STATIC EQUILIBRIUM AND STRESSES

Lecture 2

MM203

Mechanics of Materials

Questions

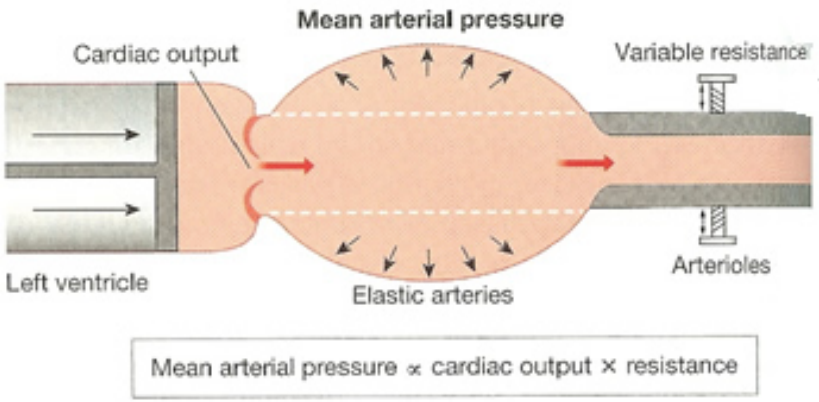
- Why buildings and bridges fall down?
- Why machinery and aero planes sometimes break?
- How worms came to be the shape they are?
- Why a bat can fly in a rose-bush without tearing its wings?

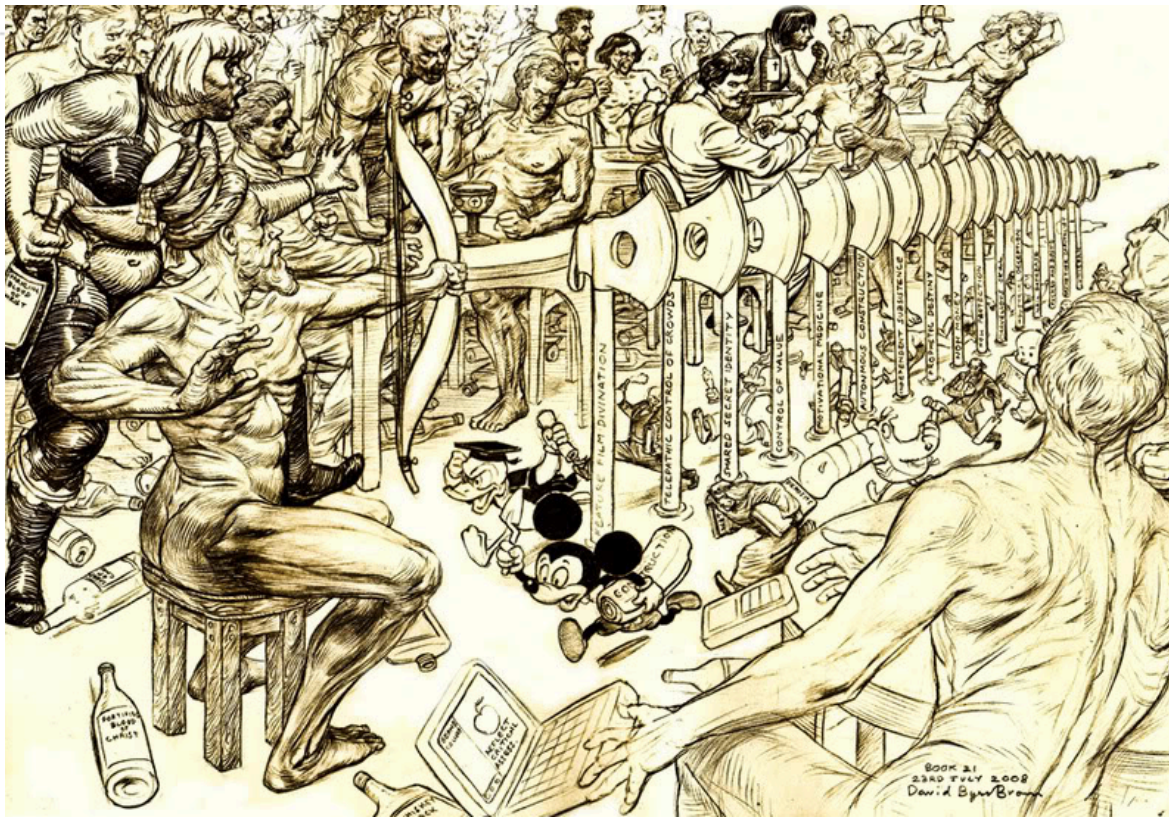

- Everything is a structure of one kind or the other.

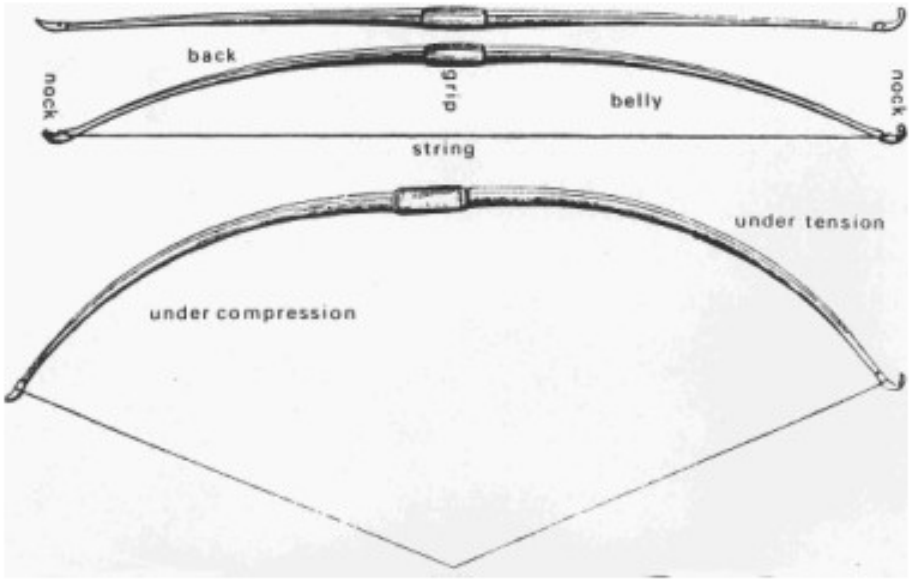
Essential lessons: How do these structures fail or not fail.

The need to be strong and to support various necessary loads has influenced the development of all sorts of creatures and devices –including man.

