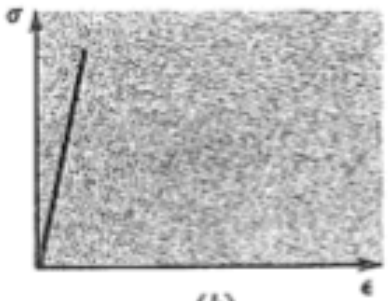
FAILURE CRITERIA

Mechanics of Materials

MM203

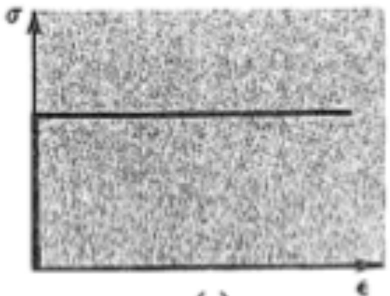
(*a*)

Rigid material.



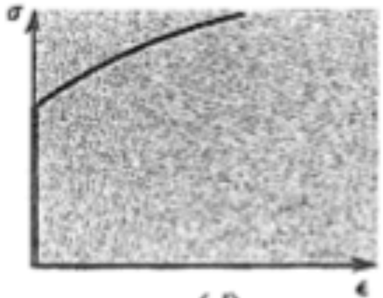
(*b*)

Linearly *elastic* material.



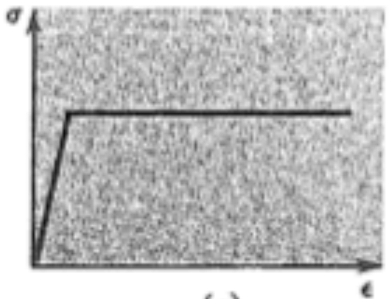
(*c*)

Perfectly plastic material
(non-strain-hardening).



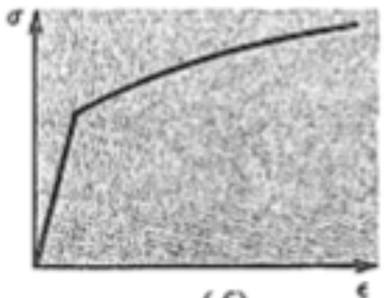
(*d*)

Rigid-plastic material
(strain-hardening).



(*e*)

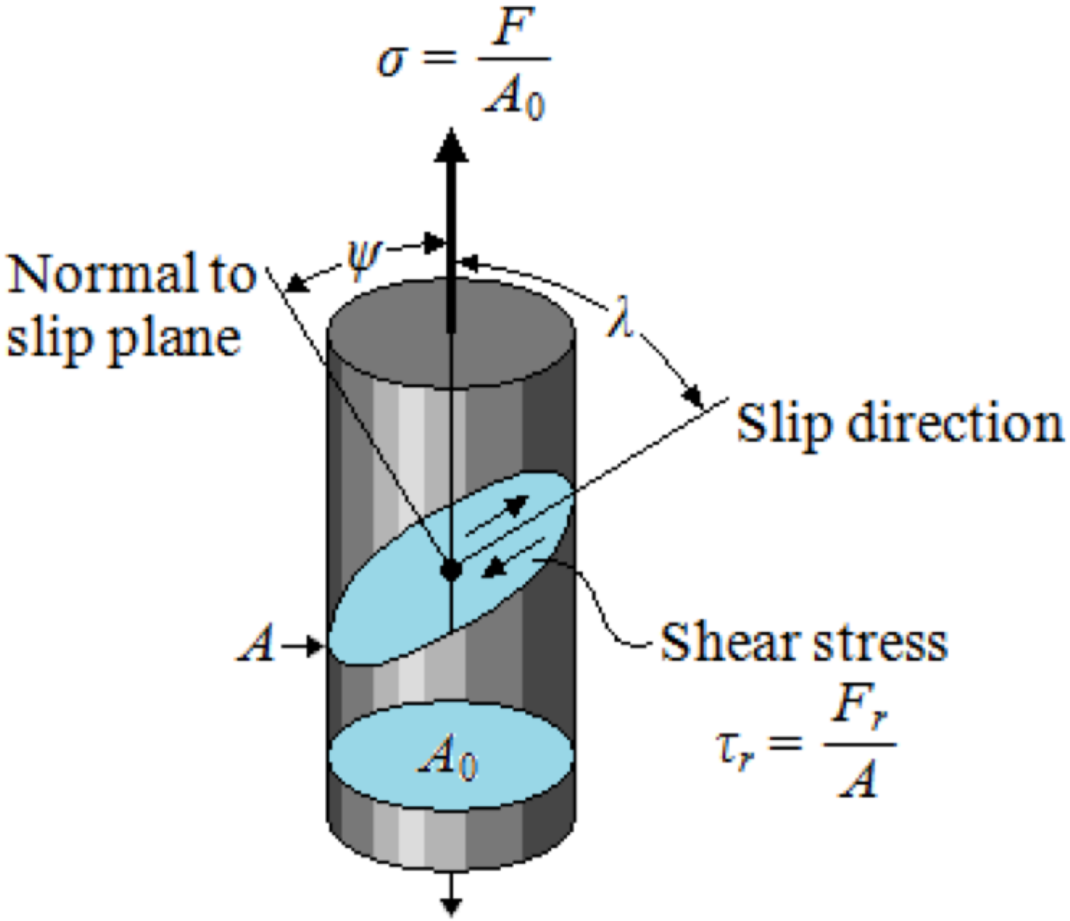
Elastic–perfectly plastic material
(non-strain-hardening).

