Tutorial 3

1) Give an example of a polynomial $P_{2n+2}(x)$ of degree $2n+2$ such that Gaussian Quadrature (with $n+1$ nodes) is not exact for $P_{2n+2}(x)$

2) Approximate $\int_0^1 e^{-x^2}\,dx$

by
1) Trapezoidal rule $h = 0.25$
2) Simpsons rule $h = 0.5$
3) Gaussian Quadrature with 3 nodes.

3) Suppose $f''(x) \geq 0 \quad \forall x \in [a,b]$.
Let α = approximation of $\int_a^b f(x)\,dx$ by composite Mid point rule
β = approximation of $\int_a^b f(x)\,dx$ by composite trapezoidal rule.
Let $I = \int_a^b f(x)\,dx$ (exact value)
Show I is between α and β

4) Consider $I = \int_0^1 \sin(x^3)\,dx$

a) How many subdivisen of the interval ~~I~~ $= [0,1]$ is needed so that Trapezoid rule gives an error of 10^{-4} (or less)

b) (Same Question as b) for Simpson's rule

5) ~~Find~~ Approximate $\int_0^1 \frac{\cos x}{\sqrt{x}}\,dx$ by Taylor series method