STRESS TRANSFORMATION AND MOHR CIRCLE

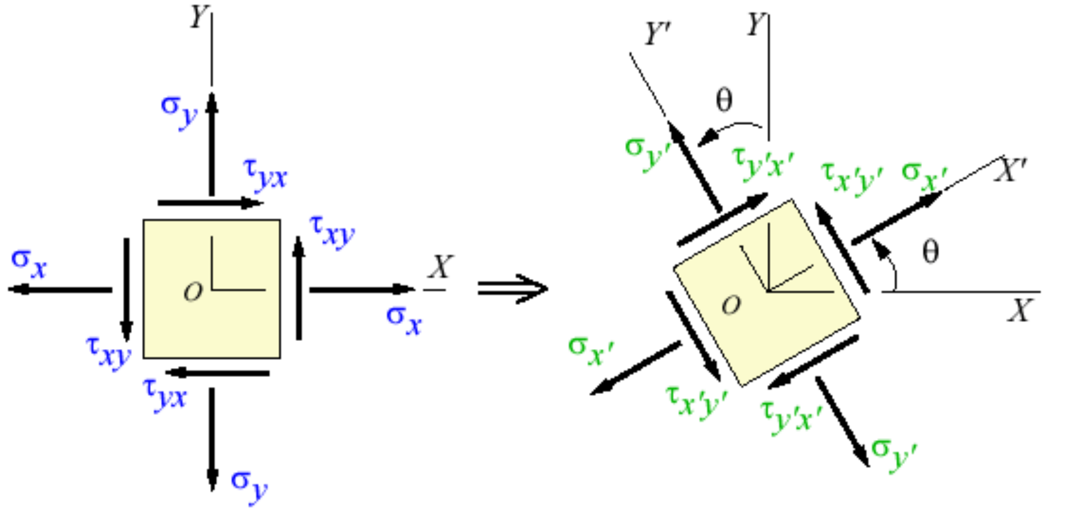
Lecture 6

28.01.2020

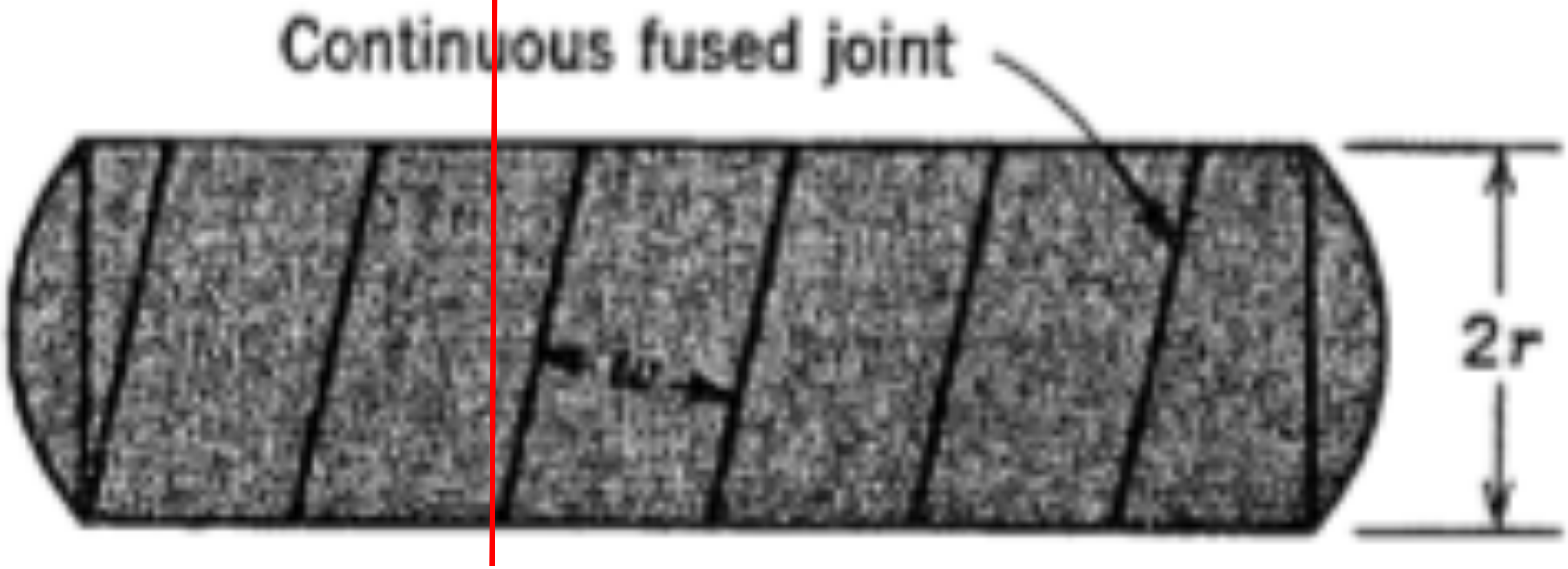
