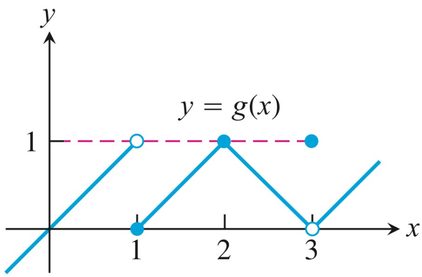
Tutorial 1 (SVC) to be discussed on October 21, 2024

Q1. For the function $g(x)$ graphed here, find the following limits or explain why they do not exist.

a. $\lim_{x \to 1} g(x)$ **b.** $\lim_{x \to 2} g(x)$ **c.** $\lim_{x \to 3} g(x)$

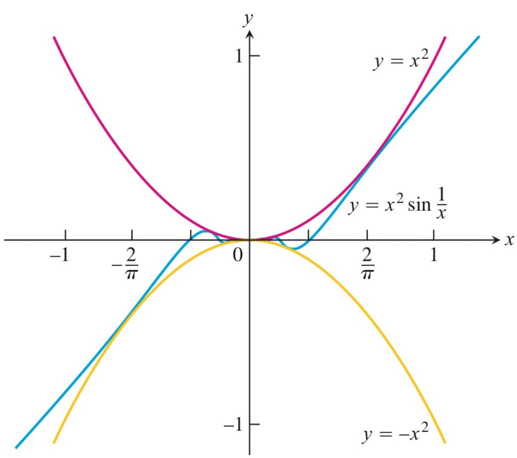


Q2. Evaluate the following limit:

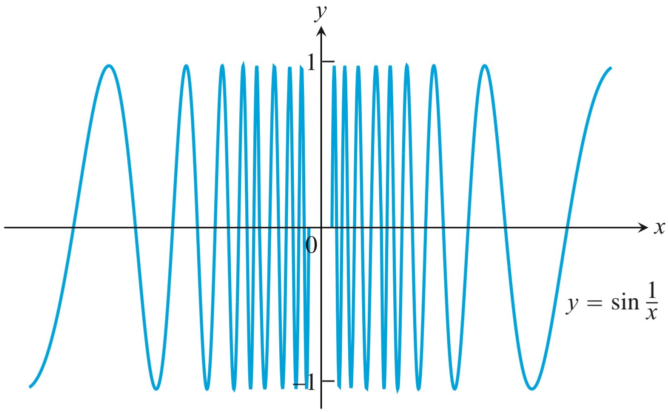
$$\lim_{t \to -1} \frac{t^2 + 3t + 2}{t^2 - t - 2}$$

Q3. Prove that

$$\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$$



Q4. Show that $y = \sin(1/x)$ has no limit as x approaches zero from either side



Q5. Evaluate the following limit:

$$\lim_{r \to \infty} \frac{r + \sin r}{2r + 7 - 5 \sin r}$$

Q6. Evaluate

$$\lim \left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}} \right) \text{ as}$$

a. $x \to 0^+$

b. $x \to 0^-$

c. $x \to 1^+$

d. $x \to 1^-$

Q7. At what points are the following functions continuous ?

1. $y = \tan \frac{\pi x}{2}$

2. $y = \frac{\sqrt{x^4 + 1}}{1 + \sin^2 x}$

Q8. For what value of b is

$$g(x) = \begin{cases} x, & x < -2 \\ bx^2, & x \geq -2 \end{cases}$$

continuous at every x?

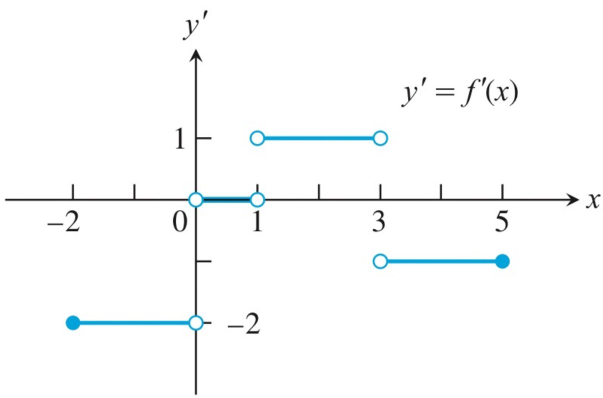
Q9. Show that the line $y = mx + b$ is its own tangent at any point $(x_0, mx_0 + b)$.

Q1. Use the following information to graph the function f over the closed interval $[-2, 5]$.

i) The graph of f is made of closed line segments joined end to end.

ii) The graph starts at the point $(-2, 3)$.

iii) The derivative of f is the step function in the figure shown here.



Q2. **a.** Let $f(x)$ be a function satisfying $|f(x)| \leq x^2$ for $-1 \leq x \leq 1$. Show that f is differentiable at $x = 0$ and find $f'(0)$.

b. Show that

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is differentiable at $x = 0$ and find $f'(0)$.

Q3. **Lunar projectile motion** A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec (about 86 km/h) reaches a height of $s = 24t - 0.8t^2$ meters in t sec.

a. Find the rock's velocity and acceleration at time t. (The acceleration in this case is the acceleration of gravity on the moon.)

