CG - Raghuraj Singh

UNIT-1

Pixel :→ The pixel is the smallest addressable screen element.

Frame Buffer :→ We may wish to place the intensity value for all pixels in to an array in our computer memory. This array can determine the intensity at which each pixel should be displayed. The array which contains an internal representation of the image is called the framebuffer. It collects and stores pixel value for use by the display devices.
"A frame buffer is a memory area in which a picture is stored in the form of pixel."

Vector Generation :→ The process of turning on the pixels for a line segment is called vector generation i.e. if we have to draw the line, for that we have to set intensity of pixels that lies on it.

Line Segments :→ If two endpoints p_1 and p_2 lies on a line then this is called a line segment. for example below line $p_1 p_2$ is a line segment.

Why Computer Graphics ?

Suppose we have to show the performance of some sugar factory related with profit since from 1965. We required large number of pages to store this huge information related with financial numerical and statistical information. Now for common people it requires a lot of time to understand it.

There is an alternative to show or represent this with the help of graphical tools such as bar charts or pie diagram. i.e. we can express our data in pictorial form. Hence any person can understand it at a glance. Thus computer graphics is very useful in business to represent their data, market share and profit margin.

There are many s/w packages and graphics languages. Many display devices are developed. The problem for all such is that the same program may not work on another installation. In the late 1970 the CORE System was developed. This system provides a standardized set of commands to control the construction and display of graphic images. The CORE system defined basic graphics primitive from which more complex or special purpose graphics routine could be built. The CORE system contained mechanism for describing and displaying both two dimensional and three dimensional structures.

A second standard called the graphics kernel system was developed. The GKS was heavily influenced by core. The GKS standard did contain primitive for imaging areas and colours. It introduced the concept of workstation. Another graphics standard is the programmers hierarchical interactive graphics standard (PHIGS).

takes the input and output functions and viewing model from CORE and GKS, but it is a programmers toolbox intended for programming graphics applications.

Two another graphics standards —

CGM (Computer Graphics Metafile) :→

It is a file format for picture information that allows device independent Capture, storage and transfer.

CGI (Computer Graphics Interface)

It is a Companion standard which provides a procedural interface for the CGM primitives.

Video Controller :→ A video controller is often referred as a video adapter or graphics card which is an important hardware Component. It is used to generate graphics information on display device such as monitor. Now a days Computer do not have these Video Cards but instead of this Motherboards are itself having graphics processing units which handles the task of video adapters.

Video adapter :→ A video adapter or display adapter is an integrated Circuit Card in a Computer that provides digital to analog Converter, Video RAM and a Video Controller so that data Can be sent to a Computer display. Display adapters are Characterized by,

1) Resolution
2) Color Depth
3) Refresh Rate
4) Acceleration

1) Resolution :→ Resolution refers to the maximum number of points that Can be displayed without overlap on a CRT is referred to as the resolution. A more precise definition of resolution is the number of points per Centimeter that can be plotted horizontally and vertically. Four resolution are in Common use today.

640 x 480, 800 x 600, 1024 x 768, 1280 x 1024.

Computer display generates Color by Combining amount of Red, Green and Blue. These colors are controlled by 3 wires in the display Cable. Each has a variable amount of Voltage represented by a number from 0 to 255. This produces upto 16 million possible Colors.

2) Color Depth (No. of Colors):→ Color depth is determined by the no. of bits assigned to hold Color Value.

- 1 bit — 2 Colors (black and white)
- 4 bits — 16 Colors
- 8 bits — 256 Colors
- 16 bits — 32,000 Colors.

3) Refresh Rate:→ Refresh Rate is nothing but the speed by which a particular dot on the screen is getting printed.

4) Accelerator:→ Accelerator chip is an Integrated chip existing on the display adapter. It is used to draw readymade shapes like drawing

ॐ

es :→ A point Can be said as a position in a plane, and a point Can be specified with an ordered pair of numbers (x, y), where x is horizontal distance from the origin and y is vertical distance from origin.
If we Connect two points then the results will be a line. Suppose two points (x_1, y_1) and (x_2, y_2) used to Specify a line, then we say an another point (x, y) is on the line if and only if it satisfies the line equations:, which is given below.

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \longrightarrow (1)$$

equation (1) shows that the slope between any point on the line and (x_1, y_1) is the same as the Slope between (x_2, y_2) and (x_1, y_1).
Now solve the equation for ~~line~~ . y.

$$(y-y_1)(x_2-x_1) = (y_2-y_1)(x-x_1)$$

$$(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y = \frac{y_2-y_1}{x_2-x_1}(x-x_1) + y_1$$

$$= \frac{y_2-y_1}{x_2-x_1}x - \frac{y_2-y_1}{x_2-x_1}x_1 + y_1$$

$$y = mx + y_1 - mx_1 \longrightarrow (2)$$

We know the standard equation of line is

$$y = mx + b \longrightarrow (3)$$

equation (3) is called as slope-intercept equation form of the line. Comparing equation (2) & (3) then

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

and

$$b = y_1 - mx_1$$

where slope 'm' is the change in height divided by the change in width for two points on the line. Intercept 'b' is the height at which the line crosses the y-axis.

General form of the equation:→

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$$

$$(x_2 - x_1)y - (x_2 - x_1)y_1 = (y_2 - y_1)x - (y_2 - y_1)x_1$$

$$(x_2 - x_1)y - y_1 x_2 + \cancel{x_1 y_1} = (y_2 - y_1)x - y_2 x_1 + \cancel{x_1 y_1}$$

$$(y_2 - y_1)x - (x_2 - x_1)y + x_2 y_1 - x_1 y_2 = 0$$

$$\boxed{rx + s \cdot y + t = 0}$$

where

$$r = y_2 - y_1$$
$$s = -(x_2 - x_1)$$
$$t = x_2 y_1 - x_1 y_2$$

hat is Computer Graphics :→

Computer is an information processing machine, a tool for storing, manipulating and correlating data. Using Computers we are able to generate or collect and process information in a scope never before possible. This information can help us in a lot of way like

- Making decisions.
- Understand our world
- and Control its operations.

But as the volume of information increases a problem arises. How can this information be effectively and efficiently transferred between human and machine. Computer graphics strikes directly at this problem. "It is a study of technique to improve communication between human and machine". Giving the computer the ability to express its data in pictorial form can greatly increases its ability to provide information to the human user.

Computer Graphics allows communication through picture, charts and graphs to the people. A huge database can be represented by pictures like bar charts, pie charts etc. Computer Graphics is a field which deals with picture and images. One can find many definition of Computer graphics of which few are given below.

Computer Graphics is the use of a Computer to define, store, manipulate and present pictorial output. The Computer presents stored info. to an ... character

1: Computer Graphics is a pictorial representation the image and manipulation of data by a Computer.