MM203

Mechanics of Materials

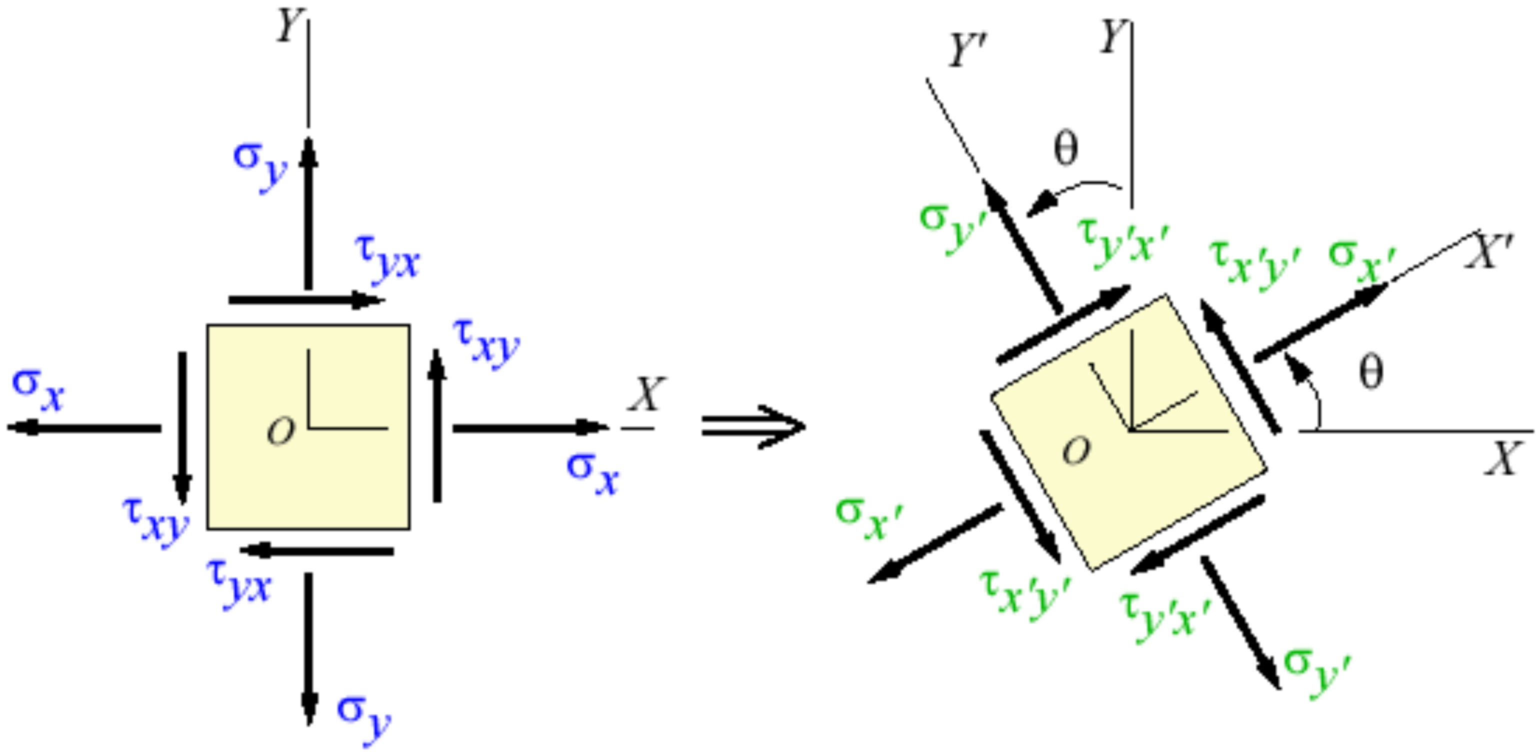
Maximum shear stress that can be tolerated: τ_{crit}



