STATIC EQUILIBRIUM AND STRESSES PART II

Lecture 3 ,4 and 5

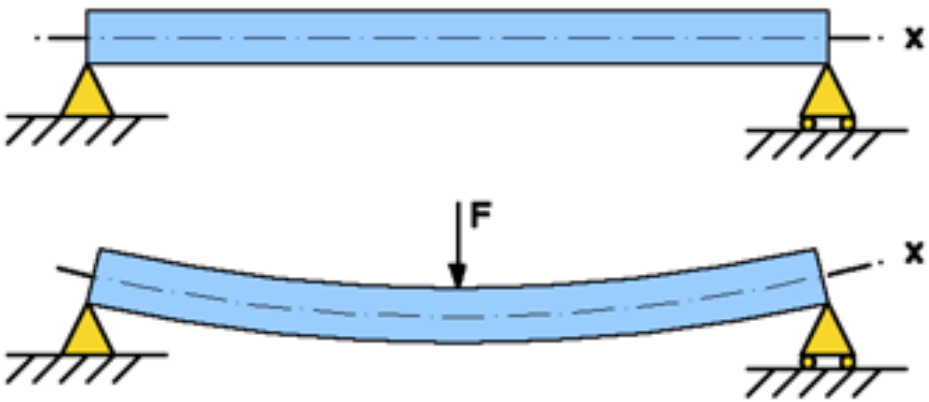
MM203

Mechanics of Materials

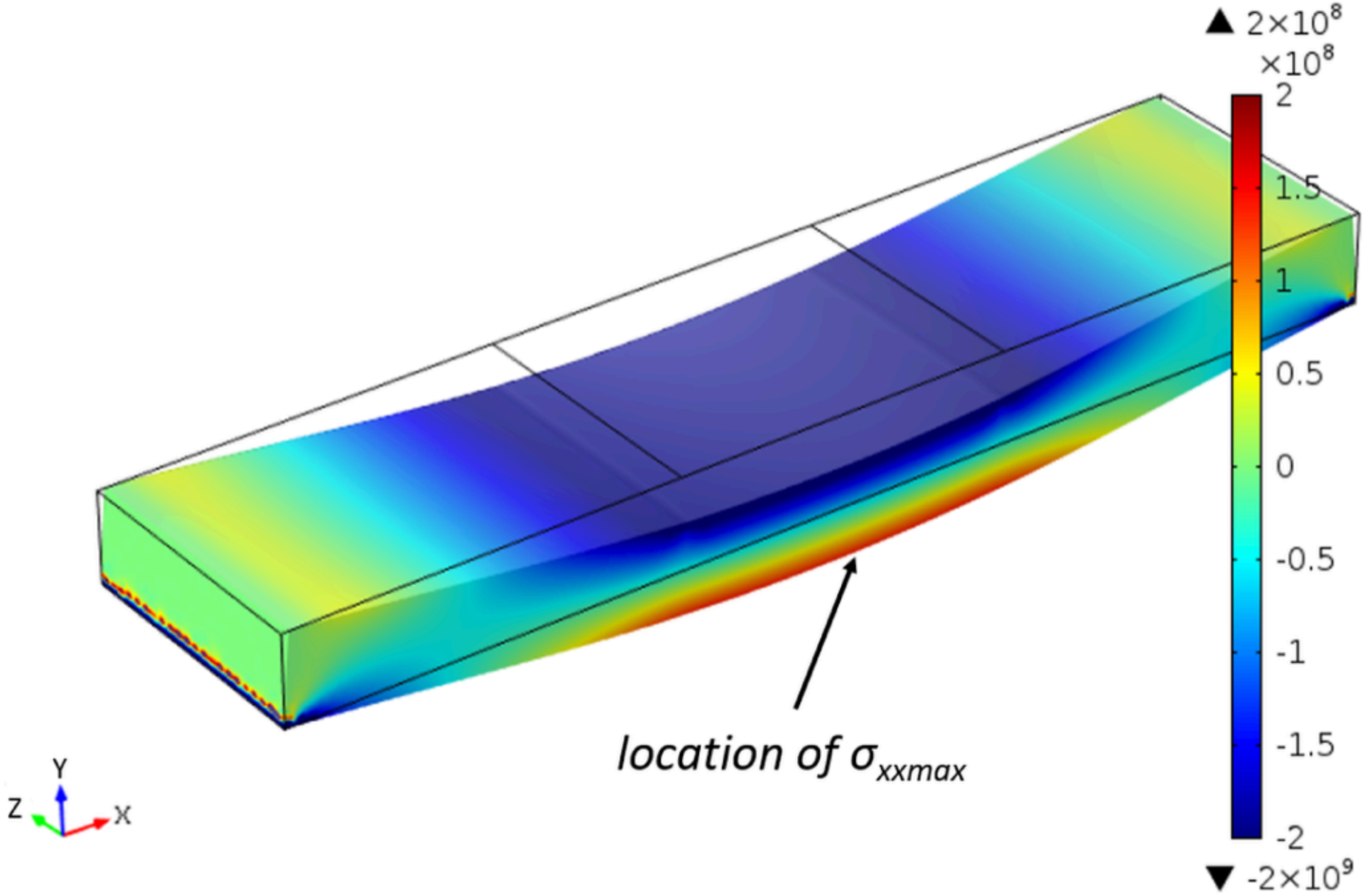
Recap

- Different kind of structures.
- Free body diagram and forces and moments applied by surroundings.
- Development of theory of elasticity and concepts of stresses and strains.
- Requirements to define stress: Force and Area.
- Shear stress and normal stress.
- Constitutive equations.