2. Computer Graphics is the use of Computer hardware and software to create, manipulate and present picture and images.

3. Computer Graphics is a process of generation of images of virtual scenes using Computer hardware

Thus Computer Graphics is very useful in business to represent their data, market share and profit margin.

Types of Computer Graphics :→

There are two types of Computer Graphics

1. Raster Graphics or Bit mapped Graphics
2. Random Graphics or Vector Graphics.

1. Raster Graphics or Bit-mapped Graphics

Raster or Bit-mapped Graphics are stored and held as collection of bits in memory locations corresponding to pixel on the screen. Bit mapped graphics treat images as collection of dots rather than as shapes. Within a Computer a bit mapped graphics is represented as an array of bits that describe characteristics of the endividual pixels making the images. For example

2. **Vector Graphics** :→ Vector Graphics is a method of generating images that uses mathematical description to determine the position, length and direction in which lines are to be drawn. In vector graphics objects are treated as collection of lines, rather than pattern of endirisual dots. For example.

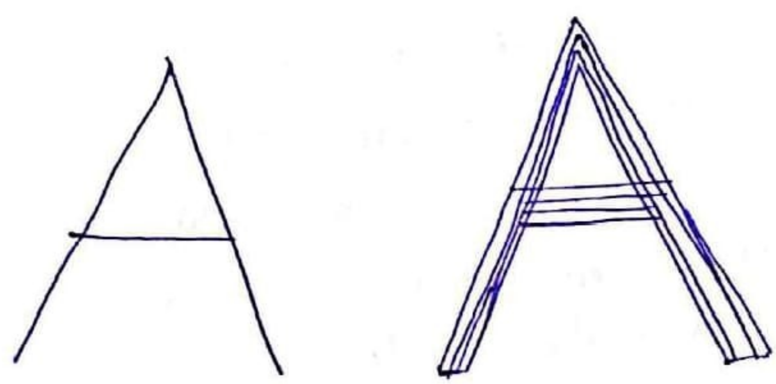
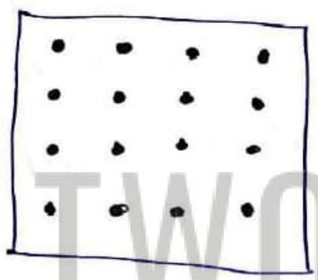


Image Types Image Can be Classified as follows

1. Monochrome Image.
2. Grey scale Image.
3. Colour (24-bit) Images
4. Half-toned Images.

1. Monochrome Images :→

In monochrome Image each pixel is stored as a single bit (0 or 1). Here 0 represents black while 1 represents white. It is a black and white Image in the strictest sense. These Image are also called bit mapped images. In such Images, we have only black and white pixels and no other shades of gray.



2. Gray Scale Image

Here each pixel is usually stored as a byte (8-bits). Due to this, each pixel can have values ranging from 0 (black) to 255 (white). Gray Scale images, as the name suggests have black, white and various shades of gray presents in the image.

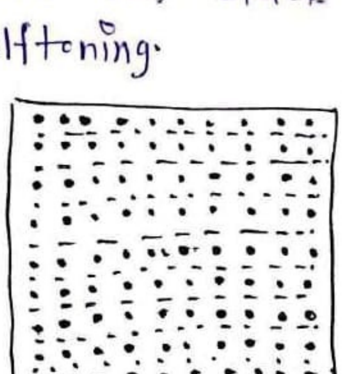
3. Colour Images _ (24 bit)

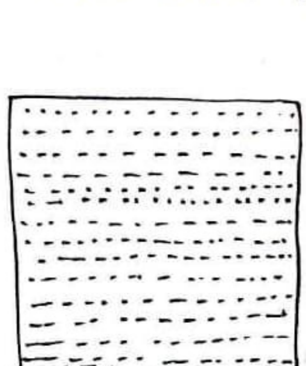
Colour Images are based on the fact that a Variety of Colours Can be generated by mixing the three primary colours. Red, Green and Blue in proper proportions. In Colour age each pixel is Composed of RGB values and each of these colours require 8-bits (one byte) for its representation. Hence each pixel is represented by 24-bits [R (8-bits), G (8-bits), B (8-bits)]. A 24 bit Colour image supports 16,777,216 different Combination of colors.

4. Half Toning :→

It is obvious that gray scale image definitely looks better than the monochrome image as it utilizes more gray levels. Most of the printers are all bilevel devices that is they have only a black cartidge and can only produce two levels (black on a white background). In fact most of the printing Jobs are done using bilevel devices.

You have all read newspaper at some point of time the images do look like gray level images. But all the images generated are basically using black Colour. Inspite of this we do get an illusion of seeing gray levels. The technique to achieve gray levels from only black and white level is called halftoning.





Application of Computer Graphics :→

Computer Graphics is greatly benifit in every field. Some of the major application area of its use are.-

① Computer-Aided Design (CAD)

A major use of Computer graphics is in design process, particularly for engineering and architectural systems, but almost all products are now Computer designed. CAD methods are now routinely used in the design of building, automobiles, aircraft, watercraft, spacecraft, ~~textiles~~ and many-many other products.

For some design applications, objects are first displayed in wireframe outline form that shows the overall shape and internal features of an objects. Some CAD software is capable of dynamic mathematical modeling, in which case it may be marked as CADD (Computer Aided Design and drafting). CAD is used in the design of tools and machinery and in the drafting and design of all types of building from small residintial types to the largest commercial and industrial structures (hospital and factories). CAD has also become an especially important technology within the scope of Computer aided technologies, with benifit such as lower product development Costs and a greatly shortened design cycle. CAD enables designer to layout and develop work on screen, print it out and save it for future editing, saving time on their drawings.

2. Electronics Design :→

We know electronic devices like radio, television and Computers are madeup of many small electronic Components. These parts are mounted on boards, Called printed Circuit design and interConnected by Copper tracks on boards. Until few years ago the design of PCB was done entirely by hand. Today this procedure is Completely Computerized. The engineers works on the video screen and designs the PCB. Once the design is Completed, the design system generates a hardcopy output.

3. Mechanical Design :→

We know machines are simply Complex arrangement of mechanical parts. Today Constructive Solid Geometry technique (CSG) is used to design different part on Computer. Today, before manufacturing any machine, engineers design that machine on Computer first and with the help of that they design it.

4. Entertainment and Animation :→

Computer graphics methods are used in making Cartoon animation films. Sometimes the graphic scenes are displayed by themselves and sometimes graphics objects are combined with actors and live scenes. Several developers have used graphics sound, animation of multimedia to Create Varietty of games.

5. Aerospace Industry :→

Aircraft designer use Computers to simulate the airflow over the wings. The air pattern around the wings are displayed on the graphics monitor. The pattern of shock waves around the aircraft give the designer an idea about the area that may Create unwanted turbulance. It is possible by using super Computer such CRAY or ETA.

