

Lecture Scribe Overleaf Code



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PDF

You are generating a lecture scribe intended as academic reference material for a reading-based / closed-notes exam.

Context (authoritative and exclusive):

The attached materials (lecture slides / lecture PDF and relevant textbook excerpts) are the only allowed source of content.

Do not use external knowledge, prior training, intuition, or examples not explicitly present in the provided context.

Primary Objective:

Produce a faithful, exam-ready reconstruction of what was taught in the lecture such that a student can rely solely on this document for revision.

Reasoning Requirement (Chain-of-Thought):

While writing the scribe, reason step by step through:

Definitions and notation

Assumptions and conditions

Theorems or stated results

Proofs or derivations

Worked examples

Make all logical steps explicit exactly as they appear or are implied in the context, so that the reasoning can be followed during exam preparation.

Content Requirements:

The scribe must include, where present in the context:

Formal definitions and notation

Explicit assumptions and constraints

Statements of results, propositions, or theorems

Step-by-step proofs or derivations

Worked examples with intermediate steps

Clear logical flow matching the lecture structure

Strict Constraints:

Do not introduce new material, intuition, shortcuts, or alternative explanations

Do not simplify, reinterpret, or extend beyond the lecture/textbook

Do not add examples, remarks, or insights not present in the context

Do not provide multiple solution methods

Do not summarize at a high level; maintain formal academic tone

Output Style:

Formal, structured, academic writing

Clear sectioning following the lecture order

Explicit reasoning steps included where appropriate

No conversational language

Verification Standard:

The output should answer:

"If a student studies only this scribe, can they reliably

reconstruct the lecture content and revise for the exam?"

Generate the lecture scribe accordingly.

give overleaf (latex) code

Below is a **faithful, exam-ready lecture scribe** written strictly from the attached **Lecture 7 slides** for **CSE400 – Fundamentals of Probability in Computing**. It is provided as **complete Overleaf (LaTeX) source code**, suitable for direct compilation.

Scope notice (important for exam integrity):

The uploaded slides contain full material for **CDF and PDF**, including definitions, properties, derivations, and worked examples.

The slides **only introduce headings** for *Expectation of RVs and Moments* without definitions, formulas, or examples.

Accordingly, this scribe **does not invent or reconstruct missing content** for those sections and preserves the lecture exactly as provided.

✓ Overleaf (LaTeX) Code — Lecture Scribe

latex

```
\documentclass[12pt]{article}

\usepackage{amsmath, amssymb}
\usepackage{geometry}
\usepackage{setspace}
\usepackage{hyperref}

\geometry{margin=1in}
\onehalfspacing

\title{\textbf{CSE400: Fundamentals of Probability in Computing}}
Lecture 7: Expectation, CDFs, PDFs and Problem Solving}
\author{Dhaval Patel, PhD\\
Associate Professor, CSE\\
Ahmedabad University}
\date{January 27, 2025}
```

```
\begin{document}
```

```
\maketitle
```

```
\tableofcontents
```

```
\newpage
```

```
%-----
```

```
\section{Cumulative Distribution Function (CDF)}
```

```
\subsection{Definition}
```

Let X be a random variable.

The **Cumulative Distribution Function (CDF)** of X is defined as:

```
\[
```

$$F_X(x) = \Pr(X \leq x), \quad -\infty < x < \infty$$

```
\]
```

The behavior of $F_X(x)$ determines most of the information about the random experiment described by the random variable X .

```
\subsection{Properties of the CDF}
```

The CDF $F_X(x)$ satisfies the following properties:

```
\begin{enumerate}
```

```
  \item \textbf{Bounds:}
```

```
  \[
```

$$0 \leq F_X(x) \leq 1$$

```
  \]
```

```
  \item \textbf{Limits at Infinity:}
```

```
  \[
```

$$F_X(-\infty) = 0, \quad F_X(\infty) = 1$$

```
  \]
```

```
  \item \textbf{Monotonicity:}
```

For $x_1 < x_2$,

```
  \[
```

$$F_X(x_1) \leq F_X(x_2)$$

```
  \]
```

\item \textbf{Probability Over an Interval:}

For $x_1 < x_2$,

\lceil

$$\Pr(x_1 < X \leq x_2) = F_X(x_2) - F_X(x_1)$$

\rceil

\end{enumerate}

These properties are necessary for a function to be a valid CDF.