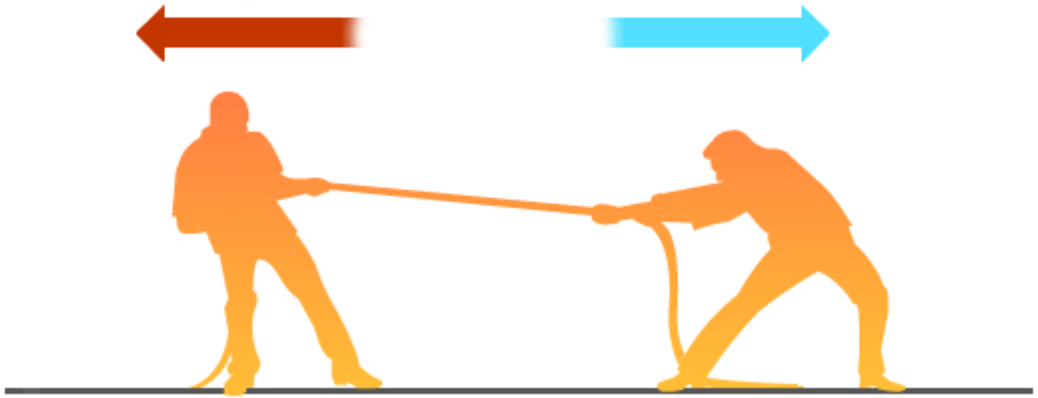
Examples

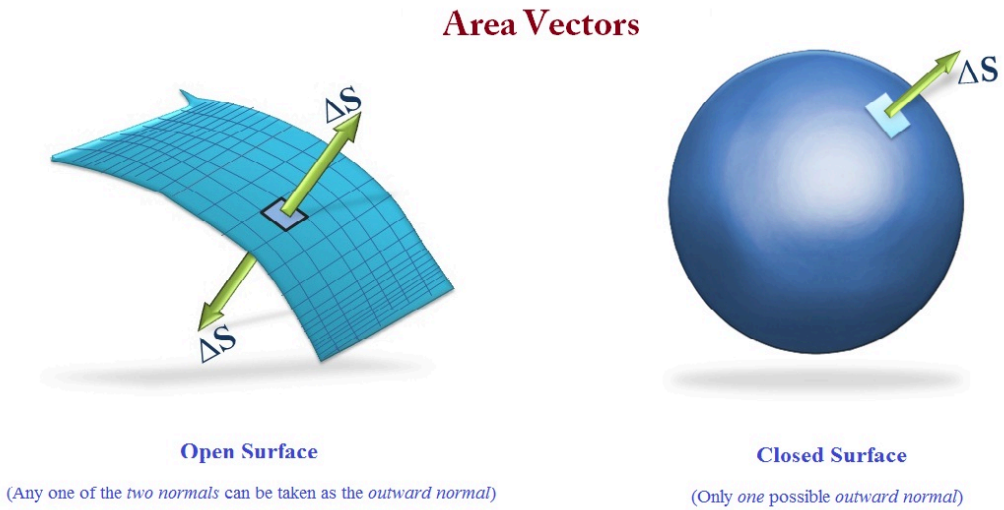


Stresses

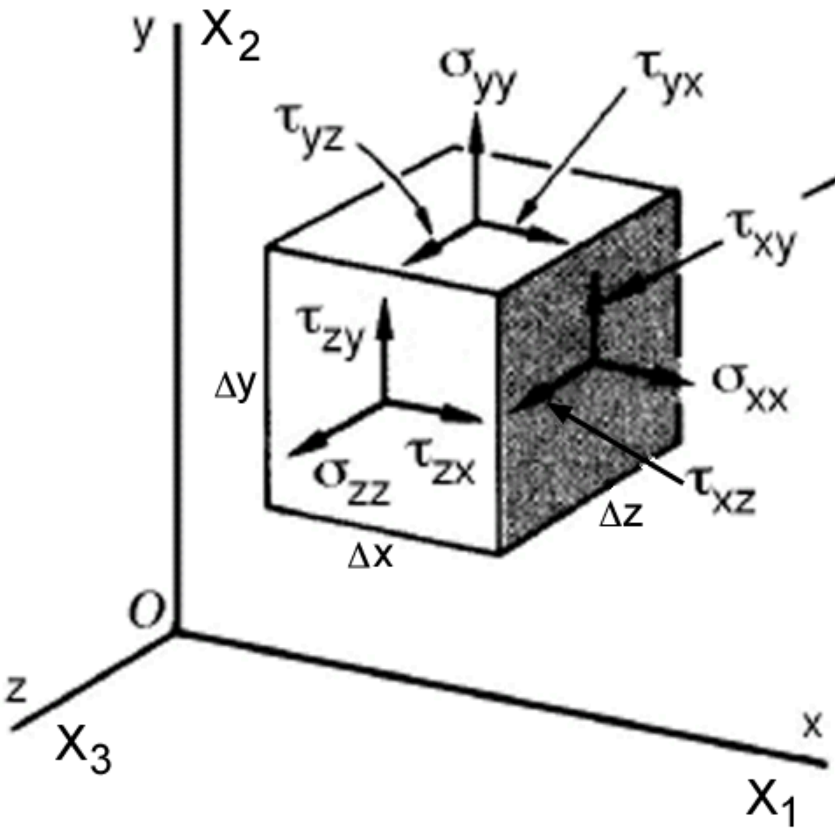


How to represent a general state of stress.





Tensor Representation

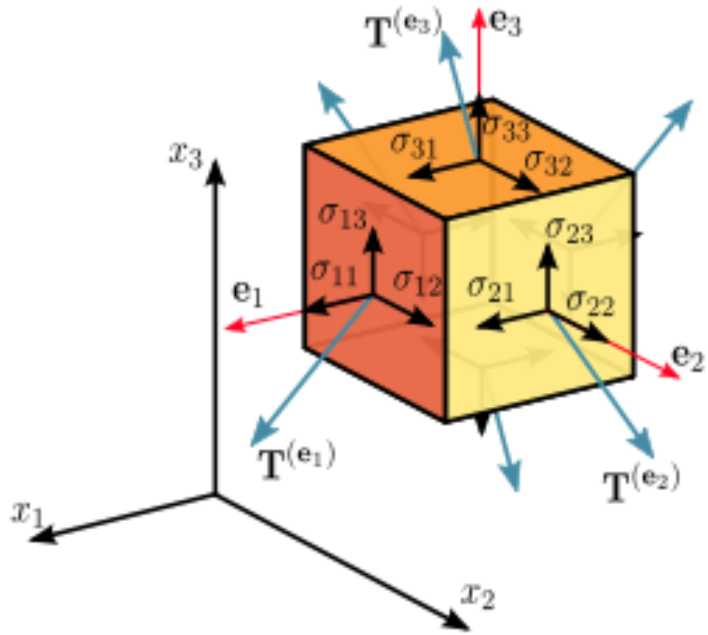


$$\begin{Bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{Bmatrix}$$

σ_{ij} where (i and j = 123)

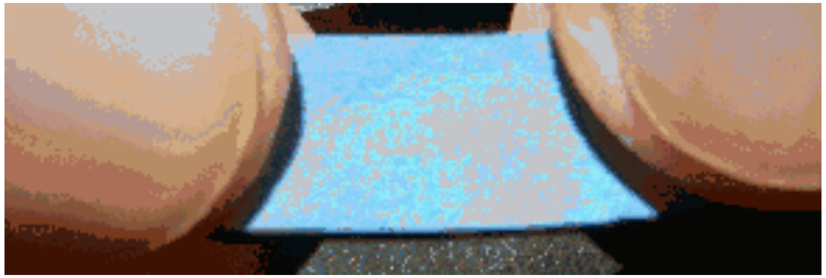
How to calculate the stresses in the bridge



Strain Tensor

- Write the strain tensor for a general state of stress.

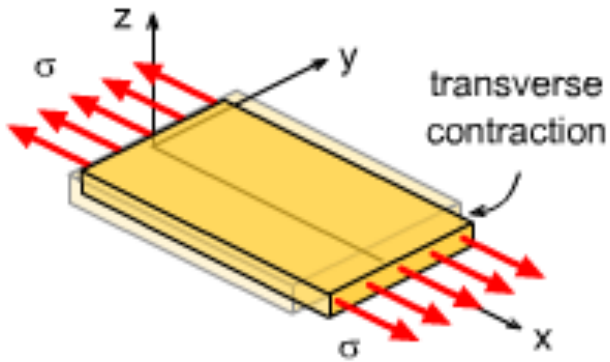
$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$



Poisson's Ratio

- When loaded axially, what happens in lateral direction.

$$\nu = -\frac{d\varepsilon_{\mathrm{trans}}}{d\varepsilon_{\mathrm{axial}}} = -\frac{d\varepsilon_{\mathrm{y}}}{d\varepsilon_{\mathrm{x}}} = -\frac{d\varepsilon_{\mathrm{z}}}{d\varepsilon_{\mathrm{x}}}$$



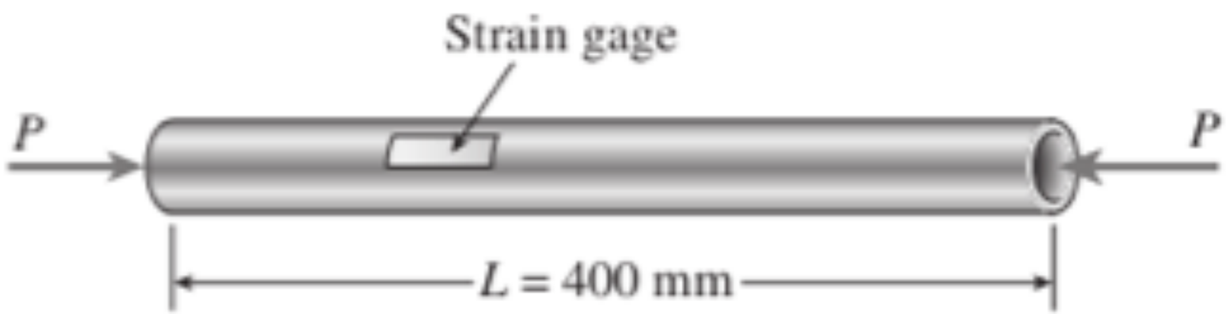
Generate State of Strain for Uniaxial Loading

- Write the general state of strain for uniaxial loading.

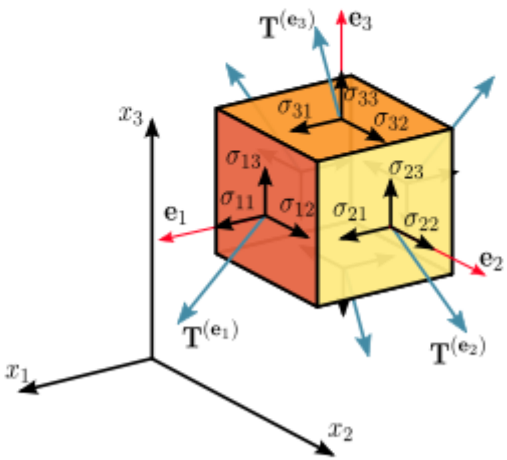
Problem 1 A circular aluminum tube of length $L = 400$ mm is loaded in compression by forces P (see figure). The outside and inside diameters are 60 mm and 50 mm, respectively. A strain gage is placed on the outside of the bar to measure normal strains in the longitudinal direction.

(a) If the measured strain is $\epsilon = 550 \times 10^{-6}$, what is the shortening δ of the bar?

(b) If the compressive stress in the bar is intended to be 40 MPa, what should be the load P?



