MLE : $\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{\theta}_1)^2$$

(2.) $L(\theta | x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} {}^{m}C_{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$

Taking log on both sides.

$$l(\theta | x_1, x_2, \ldots x_n) = \sum_{i=1}^{n} \left[\log \binom{m}{x_i} + x_i \log(\theta) + (m - x_i) \log(1-\theta) \right]$$

$$\frac{dl}{d\theta} = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \left(\frac{m - x_i}{1-\theta} \right) \right] = 0$$

Solving for θ

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m}{1-\theta} \right] = 0$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{nm}{1-\theta} = 0$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{nm}{1-\theta}$$

$$(1-\theta) \sum_{i=1}^{n} x_i = \theta nm$$

$$\boxed{\theta = \frac{\sum x_i}{\sum x_i + nm}}$$

1. Normal distribution MLE derivation

$$f(x) = \frac{1}{\sqrt{2\pi\theta_2}} e^{\left(\frac{-(x_i-\theta_1)^2}{2\theta_2}\right)}$$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\left(\frac{-(x_i-\theta_1)^2}{2\theta_2}\right)}$$

Taking natural logarithm.

$$l(\theta_1, \theta_2) = \frac{n}{2}\log(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Differentiating w.r.t. θ_1 and θ_1.

w.r.t θ_1

$$\frac{dl}{d\theta_1} = \frac{1}{\theta_1}\sum_{i=1}^{n}(x_i-\theta_1) = 0$$

$$\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

w.r.t θ_2

$$\frac{dl}{d\theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2}\sum_{i=1}^{n}(x_i-\theta_1)^2 = 0$$

solving for θ_2.

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$