Stresses at given coordinate system   Stresses transformed to another coordinate

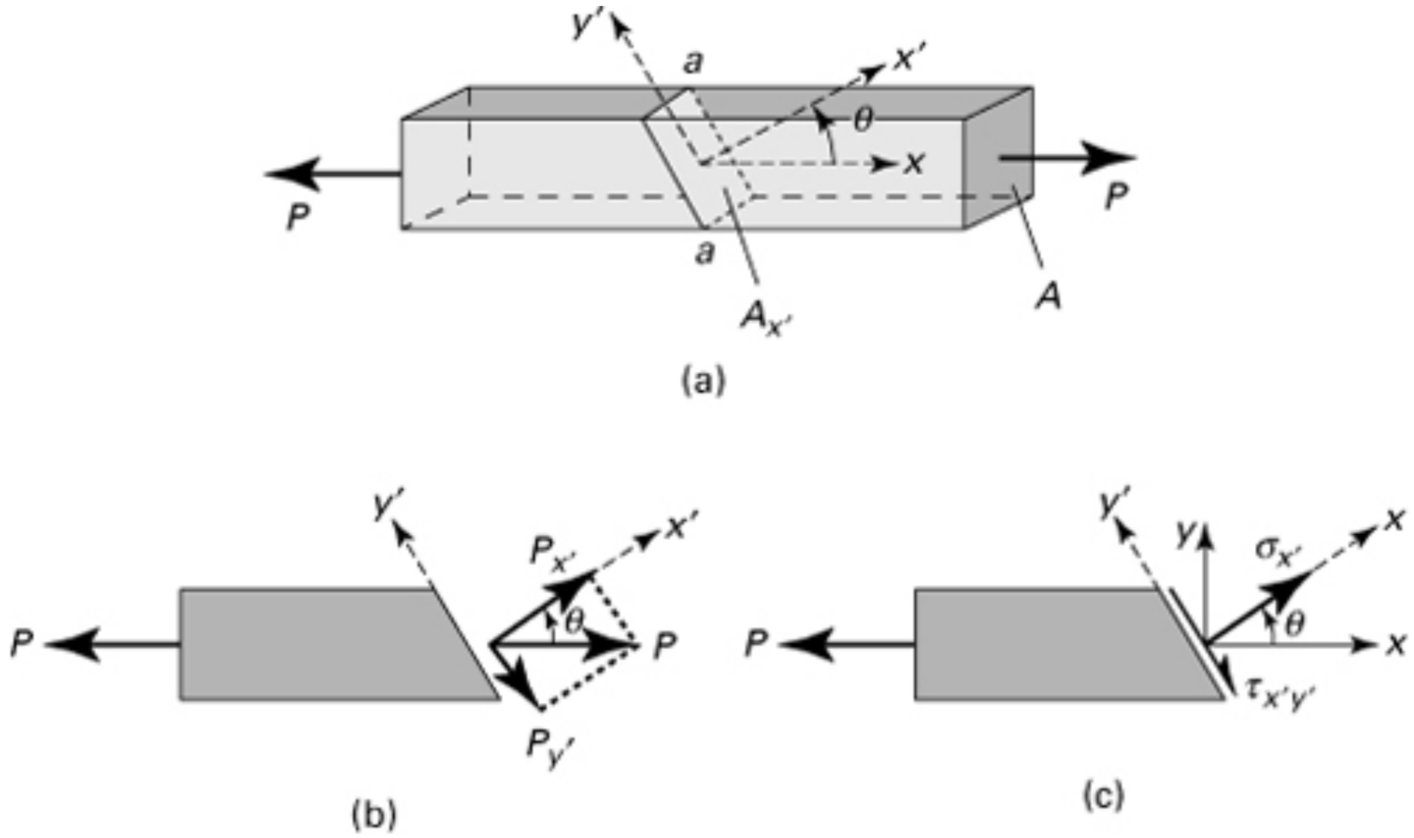


Stress Transformation