What is the strain in terms of stresses



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

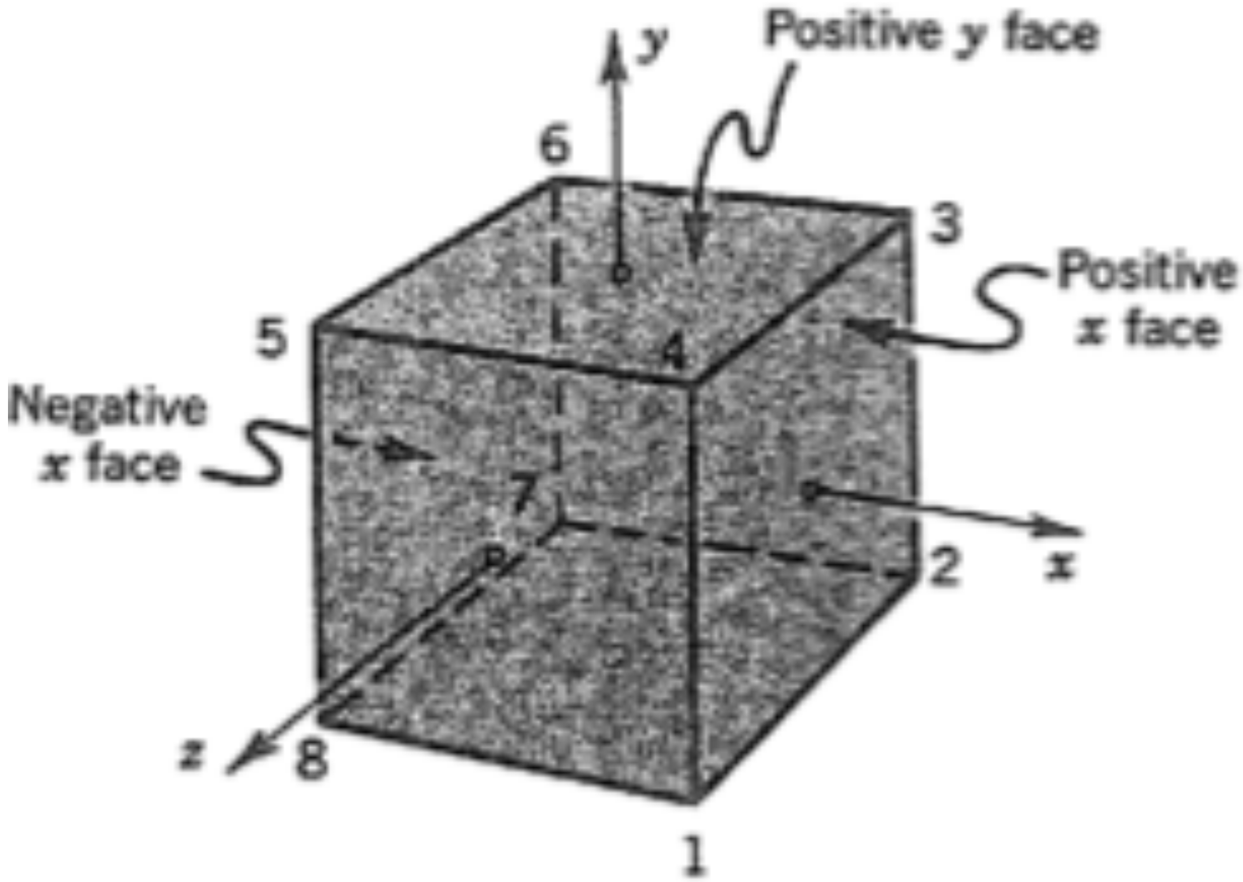
$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

Volume change

Isotropic Material

$$G = \frac{E}{2(1+\nu)}$$

Sign convention



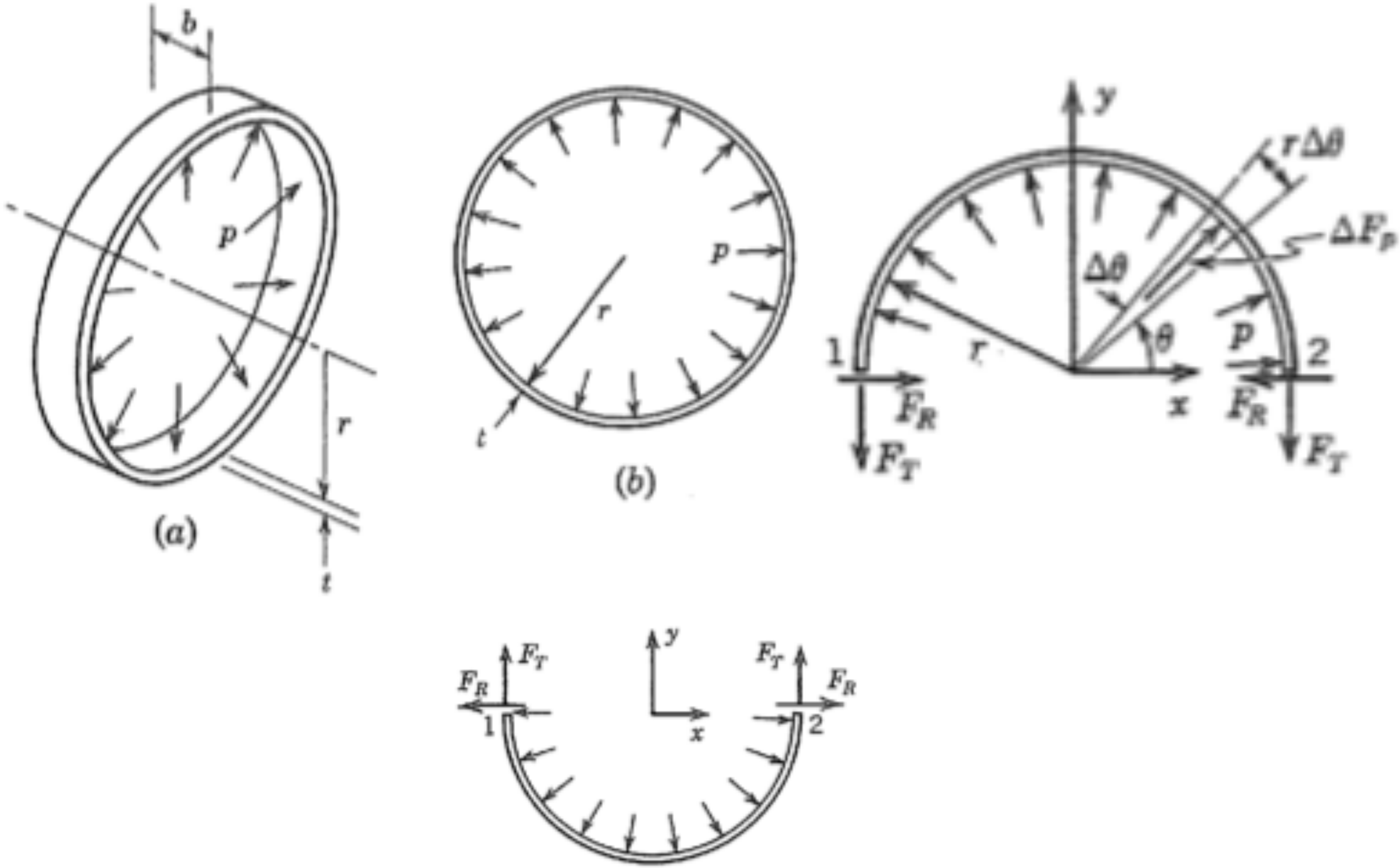
Three aspects of problem-solving

- Principles of equilibrium.
- Geometrical compatibility.
- Relations between force and deformation.

Plane fuselage

Aloha Airlines

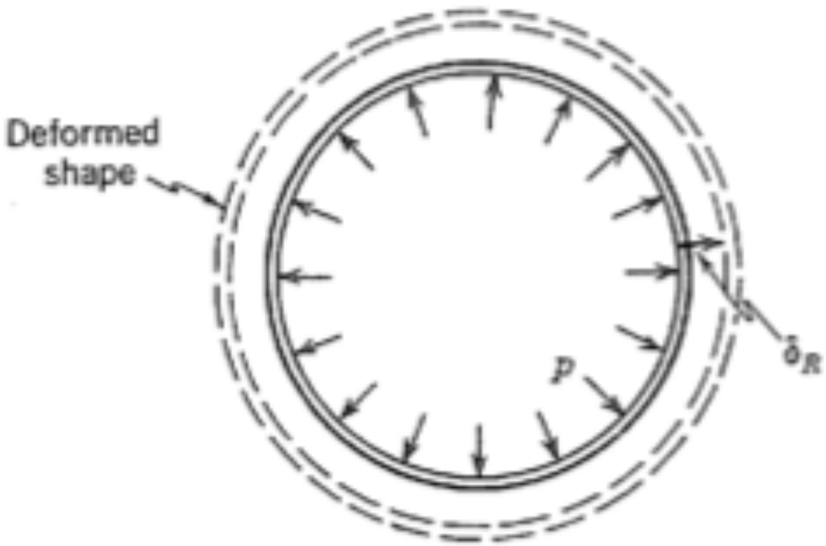
Hoop Stress



b
p
r
t
(a)
p
r
t
(b)
y
$r\Delta\theta$
ΔF_p
$\Delta\theta$
θ
r
1
2
p
F_R
F_T
x
F_R
F_T
y
x
F_T
F_R
1
F_T
F_R
2

Three aspects of problem-solving

- Principles of equilibrium.
- Geometrical compatibility.
- Relations between force and deformation.