%-----

\subsection{Example 1: Validity of CDFs}

Determine whether each of the following functions is a valid CDF.

\subsubsection*(a)

\lceil

$$F_X(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}(x)$$

\rceil

\textbf{Verification:}

\begin{itemize}

\item $\tan^{-1}(x)$ is monotone increasing.

\item $\tan^{-1}(-\infty) = -\frac{\pi}{2}$ and $\tan^{-1}(\infty) = \frac{\pi}{2}$.

\end{itemize}

Thus,

\lceil

$$F_X(-\infty) = 0, \quad F_X(\infty) = 1$$

\rceil

Hence, this is a valid CDF.

\subsubsection*(b)

\lceil

$$F_X(x) = \left[1 - e^{-x}\right]u(x)$$

\rceil

where $u(x)$ is the unit step function.

\textbf{Verification:}

\begin{itemize}

\item $F_X(x)=0$ for $x<0$.

- $F_X(x)$ is non-decreasing.
- $\lim_{x \rightarrow \infty} F_X(x) = 1$.

Hence, this is a valid CDF.

(c)

$$F_X(x) = e^{-x^2}$$

This function does not satisfy the limit condition:

$$\lim_{x \rightarrow \infty} F_X(x) \neq 1$$

Hence, it is **not** a valid CDF.

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Example 2: Probability Computation Using a CDF

Suppose a random variable has CDF:

$$F_X(x) = (1 - e^{-x})u(x)$$

Compute the following probabilities.

(a) $\Pr(X > 5)$

$$\Pr(X > 5) = 1 - \Pr(X \leq 5) = 1 - F_X(5)$$

$$= 1 - (1 - e^{-5}) = e^{-5}$$

(b) $\Pr(X < 5)$

$$\Pr(X < 5) = F_X(5) = 1 - e^{-5}$$

\]

\subsubsection*{(c) $\Pr(3 < X < 7)$ }

\[

$$\Pr(3 < X < 7) = F_X(7) - F_X(3)$$

\]

\[

$$= (1 - e^{-7}) - (1 - e^{-3}) = e^{-3} - e^{-7}$$

\]

\subsubsection*{(d) $\Pr(X > 5 \mid X < 7)$ }

Using conditional probability:

\[

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

\]

\[

$$\Pr(X > 5 \mid X < 7) = \frac{\Pr(5 < X < 7)}{\Pr(X < 7)}$$

\]

\[

$$= \frac{F_X(7) - F_X(5)}{F_X(7)}$$
$$= \frac{(1 - e^{-7}) - (1 - e^{-5})}{1 - e^{-7}}$$

\]

%-----

\section{Probability Density Function (PDF)}

\subsection{Definition}

For a continuous random variable X , the **Probability Density Function (PDF)** is defined as:

\[

$$f_X(x) = \lim_{\epsilon \rightarrow 0} \frac{\Pr(x \leq X < x + \epsilon)}{\epsilon}$$

\]

\subsection{PDF--CDF Relationship}

For a continuous random variable:

$$\Pr(x \leq X < x + \epsilon) = F_X(x + \epsilon) - F_X(x)$$

Substituting into the definition:

$$f_X(x) = \lim_{\epsilon \rightarrow 0} \frac{F_X(x + \epsilon) - F_X(x)}{\epsilon}$$

Thus,

$$\boxed{f_X(x) = \frac{d}{dx} F_X(x)}$$

Inverse Relationship

Conversely, the CDF can be obtained from the PDF by integration:

$$F_X(x) = \int_{-\infty}^x f_X(t) dt$$

Hence:

- The PDF is the derivative of the CDF.
- The CDF is the integral of the PDF.

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Expectation of Random Variables

The lecture outline introduces the following topics:

- Expectation of random variables
- Expectation of a function of a random variable
- Linear operation with expectation
- n^{th} moments and central moments (variance, skewness, kurtosis)

Note:

The provided lecture slides do not include definitions, formulas, derivations, or examples for these topics.

Therefore, no additional content is reconstructed here.

`\end{document}`

If you want, I can next:

- Split this into **exam-oriented short notes**, or
- Add **margin comments** highlighting what is typically tested, or
- Format it exactly to your department's **scribe submission standard** (font, spacing, cover page).