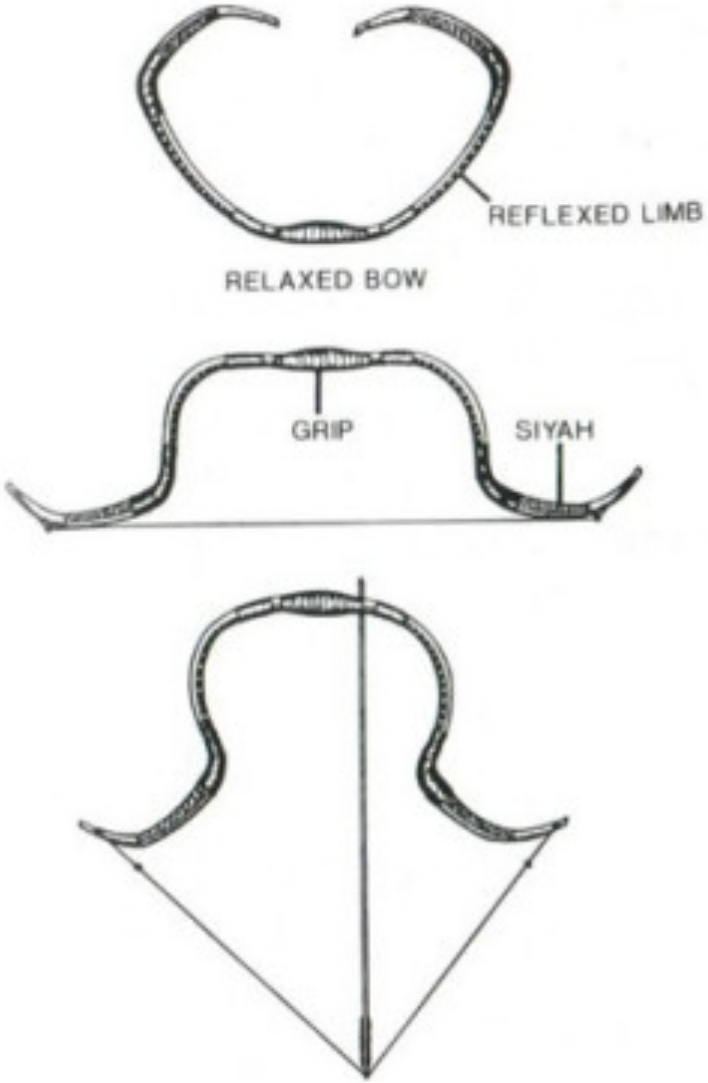
- How do our arteries work?
- Why are sailing ships rigged the way they are?
- Why did the bow of Odysseus have to be so hard to string?
- Why did the ancients take the wheels off their chariots at night?







Sketches of traditional long bow in unstrung, strung and extended conditions



Sketches of palintonos in unstrung, strung and extended conditions

3rd century BC

1436

80 ton cupola on top of a hollow structure made of granite: 1010 AD years old

Thanjavur Big temple, 80 tone dome atop the tower. www.thehindu.com

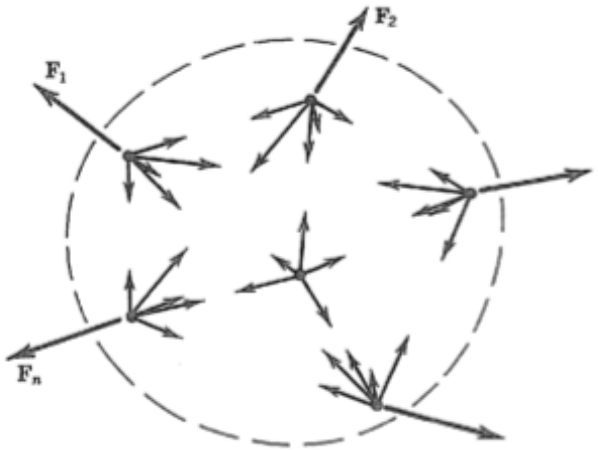
No modern equipments.

- Placing a cupola on a hollow structure.
- Lifting 80 ton cupola on top of a 216 feet tower.
- Granite without any binding material.
- Cutting and carving granite.
- Building the temple in 7 years.

