DEFORMATION AND STRAINS

MM203 Mechanics of Materials

Lecture 7

20.01.2020

Recap

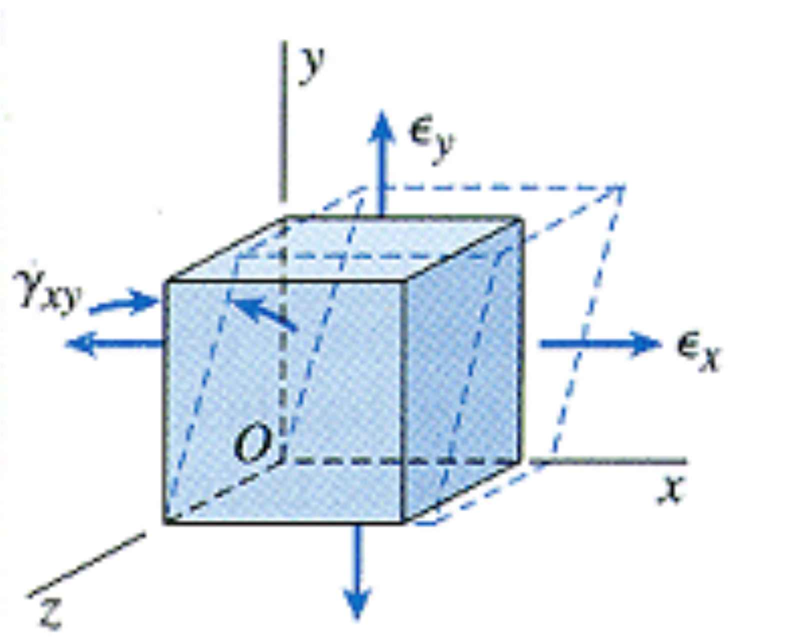
- Static equilibrium.
- True for a rigid as well a deformable body.
- Conditions for geometrical compatibility imposed by the deformation of a continuous body.
- No voids are created in the body.
- Independent of the equilibrium establishment till now.

- Displacement of a body composed of 2 parts: (1) translation of the body/or rotation as a whole(rigid body motion), (2) motion of points relative to each other.
- Motion of points of a body relative to each other: deformation.
- Displacements associated with rigid body motion can be small or large.
- Displacements associated with deformation usually are small.
- Since the body is in static equilibrium, not worry about rigid body motion.

Plane strain

- Deformation is confined to the plane.

Plane strain



$\tau_{xz} = 0 \qquad \tau_{yz} = 0$

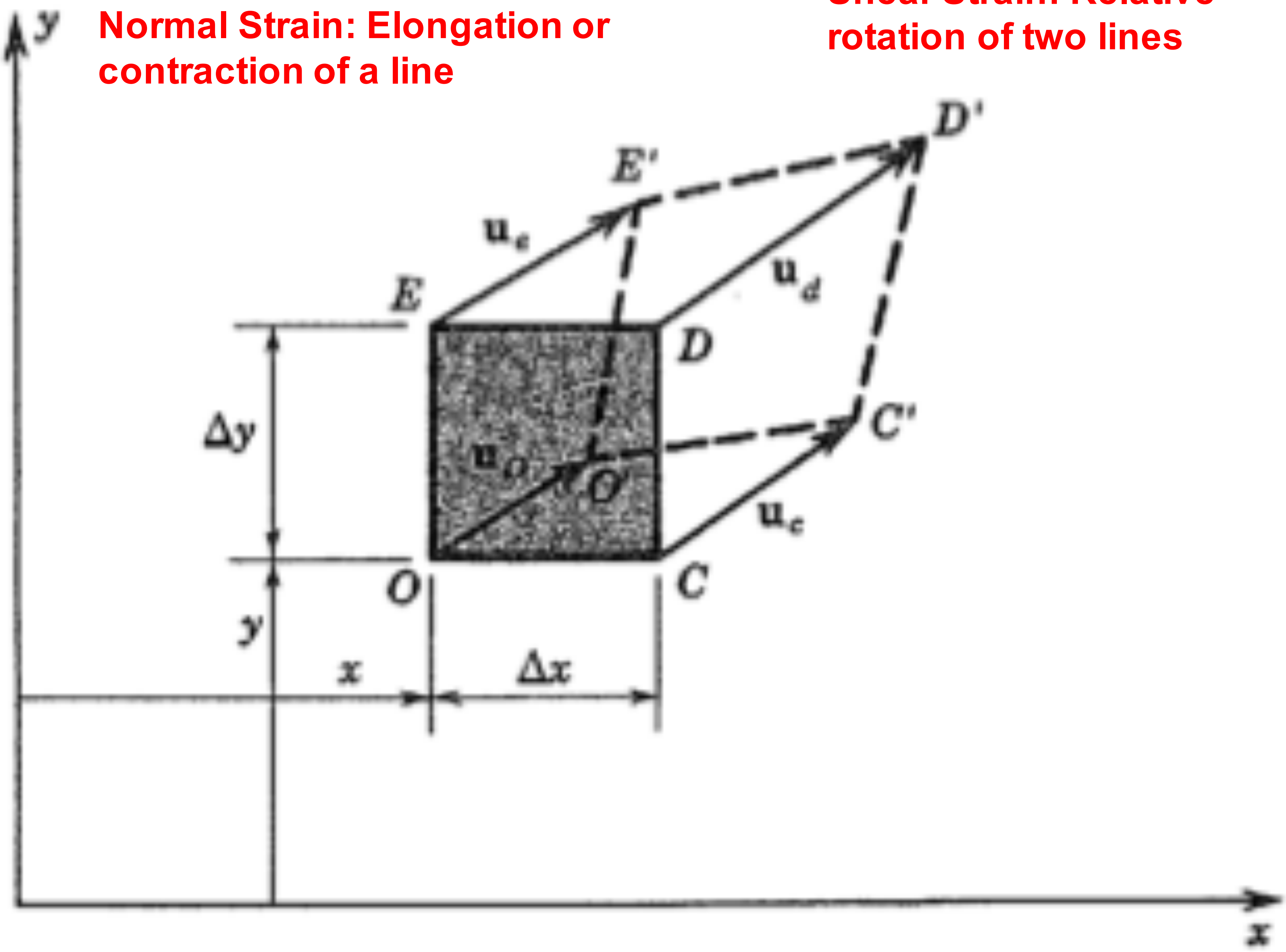
σ_x, σ_y, σ_z, and τ_{xy} may have nonzero values

