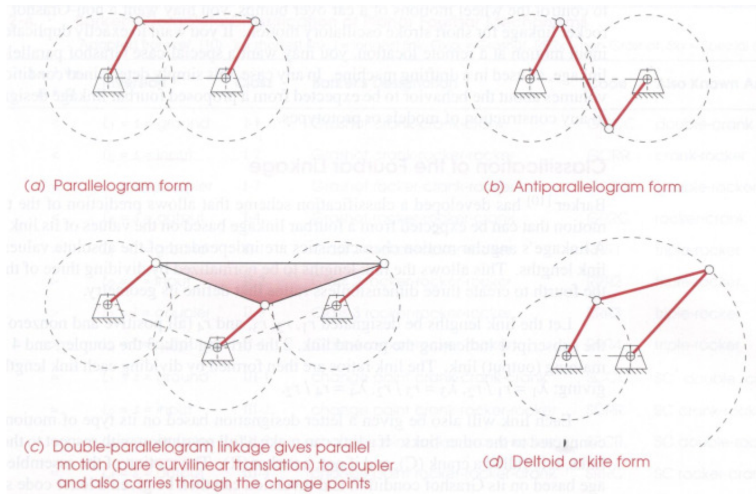
ME220 Theory of Machines and Machine Design

Lec 6 – 27 Jan 2020

Special Grashof Case

$S + L = P + Q$:



(a) Parallelogram form

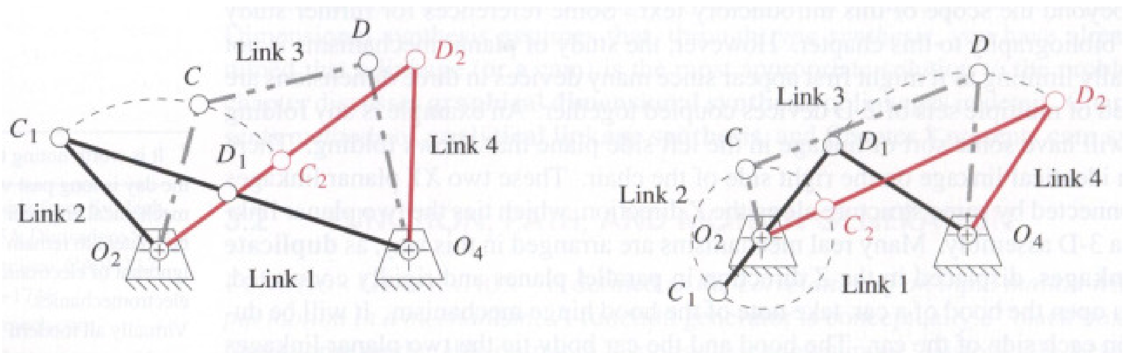
(b) Antiparallelogram form

(c) Double-parallelogram linkage gives parallel motion (pure curvilinear translation) to coupler and also carries through the change points

(d) Deltoid or kite form

For parallelogram mechanisms, there is a configuration when all links become collinear. This configuration is called change point. At change points the parallelogram configuration can switch between parallelogram and anti-parallelogram configurations un-predictably. To avoid this and third parallel crank with a slight phase shift is added to the linkage as shown in Fig (c) above. This phase shift ensures that in no configuration all three cranks become collinear and the non-collinear crank is always able to drive the remaining cranks out of the change point in the same direction.

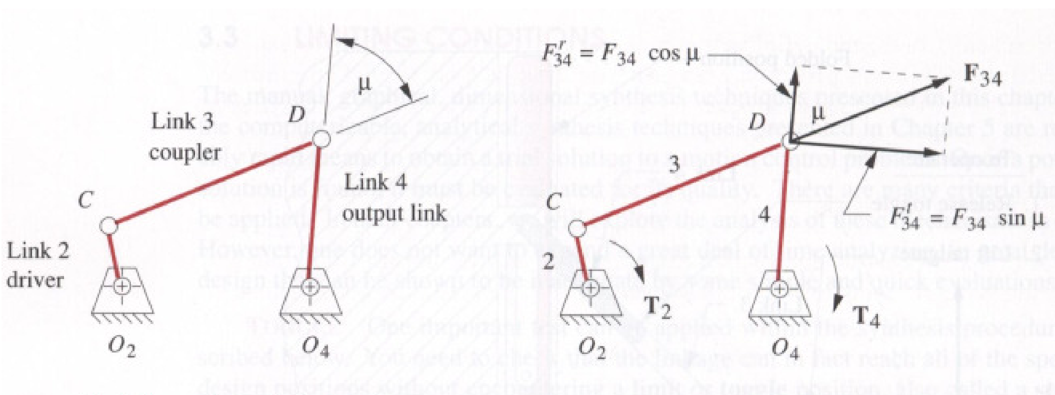
Limiting Conditions: Toggle/Stationary Points



