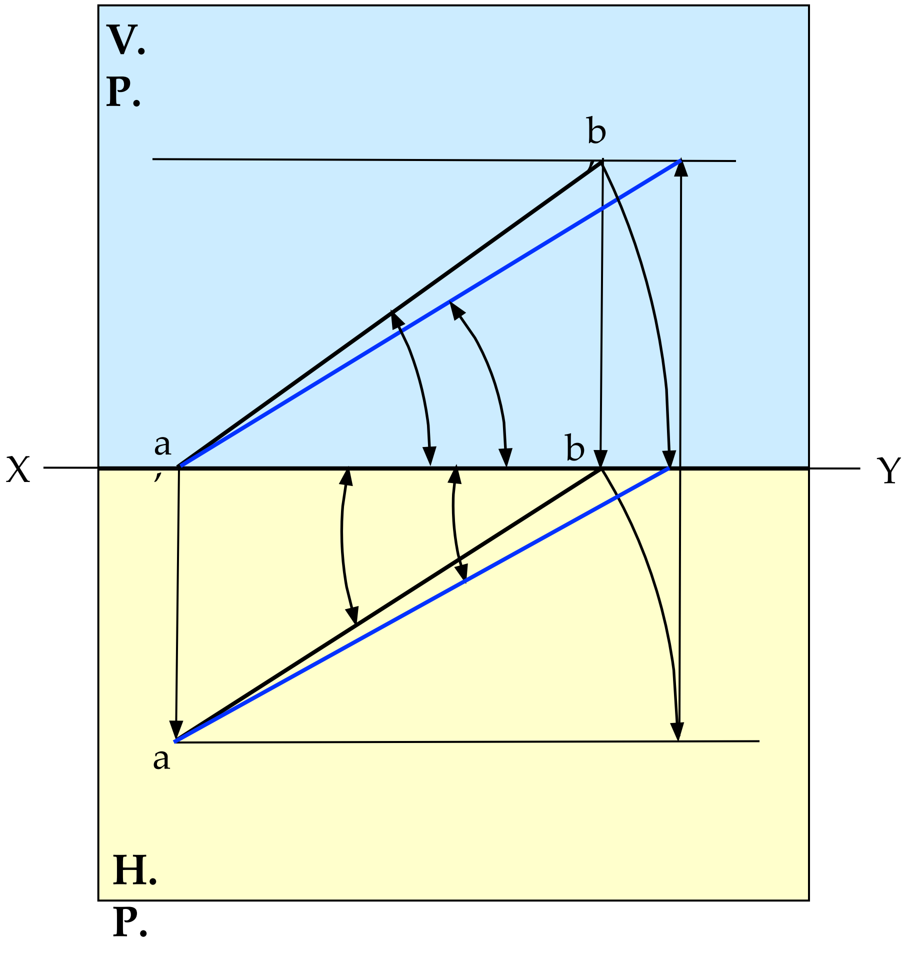
Projection of straight lines

A room is 4.8 m x 4.2 m x 3.6 m high. Determine graphically the distance between a top corner and the bottom corner diagonally opposite to it