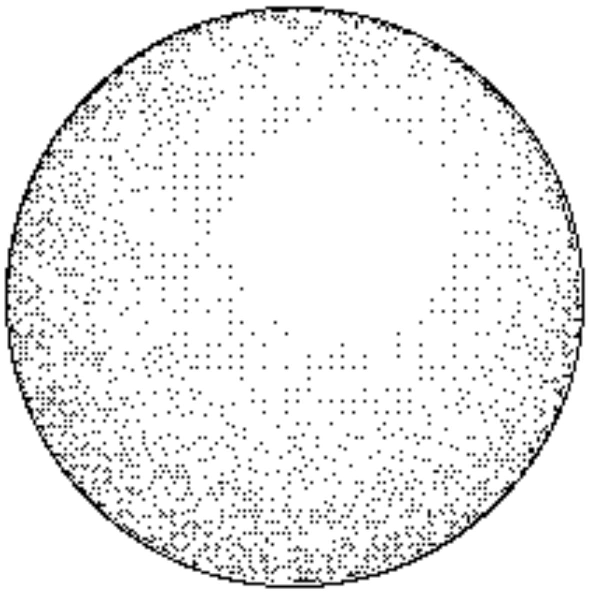
(e)

Three aspects of problem-solving

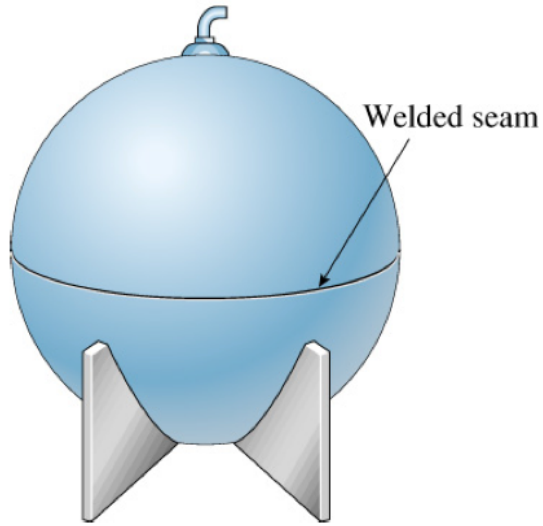
- Principles of equilibrium.
- Geometrical compatibility.
- Relations between force and deformation.

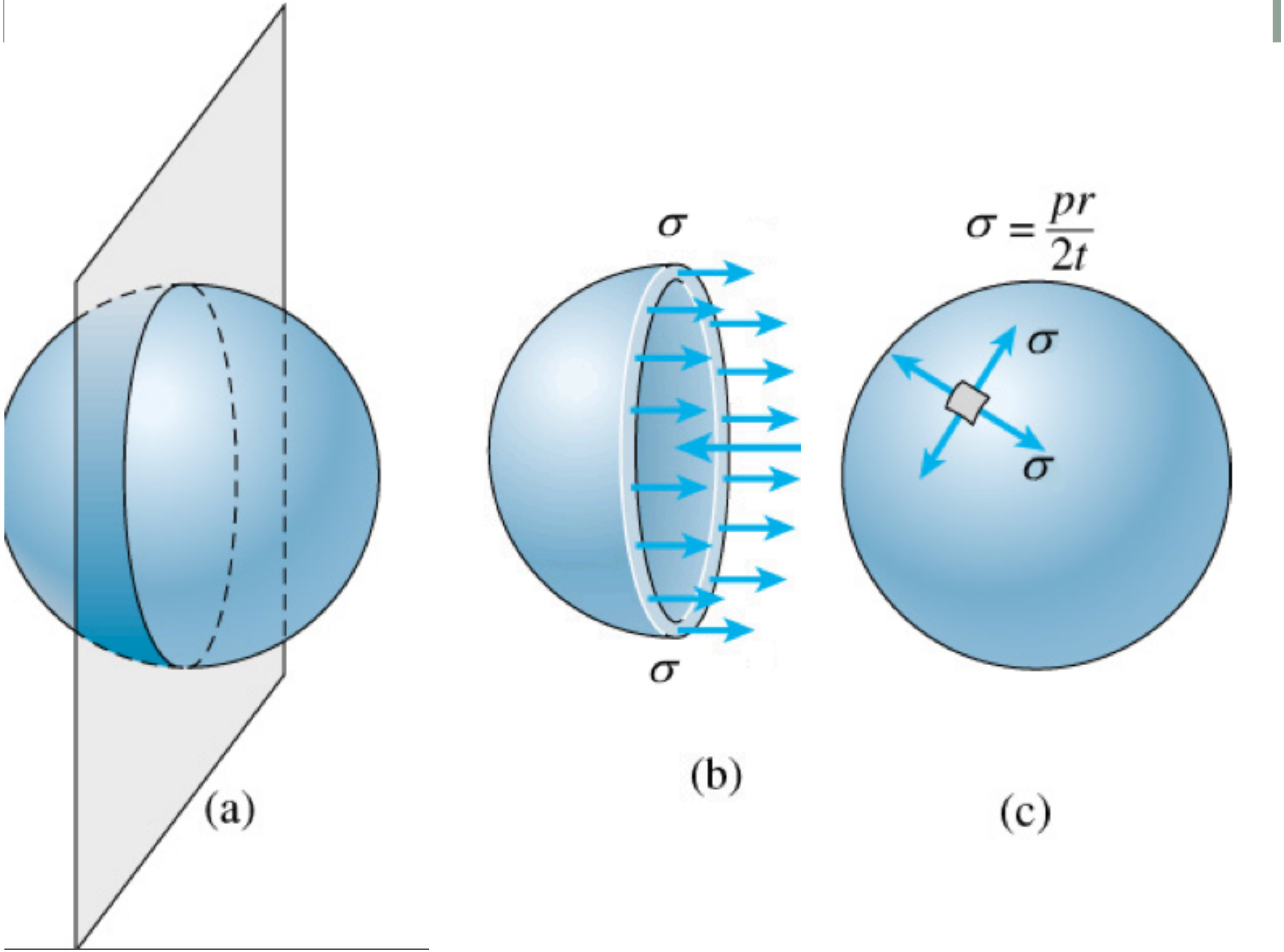
Problem

- What are the stresses in a sphere if the pressure is p?



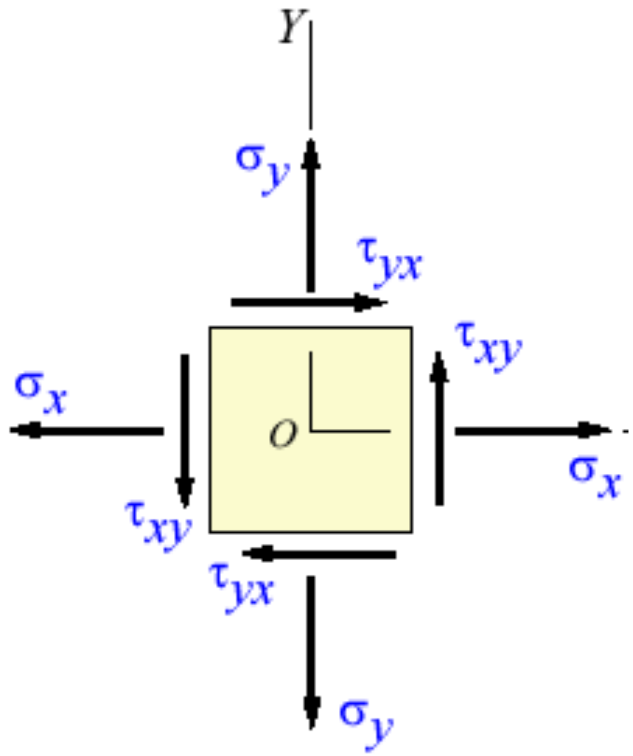
Another common pressure vessel shape is the sphere. Storage containers for high pressure gasses are often spherical. Also, when we blow-up balloons they often take the shape of a



