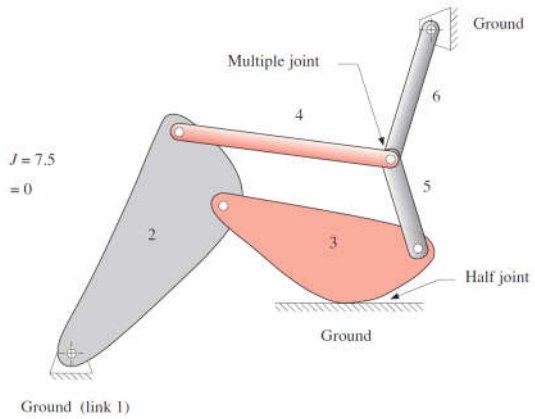
ME220 Theory of Machines and Machine Design

Lec 3 – 16 Jan 2020

Calculation of Degrees-of-Freedom



Please attempt to calculate of number of DOFs of the mechanism below by inspection or any method you can think of!!

Determining Degrees-of-Freedom

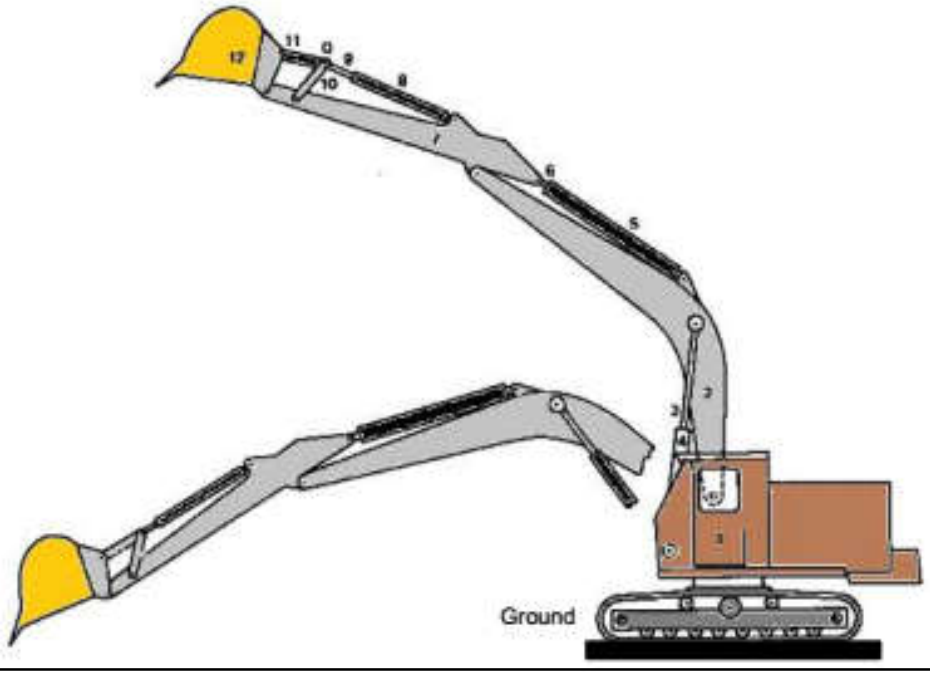
- Gruebler's Equation for Planar Mechanisms: M (or DOF) = 3L – 2J – 3G

  M/DOF : Degrees of freedom
  L : Number of Links
  J : Number of Joints (counting half joints as 1/2 and full joints as 1)
  G : Number of Grounded Links

- Kutzbach's Modification (called Kutzbach's Equation): M (or DOF) = $3(L-1) - 2J_1 - J_2$

  M/DOF : Degrees of freedom
  L : Number of Links
  J_1 : Number of 1 DOF (full joints)
  J_2 : Number of 2DOF (half joints)