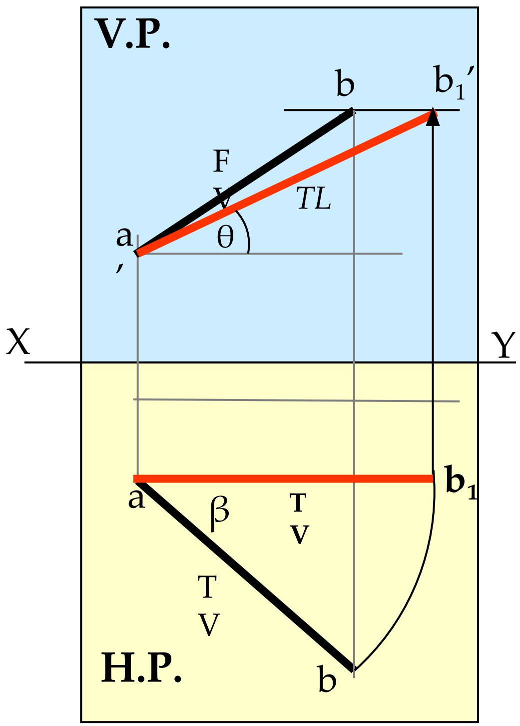
Projection of straight lines



Projection of straight lines

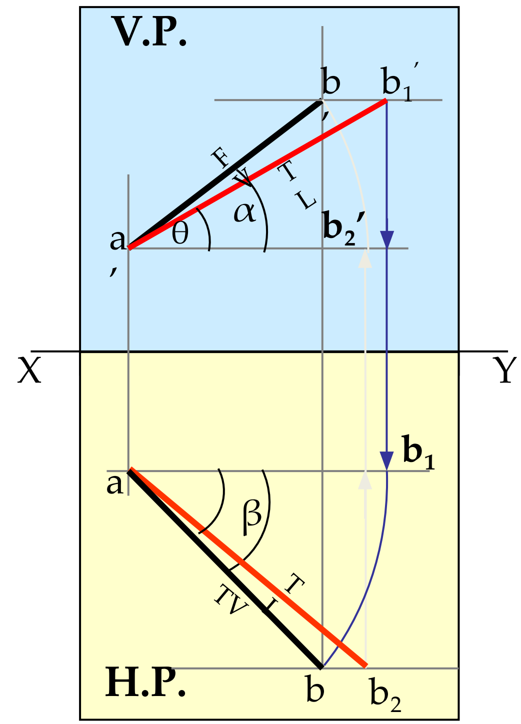
True length and views

If FV and TV are known



True length and true inclination with respect to HP

If true length and true inclinations are known



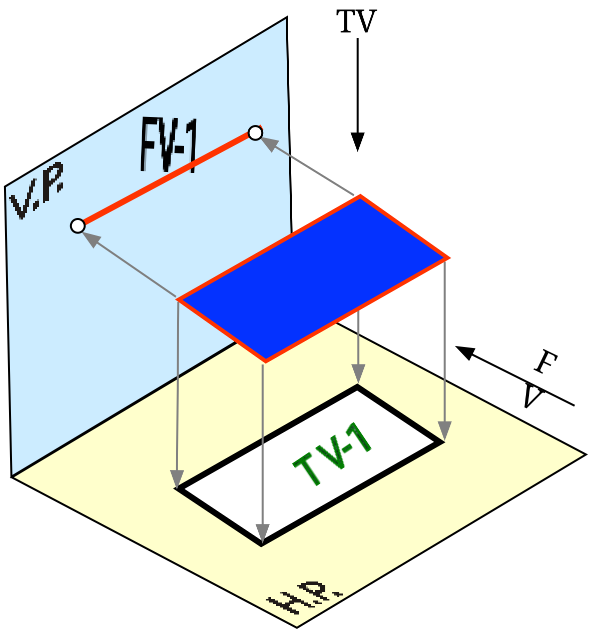
Locate TV & FV