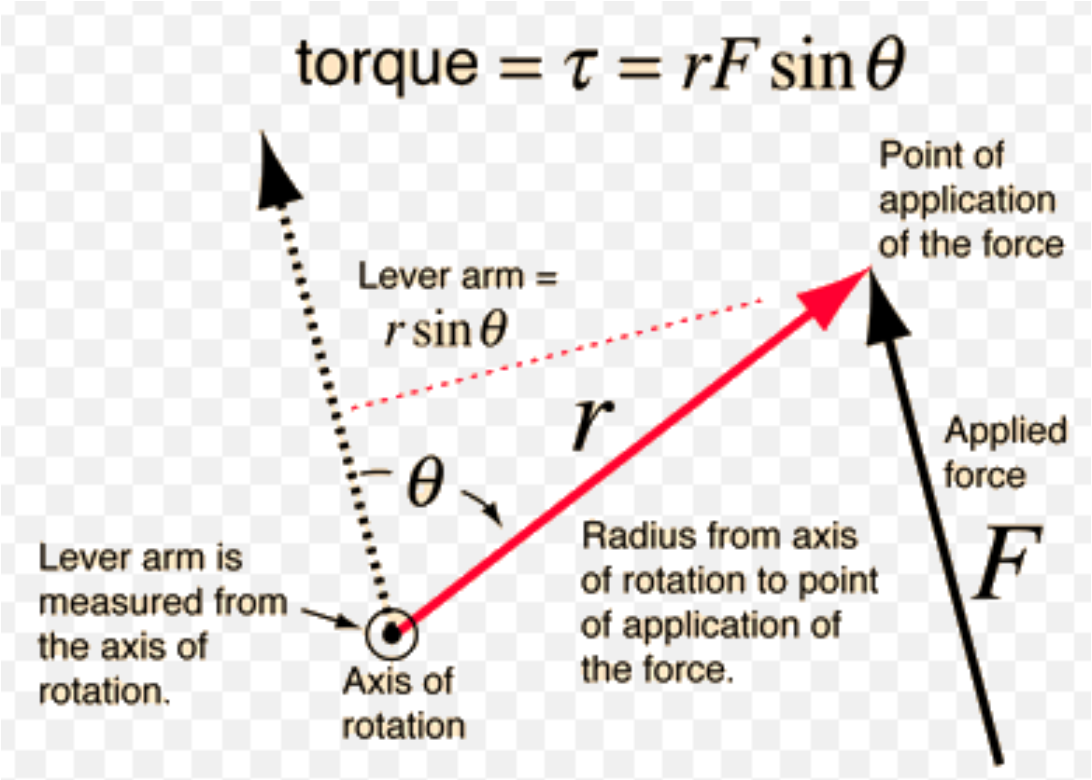
Static Equilibrium

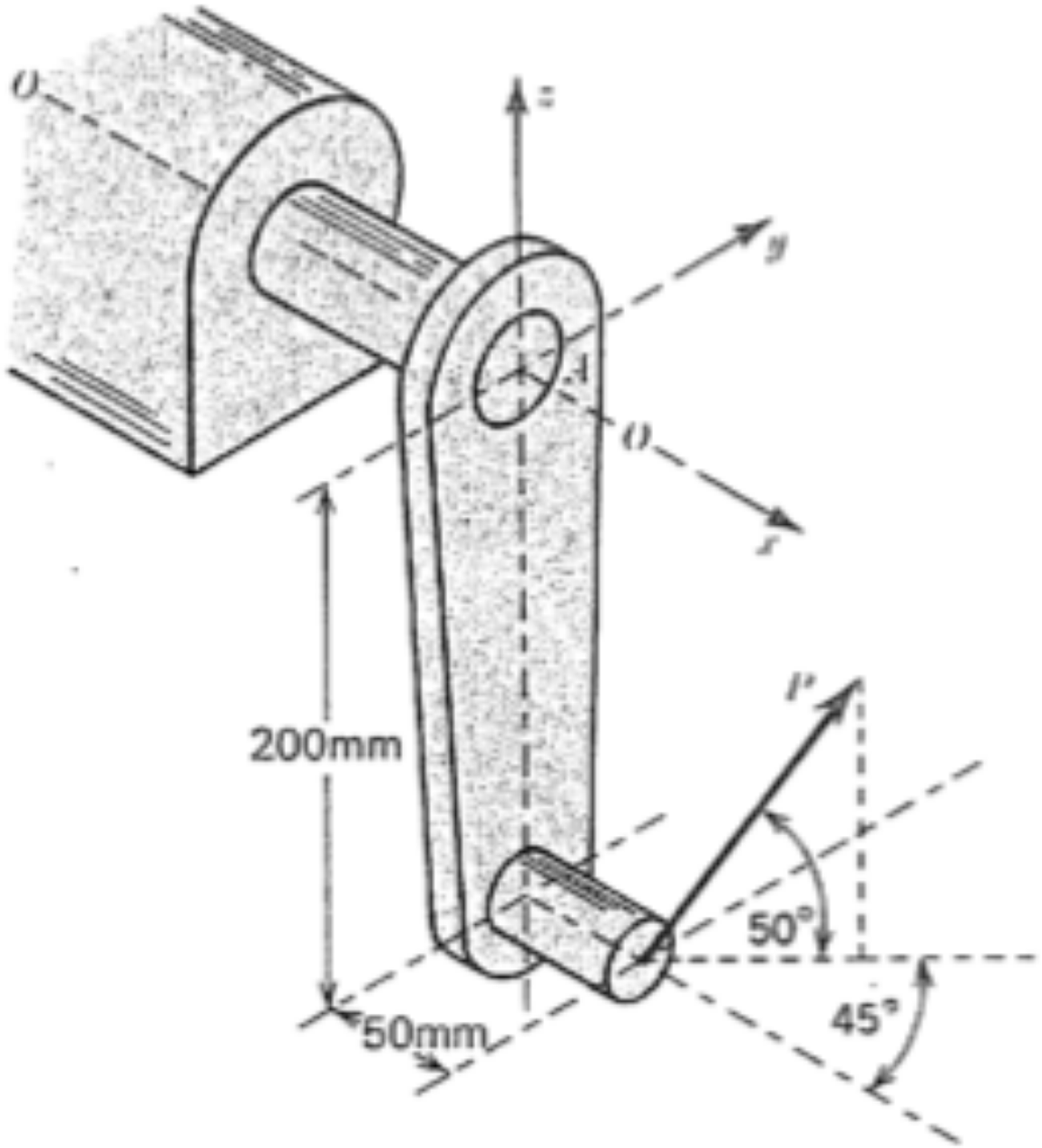
- The vector sum of external forces must be 0.
- $\Sigma F=0$

- The sum of all torques due to all external forces about any axis must be 0.
- $\Sigma M=0$