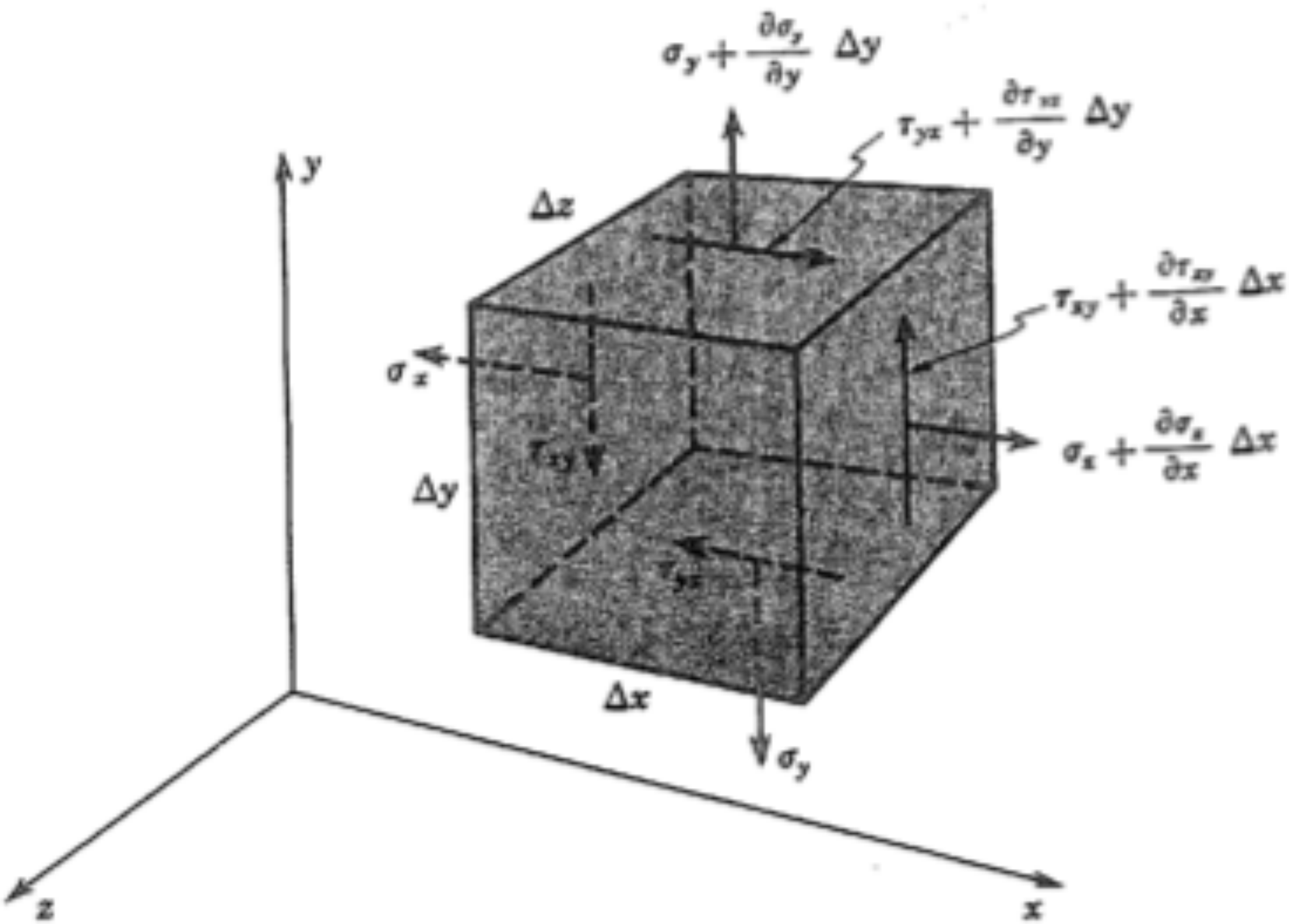

σ
σ
$\sigma = \frac{pr}{2t}$
σ
σ
(a)
(b)
(c)

Plane stress

- Pull on a long thin wire of uniform section along the x-axis.
- Thin sheet being pulled by forces in the plane of the sheet.
- No variation a



Equilibrium



$$\Sigma \mathbf{M} = \left\{ (\tau_{xy}\Delta y\, \Delta z)\frac{\Delta x}{2} + \left[\left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}\Delta x\right)\Delta y\, \Delta z\right]\frac{\Delta x}{2} \right.$$

$$\left. - (\tau_{yx}\Delta x\, \Delta z)\frac{\Delta y}{2} - \left[\left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\Delta y\right)\Delta x\, \Delta z\right]\frac{\Delta y}{2}\right\}\mathbf{k} = 0$$

$$\Sigma F_x = \left(\sigma_x + \frac{\partial \sigma_x}{\partial x}\Delta x\right)\Delta y\, \Delta z + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\Delta y\right)\Delta x\, \Delta z$$

$$- \sigma_x \Delta y\, \Delta z - \tau_{yx}\Delta x\, \Delta z = 0$$

$$\Sigma F_y = \left(\sigma_y + \frac{\partial \sigma_y}{\partial y}\Delta y\right)\Delta x\, \Delta z + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}\Delta x\right)\Delta y\, \Delta z$$

$$- \sigma_y \Delta x\, \Delta z - \tau_{xy}\Delta y\, \Delta z = 0$$

$$\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}\frac{\Delta x}{2} - \tau_{yx} - \frac{\partial \tau_{yx}}{\partial y}\frac{\Delta y}{2} = 0$$

$$\tau_{yx} = \tau_{xy}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

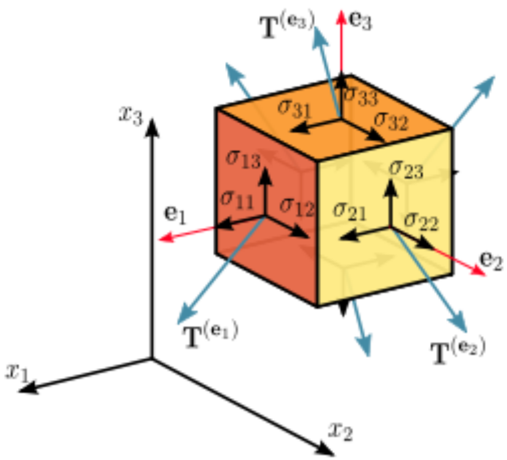
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0$$

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} + \frac{\partial \sigma_{31}}{\partial x_3} = 0$$

$$\frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{32}}{\partial x_3} = 0$$

$$\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} = 0$$

What is the strain in terms of stresses



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

$$\sigma = E\varepsilon$$

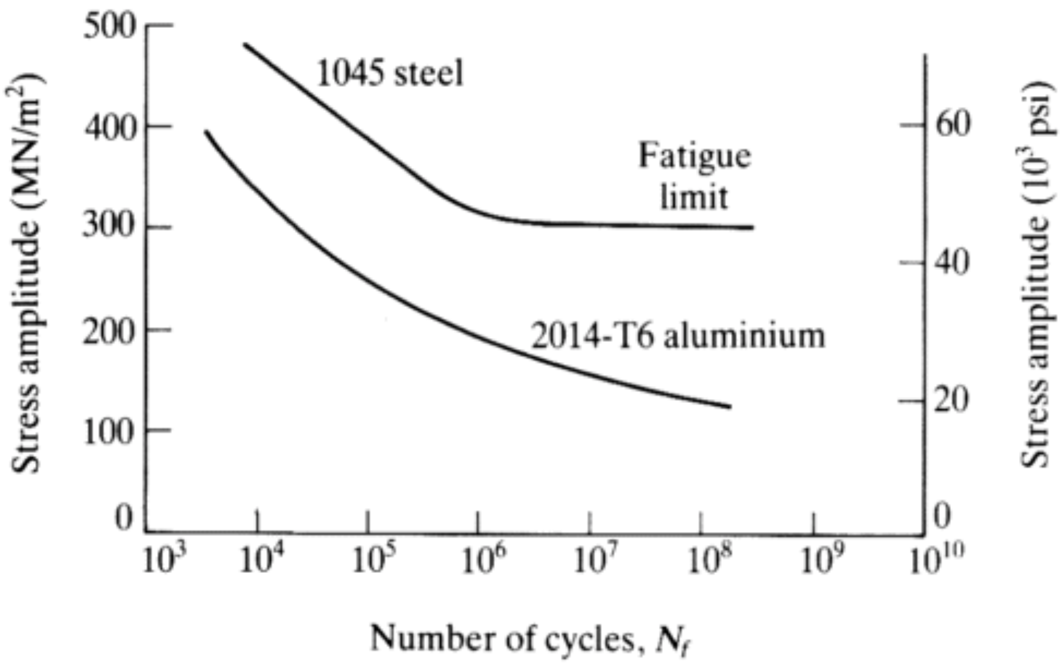
Only applicable in the specific situation of uniaxial stress (along the force) dependence on the strain in the direction of the force.