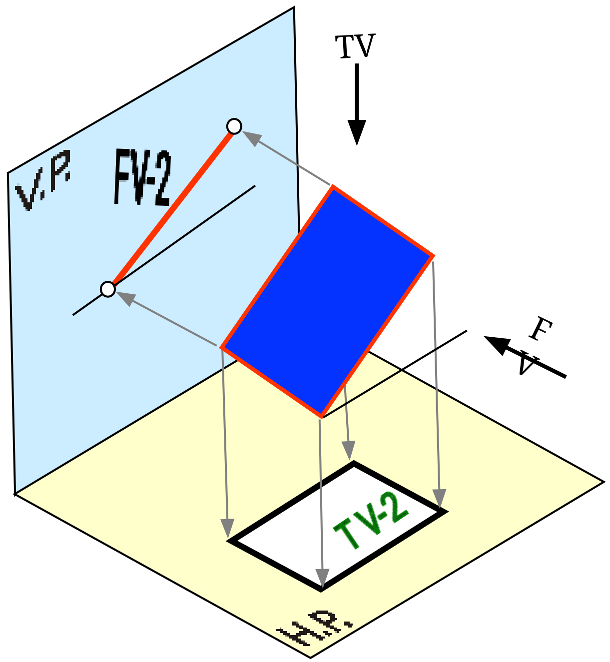
Projection of planes

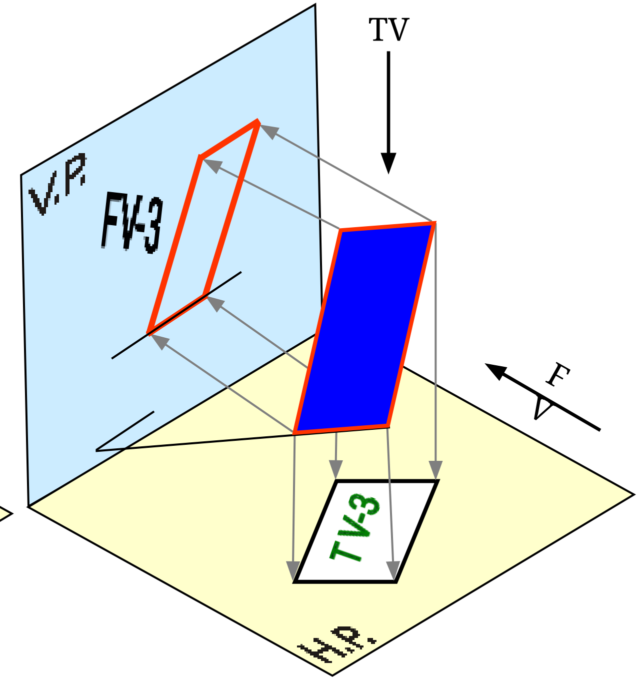
Two main types of planes

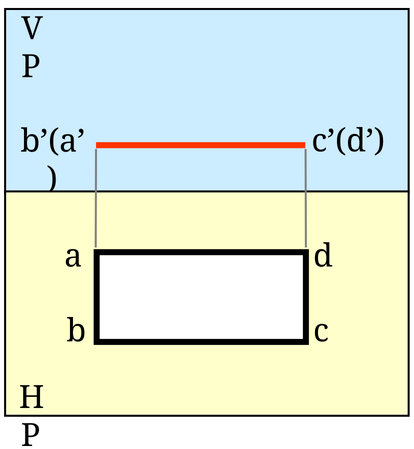
- Perpendicular planes
  - Perpendicular to both reference plane
  - Perpendicular to one and parallel to other
  - Perpendicular to one and inclined to other
- Oblique planes
  - Inclined to both the reference planes