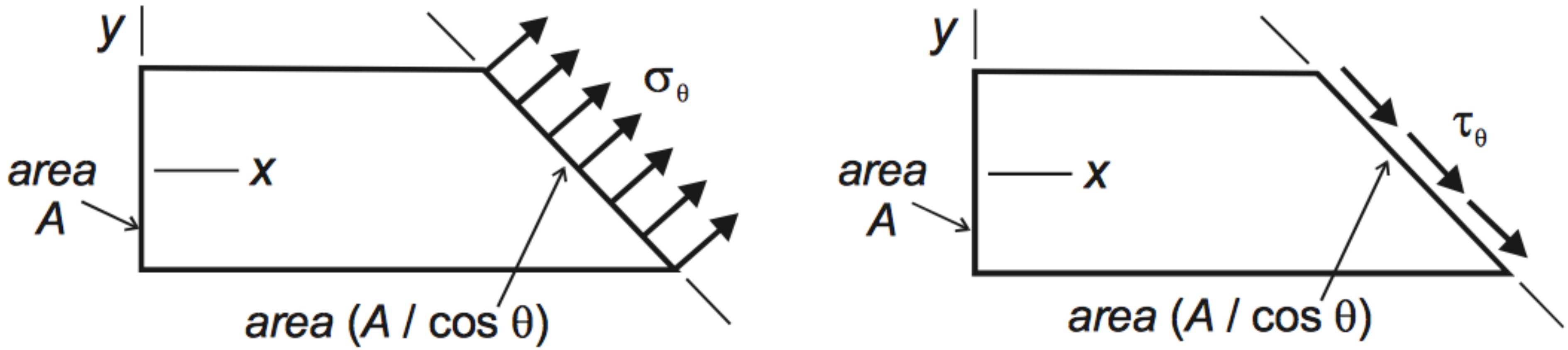


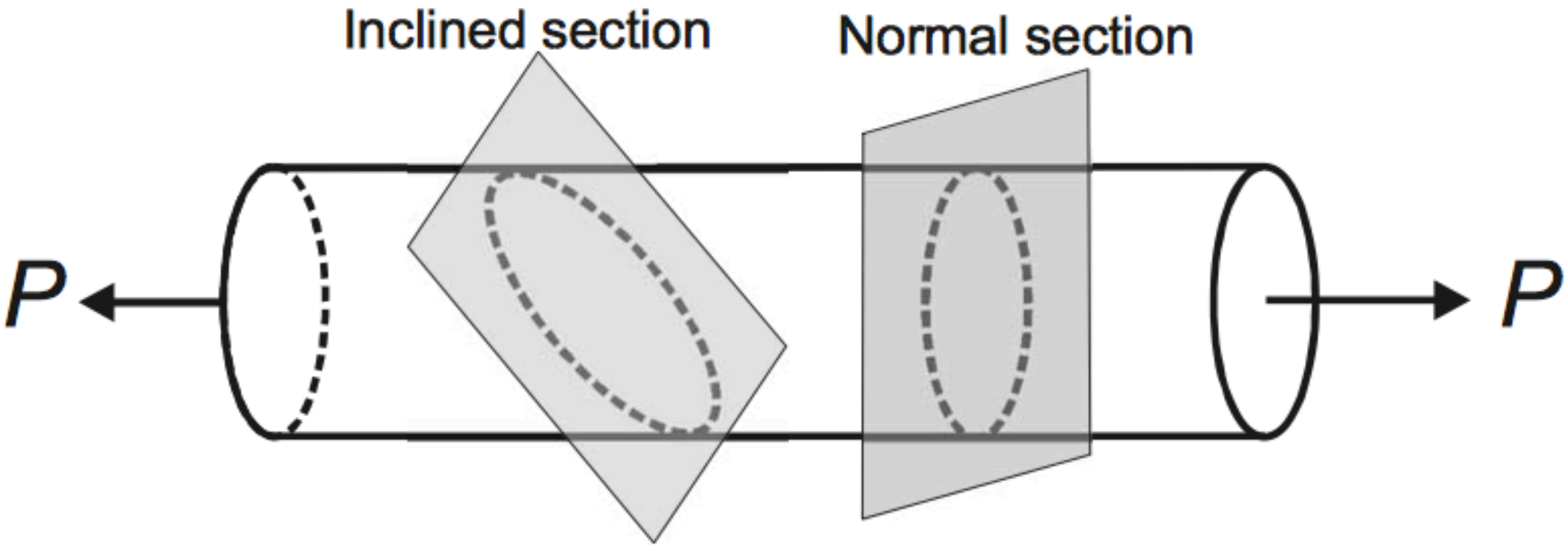
Stresses at given coordinate system   Stresses transformed to another coordinate

