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PARAMETER ESTIMATION

Q1 Random sample $(X_1, X_2, X_3, \ldots X_n)$

$\mu = \theta_1$

$\sigma^2 = \theta_2$

Likelihood fn $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$

take log

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[\frac{-1}{2} \ln(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2} \right]$$

→ Differentiate wrt θ_1

$$\frac{d \ln L(\theta_1, \theta_2)}{d\theta_1} = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$x_i - n\theta_1 = 0$$

$$\text{mean} = \theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}$$

→ Differentiate wrt θ_2

$$\frac{d \ln L(\theta_1, \theta_2)}{d\theta_2} = \sum_{i=1}^{n} \left[\frac{-1}{2\theta_2} + \frac{(x_i-\theta_1)^2}{2\theta_2^2} \right] = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Q2 Binomial distribution $B(x, \theta)$

$p = \theta$, $q = 1 - \theta$

$$f(x_i; n, \theta) = {}^{n}C_{x} \, \theta^{x} (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{x_i} \, \theta^{x_i} (1-\theta)^{n-x_i}$$

take log

$$\ln L(\theta) = \sum_{i=1}^{n} \left[\ln {}^{n}C_{x_i} + x_i \ln \theta + (n - x_i) \ln (1-\theta) \right]$$

diff wrt θ

$$\frac{d \ln L(\theta)}{d\theta} = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n - x_i}{1-\theta} \right] = 0$$

$$\sum_{i=1}^{n} \left[\frac{(1-\theta) x_i - \theta (n - x_i)}{\theta (1-\theta)} \right] = 0$$

$$\sum_{i=1}^{n} \left[(1-\theta) x_i - (n - x_i) \theta \right] = 0$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{n \cdot m} \rightarrow \text{MLE}$$