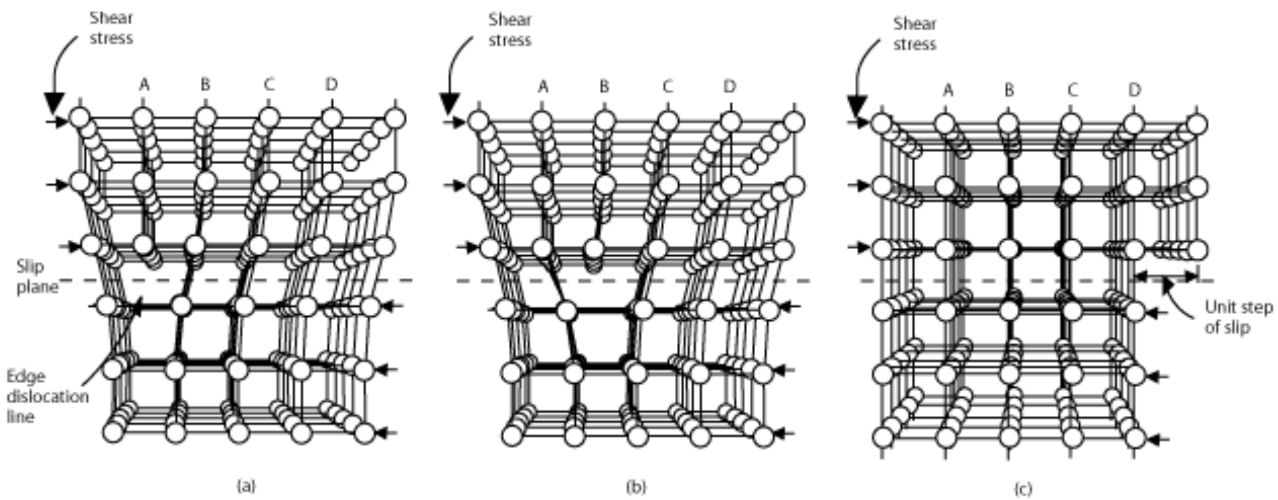


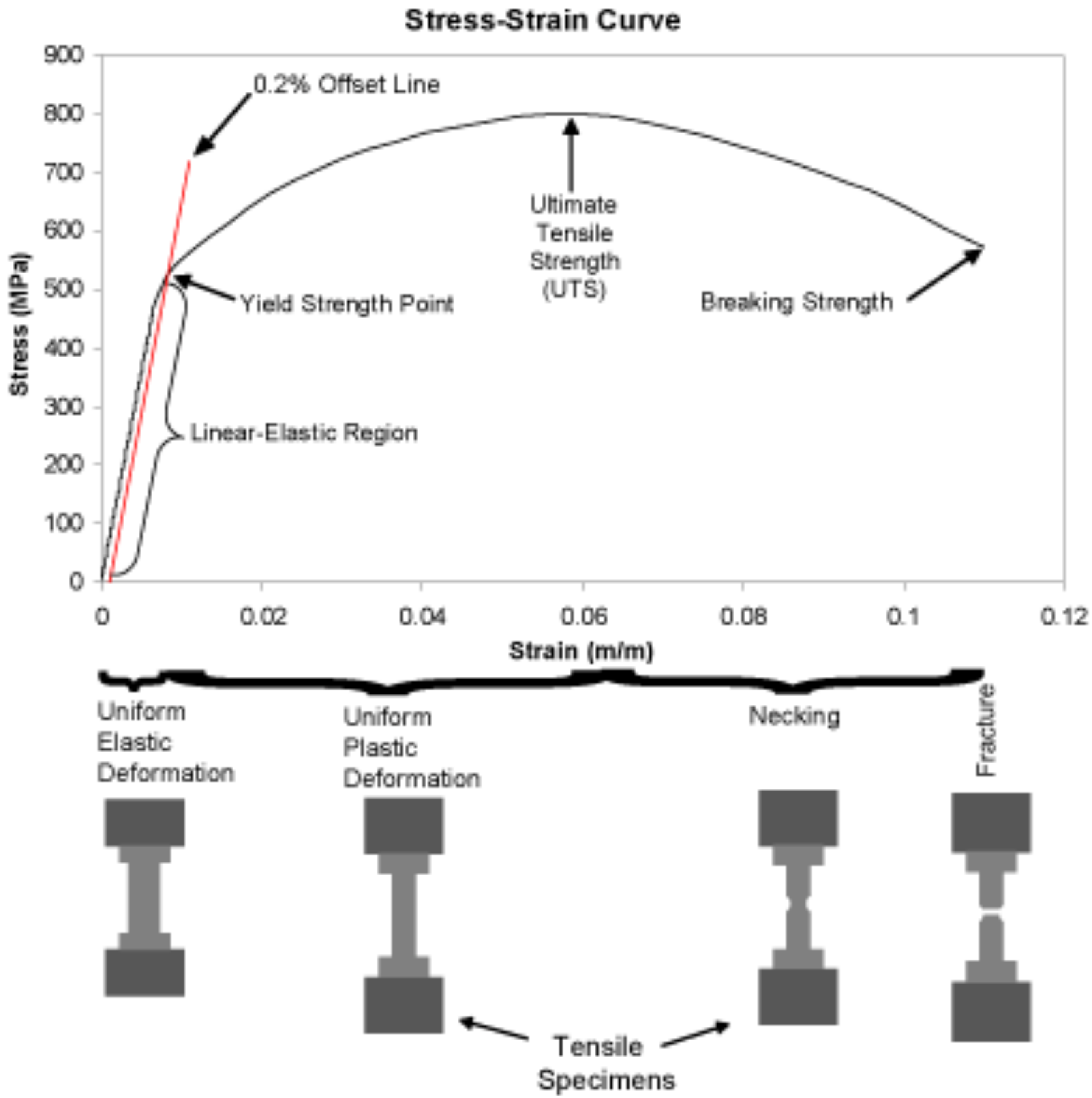
(*f*)

Elastic-plastic material
(strain-hardening).

