ME220 Theory of Machines and Machine Design

Lec 4 – 20 Jan 2020

Number Synthesis

Number Synthesis refers to determination of the number and order of links and joints necessary to produce motion of a particular DOF (Link Order = Nodes per Link).

Hypothesis: If all joints are full joints, an odd number of *DOF* requires an even number of links and vice versa.

Proof: **Given:** All even integers can be denoted by $2m$ or by $2n$, and all odd integers can be denoted by $2m-1$ or by $2n-1$, where n and m are any positive integers. The number of joints must be a positive integer.

Let : L = number of links, J = number of joints, and $M = DOF = 2m$ (i.e., all even numbers)

Then: rewriting Gruebler's equation to solve for J,

$$J = \frac{3}{2}(L-1) - \frac{M}{2}$$ Eq1

Try: Substituting $M = 2m$, and $L = 2n$ (i.e., both any even numbers):

$$J = 3n - m - \frac{3}{2}$$

This cannot result in J being a positive integer as required.

Try: $M = 2m-1$ and $L = 2n-1$ (i.e., both any odd numbers):

$$J = 3n - m - \frac{5}{2}$$

This also cannot result in J being a positive integer as required.

Try: $M = 2m-1$, and $L = 2n$ (i.e., odd-even):

$$J = 3n - m - 2$$

This is a positive integer for $m \geq 1$ and $n \geq 2$.

Try: $M = 2m$ and $L = 2n-1$ (i.e., even-odd):

$$J = 3n - m - 3$$

This is a positive integer for $m \geq 1$ and $n \geq 2$.

Single-DOF Mechanisms are extremely common in applications. So lets try analyzing different combinations possible to construct 1-DOF Mechanisms using only single DOF joints..

For one-*DOF* mechanisms, we can only consider combinations of 2, 4, 6, 8 . . . links. Letting the order of the links be represented by:

B = *number of binary links*
T = *number of ternary links*
Q = *number of quaternaries*
P = *number of pentagonals*
H = *number of hexagonals*

the total number of links in any mechanism will be:

$$L = B + T + Q + P + H + \cdots$$ Eq2

Since *two link nodes* are needed to make *one joint*:

$$J = \frac{nodes}{2}$$

and

$$nodes = order\ of\ link \times no.\ of\ links\ of\ that\ order$$

then

$$J = \frac{(2B + 3T + 4Q + 5P + 6H + \cdots)}{2}$$ Eq3

Number Synthesis

Substituting expressions for L and J above in Gruebler's equation we get

$$M = 3(B+T+Q+P+H-1) - 2\left(\frac{2B+3T+4Q+5P+6H}{2}\right)$$

$$M = B - Q - 2P - 3H - 3$$

Note what is missing from this equation! The ternary links have dropped out. The *DOF* is independent of the number of ternary links in the mechanism. But because each ternary link has three nodes, it can only create or remove 3/2 joints. So we must add or subtract ternary links in pairs to maintain an integer number of joints. *The addition or subtraction of ternary links in pairs will not affect the DOF of the mechanism.*

Combining Eq 1 & 3

$$\frac{3}{2}(L-1) - \frac{M}{2} = \frac{(2B+3T+4Q+5P+6H)}{2}$$

$$3L - 3 - M = 2B + 3T + 4Q + 5P + 6H \qquad \textbf{Eq4}$$

Combining Eq 4 & 2 to eliminate B

$$L - 3 - M = T + 2Q + 3P + 4H \qquad \textbf{Eq5}$$

We will now solve equations 2 and 5 simultaneously (by progressive substitution) to determine all compatible combinations of links for *DOF* = 1, up to eight links. The strategy will be to start with the smallest number of links, and the highest-order link possible with that number, eliminating impossible combinations.

(Note: *L must be even for odd DOF.*)

CASE 1. $L = 2$

$$L - 4 = T + 2Q + 3P + 4H = -2$$

This requires a negative number of links, so $L = 2$ is impossible.

CASE 2. $L = 4$

$$L - 4 = T + 2Q + 3P + 4H = 0; \qquad \text{so: } T = Q = P = H = 0$$

$$L = B + 0 = 4; \qquad B = 4$$

The simplest one-*DOF* linkage is four binary links—the **fourbar linkage**.

CASE 3. $L = 6$

$$L - 4 = T + 2Q + 3P + 4H = 2; \qquad \text{so: } P = H = 0 \qquad (2.7c)$$

T may only be 0, 1, or 2; Q may only be 0 or 1

If $Q = 0$ then T must be 2 and:

$$L = B + 2T + 0Q = 6; \qquad B = 4, \qquad T = 2 \qquad (2.7d)$$

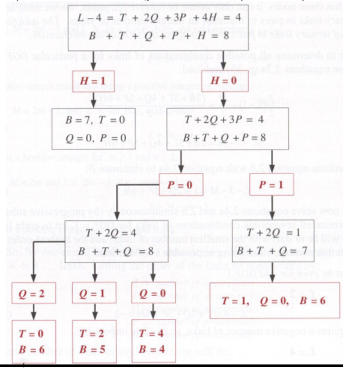
If $Q = 1$, then T must be 0 and:

$$L = B + 0T + 1Q = 6; \qquad B = 5, \qquad Q = 1 \qquad (2.7e)$$

There are then two possibilities for $L = 6$. Note that one of them is in fact the simpler fourbar with two ternaries added as was predicted above.

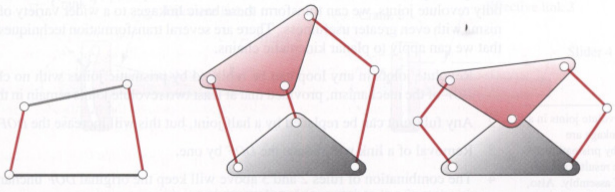
CASE 4. $L = 8$

With 8 links it turns out that there are many feasible combinations. We will not be using this for the purpose of this course (just a summary chart is provided below).



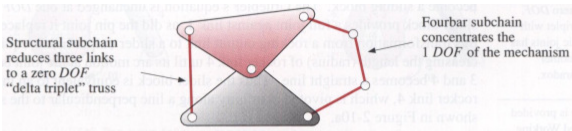
Isomers

Just like isomers of molecules are formed by combining the same atoms in different ways. Isomers of linkages are formed by combining the same set of links and joints in different configurations.



The only fourbar isomer Stephenson's sixbar isomer Watt's sixbar isomer

All valid isomers of 1-DOF four-bar and six-bar linkages.



An invalid six-bar isomer which reduces to the simpler four-bar mechanism