6. Medical Technology :→

Today, Computerized Axial Tomography (CAT) are used to Compose the three dimensional model of the brain by taking multible X-rays of it. CAT scan

, very useful in detecting various maladies like brain tumors. Computers are also useful to generate images of delicate organs. It is useful to study the effect of drugs on tissues.

7. Office Automation and Desktop Publishing :→

The Desktop publishing is one approach towards the Office Automation. The desktop publishing on personal Computers allows the use of Computer graphics for Creation of business graphics, charts map and other pictorial features.

8. Cartography :→ Computer graphics is used to produce accurate and schematic representation of geographical and other natural phenomena from measurement data. Examples include geographics maps, weather maps, oceanographic charts, contour map and population density maps.

9. Art and Commerce :→ Computer graphics is used to produce pictures that express a message and attract attention. Personal Computers and videotext terminal in public places such as museums, transportation terminals, supermarket and hotels. Finally slide production for commercial, scientific or educational presentation

is another Cost effective use of graphics.

10. Internet :

The latest in the list of uses of Computer graphics is Internet. The internet is accessed through telephonic Communication, mostly via satellites. The Internet would be nothing without the graphics. There is tremendous variety of multimedia Content available on the web. We can listen to high quality streaming music or watch the latest movie trailers on the web.

How much time is spent scanning across each row of pixels during screen refresh on a raster system with resolution of 1280X1024 and a refresh rate of 60 frames per second.

Solution: Here resolution, 1280X 1024

That means system contains 1024 scan line and each scan line contains of 1280 pixel.

and refresh rate = 60 frame/sec.

that means 1 frame takes 1/60 sec.

$\because$ 1 frame consists of 1024 scan lines.

$\therefore$ 1024 scan lines takes to 1/60 sec.

$\therefore$ 1 scan line takes $\frac{1}{60 \times 1024}$ = 0.058 sec

= 58 msec.

Ans

Q2. Suppose we have a computer with 32 bits per word and a transfer rate of 1 mips (million instruction per second). How long would it take to fill the frame buffer of a 300 dpi (~~dot instruction per second~~) (dot per inch) laser printer with a page size of $8\frac{1}{2}$ inch by 11 inches?

Solution: Frame buffer size = $8\frac{1}{2}$ inch × 11 inch

$= 8\frac{1}{2} \times 300 \times 11 \times 300$ [$\because$ 1 inch = 300 dpi]

= 841 5000 dots

Suppose 1 dot = n bits

So, the frame buffer size = 841500 n bits

Transfer rate = 1 mips = 1×10^6 words per second

= 32×10^6 bits per second

[∵ 1 word = 32 bits]

Time require to fill frame buffer

$$= \frac{841500n}{32 \times 10^6} = 0.263n \text{ sec.}$$

if n = 4 then the time taken = 0.263×4

= 1.052 Sec.

Ans

Q3. Consider two raster system with resolution of 640×480 and 1280×1024. How many pixel could be accessed per second in each of these systems by a display controller that refreshes the screen at a rate of 60 frame/sec? What is the access time per pixel in each system?

Solution Case 1. Resolution = 640×480

No. of pixel in one frame = $640 \times 480 = 307200$

Since controller can access 60 frames in one second.

Therefore, total no. of pixel accessed = 60×307200

= 18432000 per sec.

Access time / pixel = 1 / total pixel accessed per sec.

= 5.4×10^{-8} sec/pixel. Ans

Case 2: When resolution = 1280×1024

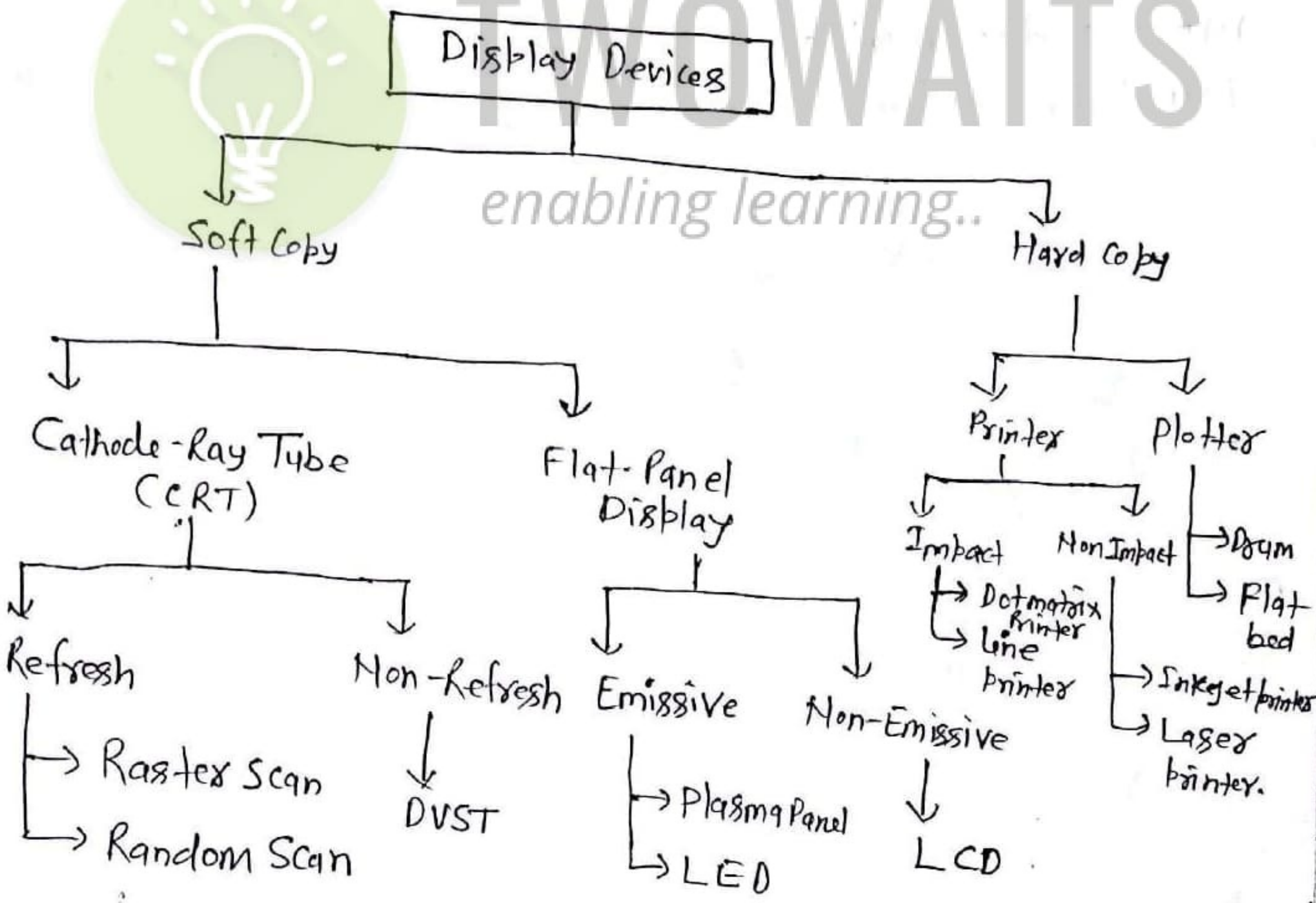
total no. of pixel accessed by the raster system = $60 \times 1280 \times 1024$

Access time / pixel = 1 / 78643200 = 1.2×10^{-8} Sec / pixel.