$\epsilon_z = 0 \qquad \gamma_{xz} = 0 \qquad \gamma_{yz} = 0$

ϵ_x, ϵ_y, and γ_{xy} may have nonzero values

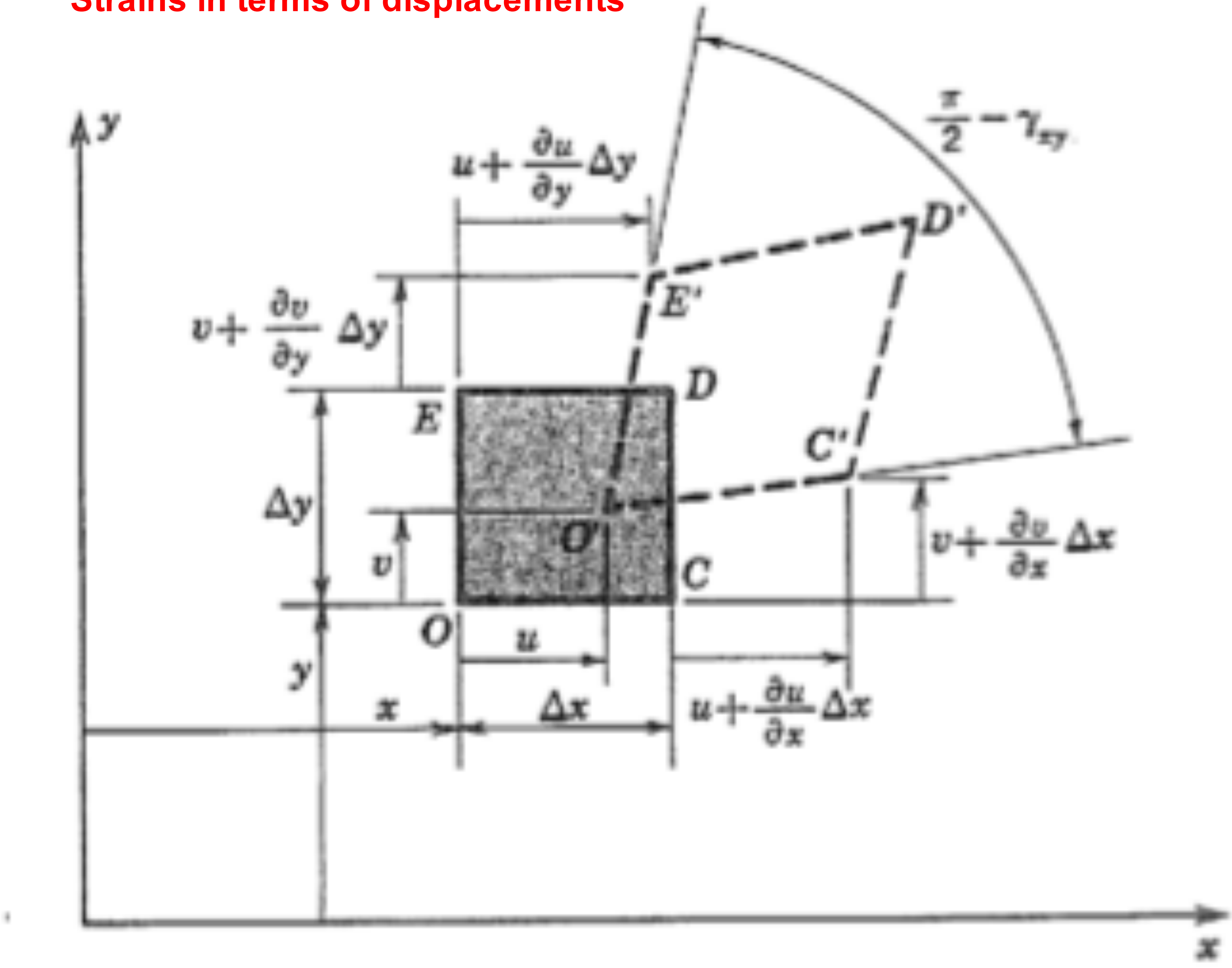
Normal Strain: Elongation or contraction of a line

