Selection Algorithms

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1 Selection Algorithms

1.0.1 QuickSelect

- Wikipedia Says:
    1. In computer science, quickselect is a selection algorithm to find the kth smallest element in an unordered list. It is related to the quicksort sorting algorithm.

2 QuickSelect Algorithm PseudoCode

The algorithm asks for the kth smallest element in an unordered list. Let the list be Array.

1. Input to our function are the Array, begin, end, and k
2. Output is the kth smallest element in the Array
3. We get the partition index by calling Partition on the Array with a particular begin and end
4. If the partition index is smaller than k, then we only need to work on the right hand of partition index.
5. If the partition index is greater than k, then we only need to work on the left hand of partition index.

```
In [115]: def partition_select(A, begin, end):
              if end <= begin:
                  return begin

              import random, math
              random_loc = int(begin + math.floor((random.random() * (end - begin + 1))))
              A[end], A[random_loc] = A[random_loc], A[end]
              x = A[end]
              i = begin - 1
              j = begin
              while j < end:
                  if A[j] <= x:
                      i = i + 1
                      A[i], A[j] = A[j], A[i]
                  j = j + 1
              assert j == end
```

```
            A[i+1], A[j] = A[j], A[i+1]
            return (i+1)

        def quick_select(A, begin, end, k):
            if end > begin:
                q = partition_select(A, begin, end)
                if q < k:
                    quick_select(A, q+1, end, k)
                elif q > k:
                    quick_select(A, begin, q, k)
                elif q == k:
                    print "kth smallest is:" + str(A[k])
                    return A[k]
```

```
In [117]: A = [0, 1, 3, 5, 4, 2, -3, -2, 11]
          quick_select(A, 0, len(A) - 1, 0)
```

```
kth smallest is:-3
```

```
In [ ]: A
```

3 Appendix: QuickSort

3.1 Let's understand QuickSort

3.1.1 Divide and Conquer Idea:

Divide:

```
The idea is to find a pivot. The pivot is an index q in the array A[p:r],
such that all elements of A[p:q-1] are less than or equal to A[q],
which is less than or equal to elements of A[q+1:r].

A[p:q-1] and/or A[q+1:r] are/is possibly empty.
```

Conquer:

```
Sort the array A[p:r] by recursive calls to Divide
```

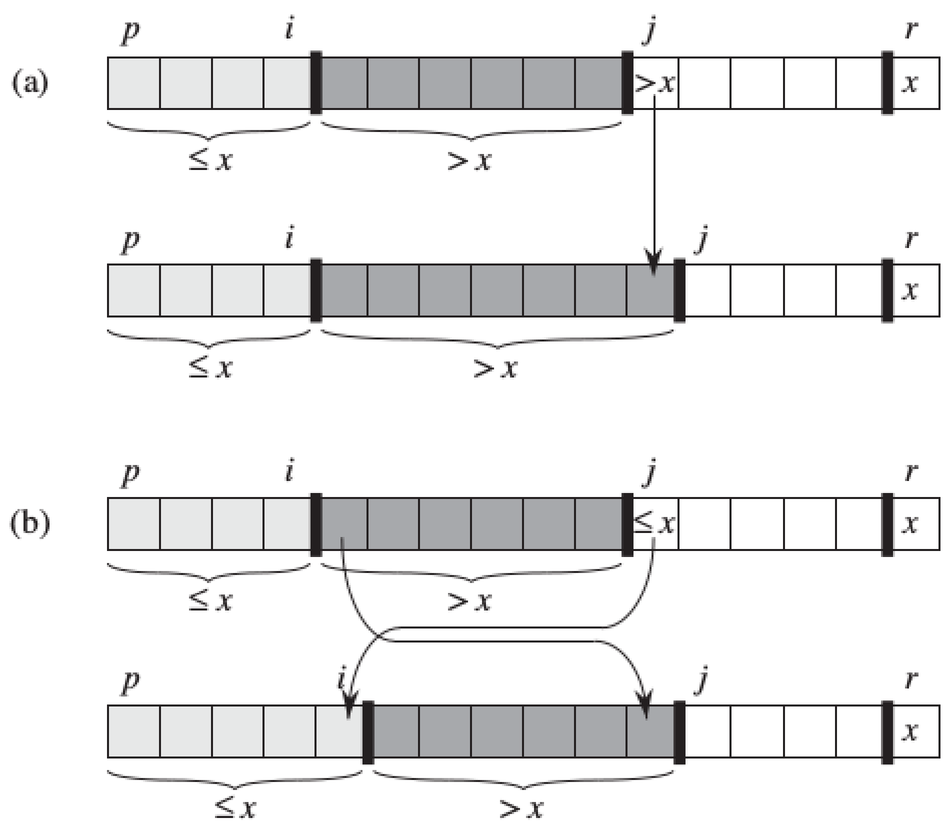
Combine

```
Merge the sorted arrays A[p:q-1] and A[q+1:r].
Because the sub arrays are sorted, the whole array is already sorted.
Combine is basically not reqired.
```