Stress Transformation



Uniaxial loading




Inclined section
Normal section
P
P

Observations on 1-D loading on oblique plane

- With initial orientation of representative element or coordinate system, there is just one non-zero stress- that is the normal stress and no other stress. (no shear stress)

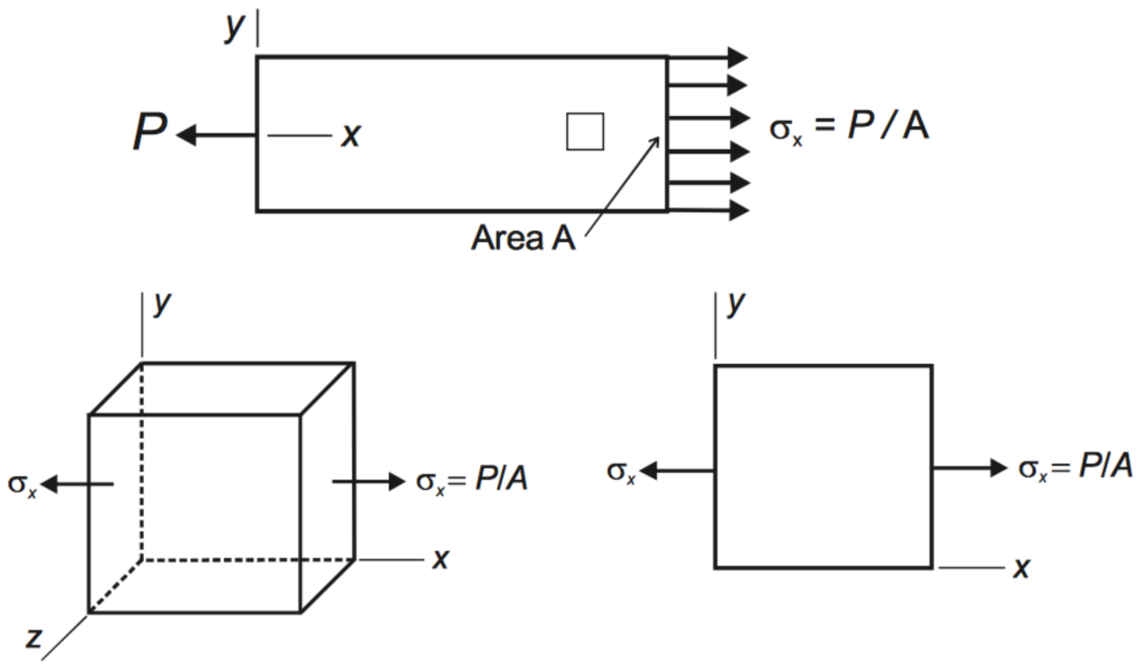
- On oblique plane or the rotated representative element/coordinate system, there are more stresses: normal+ shear stress.

- The external loading has not changed.

- The magnitude that is assigned for stress depends on the orientation of reference plane/representative element/reference coordinate system.

Stress Elements

- Stress elements are a useful way to represent stresses acting at some point on a body.
- Isolate a small element and show stresses acting on all faces.
- Dimensions are “infinitesimal”, but are drawn to a large scale.