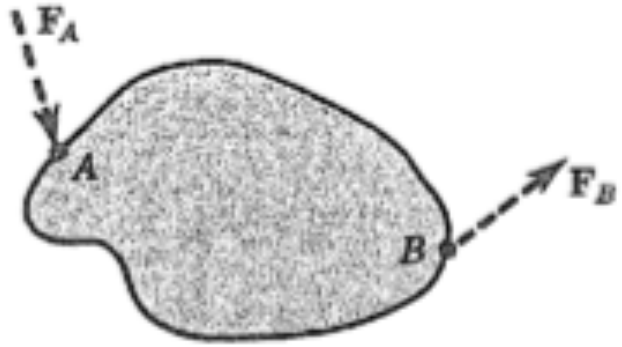


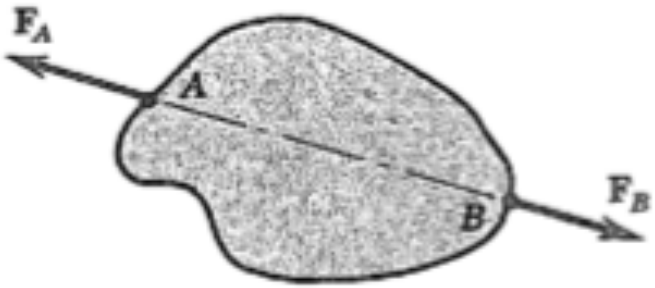
Calculating moments



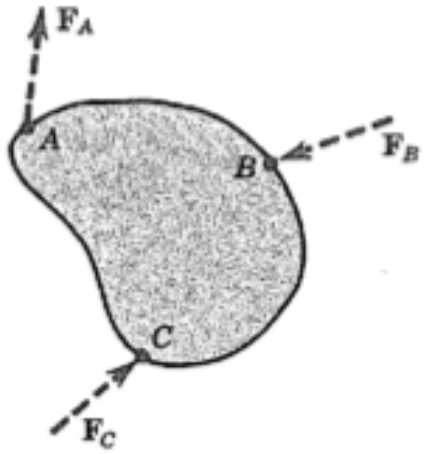

O
z
y
O
x
200mm
F
50°
45°
50mm

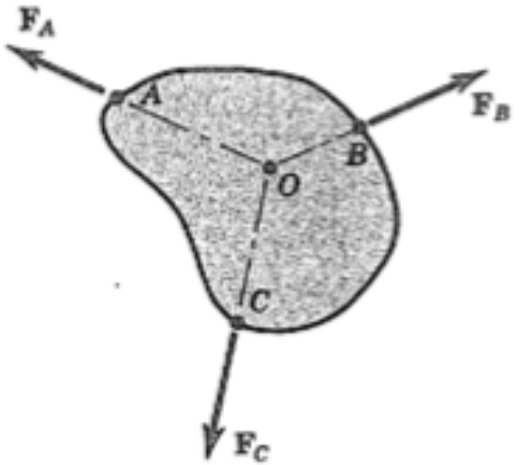
2 force member





3 force member

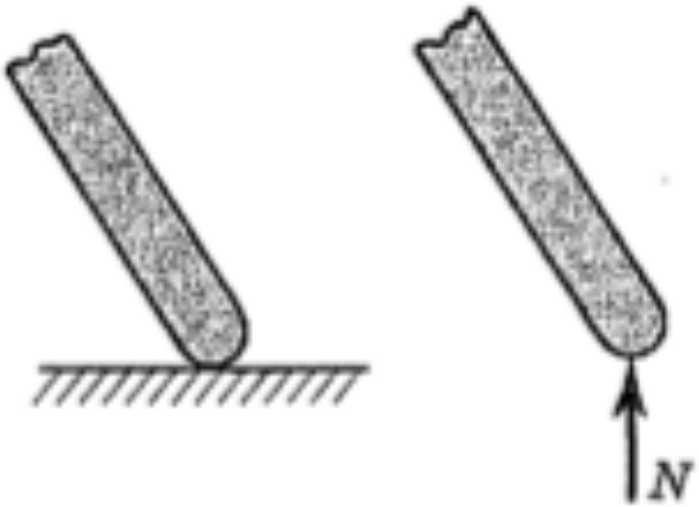




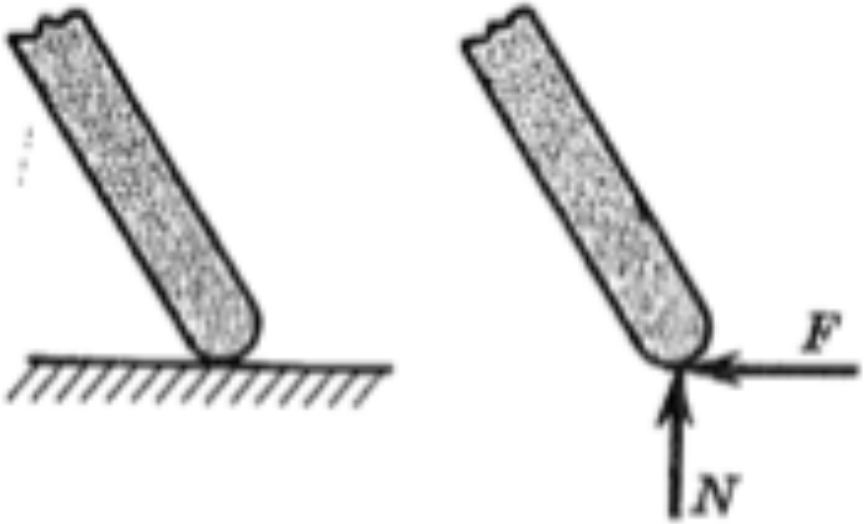
What about a 4 member system?