Plastic yielding





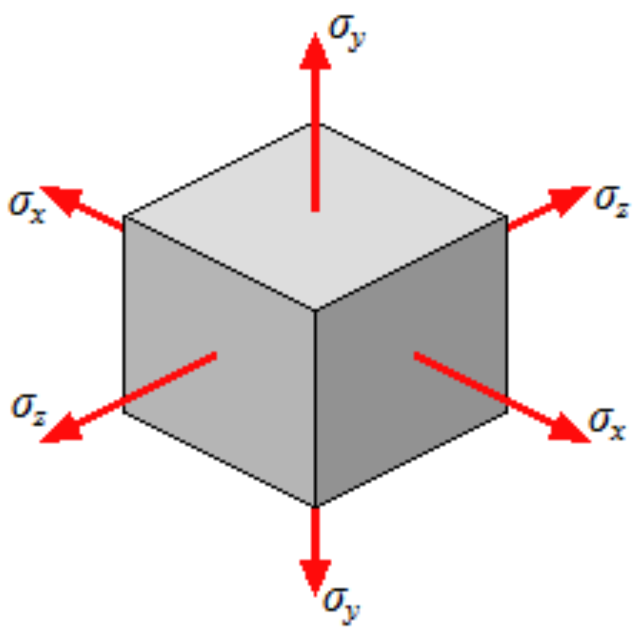


Stress-Strain Curve
900
800
700
600
500
400
300
200
100
0
Stress (MPa)
0
0.02
0.04
0.06
0.08
0.1
0.12
Strain (m/m)
0.2% Offset Line
Ultimate Tensile Strength (UTS)
Yield Strength Point
Breaking Strength
Linear-Elastic Region
Uniform Elastic Deformation
Uniform Plastic Deformation
Necking
Fracture
Tensile Specimens

- Will a material with the following states of stresses fail under the following stress situation
- $\sigma 1=200MPa$, $\sigma 2=-200MPa$
- $\sigma y=250MPa$

Failure Criteria

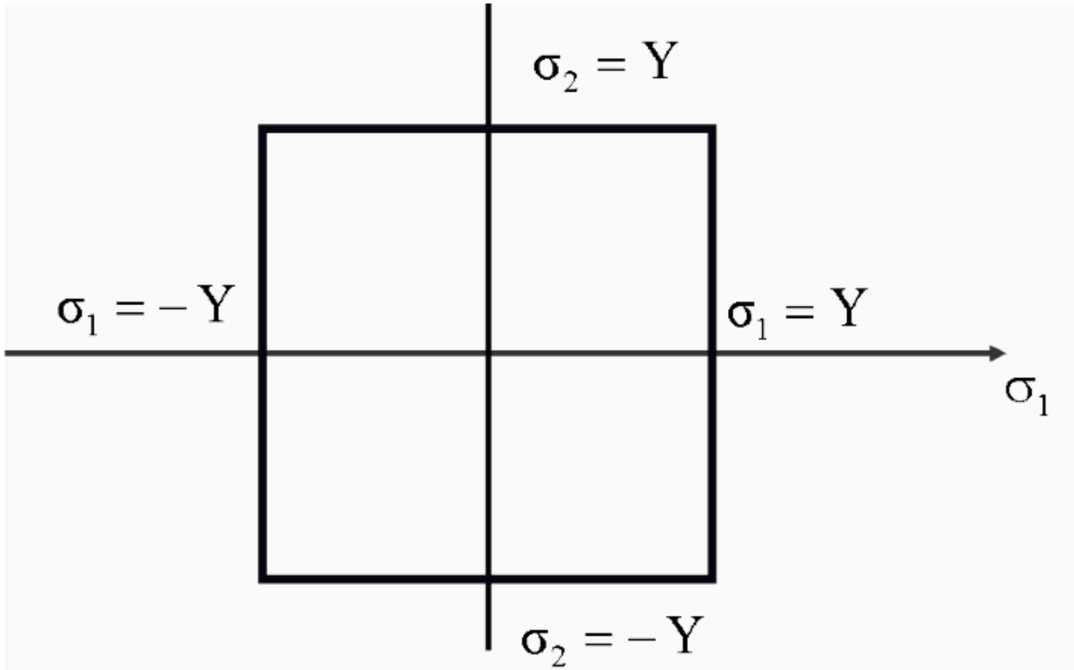
- In uniaxial state of stress, material yields when $\sigma > \sigma_y$.
- When does the material yield when a multiaxial stress state is imposed.
- Three criteria:
- Rankine (maximum principal stress)
- Tresca (maximum shear stress)
- Von Mises (maximum distortion energy)



Rankine stress criteria (maximum principal stress criteria)

- **For brittle materials**.
- A brittle material ruptures when the maximum principal stress in the specimen reaches some limiting value for the material.
- This critical value can be inferred from a tensile test as well.

$$f = \max\left(\left|\sigma_1\right|, \left|\sigma_2\right|, \left|\sigma_3\right|\right) = Y$$