Example: Calculate Degrees-of-Freedom for this Earth Digger's Arm



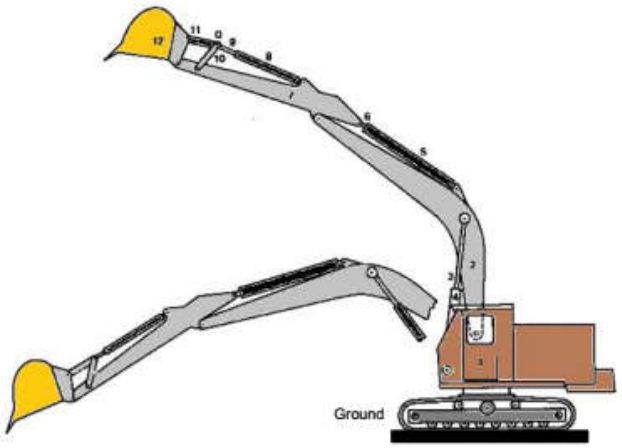
Example: Calculate Degrees-of-Freedom for this Earth Digger's Arm

M (or DOF) = $3(L-1) - 2J_1 - J_2$

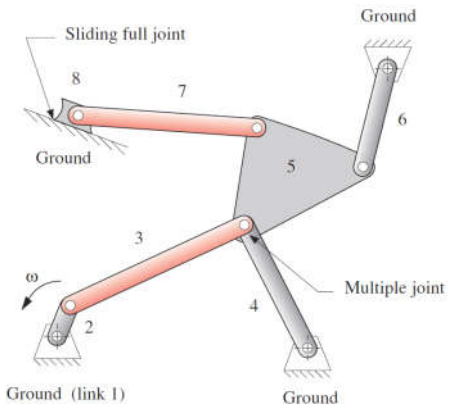
L = 12
J_1 = 12 (pin joints) + 3 (slide joints)
J_2 = 0

M (or DOF) = 3(12-1) – 2(12+3) – 0
= 33 – 30 = 3

The three prismatic joints are used as input joints by means of hydraulic joints controlled by the operator.

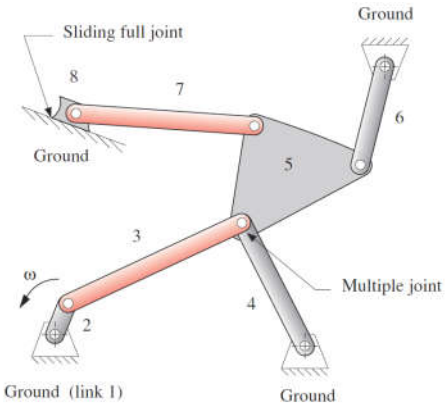


Example – DOF: Multiple Joints



Multiple joints count as one less than number of links joined at the joint.

Example – DOF: Multiple Joints

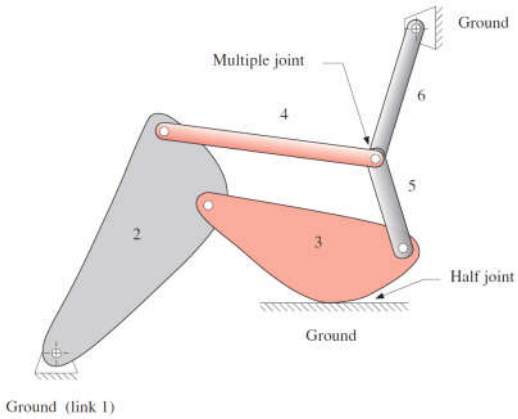


L = 8
J_1 = 10
J_2 = 0

M = 1

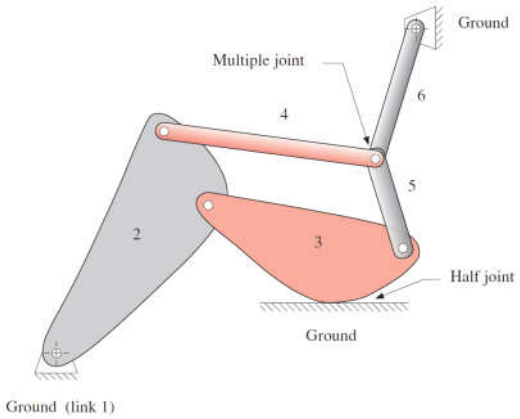
Multiple joints count as one less than number of links joined at the joint.