Free Body Diagram

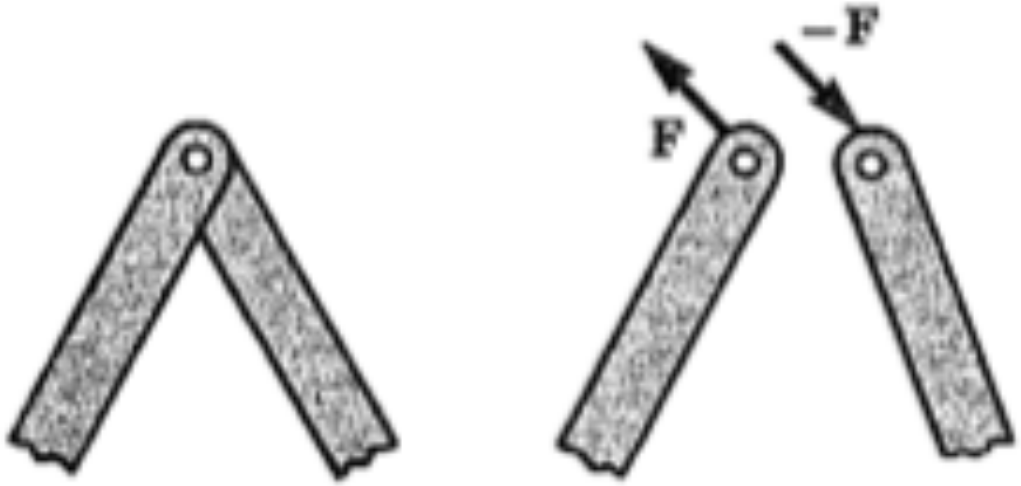
Frictionless surface



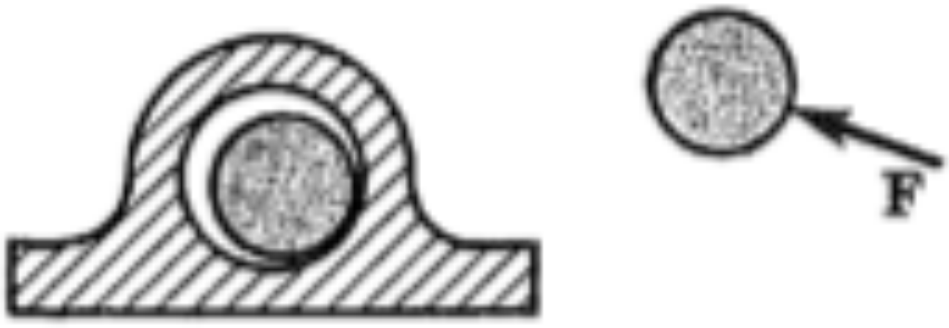
Surface with friction



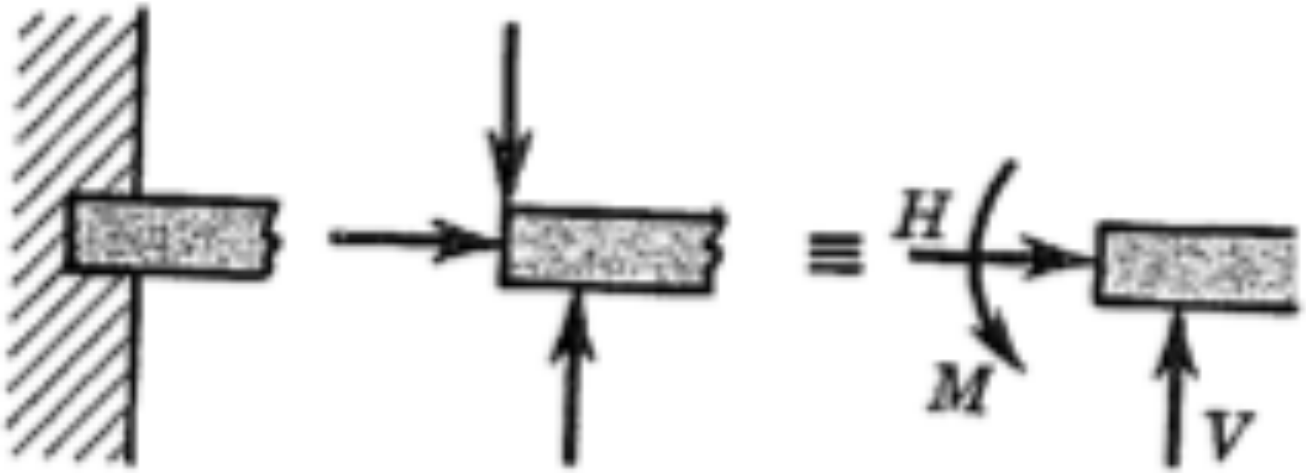
Frictionless pinned joint



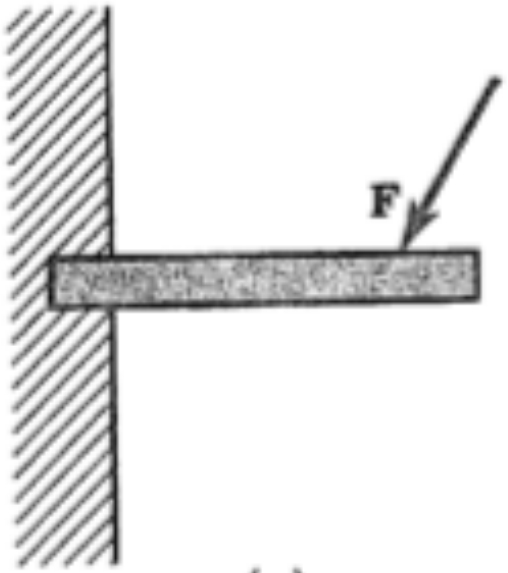
Frictionless shaft



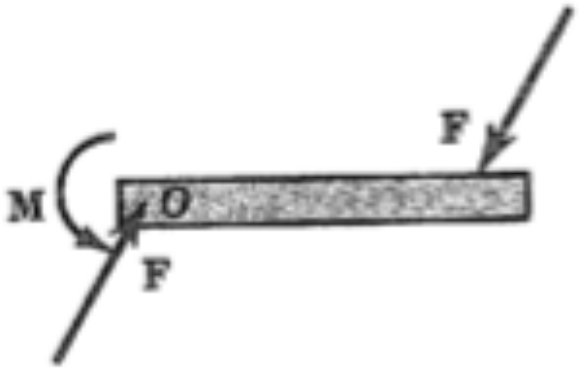
Ideal clamped support



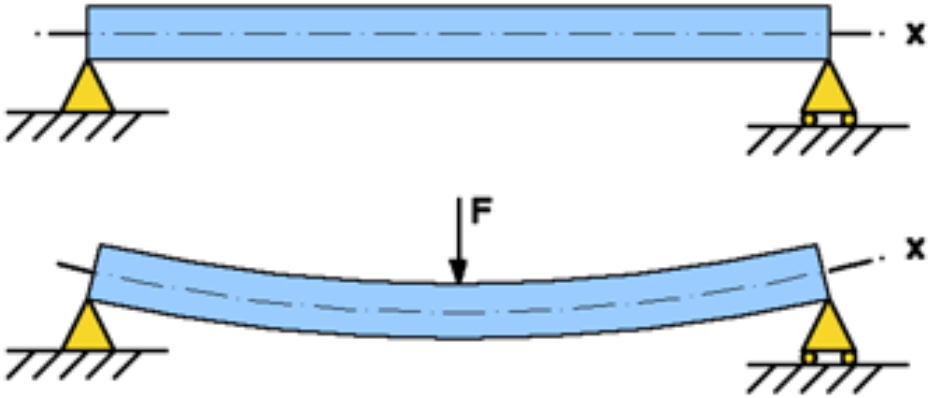
Free body diagram



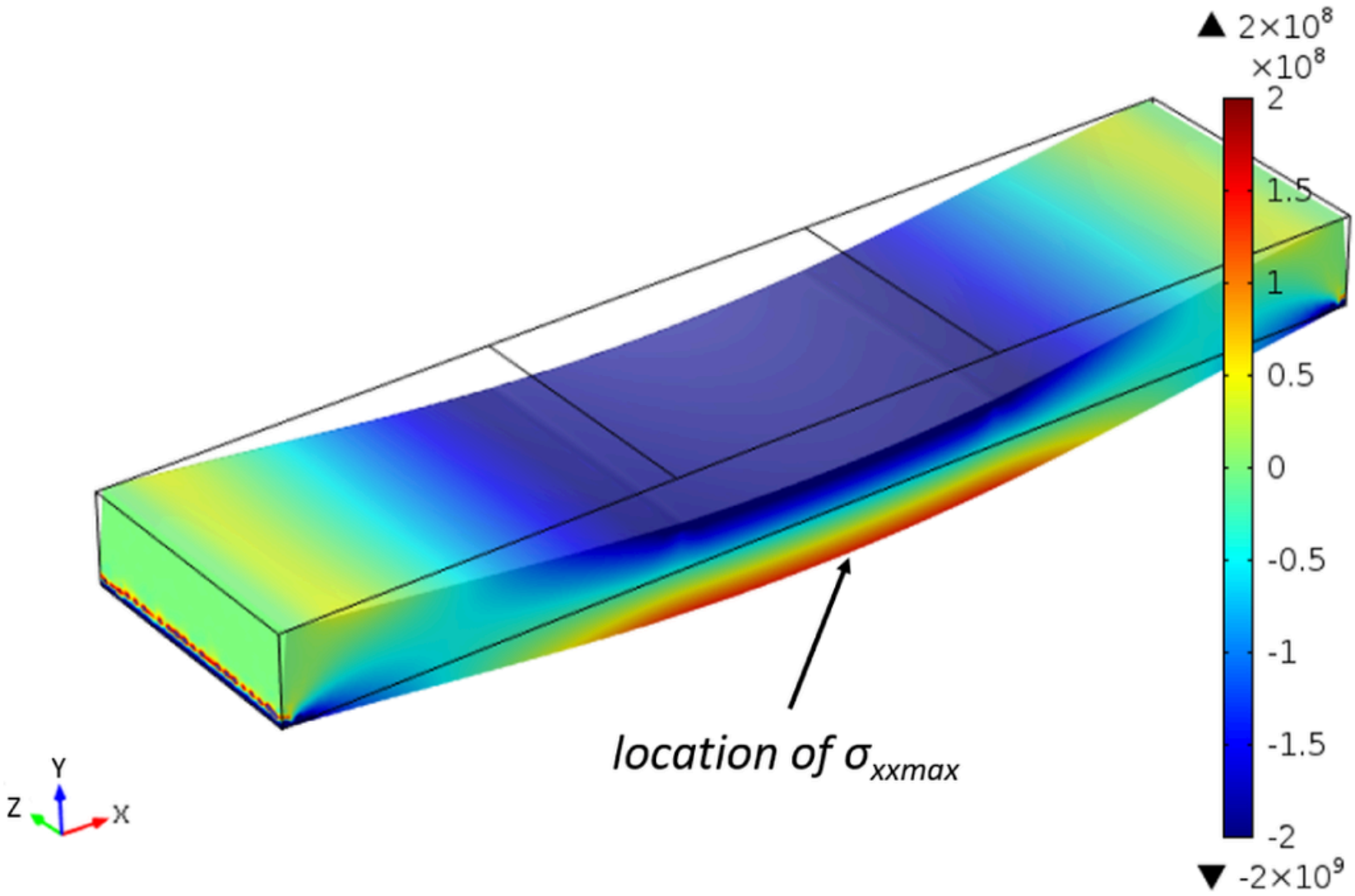
(a)



Examples

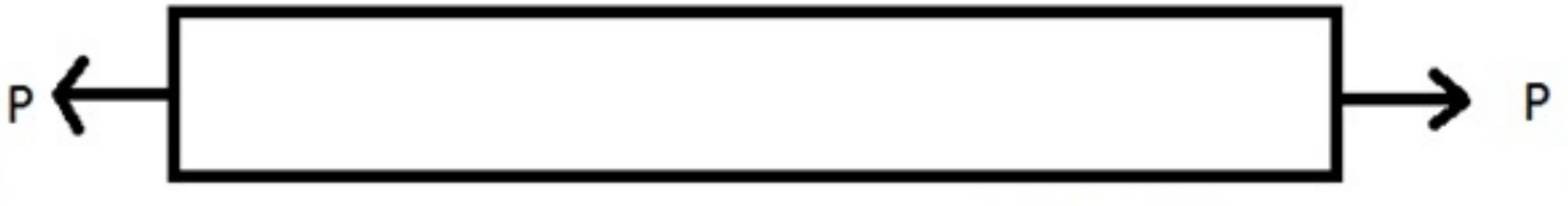


Stresses



Normal Stress

- Novelty: Will deal with stresses at different points just like pressure in a fluid.
- Deal extensively with strains and stresses.



- Observations
- We need a reference plane to define the stress.
- Need a direction in which the force is acting.

Why structures carry load?

- Why we don’t fall through the floor?
- Credit : Robert Hooke (1635-1702)

Hooke's Law: devised after multiple experiments.

- The power (read force) of any spring is in the same proportion with the tension (read extension) thereof: That is, if one power stretch or bend it one space, two will bend it two, three will bend it three, and so forward. And this is the Rule or Law of Nature, upon which all manner of Restituent or Springing motion doth proceed.