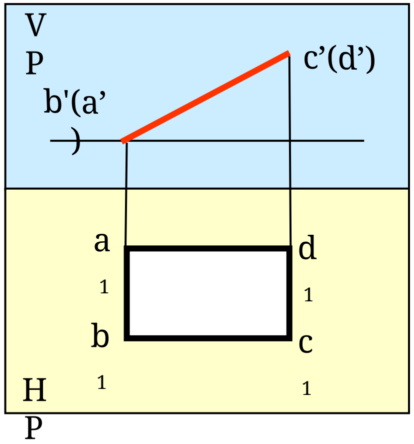
Projection of Planes

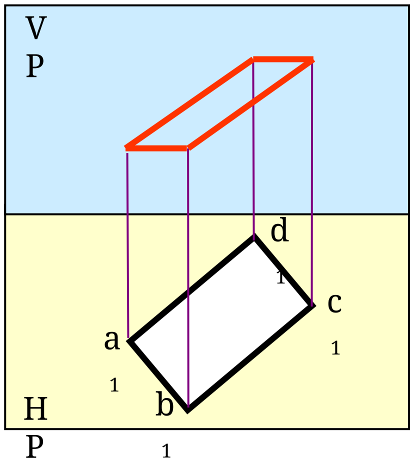












Projection of Planes

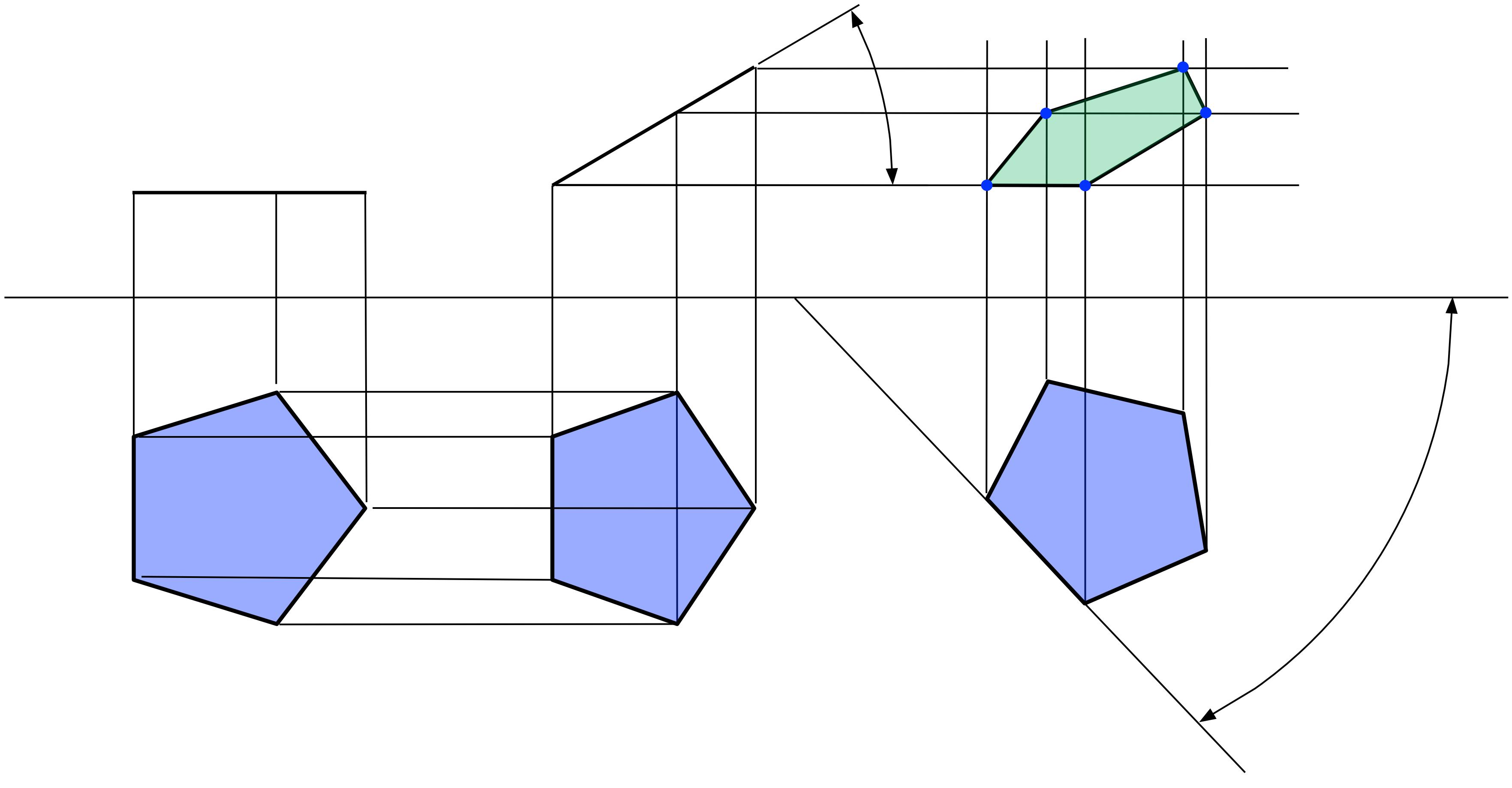
Given

- Surface inclined to one reference plane
- One of its edges or a diagonal inclined to other reference plane