3.1.2 Let's write the Partition Function

Algorithm for partition:

1. input to the algorithm: Array, p = start index for the sub array, r = end index for the sub array
    1. Initialize i = p - 1 : i is the index that tells that elements from p to i (inclusive) are smaller than Array[r]
2. Case b in the diagram beloww: Traverse the array: j=0 to j < r if Array[j] is smaller than or equal to Array[r]: then, move Array[j] to Array[i+1] and update i to i+1: (Verify in the image below that the loop invariant is satisfied) also increment j to make it ready for the next iteration iterate with new j
3. Loop has ended: j must be equal to r now: because the above loop's termination condition is that. Now i+1 is the first element that is greater than or equal to Array[r] swap (i+1)th position and rth position: At this stage, till i everything is smaller than Array[r] return i+1: i+1 is the pivot.



QuickSort Algorithm Loop Invariant

```
In [1]: def partition(array, p,r):
            print "**********Working within p:" + str(p) + " and r:" + str(r)
            # initialize data for entering the loop
```

```python
        i = p - 1
        x = 0
        if r < len(array):
            x = array[r]
        j = p
        while j < r:
            if array[j] <= x:
                i = i + 1
                print "Swap intermediate i:" + str(i) + " and j:" + str(j)
                array[i], array[j] = array[j], array[i]
                j = j + 1
            else:
                j = j + 1
        print "Swap (i + 1):" + str(i+1) + " and j:" + str(j)
        if i < j:
            array[i+1], array[j] = array[j], array[i+1]
        return i+1

    def quicksort(A, p , r):
        if p < r:
            q = partition(A, p, r)
            print A[p:q], A[q], A[q+1:r]
            quicksort(A, 0, q-1)
            quicksort(A, q+1, r)
        else:
            return
```

In [2]:
```python
A = [6, 0, 5, 13, 13, 2, 1, 2]
quicksort(A, 0, len(A) - 1)
```

```
**********Working within p:0 and r:7
Swap intermediate i:0 and j:1
Swap intermediate i:1 and j:5
Swap intermediate i:2 and j:6
Swap (i + 1):3 and j:7
[0, 2, 1] 2 [13, 6, 5]
**********Working within p:0 and r:2
Swap intermediate i:0 and j:0
Swap (i + 1):1 and j:2
[0] 1 []
**********Working within p:4 and r:7
Swap intermediate i:4 and j:4
Swap intermediate i:5 and j:5
Swap intermediate i:6 and j:6
Swap (i + 1):7 and j:7
[13, 6, 5] 13 []
**********Working within p:0 and r:6
```

```
Swap intermediate i:0 and j:0
Swap intermediate i:1 and j:1
Swap intermediate i:2 and j:2
Swap intermediate i:3 and j:3
Swap (i + 1):4 and j:6
[0, 1, 2, 2] 5 [6]
**********Working within p:0 and r:3
Swap intermediate i:0 and j:0
Swap intermediate i:1 and j:1
Swap intermediate i:2 and j:2
Swap (i + 1):3 and j:3
[0, 1, 2] 2 []
**********Working within p:0 and r:2
Swap intermediate i:0 and j:0
Swap intermediate i:1 and j:1
Swap (i + 1):2 and j:2
[0, 1] 2 []
**********Working within p:0 and r:1
Swap intermediate i:0 and j:0
Swap (i + 1):1 and j:1
[0] 1 []
**********Working within p:5 and r:6
Swap intermediate i:5 and j:5
Swap (i + 1):6 and j:6
[6] 13 []
**********Working within p:0 and r:5
Swap intermediate i:0 and j:0
Swap intermediate i:1 and j:1
Swap intermediate i:2 and j:2
Swap intermediate i:3 and j:3
Swap intermediate i:4 and j:4
Swap (i + 1):5 and j:5
[0, 1, 2, 2, 5] 6 []
**********Working within p:0 and r:4
Swap intermediate i:0 and j:0
Swap intermediate i:1 and j:1
Swap intermediate i:2 and j:2
Swap intermediate i:3 and j:3
Swap (i + 1):4 and j:4
[0, 1, 2, 2] 5 []
**********Working within p:0 and r:3
Swap intermediate i:0 and j:0
Swap intermediate i:1 and j:1
Swap intermediate i:2 and j:2
Swap (i + 1):3 and j:3
[0, 1, 2] 2 []
**********Working within p:0 and r:2
Swap intermediate i:0 and j:0
```

```
Swap intermediate i:1 and j:1
Swap (i + 1):2 and j:2
[0, 1] 2 []
**********Working within p:0 and r:1
Swap intermediate i:0 and j:0
Swap (i + 1):1 and j:1
[0] 1 []
```

```
In [3]: A
```

```
Out[3]: [0, 1, 2, 2, 5, 6, 13, 13]
```

4 Questions from CLRS:

1. 7.1-1 Using Figure 7.1 as a model, illustrate the operation of PARTITION on the array A = [13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11]