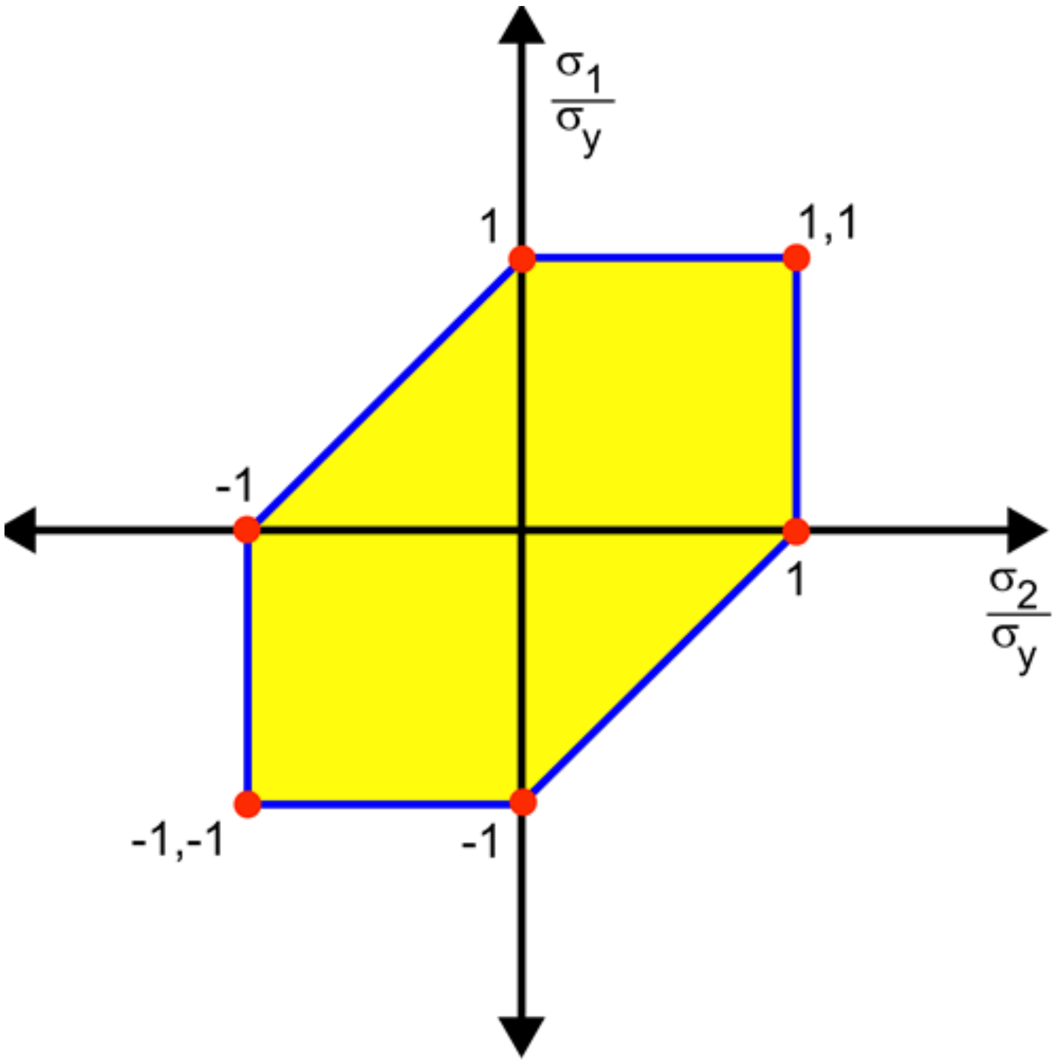
Tresca Criteria

- Yielding starts when the maximum shear stress in the material τ_{max} equals the maximum shear stress at yielding in a simple tension test τ_y.
- For a multiaxial stress state : $\tau_{max}=(\sigma_{max}-\sigma_{min})/2$.
- σ_{max}: Maximum principal stress.
- σ_{min}: Minimum principal stress.
- For yield : $\sigma_{max}-\sigma_{min}=\sigma_y$