Second

Display Devices

The display systems are often referred to as video monitor or video display unit. The display devices are also known as output devices. It is responsible for graphic display. The display system may be attached with a PC to display character, picture and video output. The most commonly used output device in a graphics system is video monitor and the operation of most video monitors is based on the Cathode-Ray Tube (CRT) design.



Suppose 1 :-

every display system has three parts.

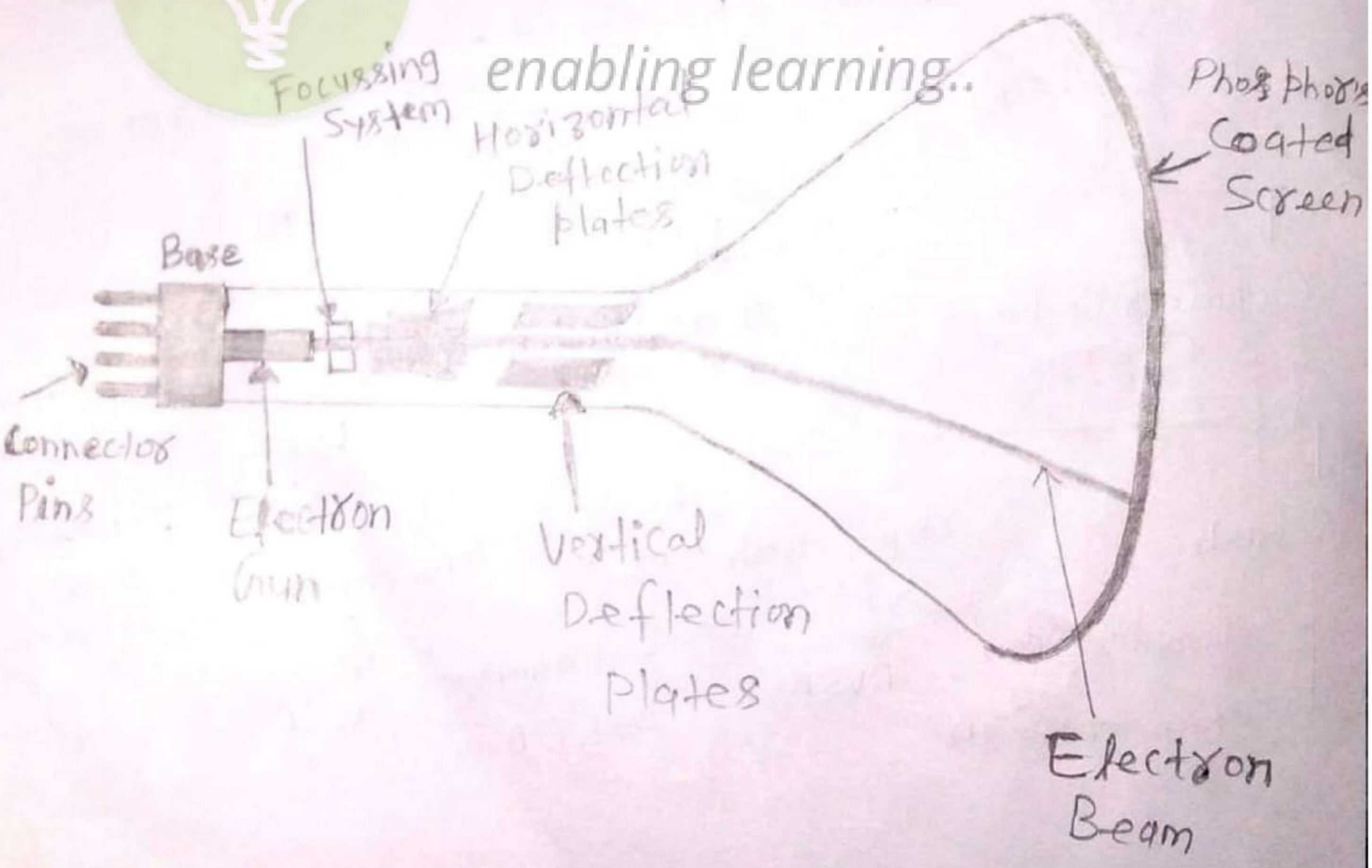
(a) Display adapter, that creates and hold the image information.

(b) Monitor, which display that information.

(c) Cable, that carries image data between display adapter and the monitor.

Cathode-Ray Tube (CRT)

It is now very common to see picture on video monitors. The operation of most video monitor is based on the standard Cathode Ray Tube design as shown in the figure.



image

...ic Operation of CRT :→

The operation of CRT is based on the concept of energizing an electron beam that strikes the phosphor coated screen at a very high speed. A beam of electron is emitted by an electron gun which pass through focussing system and deflection system that directs the beam toward specified position on the phosphor coated screen. The phosphor then emits a small spot of light at each position contacted by the electron beam.

Working :→

The cathode is heated untill electrons generates in diverging clouds which are then focussed into a beam by diverging clouds. Beam passes between two pairs of metal plates one vertical and other horizontal. A voltage difference is applied to each pair of plates according to the amount that the beam is to be deflected in each direction. An electron beam passes between each pair of plates it is bent towards the plate with the higher positive voltage.

... order of 60 to 80 frame per second. ... can display

Different kind of phosphors are available for us. Beside color, a major difference between phosphors is their persistance: how long they continue to emit light after the CRT beam is removed.

Persistance :→

→ Persistance is defined as the time it takes the emitted light from screen to decay to one tenth of its original intensity. Lower persistance phosphors require higher refresh rate to maintain a picture on the screen without flicker.

Resolution :→

→ The maximum number of points that can be displayed without overlap on a CRT is referred to as the resolution. A more precise definition of resolution is the number of points per centimeter that can be plotted horizontally and vertically, although it can be simply stated as the total number of points in each direction.

In normal CRT we have basically two types of scanning.