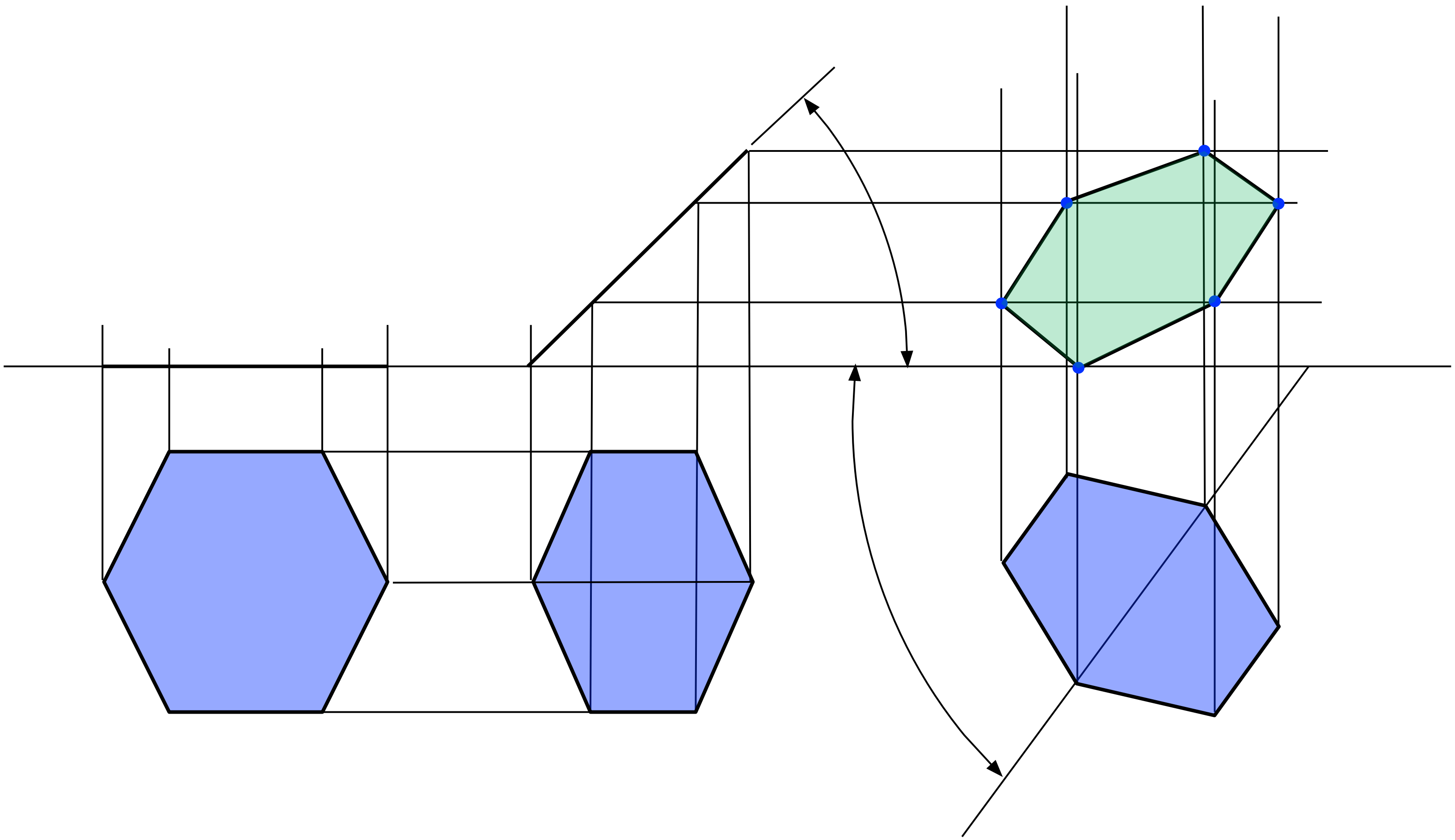
Procedure

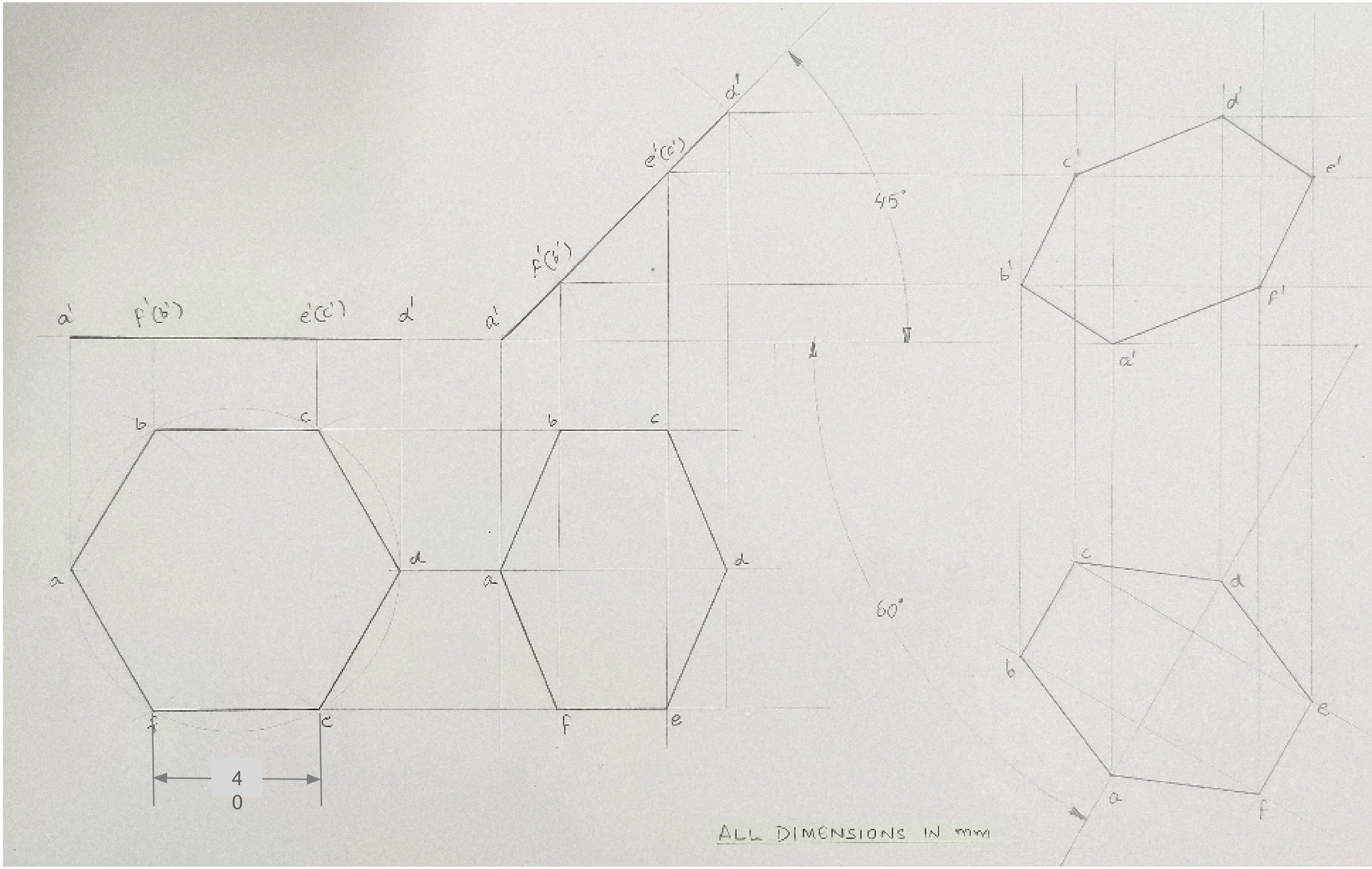
- Assume surface parallel to reference plane and draw FV and TV
- Consider surface inclination and draw 2nd FV & TV
- Consider edge inclination and draw final FV and TV