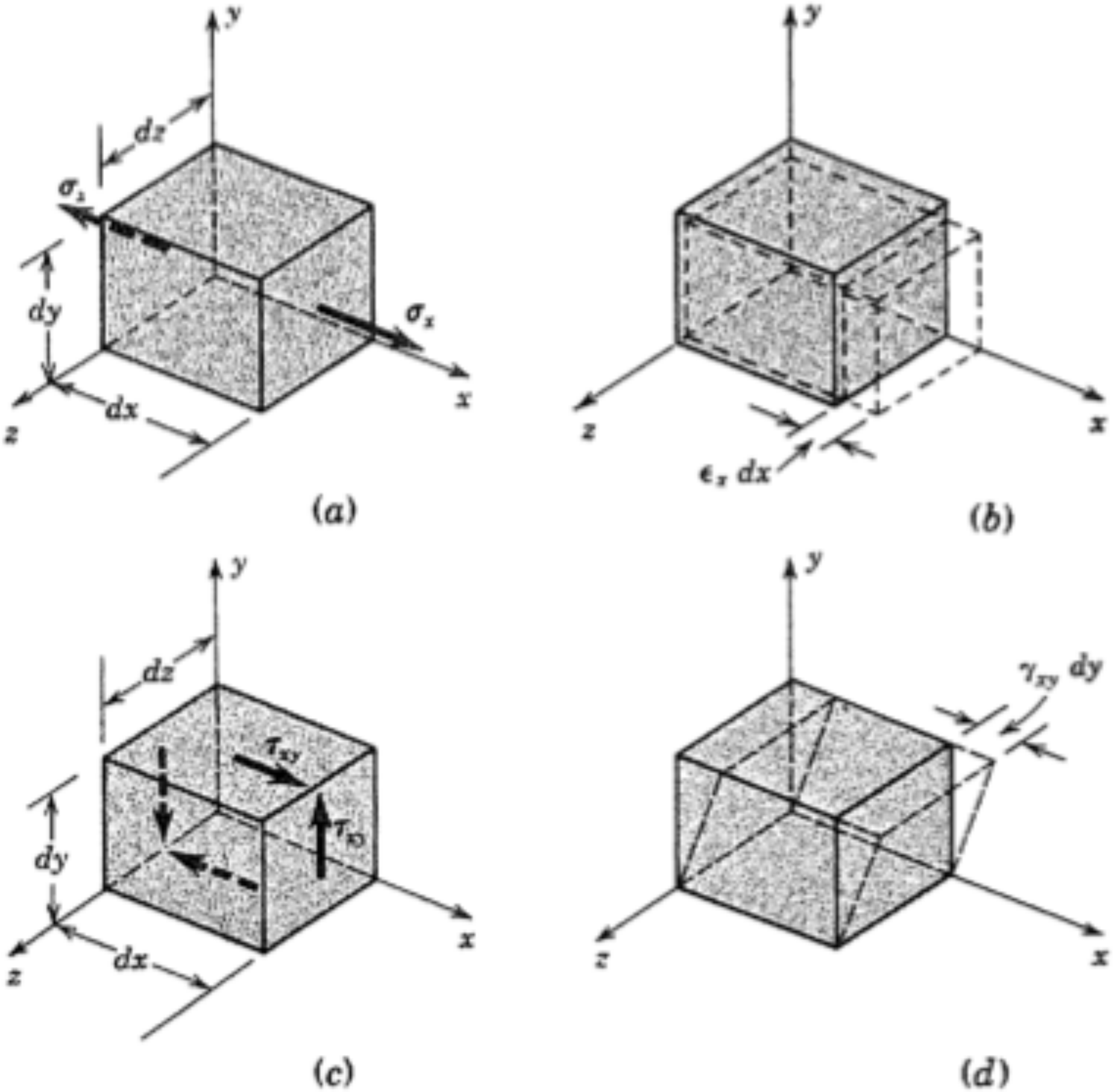
$$\left.\begin{array}{c} |\sigma_{\mathrm{I}} - \sigma_{\mathrm{II}}| = \sigma_{yield} \\ \underline{\underline{\text{or}}} \\ |\sigma_{\mathrm{II}} - \sigma_{\mathrm{III}}| = \sigma_{yield} \\ \underline{\underline{\text{or}}} \\ |\sigma_{\mathrm{III}} - \sigma_{\mathrm{I}}| = \sigma_{yield} \end{array}\right\} \text{for yield}$$

Recall $\sigma_y = 2\tau_y$

Special case: Plane Stress



Strain energy



Dilatation versus distortion

- A generalized state of stress in a material will produce 1) shape change 2) volume change.