1. Raster Scan Display
2. Random Scan Display.

Access time / pixel = 1/78643200 = 1.2 × 10⁻⁸ sec / pixel.

ster Scan Display

The most common type of graphics monitor employing a CRT is the raster scan display based on television technology. In a Raster scan display the electron beam is swept across the screen one row at a time from top to bottom. As the electron beam moves across each row, the beam intensity is turned on and off to create a pattern of illuminated spots. Picture definition is stored in a memory area called the refresh buffer or frame buffer. This area holds the set of intensity values for all the screen points. Stored intensity values are then retrieved from a refresh buffer and painted on the screen one row at a time. Each screen point is reffered to as a pixel. Home television set are examples of other system using raster scan methods.

Intensity range for pixel positions depend on the capability of the raster system. In a simple black and white system, each screen point is either ON or OFF. So only one bit per pixel is needed to control the intensity of screen positions. For a bilevel system, a bit value of 1 indicates that the electron beam is to be turned on at that position and a value of 0 indicates that the beam intensity is turned OFF. Refreshing on raster scan display is carried out at the rate of 60 to 80 frame per second.

(2) Random Scan display when operated as random-scan display unit, a CRT has the electron beam directed only to the parts of the screen where a picture is to be drawn. Random scan monitors draw a picture one line at a time and for this reason are also referred to as vector display. Refresh rate on a random-scan system depends on the number of lines to be displayed. Picture definition is stored as a set of line drawing commands in an area of memory referred to as the refresh display file. Random scan system are designed for line drawing applications and cannot display realistic shaded scenes.

Q: Give the difference between Raster Scan System and Random System.

Ans

Base of Difference	Raster Scan System	Random Scan System
Electron Beam	The electron beam is swept across the screen, one row at a time, from top to bottom.	The electron beam is directed only to the parts of screen where a picture is to be drawn.
Resolution	Its resolution is poor because raster system in contrast produces zig-zag lines that are plotted as discrete point sets.	Its resolution is good because this system produces smooth lines drawings because CRT beam directly follows the line path.

Picture Definition	Picture definition is stored as a set of intensity values for all screen points, called pixel in a refresh buffer area.	Picture definition is stored as a set of line drawings instruction in a display file.
Realistic Display	The capability of this system to store intensity values for pixel makes it well suited for the realistic display of scenes contain shadow and color pattern.	These systems are designed for line-drawings and can't display realistic shaded scenes.
Draw an Image	Screen points/pixel are used to draw an Image.	mathematical functions are used to draw an Image.

Color CRT Monitors :→

There are two way of getting color displays—

1 Beam Pentration method.

2. Shadow mask method.

1. Beam Pentration method :→ The beam pentration method for displaying color pictures has been used with random scan monitors. Two layers of phosphor usually red and green are coated onto the inside of CRT screen, and the display color depends on how far the electron beam pentrates into the phosphor layers.

A beam of slow electrons excites only the outer red layer. A beam of very fast electrons penetrates through the red layer and excites the inner green layer. At Intermediate beam speeds combination of red and green light are emitted to show two additional colors yellow and orange. Beam penetration has been an inexpensive way to produce color in random scan monitors, but only four colors are possible, and the quality of pictures is not as good as with other methods.

2) Shadow-mask methods :→

Shadow mask method are commonly used in raster scan systems because they produce a much wider range of colors than the beam penetration method. A shadow mask CRT has three phosphor colors dots at each pixel position. One phosphor dot emits a red light, another emits a green light, and the third emits a blue light. This type of CRT has three electron guns, one for each color dot, and a shadow mask grid just behind the phosphor-coated screen. Figure as shown below illustrates delta-delta shadow mask method commonly used in color CRT systems. The color guns are arranged so that the individual beam coverage intersect at the shadow mask. Upon passing the hole in the shadow mask the red beam, for example is prevented or masked from intersecting either the green or blue phosphor dot.

Advantage of shadow-mask method

1. This method is commonly used in raster scan system as they produce a much wider range of colors than the beam pentration method.

2. We can even have an in-line arrangement of RGB color dots on the screen along one scan line and not only a triangular pattern. This pattern is used in higher-solution color CRTs.

Disadvantage of shadow mask method

1. It is very difficult to converge all three electron beams on same hole in shadow mask. This is so because the screen is a limited area.

2. They have poor resolution.

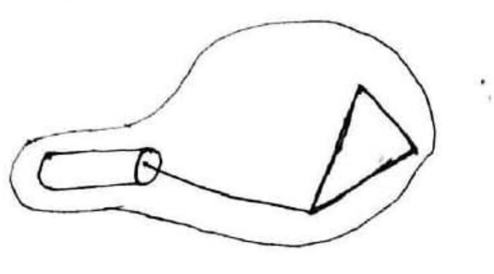


Fig: Beam Pentration Random scan system

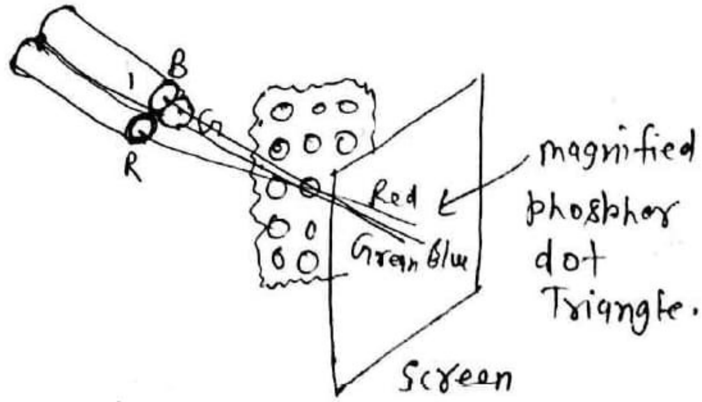


Fig: Delta-Delta Shadow mask CRT.

Advantage of Beam Pentration CRT :→

1. Half Cost as Compared to that of shadow mask CRT.
2. It is an inexpensive way to produce Color in random - scan monitors.
3. Its resolution is better.

Disadvantage of Beam Pentration Method

It Consumes significant amount of accelerating potential in order to switch Color.
When accelerating potential changes. A problem arises at the time of switching of colors as it is time Consuming.

Advantage of DDA Algorithm :→

1. It is simple to understand.
2. It is a faster method than direct use of line equation $y = mx + b$
3. It requires no special skills for implementation.

Disadvantage of DDA Algorithm :→

1. It is orientation dependent, so the end point accuracy is poor.
2. A floating point addition is still needed in determining each successive point which is time Consuming.

Line Generation Algorithm :→

There are two vector generation Algorithm.

1. DDA (Digital Differential Analyzer) Algorithm.
2. Bresenham's Algorithm.

1. DDA Algorithm :→

The DDA is a scan-conversion line algorithm based on calculating either dy or dx. In this algorithm we use floating point airthmatic. Which make DDA Algorithm slower. But due to floating point airthmetic it is more accurate method than Bresenham's algorithm. The name comes from the fact that we use the same technique as a numerical method for solving differential equation.

