

L11\_S2\_A.pdf

PDF

Generate a scribe for exam reference material from the given lecture slides. The scribe must reconstruct what is taught in the lecture slides so that a student can rely on the scribe alone for revision for a closed books examination.

Use only the given material that is the lecture slides and no extra content. The scribe should not introduce any new concepts, interpretation, explanations, examples, solved problems, proofs or derivations. Do not exceed the scope of the given content in any way.

Using such an approach, an attempt will be made to write the scribe where the reasoning is also explicit within the scribe itself while writing it in such a way that all the definitions, assumptions, notations, derivations, proofs, etc., are reasoned step by step as they are found within the lecture material itself. The reasoning for all this should emerge only from the given material itself.

The Scribe must include step by step proofs and derivations, with examples given in the slides only. The scribe should be structured in a way that all logical dependencies between the ideas have to be clear so that it reflects and follows the flow of the lecture.

The lecture must maintain formal academic tone without any creative writing or commentary on the material. The scribe should be clearly structured in a section sub-section format and with bullet points highlighting the important points and making it clear for revision.

The scribe should avoid any kind of summaries interpretations, explanations and but should contain all examples, questions and proofs and derivations and definitions provided in the lecture slides written in an academic format.

Do not explain how the scribe is written. Do not comment on the process. No additions are necessary, as the material is already provided in the context given above. The final output should resemble a lecture scribe that is intended for direct submission.



# CSE400 – Fundamentals of Probability in Computing

## Lecture 11: Transformation of Random Variables

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## 1. Transformation of Random Variables

### 1.1 Objective

- To learn transformation techniques for random variables.
  - To determine the distribution of a transformed random variable.
  - To extend the idea to functions involving two random variables.
  - To derive the distribution in the illustrative case:
    - $Z = X + Y$
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## 2. Transformation of a Random Variable

Let:

- $X$  be a random variable.
- A new random variable  $Z$  be defined as a function of  $X$ , i.e.,

$$Z = g(X)$$

The objective is:

- To determine the distribution of  $Z$  from the known distribution of  $X$ .
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### 2.1 General Method (CDF-Based Approach)

To find the distribution of  $Z$ :

1. Define:

$$F_Z(z) = P(Z \leq z)$$

2. Since  $Z = g(X)$ , substitute:

$$F_Z(z) = P(g(X) \leq z)$$

3. Express the event  $g(X) \leq z$  in terms of  $X$ .
4. Use the known distribution of  $X$  to compute the probability.
5. Once  $F_Z(z)$  is obtained, differentiate (if continuous) to obtain the PDF:

$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

### 3. Function of Two Random Variables

Let:

- $X$  and  $Y$  be two random variables.
- Define a new random variable:

$$Z = g(X, Y)$$

The objective is:

- To determine the distribution of  $Z$  from the joint distribution of  $X$  and  $Y$ .

#### 3.1 Joint Distribution Framework

If:

- $f_{X,Y}(x, y)$  is the joint PDF of  $X$  and  $Y$ ,

Then:

- Probabilities involving  $Z$  are computed using the joint distribution.

#### 3.2 CDF of $Z$

To determine the distribution of  $Z$ :

1. Define:

$$F_Z(z) = P(Z \leq z)$$

2. Substitute:

$$F_Z(z) = P(g(X, Y) \leq z)$$

3. Express the event  $g(X, Y) \leq z$  as a region in the  $(x, y)$ -plane.
4. Compute:

$$F_Z(z) = \iint_{g(x,y) \leq z} f_{X,Y}(x,y) dx dy$$

5. If continuous, differentiate:

$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

## 4. Illustrative Example: $Z = X + Y$

Let:

$$Z = X + Y$$

Objective:

- To derive the distribution of  $Z$ .

### 4.1 CDF of $Z$

By definition:

$$F_Z(z) = P(Z \leq z)$$

Substitute:

$$F_Z(z) = P(X + Y \leq z)$$

This corresponds to:

- The set of all  $(x, y)$  such that:

$$x + y \leq z$$

Thus:

$$F_Z(z) = \iint_{x+y \leq z} f_{X,Y}(x,y) dx dy$$

### 4.2 Derivation of the PDF

To obtain the PDF:

$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

Using the integral expression:

$$F_Z(z) = \iint_{x+y \leq z} f_{X,Y}(x,y) dx dy$$

Differentiating with respect to  $z$  yields:

$$f_Z(z) = \int_{-\infty}^{\infty} f_{X,Y}(x, z-x) dx$$

This provides the distribution of  $Z = X + Y$ .

## 5. Logical Flow of the Lecture

- Start with transformation of a single random variable.
- Extend to functions involving two random variables.
- Use the cumulative distribution function as the primary method.
- Convert probability statements into integrals over appropriate regions.
- Differentiate to obtain the PDF.
- Apply the method to the specific case:

$$Z = X + Y$$

## 6. Key Formulas

### Single Random Variable

$$F_Z(z) = P(g(X) \leq z)$$

$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

### Two Random Variables

$$F_Z(z) = \iint_{g(x,y) \leq z} f_{X,Y}(x,y) dx dy$$

Special Case:  $Z = X + Y$

$$F_Z(z) = P(X + Y \leq z)$$

$$f_Z(z) = \int_{-\infty}^{\infty} f_{X,Y}(x, z - x) dx$$

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End of Lecture 11 Scribe