- Robert Hooke

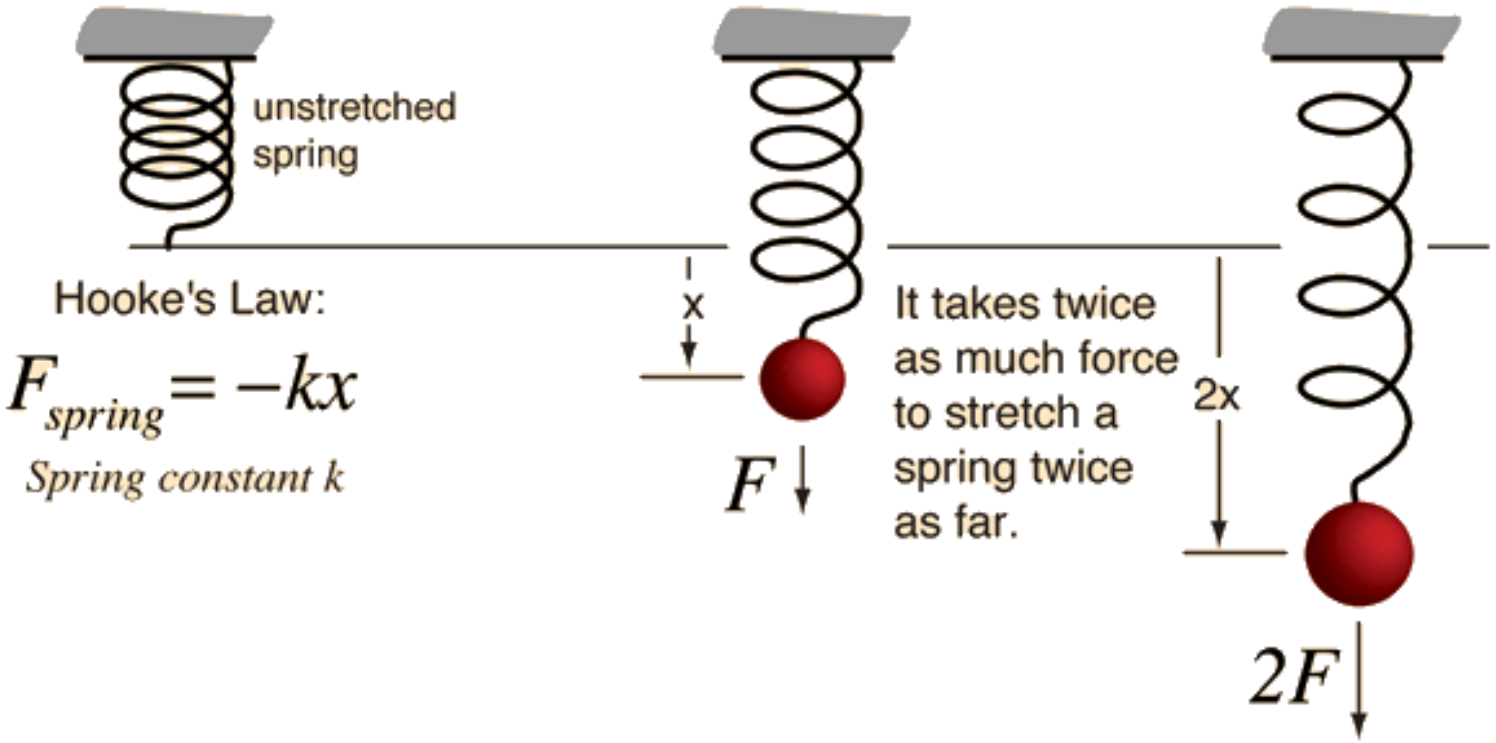
How elasticity got bogged down

- 120 years after Hooke's death: no progress.
- Newton (1642-1727)
- Mistake: Considering the structure as a whole rather than forces and extensions at a point.
- Concept of the elastic conditions at a specified point inside a material: stress and strain.
- Put forward: Augustin Cauchy (1789-1857) in **1822**.

- **Galileo** (1564-1642):
- Other things being equal, a rod which is pulled in tension has a strength which is proportional to its cross-sectional area. Thus, if a rod of two square centimeters cross-section breaks at a pull of 1000 kg, then one of four square centimeters cross-section will need a pull of 2000 kg force in order to break it, and so on.

- It took nearly 200 years to divide the breaking load by the area of the fracture surface, so as to get the breaking stress. (500kg per square cm)

Hooke's Law



$$\sigma = E\varepsilon$$

Young (1773-1829)

$$\sigma = E\varepsilon$$

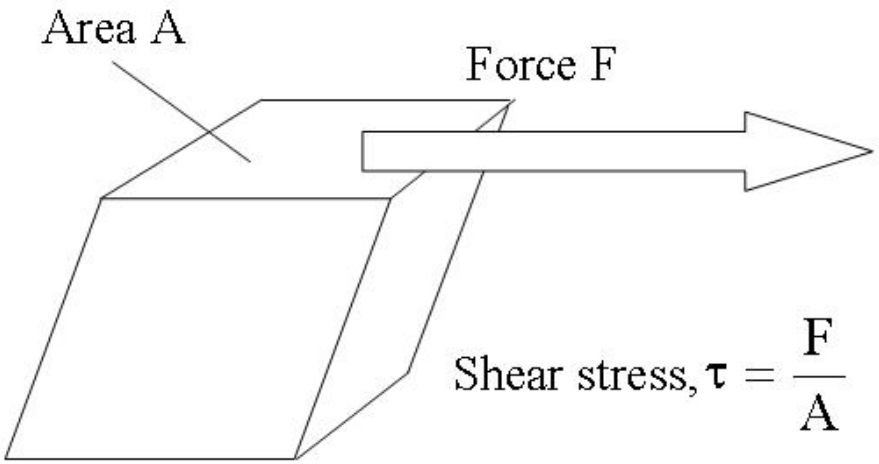
- How much a column of a material is expected to shorten under its own weight.

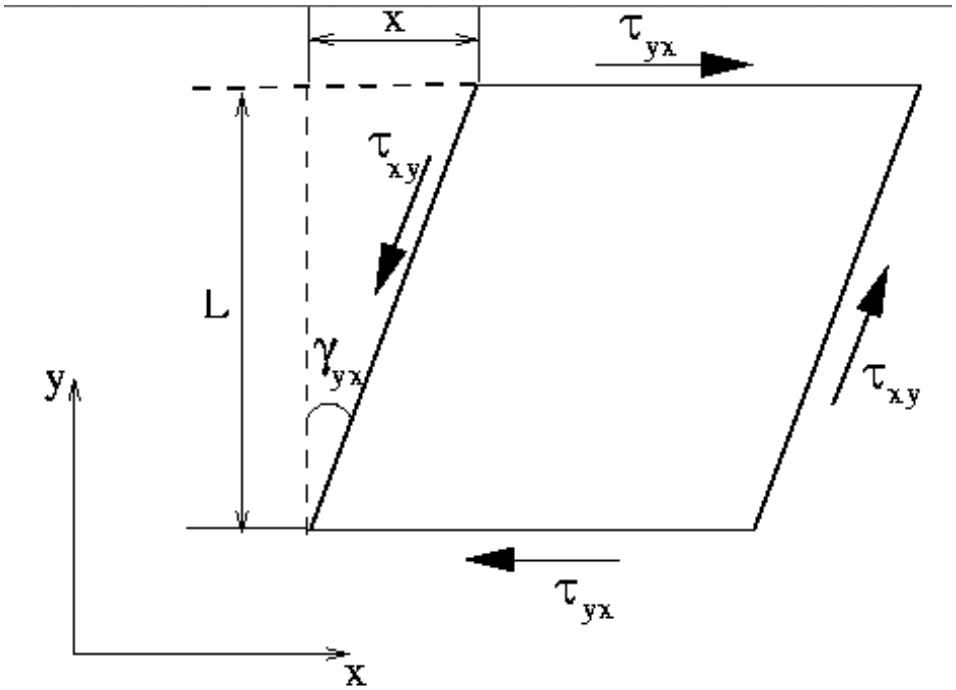
- Young's own definition of his modulus, published in **1807** is: " The modulus of elasticity of any substance is a column of the same substance, capable of producing a pressure on its base which is to the weight causing a certain degree of compression as the length of the substance is to the diminuition of its length".