$$\mathbf{T}_3 = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

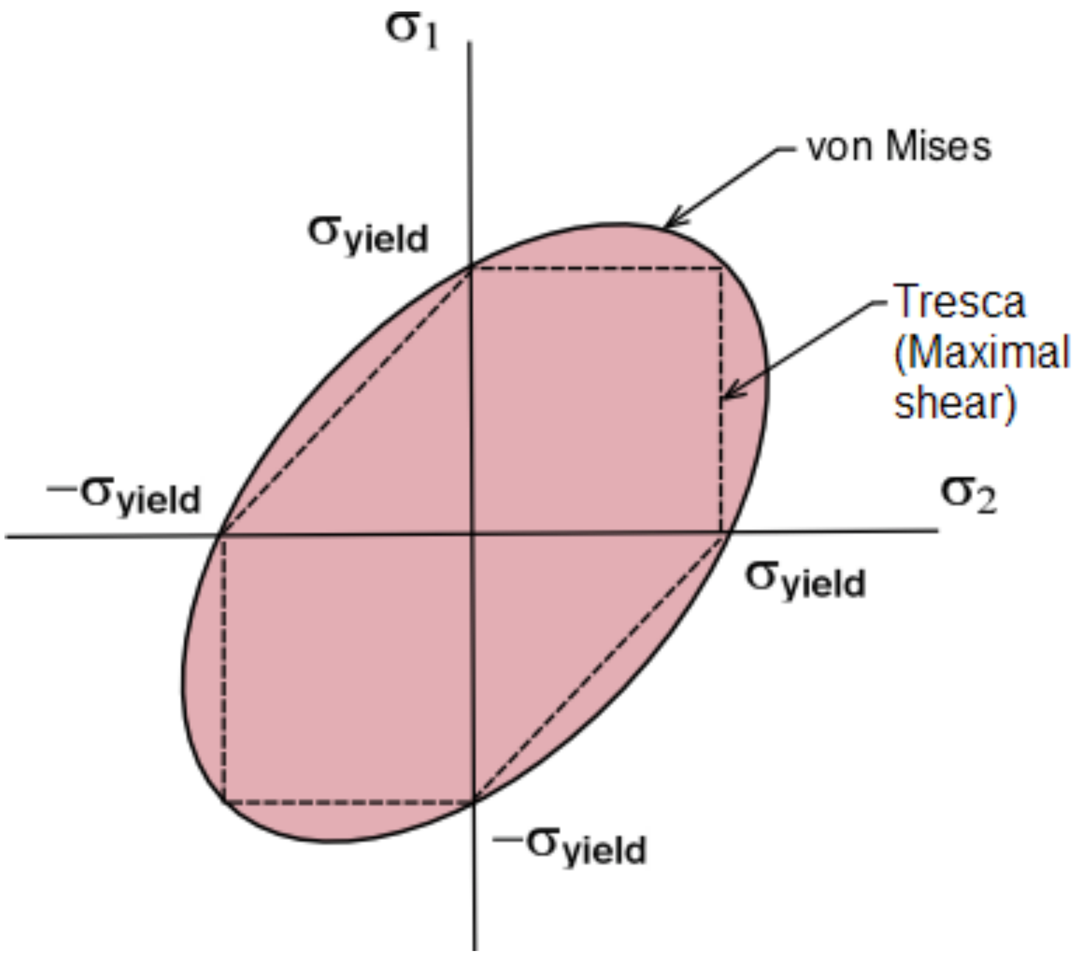
Hydrostatic State of Stress

- $(\sigma_X+\sigma_Y+\sigma_Z)/3=\sigma_H$

Von-Mises Energy Failure Criteria

- A ductile solid will yield when the distortion energy density reaches a critical value for that material.
- This should be true for uniaxial stress state also, the critical value of the distortional energy can be estimated from the uniaxial test.
- Von-Mises Criteria: Material under multiaxial loading will yield when the distortional energy is equal or greater than the critical value of the material.

Yield surface



- Will a material with the following states of stresses fail under the following stress situation
- $\sigma 1=200MPa$, $\sigma 2=-200MPa$
- $\sigma y=250MPa$
- Provide the result while using Tresca criteria versus Rankine.