Example – DOF: Higher Pair



Higher pair allows motion in 2 DOF
Link 3 can roll and slide

Example – DOF: Higher Pair

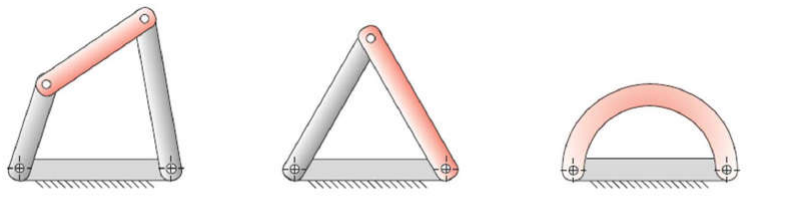


L = 6
J_1 = 7 (or 7.5 counting the higher pair as half and taking J_2 =0)
J_2 = 1

M = 0

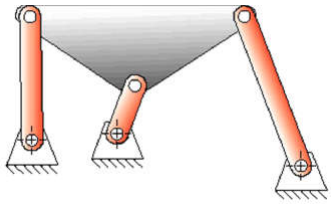
Higher pair allows motion in 2 DOF
Link 3 can roll and slide

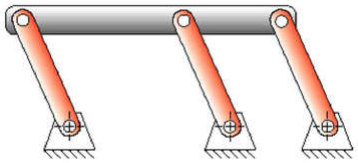
Mechanisms and Structures



- If DOF > 0, the assembly of links is a mechanism and will exhibit relative motion
- If DOF = 0, the assembly of links is a structure and will exhibit no motion
- If DOF < 0, then the assembly is a preloaded structure, no motion is possible and stresses are present

Limitations of Gruebler's/Kutzbach's Equation



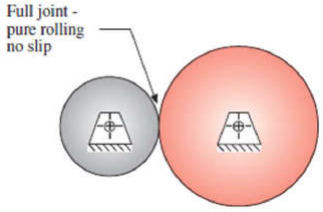


- Find the degrees of freedom of these mechanisms
  - Using Gruebler's/Kutzbach's Equation, and also
  - By Inspection
  - From the perspective of Gruebler's/Kutzbach's Equation both mechanisms above ar similar and have 0-DOF. However, the mechanism on the left is indeed had 0-DOF but the parallelogram mechanism on the right has 1-DOF.

Gruebler criterion does not include geometry!
We need to be careful and use inspection to verify the prediction

Another Anomaly: Rolling cylinders

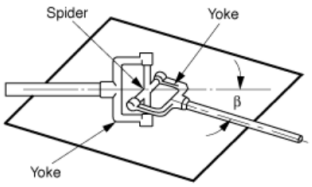
- Number of links: 3
- If no slip is allowed:
  - Number of joints that allow single DOF: 3
  - Gruebler's equation
    DOF = 3(3-1) – 2*3
    = 0
  - But we know that the mechanism has 1 DOF

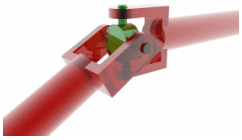


Special Geometric Condition: Length of ground link is exactly the sum of other two links

DOF in Spatial Mechanisms

- M/DOF = $6(L-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5$
  - L: Number of Links
  - J_n: Number of Joints with n DOFs



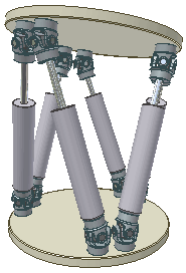


Universal Joint (2-DOF)

- Example – Stewart Platform

6 – Cylindrical (2-DOF) Joints (hydraulic/pneumatic cylinders)
12 – Universal Joints (2-DOF)
$J_2 = 12+6 = 18$
L (Rigid Links) = 12+2 = 14

M = 6(14-1) – 4(18) = 78 – 72 = 6 DOFs
This implies all six DOFs of the top platform (output) can be controlled using six inputs chosen to be linear displacements of cylindrical joints (hydraulic actuators)



Stewart Platform