Sol^n 1.

Consider a random sample $(X_1, X_2, X_3, --, X_n)$

$\mu = \theta_1$ (mean) and $\sigma^2 = \theta_2$ (variance)

Likelihood function $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(-\frac{(x_i-\theta_1)^2}{2\theta_2}\right)$

To maximise take log on both sides

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[\frac{-1}{2} \ln(2\pi\theta_2) - \frac{(x_i - \theta_1)^2}{2\theta_2} \right]$$

i) Differentiate w.r.t θ_1 (for θ_1)

$$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$x_i - n\theta_1 = 0$$

$$\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n} \quad [\text{Mean}]$$

ii) Differentiate w.r.t θ_2 (for θ_2)

$$\frac{d}{d\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[\frac{-1}{2\theta_2} + \frac{(x_i - \theta_1)^2}{2\theta_2^2} \right] = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2 \quad [\text{Variance}]$$

classmate

Date

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Sol^n (2) Binomial distribution $B(n,\theta)$ $p=\theta, q=1-\theta$

pmf (probability mass function)

$$f(x;n,\theta) = {}^nC_x \theta^x (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^nC_{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$$

Taking log on both sides

$$\ln L(\theta) = \sum_{i=1}^{n} \left[\ln {}^nC_{x_i} + x_i \ln\theta + (n-x_i)\ln(1-\theta)\right]$$

differentiate w.r.t θ

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta}\right] = 0$$

Find θ

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta}\right] = 0$$

$$\sum_{i=1}^{n} \left[\frac{(1-\theta)x_i - \theta(n-x_i)}{\theta(1-\theta)}\right] = 0$$

$$\sum_{i=1}^{n} (1-\theta)x_i - (n-x_i)\theta = 0$$

$$\theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i . m$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{n.m}} \rightarrow \text{MLE}$$