

1. Prove following properties of D.T.F.T:

(i) Linearity:

$$\text{if } x_1(n) \leftrightarrow X_1(e^{j\omega})$$

$$x_2(n) \leftrightarrow X_2(e^{j\omega})$$

then,

$$a x_1(n) + b x_2(n) \leftrightarrow a X_1(e^{j\omega}) + b X_2(e^{j\omega})$$

(ii) Time shifting:

$$\text{if } x(n) \leftrightarrow X(e^{j\omega})$$

then,

$$x(n - n_d) \leftrightarrow e^{-j\omega n_d} X(e^{j\omega})$$

(iii) Freqⁿ shifting:

$$\text{if } x(n) \leftrightarrow X(e^{j\omega})$$

then,

$$e^{j\omega_0 n} x(n) \leftrightarrow X(e^{j(\omega - \omega_0)})$$

(iv) Time Reversal:

$$\text{if } x(n) \leftrightarrow X(e^{j\omega})$$

then,

$$x(-n) \leftrightarrow X(e^{-j\omega})$$

(v) convolution in Time - domain:

$$x_1(n) * x_2(n) \leftrightarrow X(e^{j\omega}) \cdot X(e^{j\omega})$$

(vi) convolution in freqⁿ domain:

$$x_1(n) \cdot x_2(n) \leftrightarrow \frac{1}{2\pi} [X_1(e^{j\omega}) * X_2(e^{j\omega})]$$

(viii) Parseval's Theorem:

$$\text{if } x(n) \leftrightarrow X(e^{j\omega})$$

then

$$E_x = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \cdot d\omega$$