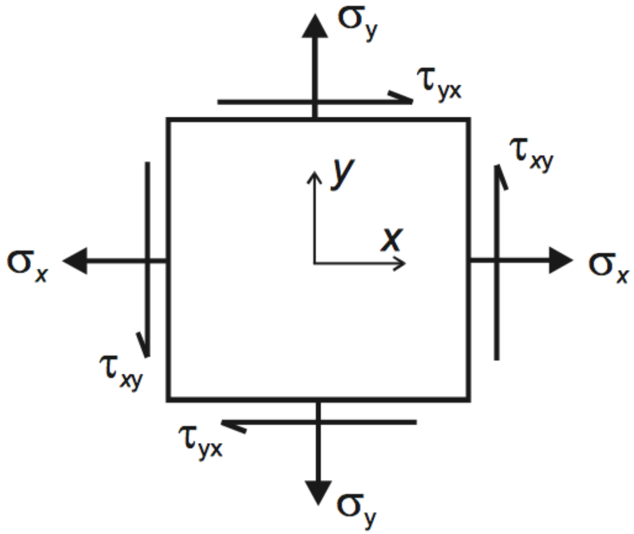


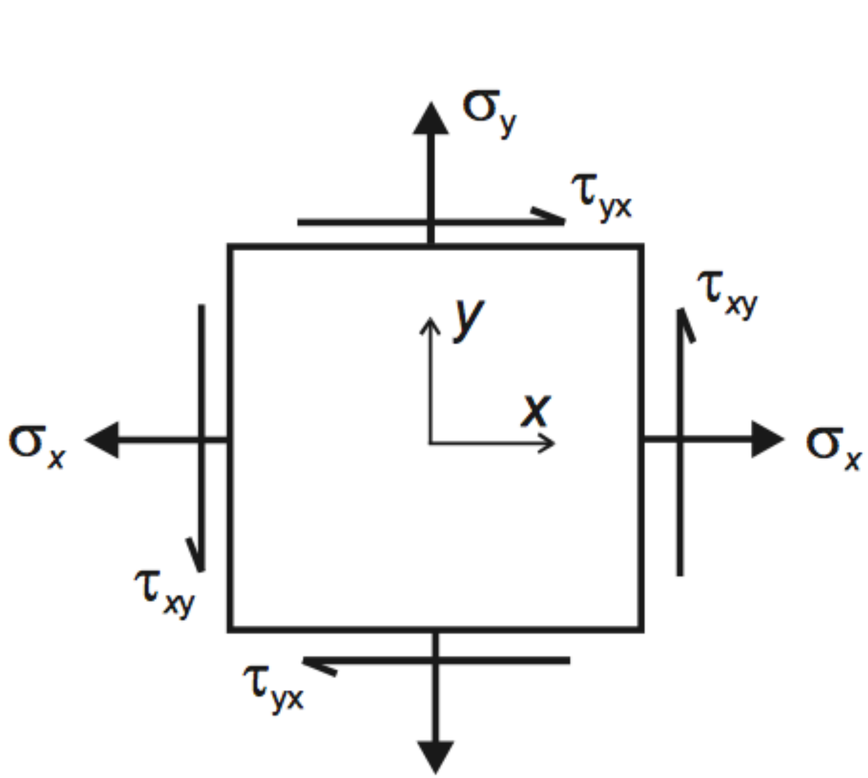
Plane stress

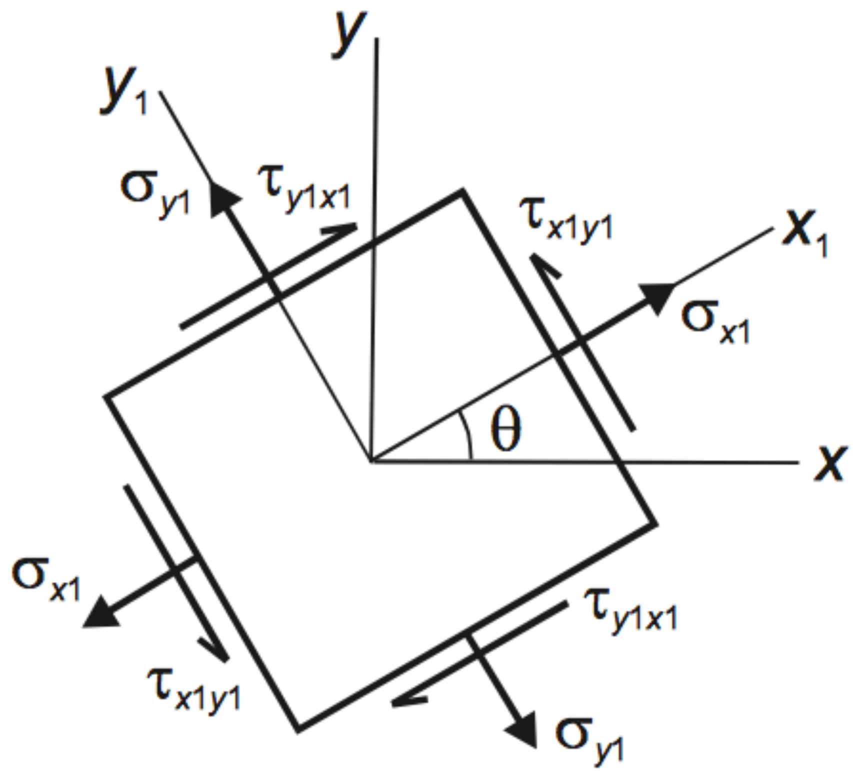
When an element is in **plane stress** in the *xy* plane, only the *x* and *y* faces are subjected to stresses ($\sigma_z = 0$ and $\tau_{zx} = \tau_{xz} = \tau_{zy} = \tau_{yz} = 0$).

Plane stress element in 2D