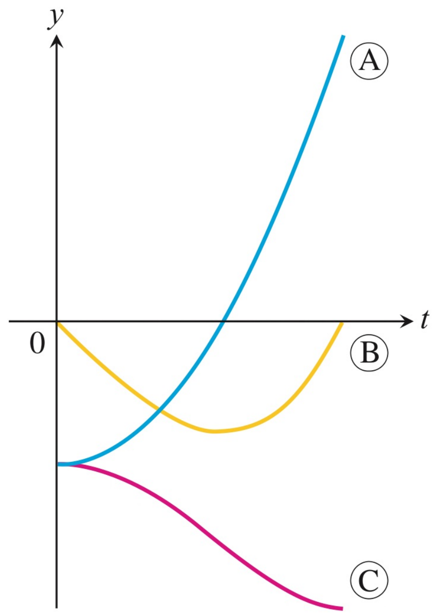
b. How long does it take the rock to reach its highest point?

c. How high does the rock go?

d. How long does it take the rock to reach half its maximum height?

e. How long is the rock aloft?

Q4. The graphs in Figure 3.22 show the position s, the velocity $v = ds/dt$, and the acceleration $a = d^2s/dt^2$ of a body moving along the coordinate line as functions of time t. Which graph is which? Give reasons for your answers.



Q5. Evaluate the following limits:

$$\lim_{\theta \to 0} \cos\left(\frac{\pi\theta}{\sin\theta}\right), \quad \lim_{x \to 0} \sec\left[\cos x + \pi \tan\left(\frac{\pi}{4 \sec x}\right) - 1\right]$$

Q6. Find dy/dt if

a) y=cot(sin(t)/t)

b) $y = 4\sin\left(\sqrt{1+\sqrt{t}}\right)$

c) $y = \left(1 + \tan^4\left(\frac{t}{12}\right)\right)^3$

d) $y = (1 + \cot(t/2))^{-2}$

Q7. **Temperature and the period of a pendulum** For oscillations of small amplitude (short swings), we may safely model the relationship between the period T and the length L of a simple pendulum with the equation

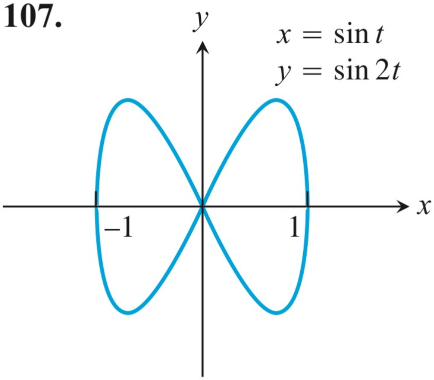
$$T = 2\pi\sqrt{\frac{L}{g}},$$

where g is the constant acceleration of gravity at the pendulum's location. If we measure g in centimeters per second squared, we measure L in centimeters and T in seconds. If the pendulum is made of metal, its length will vary with temperature, either increasing or decreasing at a rate that is roughly proportional to L. In symbols, with u being temperature and k the proportionality constant,

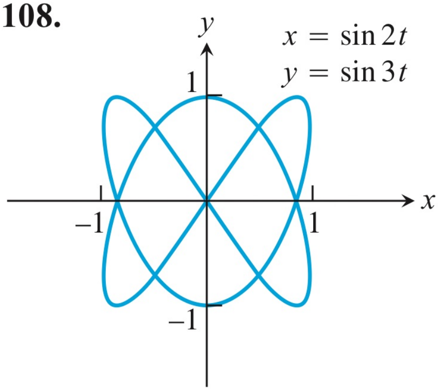
$$\frac{dL}{du} = kL.$$

Q8. The curves in Exercises 107 and 108 are called *Bowditch curves* or *Lissajous figures*. In each case, find the point in the interior of the first quadrant where the tangent to the curve is horizontal, and find the equations of the two tangents at the origin.

107.



108.



Q9. Find dy/dx if

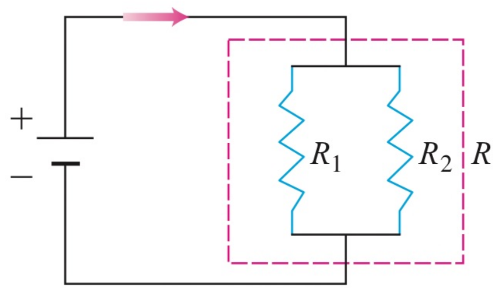
a) $$y \sin\left(\frac{1}{y}\right) = 1 - xy$$

b) $$xy = \cot(xy)$$

Q10. **Intersecting normal** The line that is normal to the curve $x^2 + 2xy - 3y^2 = 0$ at $(1, 1)$ intersects the curve at what other point?

Q11. **Resistors connected in parallel** If two resistors of R_1 and R_2 ohms are connected in parallel in an electric circuit to make an R-ohm resistor, the value of R can be found from the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$



If R_1 is decreasing at the rate of 1 ohm/sec and R_2 is increasing at the rate of 0.5 ohm/sec, at what rate is R changing when $R_1 = 75$ ohms and $R_2 = 50$ ohms?

Q12. Find the absolute maximum and minimum of the following function on the given interval.

$$g(x) = \sqrt{4 - x^2}, \quad -2 \le x \le 1$$

Q13. A drilling rig 12 mi offshore is to be connected by pipe to a refinery onshore, 20 mi straight down the coast from the rig. If underwater pipe costs \$500,000 per mile and land-based pipe costs \$300,000 per mile, what combination of the two will give the least expensive connection?