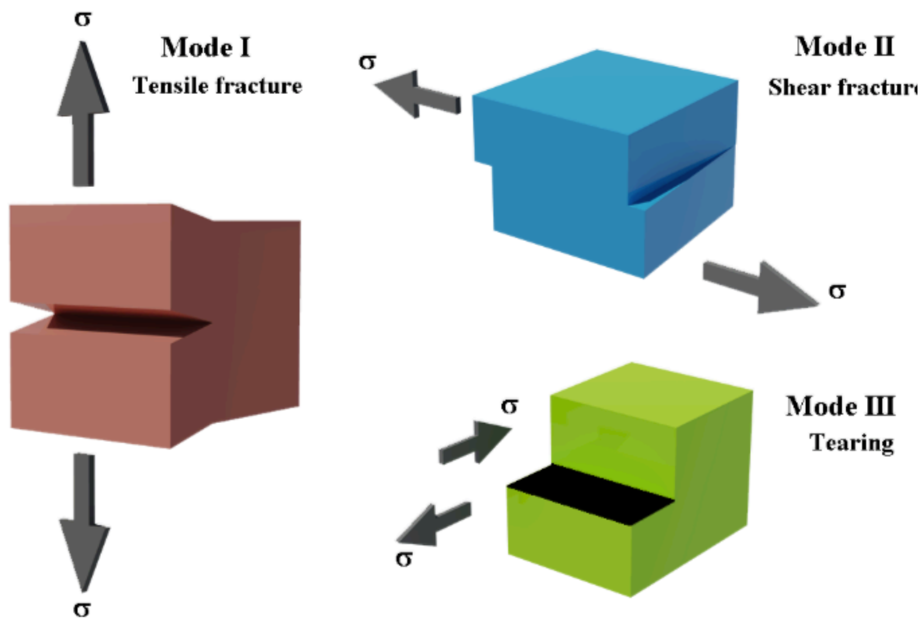
General dependence for an isotropic body

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix}$$

Ways of characterizing failure

- Yield strength
- Fracture toughness
- Fatigue strength

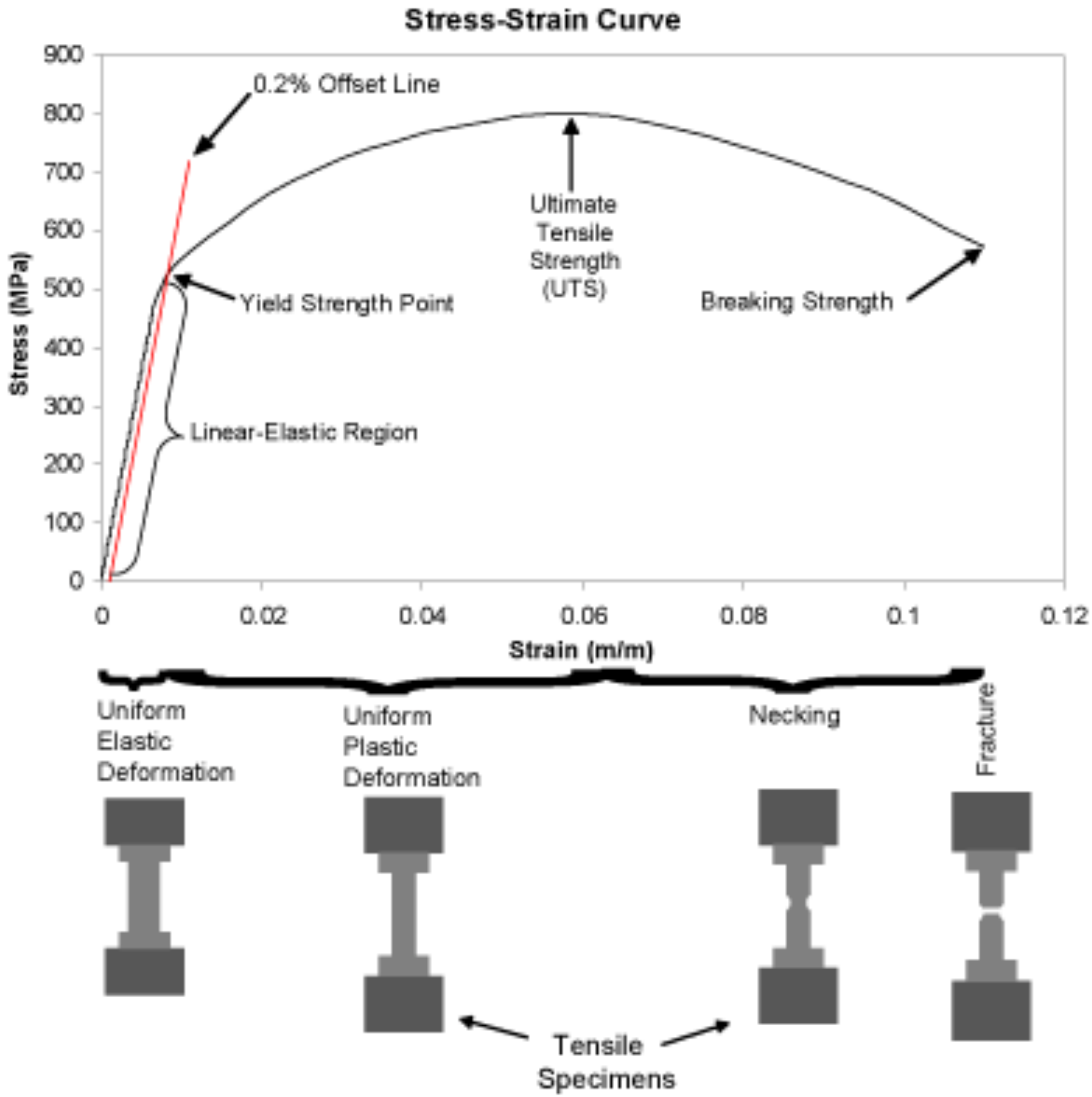




How to test mechanical properties

- Simulations from first principles.
- Experimental methods. (Tensile test)