Shear Strain: Relative rotation of two lines



- Deformation of the element in the limit that the element shrinks to zero size, and thus the strain components define the deformation at point O.
- Normal strain is positive when the line elongates.
- Shear strain is specified with respect to 2 axes with 2 subscripts to indicate these 2 axes.
- Shear strain is defined as the tangent of the change in angle between these two originally perpendicular axes.
- When the axes rotate, such that the first and third quadrants become smaller, shear strain is positive.
- When first and third quadrants become larger, shear strain is negative.
- For small shear strains (those of engineering interest are mostly less than 0.01), shear strain can be defined in terms of the angle in radians instead of tangent.

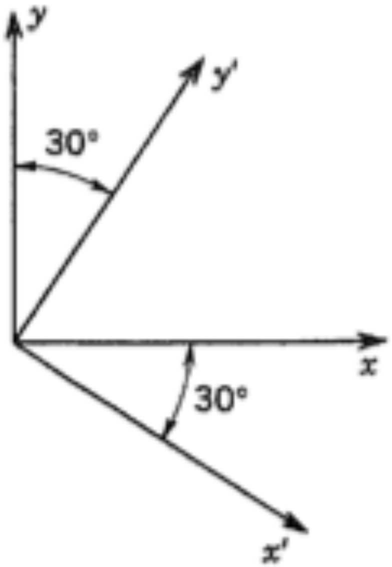
Strains in terms of displacements



Example 4.3 A sheet of metal is deformed uniformly in its own plane so that the strain components related to a set of axes xy are[1]

$$\epsilon_x = -200 \times 10^{-6}$$
$$\epsilon_y = 1000 \times 10^{-6}$$
$$\gamma_{xy} = 900 \times 10^{-6}$$

We wish to find the strain components associated with a set of axes $x'y'$ inclined at an angle of 30° clockwise to the xy set, as shown in Fig. 4.32. Also, we wish to find the principal strains and the direction of the axes on which they exist.



Similar to Mohr Circle for stress, except that x axis is for normal strains and half of shear strain is plotted on the y axis.

Steps to construct the Mohr Circle

- For the particular stress situation note down the stress acting on the elements with their correct signs.
- Draw x (σ) and y(τ) axis. Draw the center of the Mohr Circle. [$(\sigma_{xx}+\sigma_{yy})/2$, 0]
- Another point on the circle is $(\sigma_{xx}, -\tau_{xy})$ call it X.
- Second point on the circle (σ_{yy}, τ_{xy}) call it Y.
- Positive shear stress is plotted downwards at X and upwards at Y. Negative shear stress is plotted upwards at X and downwards at Y.
- Draw the circle.
- For a rotation of θ in the physical scenario, rotation of 2θ is required in Mohr Circle. Maximum rotation is 180 degrees in the circle at which stress states coincide with original ones.
- Stress components with respect to rotated X'Y' axes can be determined from the corresponding X'Y' diameter.
- To determine the stresses after rotation by Φ, make a diameter at an angle 2Φ in the same sense (clockwise vs. anticlockwise).
- Find the x and y coordinates of these 2 points, they will give ($\sigma_{x'x'}$, $\sigma_{y'y'}$ and $\tau_{x'y'}$)
- $2\theta_p$ is the angle made with the orientation for the principal stress state in the circle. θ is the physical angle.
- $\sin 2\theta_p=\tau_{xy}$/radius of circle.