In DDA algorithm there are some facts to keep in mind as given below.

Note : As we know that $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Case-I When Slope $m \leq 1$

if $\Delta x = 1$ (we sample at unit x intervals)

then $x_{i+1} = x_i + 1$

$y_{i+1} = y_i + m$

Case-II When Slope, $m > 1$

Then we sample at unit y intervals, $\Delta y = 1$

Then $x_{i+1} = x_i + \frac{1}{m}$

$y_i = y_i + 1$

→ when the slope is changed the role of x and y are Interchanged.

DDA Algorithm :→

Step1: Read the end point coordinates (x_1, y_1) and (x_2, y_2) for a line.

Step2: $dx = x_2 - x_1$

$dy = y_2 - y_1$

Step3: if abs(dx) > abs(dy) then

steps = dx

otherwise

steps = dy

Step4: $x_{inc} = \frac{dx}{length}$

$y_{inc} = \frac{dy}{length}$

Step5: putpixel (x, y, color)

Step6 $x = x + x_{inc}$

$y = y + y_{inc}$

putpixel (round (x), round (y), color)

Step7: Repeat step 6 until $x = x_2$ and $y = y_2$.

Q1. Consider the line from (1,1) to (5,6). Use DDA Algorithm to draw this line.

Solution: Step1

(1,1) — (5,6)

$x_1 = 1$, $y_1 = 1$

$x_2 = 5$, $y_2 = 6$

Step2: $dx = x_2 - x_1 = 5 - 1 = 4$

$dy = y_2 - y_1 = 6 - 1 = 5$

Step3: Here $abs(dy) > abs(dx)$

$5 > 4$

So $steps = 5$

Step4: $x_{inc} = dx / length = 4/5 = 0.8$

$y_{inc} = dy / length = \frac{5}{5} = 1$

Step5 putpixel (1,1)

Step6: Points are

1. $(X, Y) =$

$X = X + X_{inc} = 1 + 0.8 = 1.8$

$Y = Y + Y_{inc} = 1 + 1 = 2$

So $(X, Y) = (1.8, 2)$

2. $X = 1.8 + 0.8 = 2.6$

$Y = 2 + 1 = 3$

$(X, Y) = (2.6, 3)$

3. $X = 2.6 + 0.8 = 3.4$

$Y = 3 + 1 = 4$

$(X, Y) = (3.4, 4)$

4. $X = 3.4 + 0.8 = 4.2$

$Y = 4 + 1 = 5$

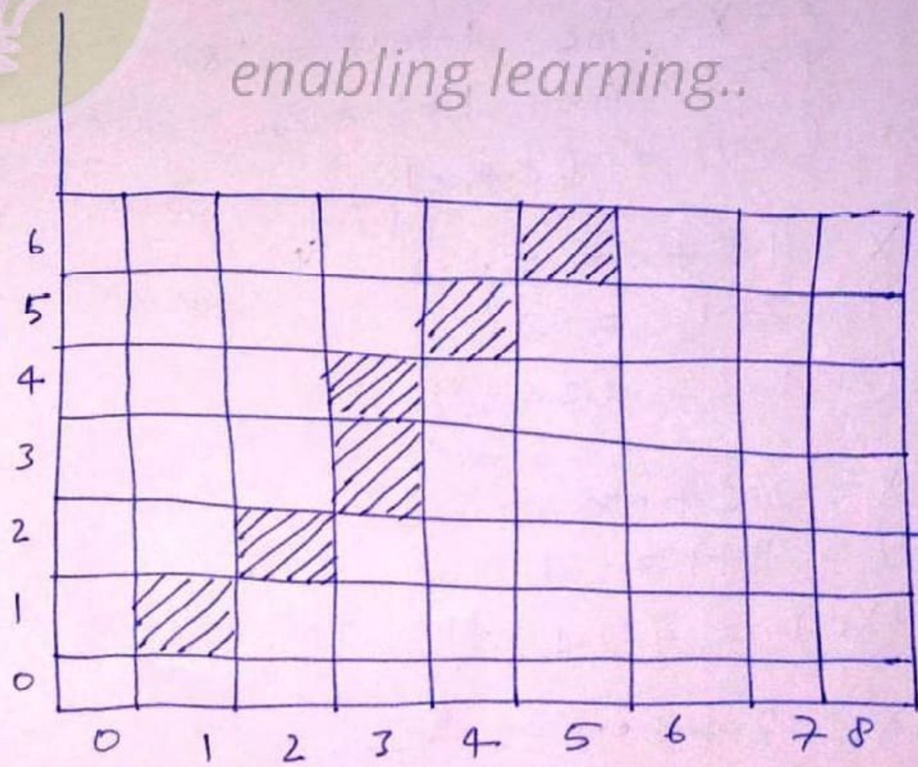
$(X, Y) = (4.2, 5)$

5. $x = 4.2 + 0.8 = 5$

$y = 5 + 1 = 6$

$(x, y) = (5, 6)$

Plot (x, y)	x	y
(1,1)	1	1
(2,2)	1.8	2
(3,3)	2.6	3
(3,4)	3.4	4
(4,5)	4.2	5
(5,6)	5	6



Doyourself :→

Q2. Draw the line from (0,0) to (−6,−6) using DDA Algorithm.

Q3 Draw the line from (0,6) to (6,) using DDA Algorithm.

Q4. Draw the line from (−1, 2) to (3, −4) using DDA Algorithm.

Q: Draw a line from (1,1) to (5,3) using DDA Algorithm.

Ans: Here

Step 1: $x_1 = 1, y_1 = 1$

$x_2 = 5, y_2 = 3$

Step 2. $dx = x_2 - x_1 = 5 - 1 = 4$

$dy = y_2 - y_1 = 3 - 1 = 2.$

Step 3: Here $abs(dx) > abs(dy)$

$4 > 2$

$Step = abs(dx) = 4$

Step 4:

