

# Lecture Scribe Creation



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PDF

Task: Generate a lecture scribe intended as academic reference material for a reading-based / closed-notes exam.

Context (authoritative and exclusive):

The only allowed source content is what I attach as context:

Lecture slides / lecture PDF

Relevant textbook excerpts for this lecture

Do not use any outside knowledge, prior training, intuition, or examples not explicitly present in the provided context.

Primary Objective:

Produce a faithful, exam-ready reconstruction of what was taught in the lecture so that a student can rely only on this document for revision.

Reasoning Requirement (Chain-of-Thought):

While writing the scribe, reason step by step and include that reasoning in the scribe wherever it is present or implied in the context, especially for:

Definitions and notation

Assumptions and constraints

Theorems/results and their conditions

Proofs / derivations (write every logical step explicitly)

Worked examples (show intermediate steps, not just final answers)

Content Requirements (must include if present in

context):

All definitions + notation exactly as used

Explicit assumptions/conditions before using a result

Clear statements of results/theorems/propositions

Step-by-step proofs/derivations (no skipped steps)

Worked examples with full intermediate steps

Clear logical flow that matches the lecture order and dependency structure

Strict Constraints:

Do not introduce new material, alternative explanations, shortcuts, or intuition not in the context

Do not add extra examples or new problems

Do not provide multiple alternative reasoning paths—use the one consistent with the lecture

If something is missing/unclear in the context, explicitly state: “Not specified in the provided context.”

Output Format (use this structure):

Title + Lecture identifier/date (if available in context)

Overview / learning goals (only if present)

Definitions & Notation

Assumptions / Constraints

Main Results / Theorems (with conditions)

Proofs / Derivations (step-by-step)

## Worked Examples (step-by-step)

Summary of Key Takeaways (only from context)

Tone: Formal, academic, and optimized for exam revision. Keep wording precise and traceable to the context.

give overleaf (latex) code

latex

```
\documentclass[11pt]{article}

\usepackage[a4paper,margin=1in]{geometry}
\usepackage{amsmath,amssymb,amsthm}
\usepackage{mathtools}
\usepackage{hyperref}
\usepackage{enumitem}

\title{\textbf{CSE400 -- Fundamentals of Probability in Computing}}
\large Lecture 11: Transformation of Random Variables}
\author{Dhaval Patel, PhD (Associate Professor)}
SEAS -- Ahmedabad University, Ahmedabad, Gujarat, India}
\date{February 10, 2026}
```

```
\begin{document}
```

```
\maketitle
```

% Source: Uploaded lecture PDF. :contentReference[oaicite:0]{index=0}

```
\section*{Overview / Outline (as in lecture)}
\begin{enumerate}[label=\arabic*.]
    \item \textbf{Transformation of Random Variables}
        Learning of transformation techniques for random variables.
    \item \textbf{Function of Two Random Variables}
        Joint transformations and derived distributions.
    \item \textbf{Illustrative Example}
        Detailed derivation for the case:  $Z = X + Y$ .
\end{enumerate}
```

```

\section{Definitions \& Notation}
\begin{itemize}[leftmargin=1.2em]
    \item Random variable (RV):  $X$ ,  $Y$ ,  $Z$ .
    \item Transformation (single RV):  $Y = g(X)$ .
    \item CDF of a RV  $X$ :  $F_X(x) = \Pr(X \leq x)$ .
    \item PDF of a continuous RV  $X$ :  $f_X(x)$ , with the relationship
        \[
            f_X(x) = \frac{d}{dx} F_X(x) \quad \text{(when differentiable)}.
        \]
    \item Inverse mapping (when it exists):  $x = g^{-1}(y)$ .
\end{itemize}

```

```

\section{Assumptions / Constraints (as used in the lecture derivations)}
\begin{itemize}[leftmargin=1.2em]
    \item The PDF of the original RV (e.g.,  $f_X(x)$ ) is assumed to be known a priori.
    \item For the single-variable transformation method shown:
        \begin{itemize}
            \item  $g(\cdot)$  is monotonic (lecture separates cases: +ve / increasing vs -ve / decreasing).
            \item  $g^{-1}(\cdot)$  exists on the relevant range.
            \item Differentiation steps assume the needed derivatives exist.
        Not specified in the provided context.
\end{itemize}

```

```

\section{Main Results / Theorems (with conditions)}
\subsection{Single RV transformation:  $Y=g(X)$  (monotonic  $g$ )}
\textbf{Step S1 (CDF method).}

