Tutorial 3 (SVC) to be discussed on October 28, 2024

Q1. Show that for any numbers a and b, the inequality $|\sin b - \sin a| \le |b - a|$ is true.

Q2. Prove that between every two zeros of $x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ there lies a zero of

$$nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + a_1.$$

Q3. Show that a cubic polynomial can have at most three real zeros.

Q4. Let $f(x) = \dfrac{x^2 - 3}{x - 2}, \quad x \neq 2$

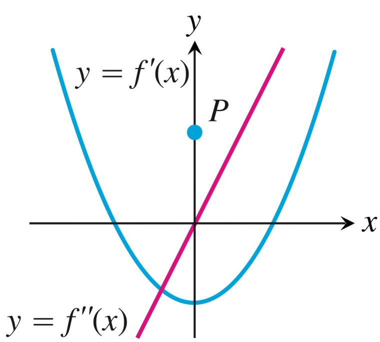
a. Find the intervals on which the function is increasing and decreasing.

b. Then identify the function's local extreme values, if any, saying where they are taken on.

c. Which, if any, of the extreme values are absolute?

Q5. Graph the function in problem 4 above.

Q6. The figure below shows the graphs of the first and second derivatives of a function $y = f(x)$. Copy the picture and add to it a sketch of the approximate graph of f, given that the graph passes through the point P.



Q7. **Temperature change** It took 14 sec for a mercury thermometer to rise from $-19°\text{C}$ to $100°\text{C}$ when it was taken from a freezer and placed in boiling water. Show that somewhere along the way the mercury was rising at the rate of $8.5°\text{C/sec}$.

Q8. Find the anti-derivative of $\dfrac{\csc\theta}{\csc\theta - \sin\theta}$

Q9. (i) Inscribe a regular n-sided polygon inside a circle of radius 1 and compute the area of the polygon for the following values of n:

a. 4 (square) **b.** 8 (octagon) **c.** 16

d. Compare the areas in parts (a), (b), and (c) with the area of the circle.

(ii) **a.** Inscribe a regular n-sided polygon inside a circle of radius 1 and compute the area of one of the n congruent triangles formed by drawing radii to the vertices of the polygon.

b. Compute the limit of the area of the inscribed polygon as $n \to \infty$.

c. Repeat the computations in parts (a) and (b) for a circle of radius r.

Q10. If $\text{av}(f)$ really is a typical value of the integrable function $f(x)$ on $[a, b]$, then the number $\text{av}(f)$ should have the same integral over $[a, b]$ that f does. Does it? That is, does

$$\int_a^b \text{av}(f)\, dx = \int_a^b f(x)\, dx?$$

Give reasons for your answer.

Q11. Use the Max-Min Inequality to find upper and lower bounds for the value of

$$\int_0^1 \frac{1}{1 + x^2}\, dx.$$

;

find upper and lower bounds for

$$\int_0^{0.5} \frac{1}{1 + x^2}\, dx \quad \text{and} \quad \int_{0.5}^{1} \frac{1}{1 + x^2}\, dx\,.$$

Add these to arrive at an improved estimate of

$$\int_0^{1} \frac{1}{1 + x^2}\, dx\,.$$