- Wrestling with an idea that was difficult to express without stress and strain.

Shear Stress

- Force is acting parallel to the surface unlike normal stress.
- Arises from the force vector component parallel to the plane.
- Does not lead to a change in length; shape distortion.

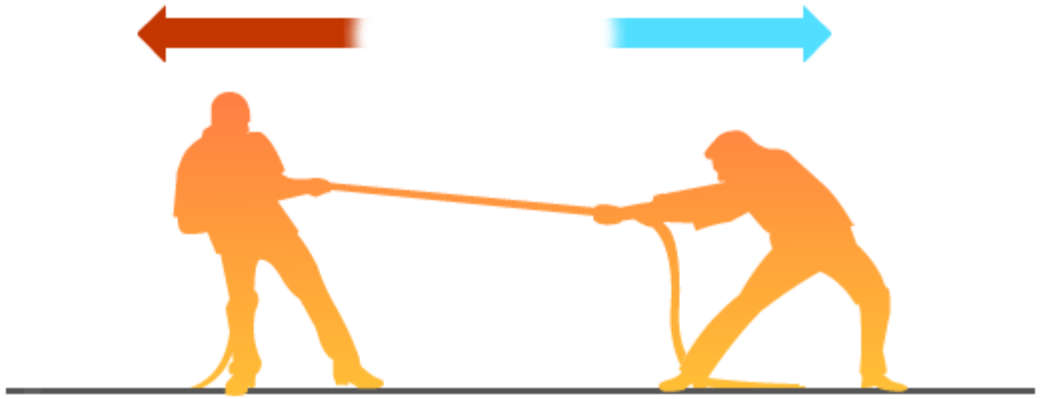


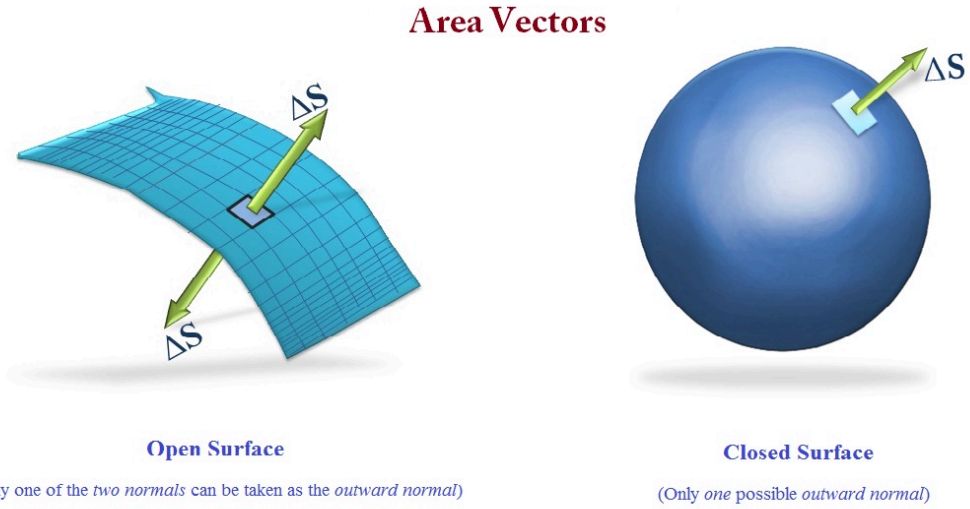


Shear Modulus or Modulus of Rigidity

- $\tau = G\gamma$

How to represent a general state of stress.