$x_{inc} = \frac{dx}{step} = \frac{4}{4} = 1$

$y_{inc} = \frac{dy}{Step} = \frac{2}{4} = 0.5.$

Step 5 Points are

1. $(x_1, y_1) = (1,1)$

2. $(x_1, y_1) =$

$x_1 = x_1 + x_{inc} = 1 + 1 = 2$

$y_1 = y_1 + y_{inc} = 1 + 0.5 = 1.5$

3. $(x_1, y_1) = (2, 1.5)$

$x_1 = 2 + 1 = 3$

$y_1 = 1.5 + .5 = 2$

$(x_1, y_1) = (3, 2)$

4. $x_1 = 3 + 1 = 4$

$y_1 = 2 + .5 = 2.5$

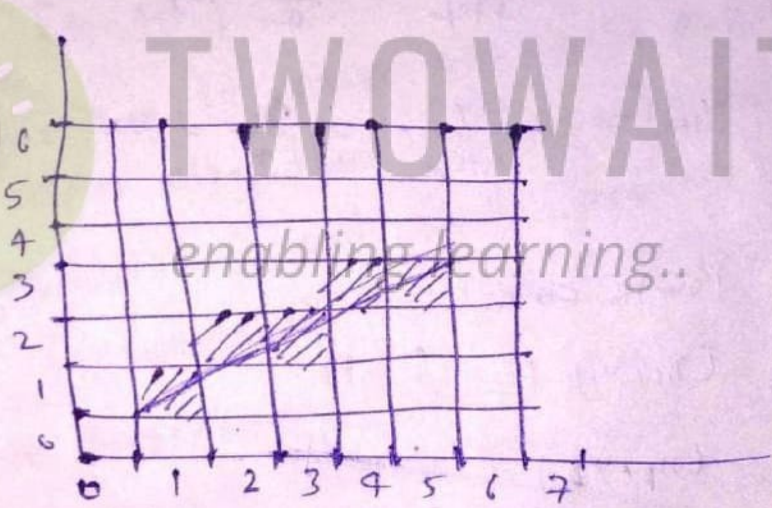
$(x_1, y_1) = (4, 2.5)$

5. $x_1 = 4 + 1 = 5$

$y_1 = 2.5 + 0.5 = 3$

$(x_1, y_1) = (5, 3)$

Plot (x, y)	x	y
(1,1)	1	1
(2,2)	2	1.5
(3,2)	3	2.0
(4,3)	4	2.5
(5,3)	5	3



Bresenham's Line Algorithm:→ The basic principle of Bresenham's line algorithm is to find the optimum raster locations to represent the straight line. To accomplish this algorithm we always increments either x or y by one unit depending on the slope of the line. Ones this is done, then increment in other varriable is found on the basis of the distance between the actual line location and the nearest pixel position.

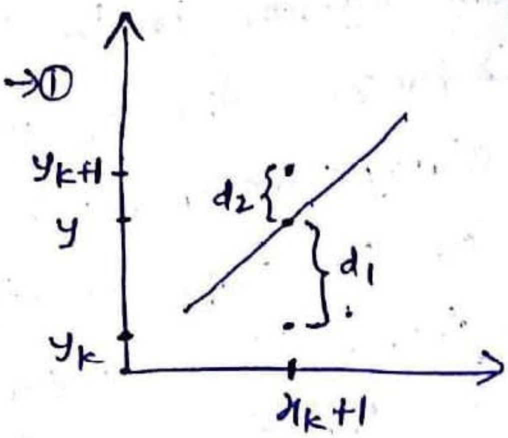
This distance is called decision varriable.

The disadvantage of DDA Algorithm is that it is very time consuming as it deals with rounding off operation and floating point arithmatic.

Bresenham algorithm is more accurate and efficient compared to DDA algorithm because it avoids the "round off operation" and scan convert the line using only incremental Integer calculation.

To illustrate Bresenham's approach, we first consider the scan conversion process for lines with positive slope less than 1. Assuming we have determined that the pixel at (x_k, y_k) is to be displayed, we next need to decide which pixel to plot in column x_{k+1}. Our choices are the pixels at positions (x_{k+1}, y_k) and (x_{k+1}, y_{k+1}). At sampling position x_{k+1}, we label vertical pixel seperations from the mathematical line path as d_1 and d_2. The y coordinate on the mathematical line at pixel column position x_k+1 is calculated as

$$y = m(x_k+1)+b \rightarrow (1)$$



Then

$$d_1 = y - y_k$$
$$= m(x_k+1)+b-y_k$$

and

$$d_2 = (y_k+1) - y$$
$$= y_k+1 - m(x_k+1) - b$$

The difference b/w these two seperation is

$$d_1 - d_2 = 2m(x_k+1) - 2y_k + 2b - 1 \rightarrow (2)$$

A decision parameter p_k for the kth step in line algorithm can be obtained by rearranging equation (2) so that it involves only integer calculations. We accomplish this by substituting $m = \Delta y/\Delta x$, where Δy and Δx are the vertical and horizontal seperation of the endpoint positions, and defining

$$p_k = \Delta x(d_1 - d_2)$$
$$= \Delta x\left[2\frac{\Delta y}{\Delta x}(x_k+1) - 2y_k + 2b - 1\right]$$
$$= 2\Delta y \cdot (x_k+1) - 2\Delta x y_k + 2\Delta x b - \Delta x$$
$$= 2\Delta y x_k + 2\Delta y - 2\Delta x y_k + 2\Delta x b - \Delta x$$
$$= 2\Delta y x_k - 2\Delta x y_k + \underline{2\Delta y + 2\Delta x b - \Delta x}$$

Note that underlined part is Constant, we call it c.

$$C = 2 \cdot \Delta y + 2 \cdot \Delta x . b - \Delta x$$

We can write expression for p_k as

$$p_k = 2 \Delta y x_k - 2 \Delta x y_k + c$$

At step k+1 the decision parameter is

$$p_{k+1} = 2 \Delta y x_{k+1} - 2 \Delta x y_{k+1} + c$$

and

$$p_{k+1} - p_k = 2 \Delta y x_{k+1} - 2 \Delta x y_{k+1} + c - (2 \Delta y x_k - 2 \Delta x y_k + c)$$

$$= 2 \Delta y (x_{k+1} - x_k) - 2 \Delta x (y_{k+1} - y_k)$$

But $x_{k+1} = x_k + 1$ so that

$$p_{k+1} - p_k = 2 \Delta y - 2 \Delta x (y_{k+1} - y_k)$$

Where the term $y_{k+1} - y_k$ is either 0 or 1, depending on the sign of parameter p_k.

When $y_{k+1} - y_k$ is 0 then

if $p_k < 0$, $p_{k+1} = p_k + 2 \Delta y$.

When $y_{k+1} - y_k$ is 1 then

if $p_k > 0$, $p_{k+1} = p_k + 2 \Delta y - 2 \Delta x$

The recursive calculation of decision parameter is performed at each integer x position starting at the left coordinate endpoint of the line. The first parameter p_0 is evaluated at starting pixel position (x_0, y_0) and with m evaluated as $\Delta y / \Delta x$.

$$p_0 = 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + 2\Delta x \cdot b - \Delta x$$

For the initial point on the line

$y_0 = m x_0 + b$ therefore

$$b = y_0 - (\Delta y / \Delta x) \cdot x_0$$

Substituting the value of b in above equation.

$$p_0 = 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + 2\Delta x [y_0 - (\Delta y/\Delta x) x_0] - \Delta x$$

$$= 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + 2\Delta x y_0 - 2\Delta y x_0 - \Delta x$$

$$= 2\Delta y x_0 - 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta x y_0 + 2\Delta x y_0 + 2\Delta y - \Delta x$$

$$\boxed{p_0 = 2\Delta y - \Delta x}$$

Bresenham's Line drawing Algorithm for $|m| < 1$

1. Input the two line endpoints and store the left end point in (x_0, y_0).

2. Load (x_0, y_0) into the frame buffer, that is plot the first point.

3. Calculate the constants Δx, Δy and $2\Delta y$ and $2\Delta y - 2\Delta x$, and obtain the starting value for decision parameter as

$$p_0 = 2\Delta y - \Delta x$$

4. At each x_k along the line, starting at $k=0$, perform the following test, if $p_k < 0$, the next point to plot is (x_k+1, y_k) and

$$p_{k+1} = p_k + 2\Delta y$$

Otherwise, the next point to plot is (x_k+1, y_k+1) and

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

5. Repeat step 4 Δx times.

Q Consider a line from (20,10) to (30,18). This line has slope of 0.8. Rasterize the line using bresenham's algorithm.

