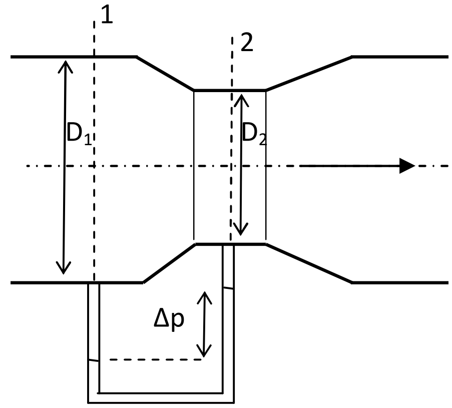
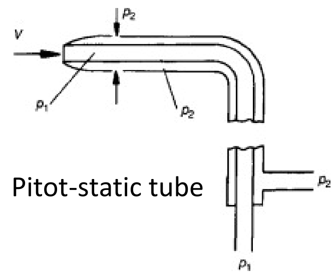
1. (a) In the Venturi meter shown, air flows at a steady rate. The flow may be assumed to be incompressible. Derive an expression for the mass flow rate as a function of pressure difference Δp (Pascals), D_1, $\beta = D_2/D_1$, and density of air ρ_{air}, kg/m^3. Discharge coefficient is unity. However there is a frictional loss $\widehat{W}_{fr}$, (J/kg) between section 1 and 2. **(3 marks)**



(b) If a **pitot-static** tube placed in the center of the narrow section 2 (throat), shows a pressure difference of 200 Pa, what is the flow rate? **(2 marks)**

density ρ_{air} = 1.18 kg/m^3, D_1 = 0.2m, $\beta = D_2/D_1$ =0.5; and $\widehat{W}_{fr}$= 50 J/kg. Pitot static is ideal: no correction required.

[Assume flow to be uniform across any cross-section].



Pitot-static tube

BONUS question IIA: Is the assumption of uniform flow reasonable? Show with numbers; μ_{air} = 20 x 10^{-6} Pa.s **(1mark)**

Overall energy balance : $\frac{V_2^2 - V_1^2}{2} + g\,(z_2 - z_1) + \frac{p_2 - p_1}{\rho} + \widehat{W}_s + \widehat{W}_{fr} = 0$**; for constant ρ**

Newtons Law: $\tau_{xz} = -\mu \frac{\partial V_z}{\partial x}$

2(a) Mass balance between sections 1 & 2

$$A_1 \rho_1 \bar{V}_1 = A_2 \rho_2 \bar{V}_2 \Rightarrow A_1 \bar{V}_1 = A_2 \bar{V}_2 \quad (\because \rho \text{ constant})$$

$$D_1^2 \bar{V}_1 = D_2^2 \bar{V}_2$$

$$\frac{D_2}{D_1} = \beta,; D_1 \left(\frac{D_2}{\beta}\right)$$

$$\bar{V}_2 = \bar{V}_1 \left(\frac{D_1}{D_2}\right)^2 = \frac{\bar{V}_1}{\beta^2}$$

Energy balance:

$$\frac{\bar{V}_2^2 - \bar{V}_1^2}{2} + \cancel{g(z_2 - z_1)}^{=0} + \frac{p_2 - p_1}{\rho} + \cancel{\hat{w}_s}^{=0} + \hat{w}_{fr} = 0$$

$$\frac{\Delta p}{\rho} = \frac{p_1 - p_2}{\rho} = \frac{\bar{V}_2^2 - \bar{V}_1^2}{2} + \hat{w}_{fr} = \frac{\bar{V}_1^2}{2}\left[\frac{1}{\beta^4} - 1\right] + \hat{w}_{fr}$$

$$\bar{V}_1 = \sqrt{\frac{2(\Delta p/\rho - w_{fr})}{\left(\frac{1}{\beta^4} - 1\right)}}$$

Volumetric flow rate =

$$\dot{Q} = \frac{\pi D_1^2}{4} \cdot \bar{V}_1 = \frac{\pi D_1^2}{4} \sqrt{\frac{2\left(\frac{\Delta p}{\rho} - \hat{w}_{fr}\right)}{\left(\frac{1}{\beta^4} - 1\right)}} = \frac{\pi D_2^2}{4} \sqrt{\frac{2\left(\frac{\Delta p}{\rho} - \hat{w}_{fr}\right)}{(1 - \beta^4)}}$$

Mass flow rate:

$$\dot{m} = \dot{Q}\rho = \frac{\pi D_1^2}{4} \rho \sqrt{\frac{2\left(\frac{\Delta p}{\rho} - \hat{w}_{fr}\right)}{\left(\frac{1}{\beta^4} - 1\right)}} = \frac{\pi D_2^2}{4} \sqrt{\frac{2\left(\frac{\Delta p}{\rho} - \hat{w}_{fr}\right)}{(1 - \beta^4)}}$$

(b) Pitot-static tube directly gives the velocity: $\frac{\bar{V}_2^2}{2} = \frac{p_{total} - p_{static}}{\rho}$

$$\rho \frac{\bar{V}_2^2}{2} = 200 \text{ Pa} \Rightarrow \bar{V}_2 = \sqrt{\frac{400}{1.18}} = 18.41 \text{ m/s}$$

$$\bar{V}_1 = 18.41 \times \beta^2 = 4.60 \text{ m/s}$$

$$Q = \frac{\pi D_2^2}{4} \cdot 18.41 = \frac{\pi \times 0.2^2}{4} \times 18.41 = 0.578 \text{ m}^3/\text{s}$$

$$\underline{\dot{m} = 0.682 \text{ kg/s}}$$

Comment: If instead Δp (shown in Fig) was given, one had use the formula in (a)

<u>BONUS</u>: Is the flow laminar or turbulent?

$$Re_{D_2} = \frac{1.18 \times 18.41 \times 0.1}{20 \times 10^{-6}} = 108619: \text{ turbulent.}$$

$$Re_{D_1} = 54280.$$

Both sections, flow is turbulent. Hence uniform assumption valid.