(a) Triple-rocker toggle positions (b) Crank-rocker toggle positions

Similar to parallelogram mechanisms, other fourbar linkages also have extreme positions determined by the collinearity of two of the moving links. For the Non-Grashof Triple Rocker shown in Fig (a) above, C_1D_1 is the toggle position reached when driven from link 2 and C_2D_2 is the toggle position reached when driven from link 4 (for both these configuration there are mirror image configurations of the once shown above that will also be toggle positions). Similar extreme configurations for a Grashof crank-rocker are shown in Fig(b) above $O_2C_2D_2$ and $O_2C_1D_1$. During these configurations, the crank rocker if driven from the crank side will keep moving but it can not be back-driven from the output side in these configurations.

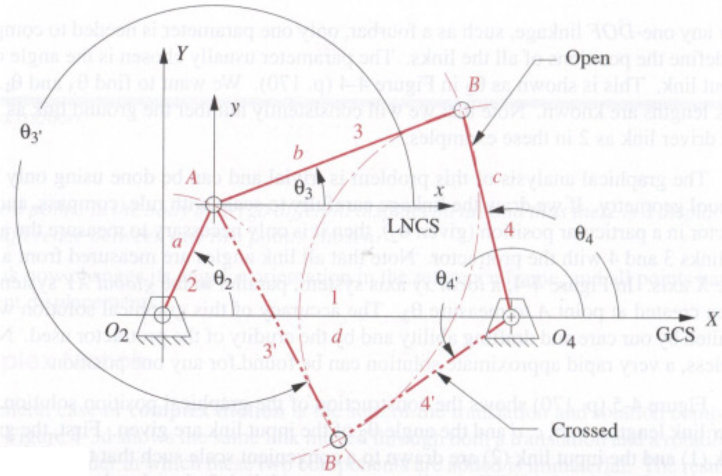
Transmission Angle



(a) Linkage transmission angle μ (b) Static forces at a linkage joint

Transmission angle captures the ease with which the coupler link is able to transmit torque to the output link. It is defined as the angle between the output link and the coupler and is usually taken as the absolute value of the acute angle of the pair of angles at the intersection of the two links (shown as μ above). It varies from some minimum to some maximum value as the linkage goes through its range of motion. The best value for this angle is 90deg and the worst value is 0deg. Usually we try to design a linkage such that this does not go below 30/40 degrees.

Position Analysis of Fourbar Linkage (Algebraic)



Problem Statement: Given the input angle and all link lengths compute the angles for the remaining two links relative to ground.

Position Analysis of Fourbar Linkage (Algebraic)

As shown in the figure on the previous page, link lengths are a,b,c, and d and joints are at A, B, O_2, and O_4.

Coordinates of A are already know to be

$$A_x = a\cos\theta_2$$
$$A_y = a\sin\theta_2$$

The coordinates of point B are found using the equations of circles about A and O_4.

$$b^2 = (B_x - A_x)^2 + (B_y - A_y)^2 \qquad \text{Eq1}$$

$$c^2 = (B_x - d)^2 + B_y^2 \qquad \text{Eq2}$$

which provide a pair of simultaneous equations in B_x and B_y.

Subtracting equation Eq2 from Eq1 gives an expression for B_x.

$$B_x = \frac{a^2 - b^2 + c^2 - d^2}{2(A_x - d)} - \frac{2A_y B_y}{2(A_x - d)} = S - \frac{2A_y B_y}{2(A_x - d)} \qquad \text{Eq3}$$

Substituting equation Eq3 into Eq2 gives a quadratic equation in B_y which has two solutions corresponding to those in the figure on previous page.

$$B_y^2 + \left(S - \frac{A_y B_y}{A_x - d} - d\right)^2 - c^2 = 0$$

This can be solved with the familiar expression for the roots of a quadratic equation,

$$B_y = \frac{-Q \pm \sqrt{Q^2 - 4PR}}{2P} \qquad \text{Eq4}$$

where:

$$P = \frac{A_y^2}{(A_x - d)^2} + 1 \qquad Q = \frac{2A_y(d - S)}{A_x - d}$$

$$R = (d - S)^2 - c^2 \qquad S = \frac{a^2 - b^2 + c^2 - d^2}{2(A_x - d)}$$

The link angles for this position can then be found from

$$\theta_3 = \tan^{-1}\left(\frac{B_y - A_y}{B_x - A_x}\right)$$

$$\theta_4 = \tan^{-1}\left(\frac{B_y}{B_x - d}\right)$$

Note that the solutions found above can be purely imaginary (complex number) in the situation when the link lengths given can not be physically connected for the input angle given.