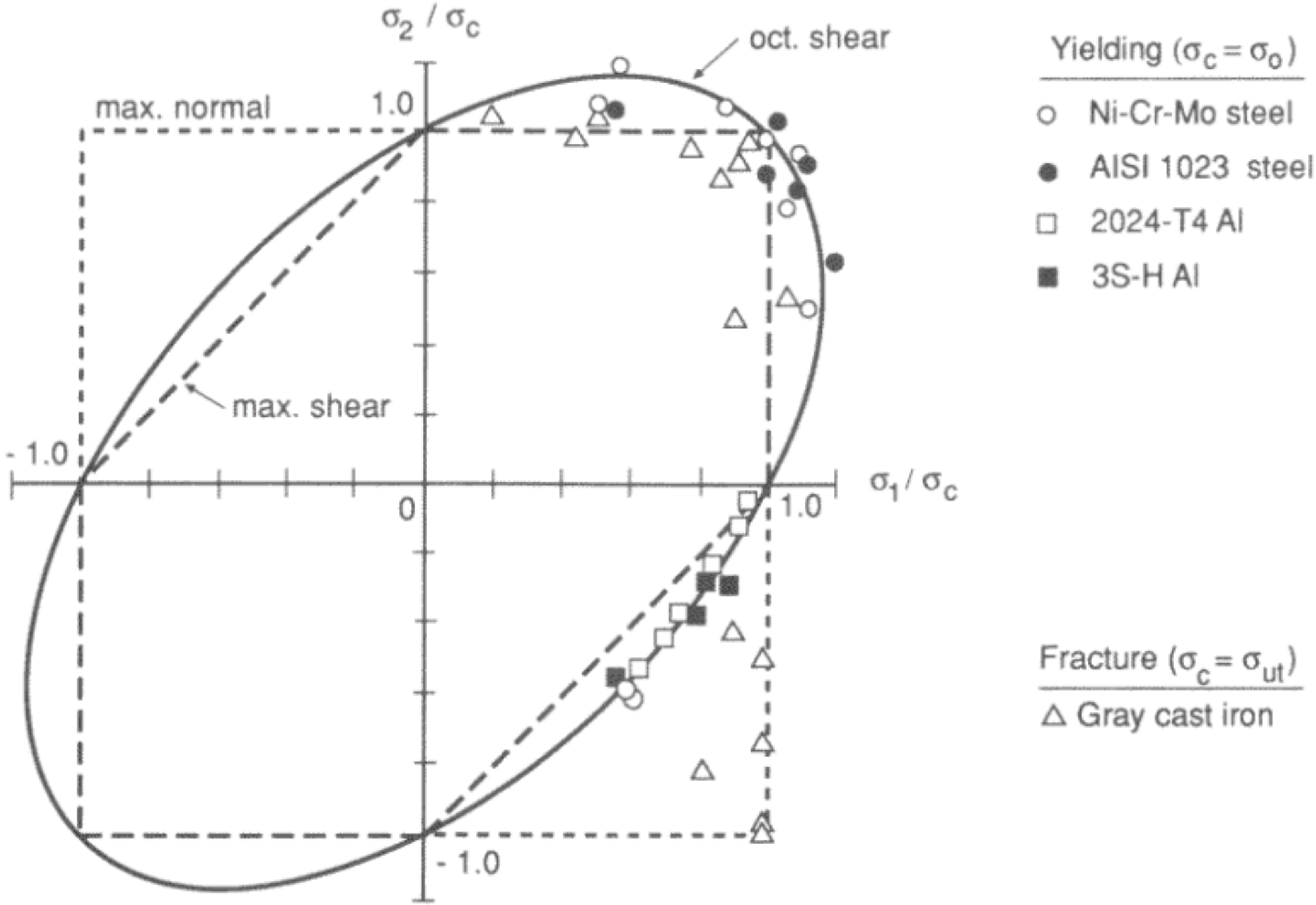


Figure 7.11 Plane stress failure loci for three criteria. These are compared with biaxial yield data for ductile steels and aluminum alloys, and also with biaxial fracture data for gray cast iron. (The steel data are from [Lessells 40] and [Davis 45], the aluminum data from [Naghdi 58] and [Marin 40], and the cast iron data from [Coffin 50] and [Grassi 49].)