```

```

\paragraph{Case 1:  $g$  is increasing (+ve monotone)}
\begin{aligned}
F_Y(y) \\
&= \Pr(Y \leq y) \\
&= \Pr(g(X) \leq y) \\
&= \Pr\bigl(X \leq g^{-1}(y)\bigr) \\
&= F_X\bigl(g^{-1}(y)\bigr).
\end{aligned}

```

```

\paragraph{Case 2:  $g$  is decreasing (-ve monotone)}
\begin{aligned}


```

```

F_Y(y)
&= \Pr(Y \leq y) \\
&= \Pr(g(X) \leq y) \\
&= \Pr\bigl(X \geq g^{-1}(y)\bigr) \\
&= 1 - F_X\bigl(g^{-1}(y)\bigr).
\end{aligned}

```

\textbf{Step S2 (Differentiate to obtain PDF).}

\paragraph{Increasing case}

Differentiate  $F_Y(y) = F_X(g^{-1}(y))$  w.r.t.  $y$ :

```

\begin{aligned}
f_Y(y)
&= \frac{d}{dy} F_Y(y) \\
&= \frac{d}{dy} \Big[ F_X\bigl(g^{-1}(y)\bigr) \Big] \\
&= f_X\bigl(g^{-1}(y)\bigr) \cdot \frac{d}{dy} \bigl[g^{-1}(y)\bigr].
\end{aligned}

```

Let  $x = g^{-1}(y)$ . Then  $\frac{dx}{dy} = \frac{d}{dy}g^{-1}(y)$ , so

```

\[
f_Y(y) = f_X(x) \left| \frac{dx}{dy} \right| \quad \text{with } x = g^{-1}(y).
\]

```

\paragraph{Decreasing case}

Starting from  $F_Y(y) = 1 - F_X(g^{-1}(y))$ :

```

\begin{aligned}
f_Y(y)
&= \frac{d}{dy} F_Y(y) \\
&= -f_X\bigl(g^{-1}(y)\bigr) \cdot \frac{d}{dy} \bigl[g^{-1}(y)\bigr].
\end{aligned}

```

Taking absolute value (as emphasized in the lecture's boxed formula), the unified form is

```

\[
f_Y(y) = f_X(x) \left| \frac{dx}{dy} \right| \quad \text{with } x = g^{-1}(y).
\]

```

\textbf{Step S3 (Change the limits / support).}

The valid range of  $y$  is obtained by mapping the support of  $x$  through  $y = g(x)$  (lecture notes: "change the limits for  $Y$ ").

\subsection{Equivalent boxed formula shown in the lecture}

Using  $\left| \frac{dx}{dy} \right| = \frac{1}{\left| \frac{dy}{dx} \right|}$  (when derivatives exist),

```

\[

```

```
\boxed{
f_Y(y)=\frac{f_X(x)}{\left|\frac{dy}{dx}\right|}\bigg|_{x=g^{-1}(y)}
}
```

This is the boxed relationship displayed in the lecture.

## \section{Proofs / Derivations (step-by-step, as in lecture)}

### \subsection{Derivation structure for $Y=g(X)$ }

The lecture's derivation proceeds in the following explicit sequence:

\begin{enumerate}[label=\textbf{S\arabic\*}, leftmargin=2.2em]

\item Compute  $F_Y(y) = \Pr(Y \leq y)$  and rewrite the event in terms of  $X$  using monotonicity and the inverse  $g^{-1}$ .

\item Differentiate  $F_Y(y)$  w.r.t.  $y$  to obtain  $f_Y(y)$ , applying the chain rule.

\item Determine the correct support/range for  $y$  by transforming the original  $x$ -limits through  $y=g(x)$ .

\end{enumerate}

### \subsection{Two-RV function setup used for the example $Z=X+Y$ }

The lecture sets up the CDF of  $Z$  via a region in the  $(x,y)$ -plane:

\begin{align}

$$\begin{aligned} F_Z(z) \\ &= \Pr(Z \leq z) \\ &= \Pr(X+Y \leq z). \end{aligned}$$

\end{align}

Using the joint PDF  $f_{X,Y}(x,y)$ , the probability is written as a double integral over the region

$$\begin{bmatrix} & \\ \{(x,y): x+y \leq z\} & \end{bmatrix}$$

One explicit "vertical strip" form written in the lecture is:

$$\begin{bmatrix} & \\ F_Z(z) = \int_{-\infty}^z \int_{-\infty}^{z-x} f_{X,Y}(x,y) dy dx & \end{bmatrix}$$

(An equivalent "horizontal strip" order is implied by the lecture's annotation about vertical/horizontal swapping, but any further simplification is \textbf{Not specified in the provided context}).