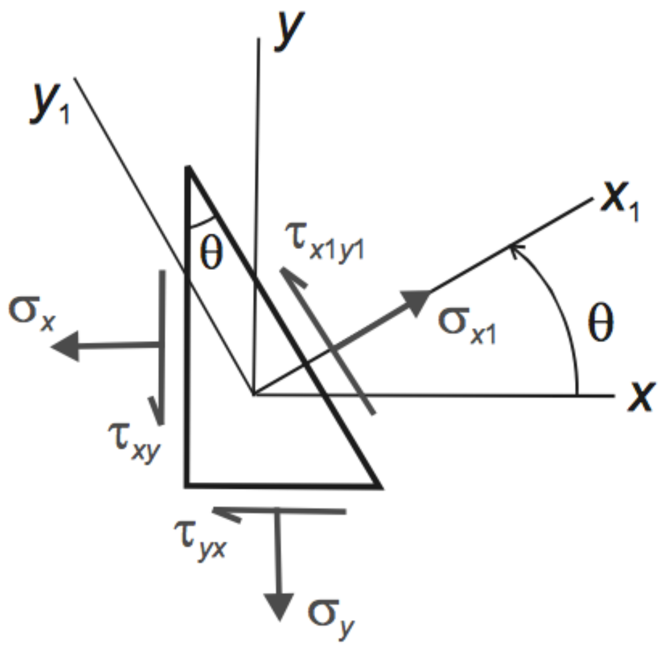
σ_x, σ_y
$\tau_{xy} = \tau_{yx}$
$\sigma_z = 0$

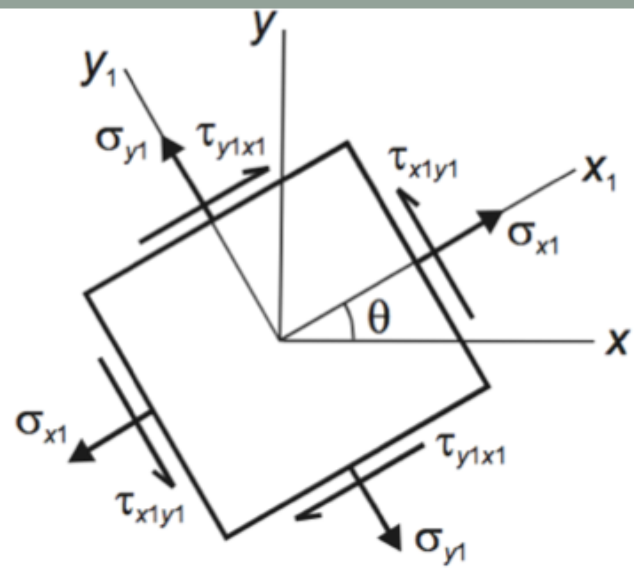






Why? A material may yield or fail at the maximum value of σ or τ. This value may occur at some angle other than $\theta = 0$. (Remember that for uni-axial tension the maximum shear stress occurred when $\theta = 45$ degrees.)