Projection of Planes

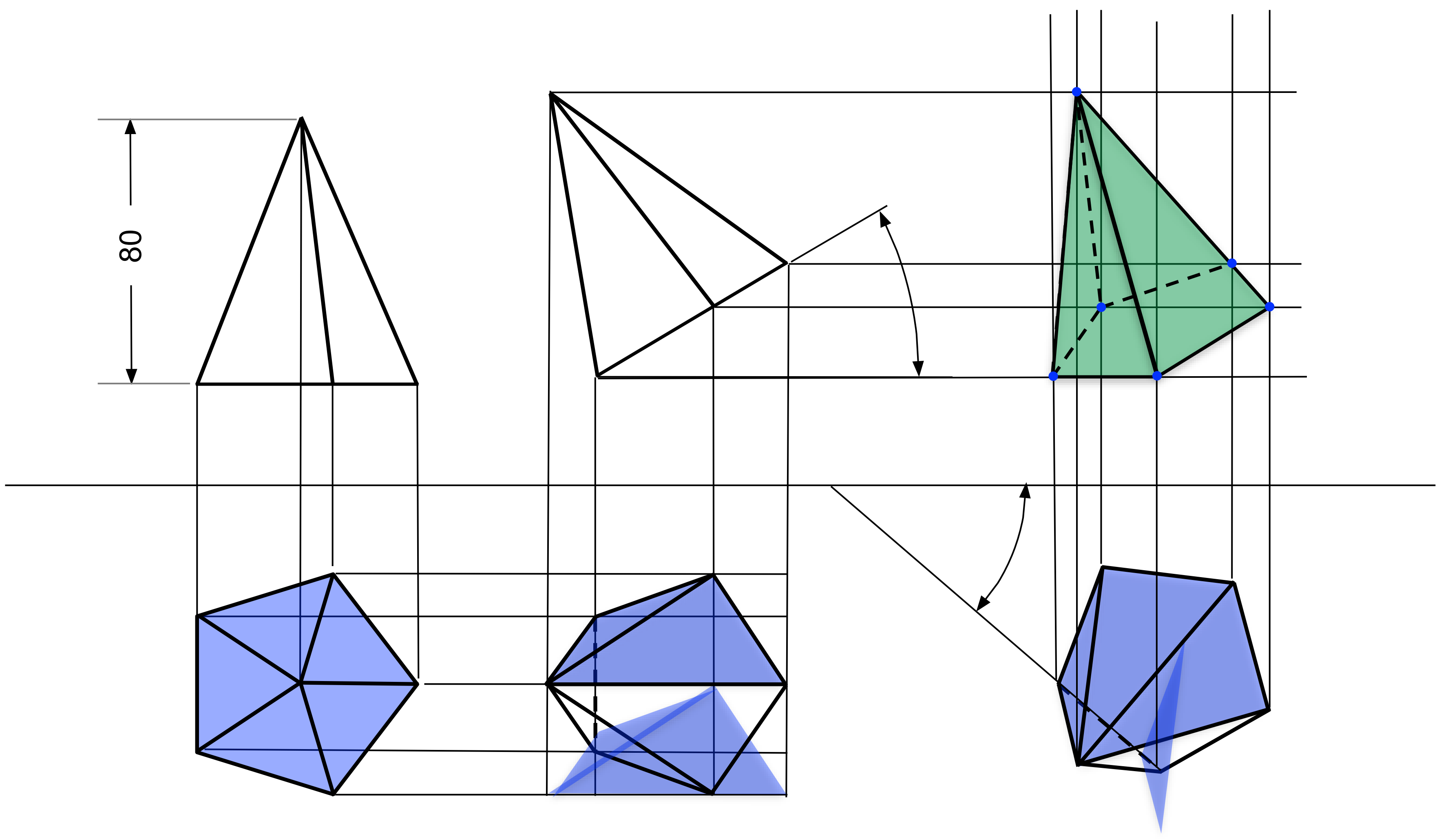


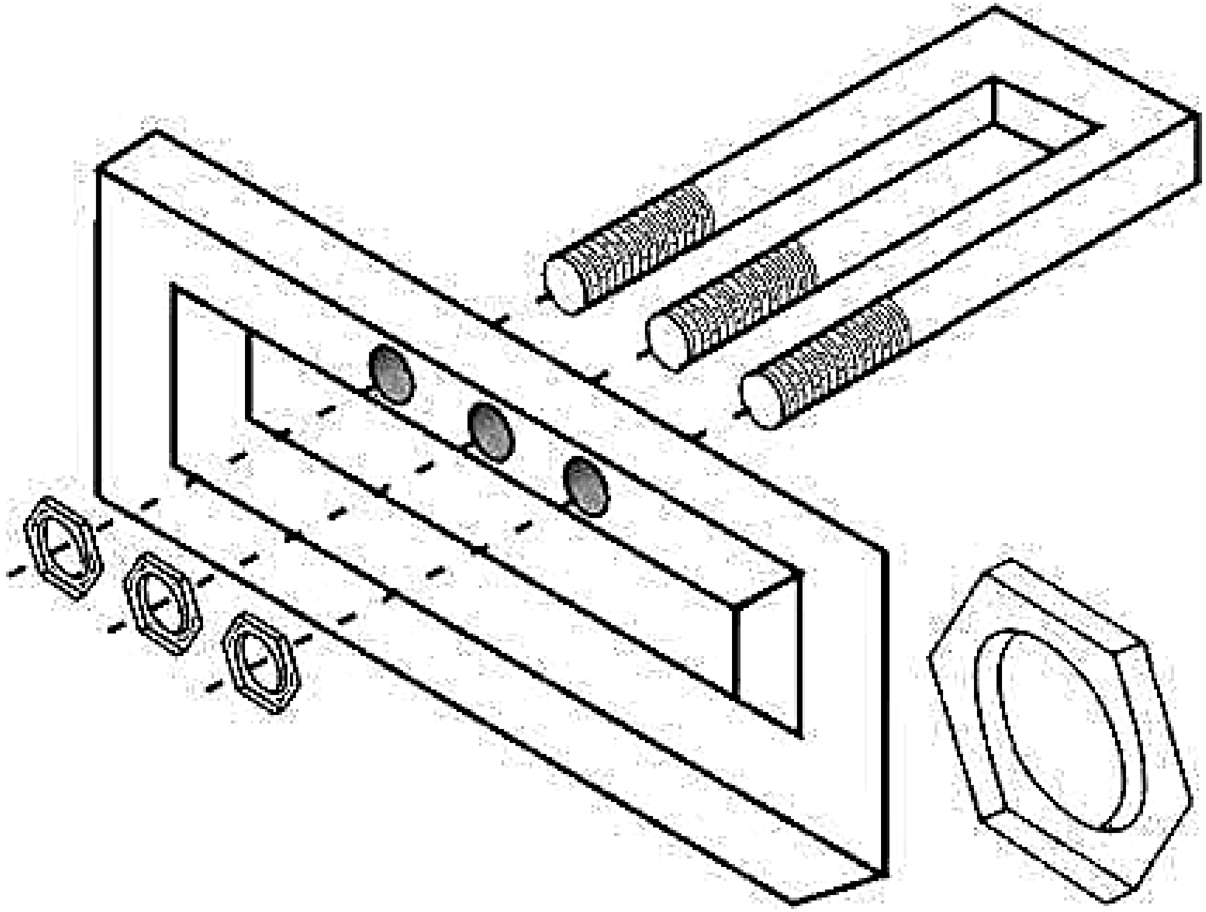
Projection of Planes




a'
f'(b')
e'(c')
d'
a'
f'(b')
e'(c')
d'
45°
c'
d'
e'
b'
f'
a'
b
c
a
d
f
c
40
b
c
a
d
f
e
60°
c
d
b
e
a
f
ALL DIMENSIONS IN mm

Projection of Solids





Thank you

https://www.goillusions.com/