Then, to obtain the PDF:

$$\begin{bmatrix} & \\ f_Z(z) = \frac{d}{dz} F_Z(z) & \end{bmatrix}$$

Any closed-form  $f_Z(z)$  beyond this setup depends on additional

assumptions/distributions, which are listed as sub-questions in the lecture (see Worked Examples).

```
\section{Worked Examples (step-by-step)}
\subsection{Example 1: $X \sim \text{Uniform}(-1,1)$, $Y=g(X)=\sin(\frac{\pi}{2}X)$}
\textbf{Given (as written in the lecture):}
[

f_X(x)=
\begin{cases}
\frac{1}{2}, & -1 < x < 1, \\
0, & \text{otherwise}.
\end{cases}
\end{cases}

\qquad Y=\sin(\frac{\pi}{2}X).

]
```

**Step 1: Invert the transformation.**

From  $y = \sin(\frac{\pi}{2}x)$ ,

```
[

x = \frac{2}{\pi}\sin^{-1}(y).
]
```

**Step 2: Compute the derivative factor.**

```
[

\frac{dx}{dy}
= \frac{2}{\pi} \cdot \frac{1}{\sqrt{1-y^2}}.
]
```

**Step 3: Apply the transformation formula.**

Using  $f_Y(y) = f_X(x) \left| \frac{dx}{dy} \right|$  with  $x = \frac{2}{\pi} \sin^{-1}(y)$ :

```
\begin{aligned}
f_Y(y) \\
&= \frac{1}{2} \cdot \left| \frac{2}{\pi} \cdot \frac{1}{\sqrt{1-y^2}} \right| \\
&= \frac{1}{\pi} \sqrt{1-y^2}.
\end{aligned}
```

**Step 4: Determine the support (change limits).**

Lecture maps endpoints:

```
[

x=-1 \Rightarrow y=\sin(-\frac{\pi}{2})=-1,
\qquad
x=1 \Rightarrow y=\sin(\frac{\pi}{2})=1.
]
```

\]

Hence,

\[

$$f_Y(y) =$$

\begin{cases}

$$\frac{1}{\pi \sqrt{1-y^2}}, & -1 < y < 1, \\ 0, & \text{otherwise}.\end{cases}$$

\end{cases}

\]

### \subsection{Example 2 (illustrative setup): $Z=X+Y$ }

The lecture explicitly states the following items to find/prove (no full solutions shown beyond the CDF-region setup):

\begin{enumerate}[label=(\roman\*)], leftmargin=2.2em]

\item Find the PDF of  $Z$ ,  $f_Z(z)$ .

\item Find  $f_Z(z)$  if  $X$  and  $Y$  are independent.

\item Let  $X \sim N(0,1)$  and  $Y \sim N(0,1)$ . Prove that  $Z \sim N(0,2)$ .

\item If  $X$  and  $Y$  are exponential distributed RVs with parameter  $\lambda$ , find  $f_Z(z)$ .

\end{enumerate}

\textbf{Derivation shown (CDF setup):}

\begin{aligned}

$$F_Z(z)$$

$$\&= \Pr(Z \leq z) \\$$

$$\&= \Pr(X+Y \leq z) \\$$

$$\&= \iint_{x+y \leq z} f_{X,Y}(x,y) dx dy \\$$

$$\&= \int_{-\infty}^z \int_{-\infty}^{z-x} f_{X,Y}(x,y) dy dx.$$

\end{aligned}

Further evaluation for cases (ii)–(iv) is \textbf{Not specified in the provided context} beyond listing these tasks.

### \section{\*Summary of Key Takeaways (only from context)}

\begin{itemize}[leftmargin=1.2em]

\item To find the distribution of a transformed RV  $Y=g(X)$  (with monotonic  $g$ ), the lecture uses the \textbf{CDF method} followed by differentiation.

\item For monotonic transformations, the resulting PDF uses the absolute derivative factor:

\[

$$f_Y(y) = f_X \bigl| (g^{-1}(y))' \bigr| \frac{dy}{dx} g^{-1}(y)$$

\quad \text{equivalently}

$$f_Y(y) = \frac{f_X(x)}{\left| \frac{dy}{dx} \right|} \Big|_{x=g^{-1}(y)}.$$

\]

\item The support of the new RV must be obtained by \textbf{changing limits} (mapping \$x\$-range through \$y=g(x)\$).

\item For \$Z=X+Y\$, the lecture sets up \$F\_Z(z)=\Pr(X+Y\leq z)\$ as a \textbf{double integral} over the half-plane} \$x+y\leq z\$ using \$f\_{X,Y}(x,y)\$, and then uses \$f\_Z(z)=\frac{d}{dz}F\_Z(z)\$.

\end{itemize}

\end{document}