$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x1y1} = -\frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

- Principal Stress: The maximum and minimum normal stresses are called principal stresses.

- In the orientation of principal stress, shear stress is 0.

Principal Stresses

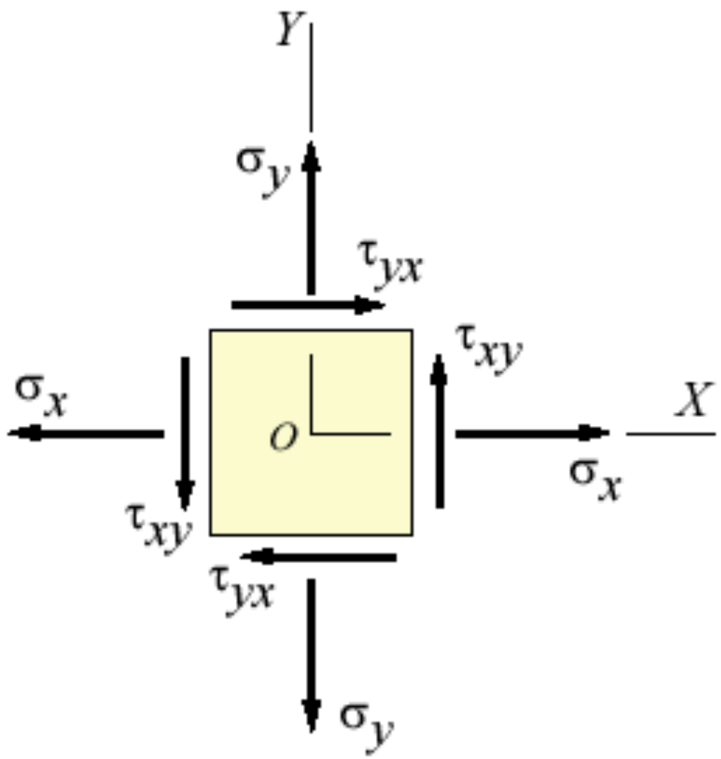
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

Principal Angles defining the **Principal Planes**

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Mohr Circle

- Geometrical representation of 2D state of stress.
- Useful for stress and strain transformations.



Steps to construct the Mohr Circle

- For the particular stress situation note down the stress acting on the elements with their correct signs.
- Draw x (σ) and y(τ) axis. Draw the center of the Mohr Circle. [$(\sigma_{xx}+\sigma_{yy})/2$, 0]
- First point on the circle is $(\sigma_{xx}, -\tau_{xy})$ call it X.
- Second point on the circle (σ_{yy}, τ_{xy}) call it Y.
- Positive shear stress is plotted downwards at X and upwards at Y. Negative shear stress is plotted upwards at X and downwards at Y.
- Draw the circle.
- For a rotation of θ in the physical scenario, rotation of 2θ is required in Mohr Circle. Maximum rotation is 360 degrees in the circle at which stress states coincide with original ones.
- Stress components with respect to rotated X'Y' axes can be determined from the corresponding X'Y' diameter.
- To determine the stresses after rotation by Φ, make a diameter at an angle 2Φ in the same sense (clockwise vs. anticlockwise).
- Find the x and y coordinates of these 2 points, they will give ($\sigma_{x'x'}$, $\sigma_{y'y'}$ and $\tau_{x'y'}$)
- $2\theta_p$ is the angle made with the orientation for the principal stress state in the circle. θ is the physical angle.
- sin $2\theta_p=\tau_{xy}$/radius of circle.

Observations

- Planes of maximum shear stress occur at 45 degrees to the principal planes.
- The maximum shear stress is equal to half the difference of the principal stresses.