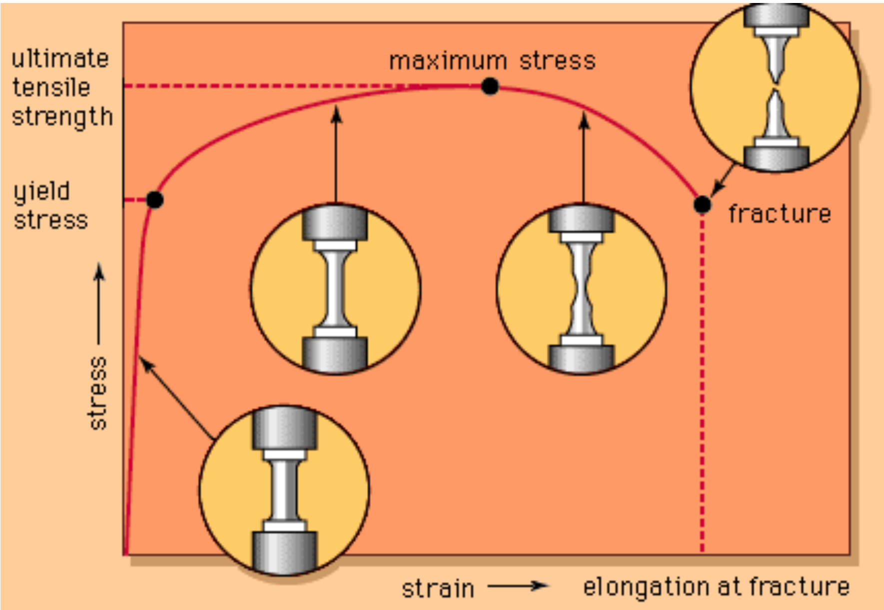
Tensile Test

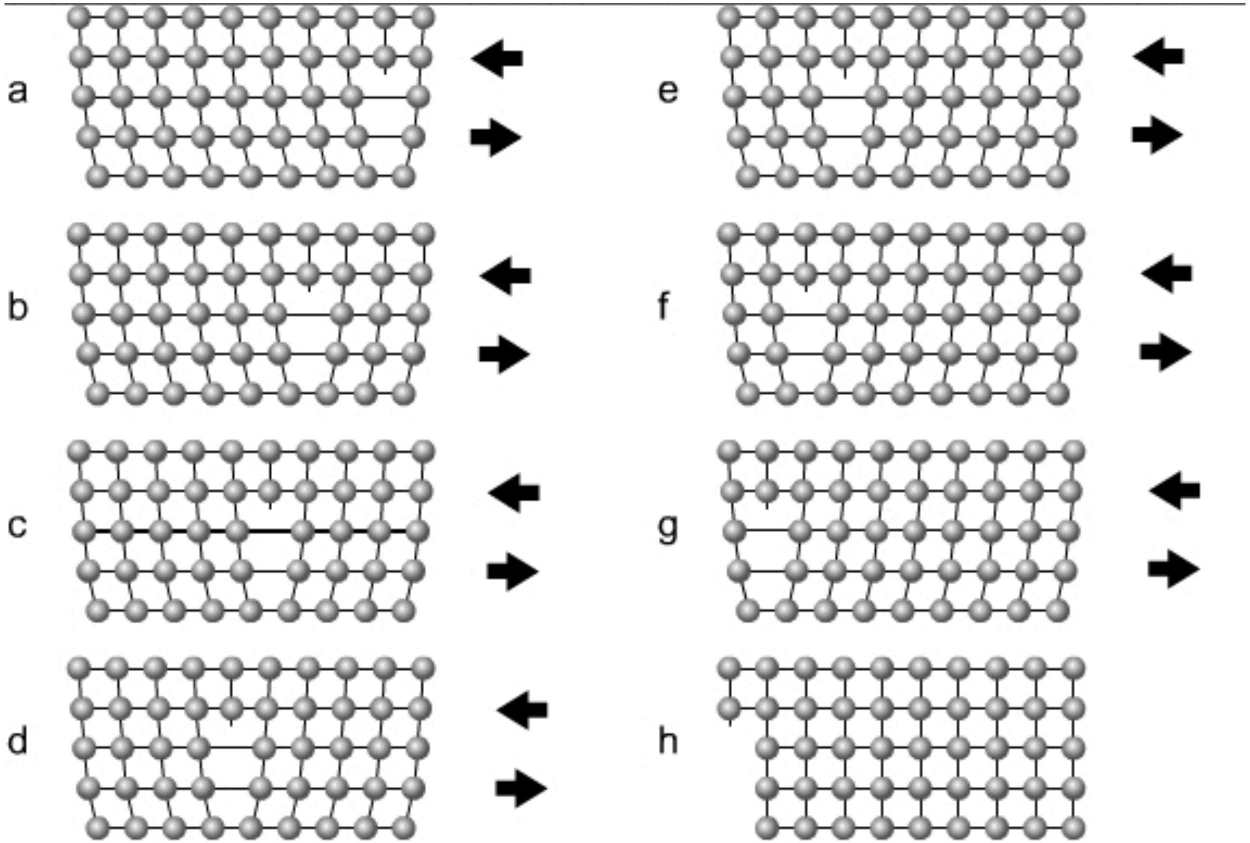


Stress-Strain Curve
900
800
700
600
500
400
300
200
100
0
Stress (MPa)
0
0.02
0.04
0.06
0.08
0.1
0.12
Strain (m/m)
0.2% Offset Line
Ultimate
Tensile
Strength
(UTS)
Yield Strength Point
Breaking Strength
Linear-Elastic Region
Uniform
Elastic
Deformation
Uniform
Plastic
Deformation
Necking
Fracture
Tensile
Specimens

