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Spring 2017

MA 573

Computational Methods of Mathematical Finance

Assignment 10 – last assignment

due on Thursday, April 20 in class

1. Price a Bermudan put option $(K - S_T)^+$ by using an implicit finite difference scheme for the Black–Scholes PDE

$$\begin{cases} -v_t(t,s) + \frac{1}{2}\sigma^2 s^2 v_{ss}(t,s) + rsv_s(t,s) &= rv(t,s) \\ v(0,s) &= (s-k)^+ \end{cases}$$

for $S_0 = 100$, $\sigma = 30\%$, $r = 2\%$, $T = 1$, $K = 115$ and quarterly possibility of early exercise (i.e., possible exercise times are $\tilde{t_1} = 0.25$, $\tilde{t_2} = 0.5$, $\tilde{t_3} = 0.75$, and $\tilde{t_4} = 1$). Use 1000 space discretization point on the interval $[0, 200]$ and implement the explicit boundary condition at 0 and a linearity boundary condition at 200. Calculate the price for 100, time discretization steps.

2. Price a American put option $(K - S_T)^+$ by using an implicit finite difference scheme for the Black–Scholes PDE

$$\begin{cases} -v_t(t,s) + \frac{1}{2}\sigma^2 s^2 v_{ss}(t,s) + rsv_s(t,s) &= rv(t,s) \\ v(0,s) &= (s-k)^+ \end{cases}$$

for $S_0 = 100$, $\sigma = 30\%$, $r = 2\%$, $T = 1$, $K = 115$. Use 1000 space discretization point on the interval $[0, 200]$ and implement the explicit boundary condition at 0 and a linearity boundary condition at 200. Calculate the price for 100 time discretization steps using

   (a) the Bermudan approximation for American options,
   (b) the Brennan-Schwartz algorithm

and compare the results.

3. An example where the linearity boundary condition will not work: consider a European power call option $\big((S_T - K)^+\big)^2$ in the Black–Scholes framework

$$\begin{cases} -v_t(t,s) + \frac{1}{2}\sigma^2 s^2 v_{ss}(t,s) + rsv_s(t,s) & = rv(t,s) \\ v(0,s) & = \big((s-k)^+\big)^2 \end{cases}$$

with $S_0 = 100$, $\sigma = 20\%$, $r = 3\%$, $T = 1$ and $K = 115$. As the payoff function is not linear but *quadratic* for large stock prices, the linearity assumption of the pricing function for large prices makes no sense. You will have to choose a finite difference approximation for the spatial derivatives v_s and v_{ss} in the row $s_{\max}$ that does not depends on v at $s_{\max+1}$, i.e., your scheme cannot longer be central but has to use one-sided derivative approximations. Specifically, use 1000 space discretization point on the interval $[0, 200]$ and implement the explicit boundary condition at 0. Calculate the option price using a Crank-Nicolson scheme with 100 time discretization steps.

4. Consider the correlated Hull-White stochastic volatility model

$$\begin{cases} dS_t & = rS_t\,dt + \sqrt{Y_t}\,dW_t^1, \qquad S_0 = s \\ dY_t & = \kappa Y_t\,dt + \xi Y_t\,dW_t^2, \qquad Y_0 = y \\ \mathbb{E}[W_t^1 W_t^2] & = \rho t. \end{cases}$$

   (a) Calculate the generator of the SDE given.

   (b) Derive the Cauchy problem for the price of a European put option in this model.

   (c) Derive a system of ODEs approximating the solution of the PDE. Calculate the $(s_{\max} \times y_{\max}) \times (s_{\max} \times y_{\max})$ matrix A such that for the $(s_{\max} \times y_{\max})$-dimensional vector v it holds that

$$\frac{d}{dt}v = Av.$$

6 points per problems