INDIAN INSTITUTE OF TECHNOLOGY, BOMBAY
Department of Metallurgical Engineering and Materials Science
MM 209: THERMODYNAMICS : 2017-18: FALL

Tutorial 4 **Date:1/09/2017**

1. Considering that air is an ideal gas and that it contains a mixture of N_2 and O_2 in 79:21 ratio, calculate the minimum work to be done to separate the 1kmol of the mixture at 300K when the pressure is kept constant.

   Work to be done to separate the mixture of N_2 and O_2 = dQ - dU

   Since we considered air to be ideal gas, change in internal energy at the same temperature should be equal to zero

   $dQ = dW = TdS$

   Change in entropy for the separation = $\Delta S_{N2} + \Delta S_{O2}$

   $\Delta S_{N2} = nR\ln W_{N2} = 1000 \times (79/100) \times 8.314 (\ln(79/100))$

   $\Delta S_{O2} = nR\ln W_{O2} = 1000 \times (21/100) \times 8.314 (\ln(21/100))$

   Minimum work needed = $dW = T(\Delta S_{N2} + \Delta S_{O2}) =$

   $= 300 \times 1000 \times 8.314 (0.79\ln(0.79) + 0.21\ln(0.21))$

   $= -1281.91kJ$

   Work to be done to separate N_2 and O_2 from its components is 1281.91kJ

2. A rigid container is divided into two compartments of equal volume by a partition. One compartment contains 1 mole of ideal gas *A* at 1 atm, and the other contains 1 mole of ideal gas *B* at 1 atm. Calculate the increase in entropy, which occurs when the partition between the two compartments is removed. If the first compartment had contained 2 moles of ideal gas *A,* what would have been the increase in entropy when the partition was removed? Calculate the corresponding increases in entropy in each of the above two situations if both compartments had contained ideal gas *A.*

In the first case, when the partition is removed the volume occupied by both A and B will be doubled.
In the case of an ideal gas, when the process is carried out at constant temperature, then
$\Delta S = nR\ln(V_f/V_i)$
For the gas A, $\Delta S_A = 1 \times 8.314 \times \ln 2$
For the gas B, $\Delta S_B = 1 \times 8.314 \times \ln 2$
Total entropy change of the system $= \Delta S_A + \Delta S_B = 8.314 \times \ln 4 = 11.52$

In the second case also when the partition is removed the volume occupied by both A and B will be doubled.

For the gas A, $\Delta S_A = 2 \times 8.314 \times \ln 2$
For the gas B, $\Delta S_B = 1 \times 8.314 \times \ln 2$

Total entropy change of the system $= \Delta S_A + \Delta S_B = 8.314 \times \ln 8 = 17.28$

In the third case, the volume occupied by 2 moles of the gas A will be the same so the total entropy change of the system = 0

In the fourth case, when the partition is removed, the entropy of mixing would be zero because on both the sides we have the same ideal gas, but there is also an expansion that has happened when the partition is removed.
Change in the entropy of 2 moles of A = $2 \times 8.314 \times \ln 2$

Now 3 moles of the gas is occupying 2 times the volume. So the volume of 3 moles of A will be decreased by two thirds.

Total change in entropy is
$= 2 \times 8.314 \times \ln 2 + 3 \times 8.314 \times \ln(2/3)$
$= 8.314 \times (\ln 32/27)$

Alternatively for the 1st case we can also do this using entropy of mixing the ideal gas

3. A body of certain mass is originally at temperature T_1, which is higher than that of a reservoir at temperature T_2. Suppose an engine operates in a cycle between the body and the reservoir until it lowers the temperature of the body from T_1 to T_2, thus extracting heat Q from the body. If the engine does work W, then it will reject heat $Q–W$ *to* the reservoir at T_2. Applying the entropy principle, show that the maximum work obtainable from the engine is

$$W_{max}=Q-T_2(S_1-S_2)$$

If the body is maintained at constant volume having constant volume heat capacity C_v = 8.4 kJ/K which is independent of temperature, and if T_1 = 373 K and T_2 = 303 K, determine the maximum work obtainable.

$\Delta S_{reservoir} = (Q-W)/T_2$
$\Delta S_{body} = S_2-S_1$
$\Delta S_{total} = (S_2-S_1) + (Q-W)/T_2$
From Clausius inequality $\Delta S_{total} \geq 0$

$(S_2-S_1) + (Q-W)/T_2 \geq 0$

$T_2(S_2-S_1) + Q - W \geq 0$

$T_2(S_2-S_1) + Q \geq W$

$W_{max} = T_2(S_2-S_1) + Q = Q-T_2(S_1-S_2)$

For the second part of the question, since the body is kept at constant volume

$(S_2-S_1) = C_v \ln(T_2/T_1)$ and $Q = C_v(T_2-T_1)$

$W_{max} = T_2(S_2-S_1) + Q = T_2 C_v \ln(T_2/T_1) + C_v(T_2-T_1)$

$W_{max} = T_2 C_v \ln(T_2/T_1) + C_v(T_2-T_1)$

Substituting the values from the given information

$W_{max} = 8.4[373 - 303 + 303\ln(303/373)] = 58.99\text{kJ}$