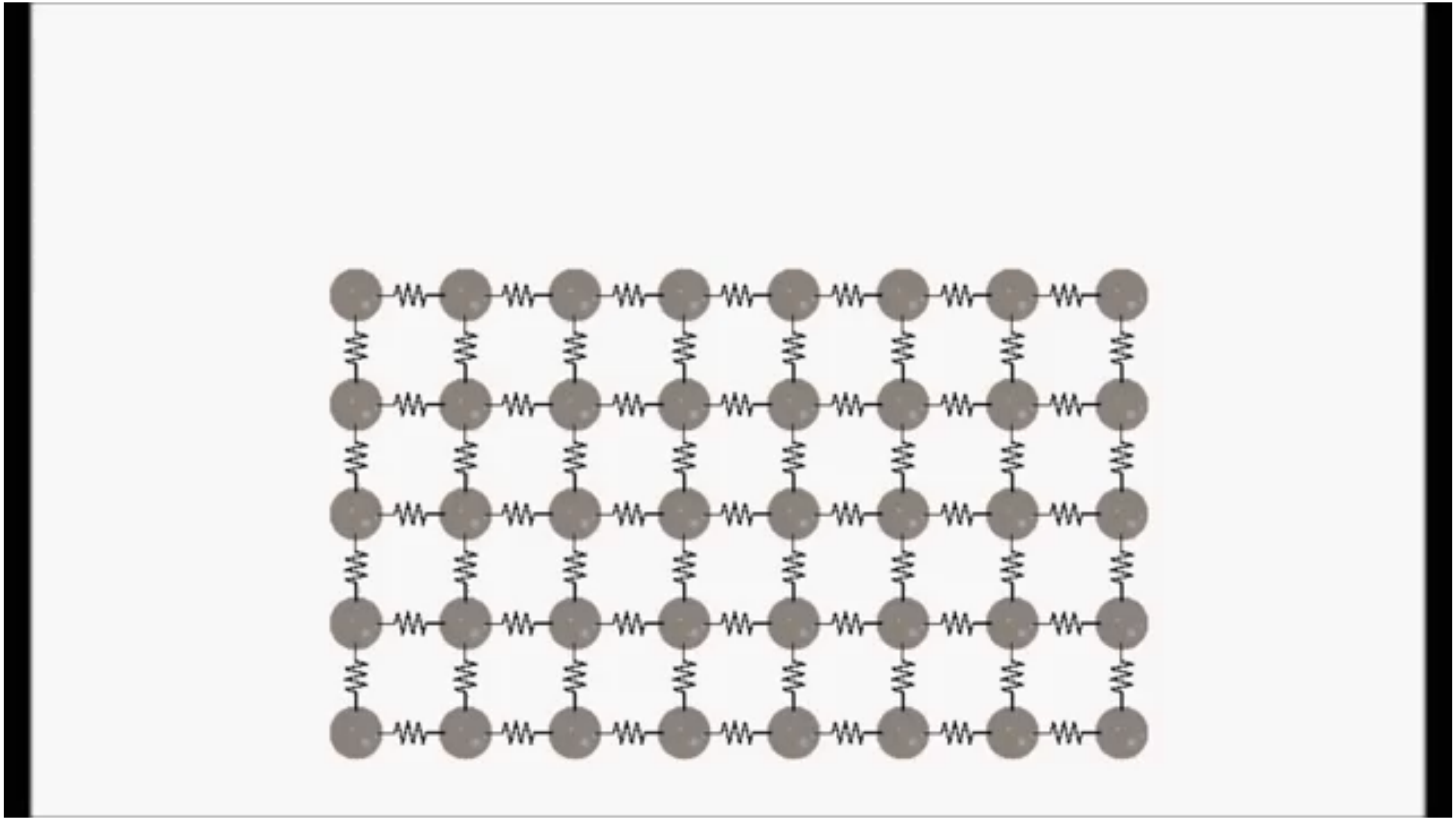
Tensile Test

Dislocations



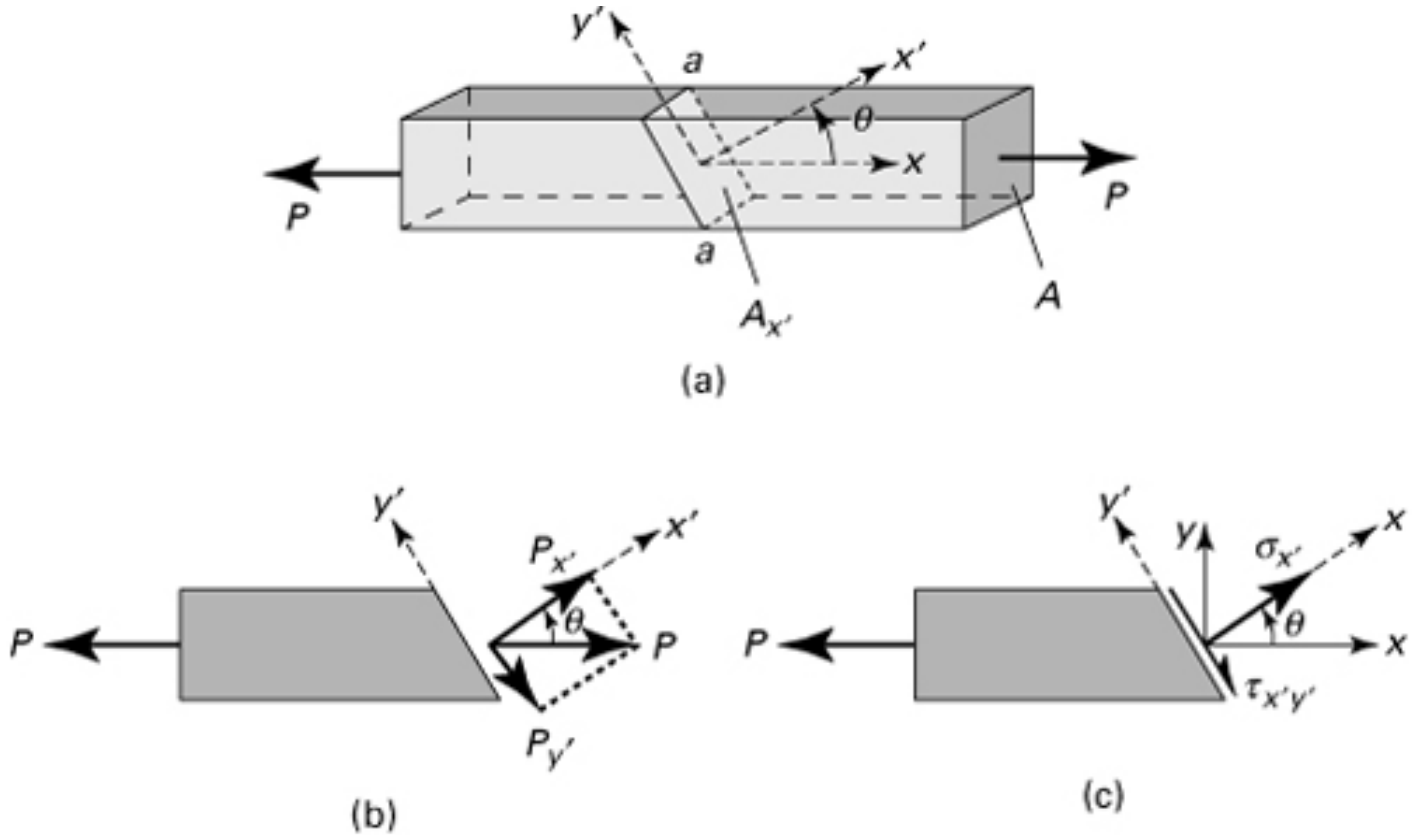


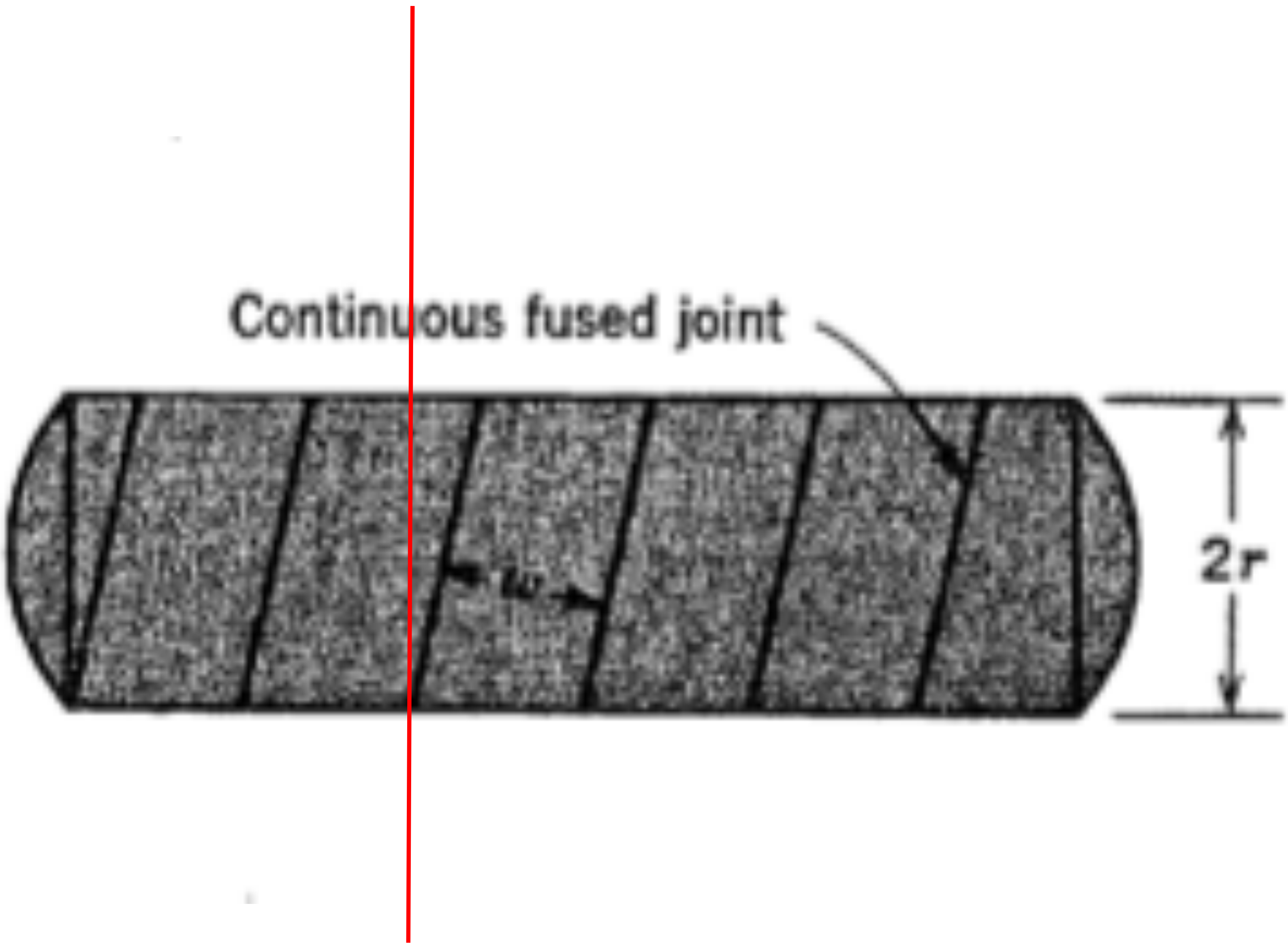
Dislocations



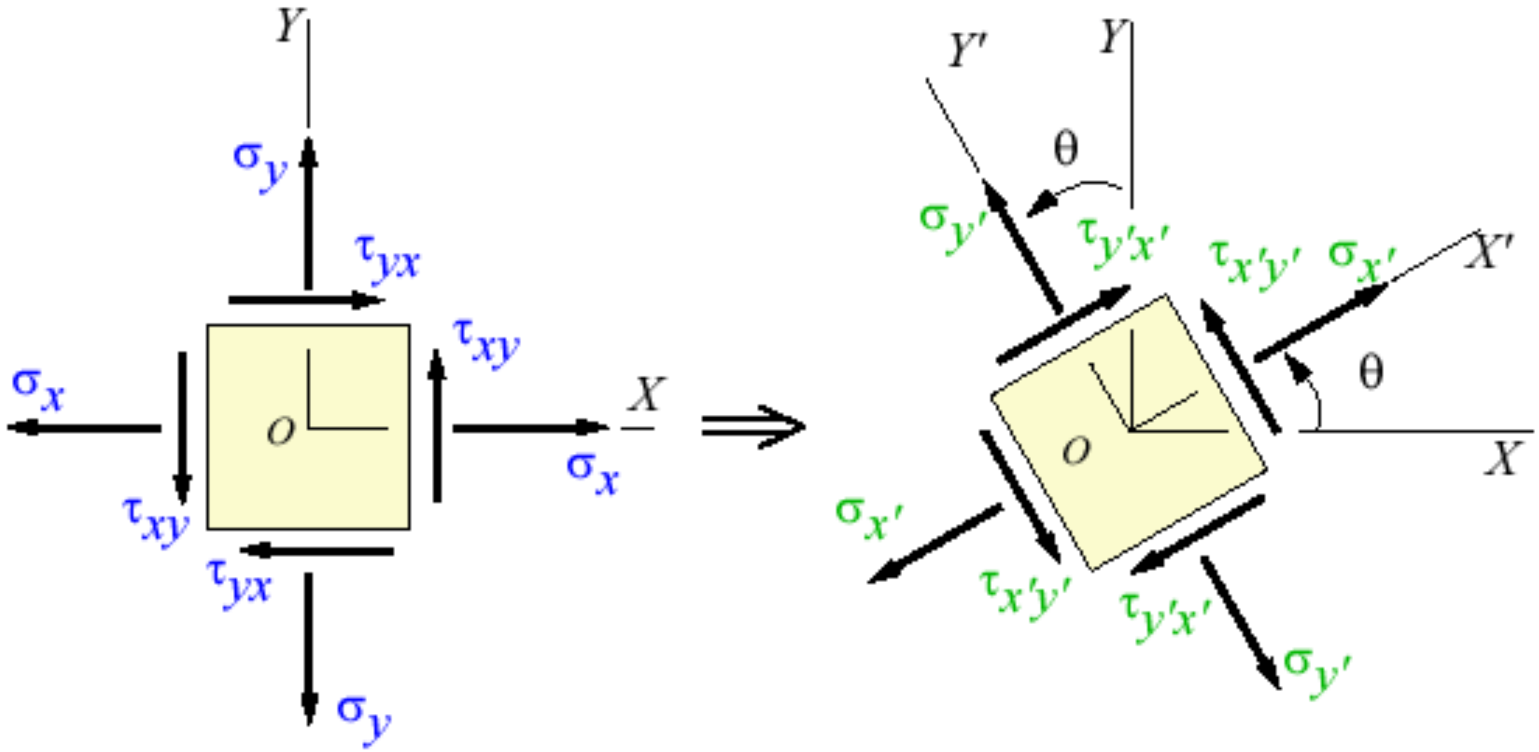
Work Hardening

Stress Transformation




Continuous fused joint
w
2r

Stress Transformation



Stresses at given coordinate system   Stresses transformed to another coordinate