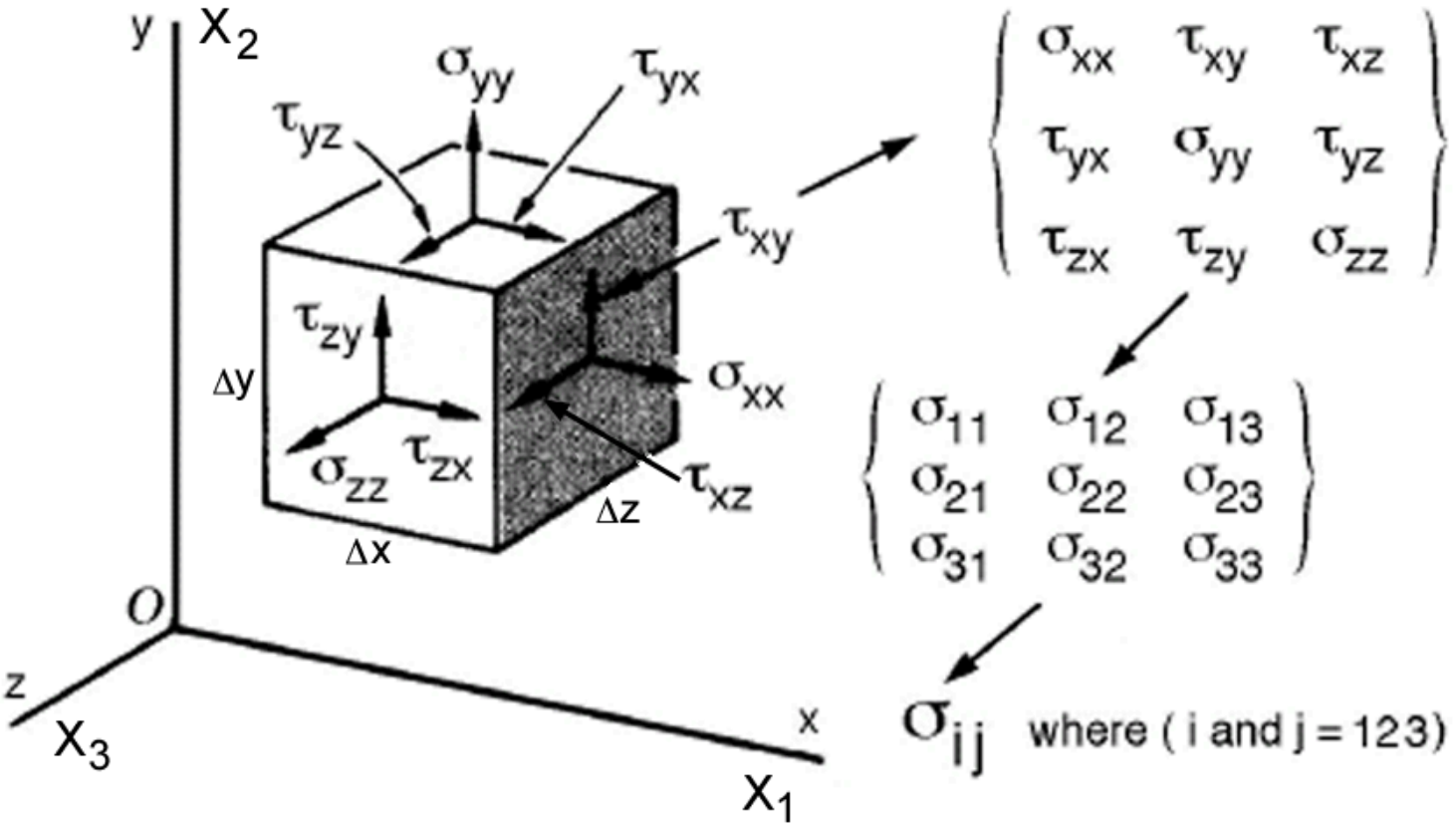
Open Surface

(Any one of the *two normals* can be taken as the *outward normal*)

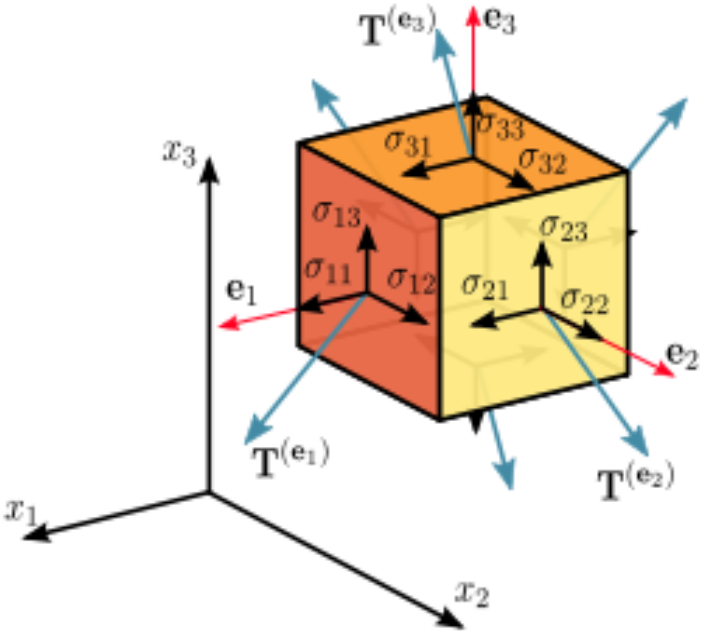
Closed Surface

(Only *one* possible *outward normal*)

Tensor Representation



How to calculate the stresses in the bridge



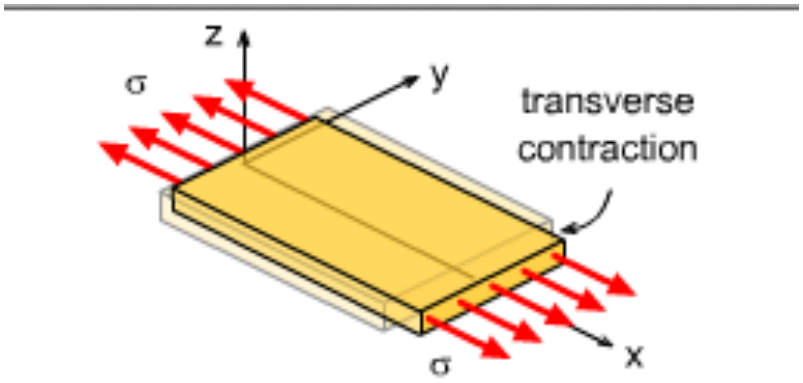
Strain Tensor

- Write the strain tensor for a general state of stress.

Poisson's Ratio

- When loaded axially, what happens in lateral direction.

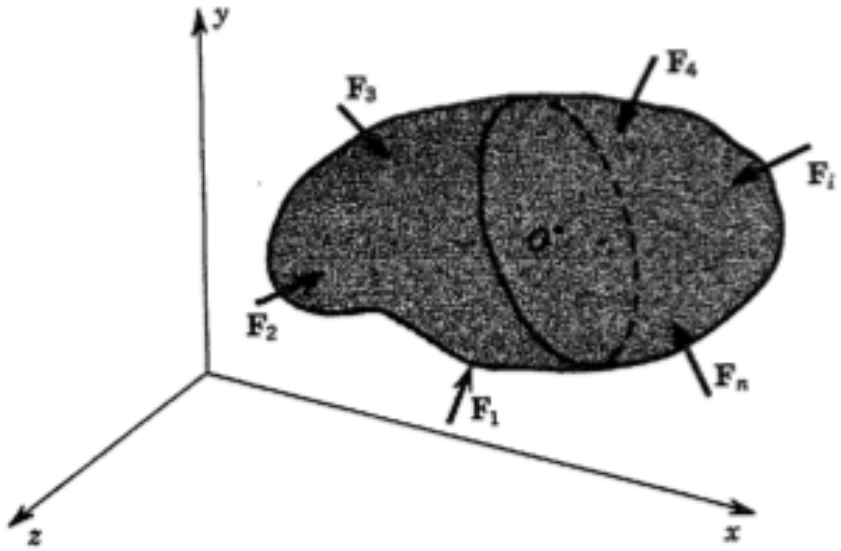
$$\nu = -\frac{d\varepsilon_{\text{trans}}}{d\varepsilon_{\text{axial}}} = -\frac{d\varepsilon_{\text{y}}}{d\varepsilon_{\text{x}}} = -\frac{d\varepsilon_{\text{z}}}{d\varepsilon_{\text{x}}}$$



Generate State of Strain

- Write the general state of strain for uniaxial loading.

Volume change


y
F3
F4
Fi
O
F2
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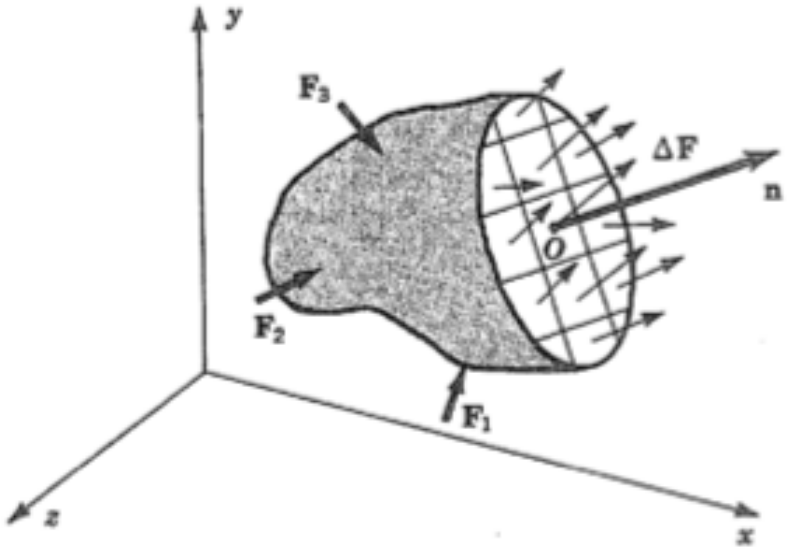
z
x


y
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