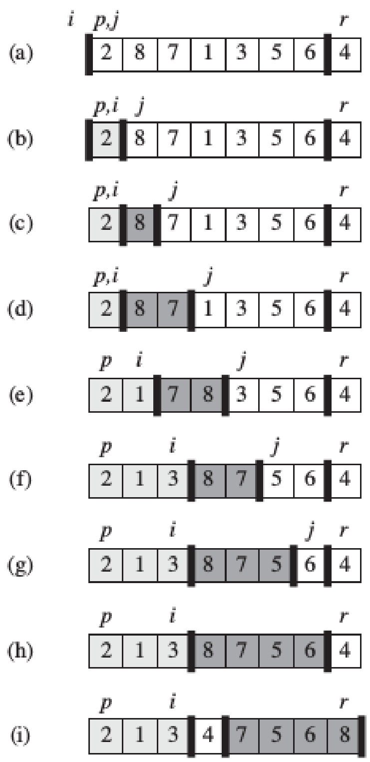


Figure 7.1 The operation of PARTITION on a sample array. Array entry $A[r]$ becomes the pivot element x. Lightly shaded array elements are all in the first partition with values no greater than x. Heavily shaded elements are in the second partition with values greater than x. The unshaded el

Answer:

When the function finishes the array looks like the following: [9, 5, 8, 7, 4, 2, 6, 11, 21, 13, 19, 12] where index of 11 is the pivot.

2. 7.1-2 What value of q does Partition function return when all elements in the array A[p..r]

have the same value? Modify Partition function so that

$$q = \left\lfloor \frac{p+r}{2} \right\rfloor$$

when all elements in the array A[p..r] have the same value.

Answer: If all the elements in the array A[p..r] are same then Partition function tries to swap each element with itself for the whole array. Partition function would return the last index of the array.

Also look at StackOverflow link

```
In [22]: def partition2(array, p, r):
             i = p - 1
             q = p
             x = array[r]
             all_equal = False
             if array[q] == x:
                 all_equal = True
             while q<r:
                 if array[q] == x:
                     i = i + 1
                     q = q + 1
                 elif array[q]<x:
                     all_equal = False
                     i = i + 1
                     array[q], array[i] = array[i], array[q]
                     q = q + 1
                 else:
                     all_equal = False
                     q = q + 1
             # assert j = r
             if all_equal == True:
                 import math
                 return math.floor((p+r)/2)
             else:
                 array[i+1], array[q] = array[q], array[i+1]
                 return i+1
```

```
In [23]: partition2([2, 2, 2, 2, 2], 0, 4)
```

```
Out[23]: 2.0
```

3. 7.1-3 Give a brief argument that the running time of Partition on a subarray of size n is $\Theta(n)$.

   Answer: To show that the running time of Partition on a subarray of size n is $\Theta(n)$, all we need to do is to show that its $\mathcal{O}(n)$ and $\Omega(n)$

   So, in the worst case scenario, the Partition algorithm has to traverse the whole array once. This proves that lower bound of Partition algorithm is $\Omega(n)$ In the best case scenario as well Partition function at least needs to increment q for each element in the array. This proves that the algorithm has an upper bound of $\mathcal{O}(n)$

4. 7.1-4 How would you modify QUICKSORT to sort into nonincreasing order?

```
In [82]: #Answer to 7.1-4:
             #In the partition function just change "<=" to ">="
         def partition_decreasing_order(array, p, r):
             i = p - 1
             if p == r:
                 return p
             x = array[r]
             q = p
             while q<r:
                 if array[q]>=x:
                     i = i + 1
                     array[q], array[i] = array[i], array[q]
                     q = q + 1
                 else:
                     q = q + 1
             #assert q = r
             array[i+1], array[q] = array[q], array[i+1]
             return i+1

         def quicksort_decreasing_order(A, p, r):
             if p < r:
                 q = partition_decreasing_order(A, p, r)
                 quicksort_decreasing_order(A, p, q-1)
                 quicksort_decreasing_order(A, q+1, r)
```

```
In [83]: A = [2, 3, 1, 6, 5, 9]
         quicksort_decreasing_order(A, 0, 5)
```

```
In [84]: A
```

```
Out[84]: [9, 6, 5, 3, 2, 1]
```

5 Unit Testing QuickSort

Get started with pytest

```
In [65]: import pytest
```

```
In [85]: def test_quicksort():
             A = [2, 3, 1, 6, 5, 9]
             p = 0
             r = len(A) - 1
             quicksort_decreasing_order(A, p, r)
             assert A == [9, 6, 5, 3, 2, 1]
```

```
In [78]: test_quicksort()
```

```
In [86]: def test_quicksort_trivial():
             A = [1]
             p = 0
             r = len(A) - 1
             quicksort_decreasing_order(A, p, r)
             assert A == [1]
```

```
In [87]: test_quicksort()
```

```
In [88]: test_quicksort_trivial()
```