Ans.

$x_1 = 20$

$x_2 = 30$ $\qquad \Delta x = x_2 - x_1 = 30 - 20 = 10$

$y_1 = 10$ $\qquad \Delta y = y_2 - y_1 = 18 - 10 = 8$

$y_2 = 18$

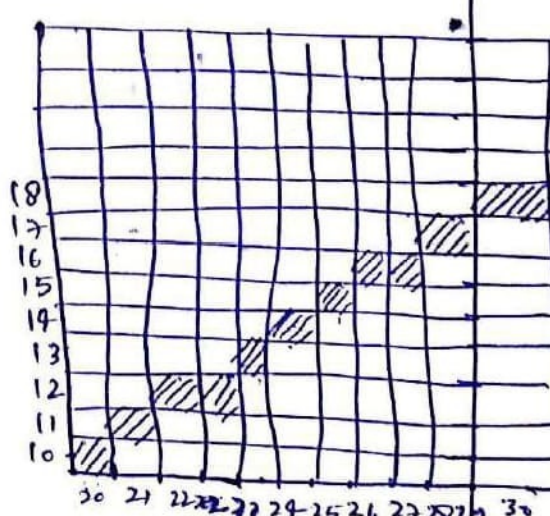
The initial decision parameter has the value

$$p_0 = 2\Delta y - \Delta x$$
$$= 2 \times 8 - 10 = 16 - 10 = 6$$

$$2\Delta y = 16, \quad 2\Delta y - 2\Delta x = -4$$

We plot the initial point $(x_0, y_0) = (20,10)$ and determine successive pixel positions along the line path from the decision parameter as

K	p_k	(x_{k+1}, y_{k+1})	
0	6	(21, 11)	
1	2	(22,12)	
2	-2	(23,12)	
3	14	(24,13)	
4	10	(25,14)	
5	6	(26,15)	
6	2	(27,16)	
7	-2	(28,16	
8	14	(29,17)	
9	10	(30,18)	



Circle - Generating Algorithms

Consider a line from (5,5) to (13,9). Use the Bresenham's algorithm to rasterize the line.

Solution:

$(x_1, y_1) = (5,5)$

$(x_2, y_2) = (13,9)$

$m = 4/8 = 1/2 = 0.5 < 1$

$p_0 = 2\Delta y - \Delta x = 2\times4 - 8 = 0$

So the next point is (6,6)

$p_1 = p_0 + 2\Delta y - 2\Delta x = 0 + 2\times4 - 2\times8 = -8$

$p_1 < 0$ So the next point to plot is (7,6)

and $p_2 = p_1 + 2\Delta y = -8 + 2\times4 = 0$

So the next point to plot is (8,7) and

$p_3 = p_2 + 2\Delta y - 2\Delta x = 0 + 2\times4 - 2\times8 = -8$

$p_3 < 0$, So the next point to plot is (9,7)

and $p_4 = p_3 + 2\Delta y = -8 + 2\times4 = 0$

So the next point to plot is (10,8) and

$p_5 = p_4 + 2\Delta y - 2\Delta x = 0 + 2\times4 - 2\times8 = -8$

$p_5 < 0$ So the next point to plot is (11,8) and

$p_6 = p_5 + 2\Delta y = -8 + 2\times4 = 0$

So the next point to plot is (12,9) and

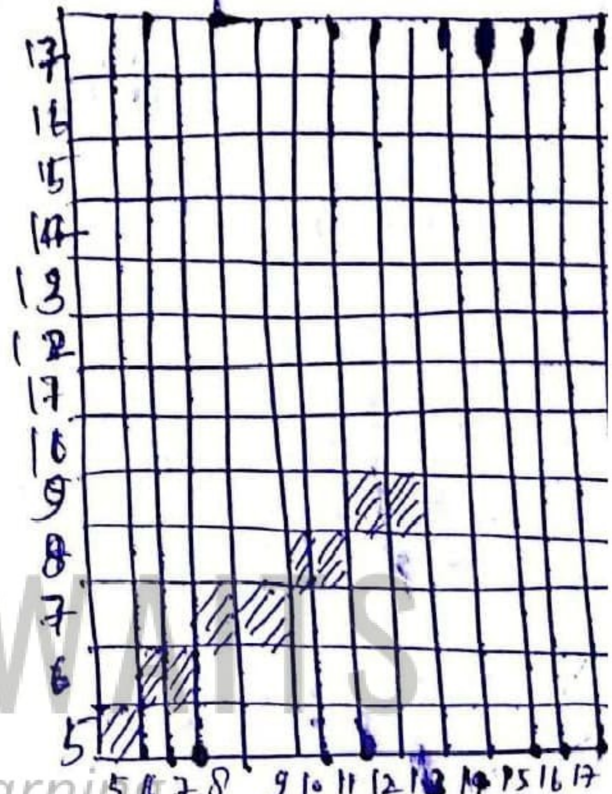
$p_7 = p_6 + 2\Delta y - 2\Delta x = 0 + 2\times4 - 2\times8 = -8$

$p_7 < 0$, So the next point to plot is (13,9)

Q ...

$(x_0, y_0) = (5,5)$

k	p_k	(x_{k+1}, y_{k+1})
0	0	(6,6)
1	-8	(7,6)
2	0	(8,7)
3	-8	(9,7)
4	0	(10,8)
5	-8	(11,8)
6	0	(12,9)
7	-8	(13,9)



Circle - Generating Algorithms

Basic Concepts in Circle Drawing

A Circle is a Symmetrical figure. It has eight way symmetry. The circle can be drawn by using the symmetry property. In a eight way symmetry property we can plot each calculated point around circle. For example, calculation of a circle point (x, y) in one octants gives the circle point shown for other seven octant just by reflection. Taking advantage of the circle symmetry in this way we can generate all pixels position around a circle by calculating only the points within the sector from $x = 0$ to $x = y$.

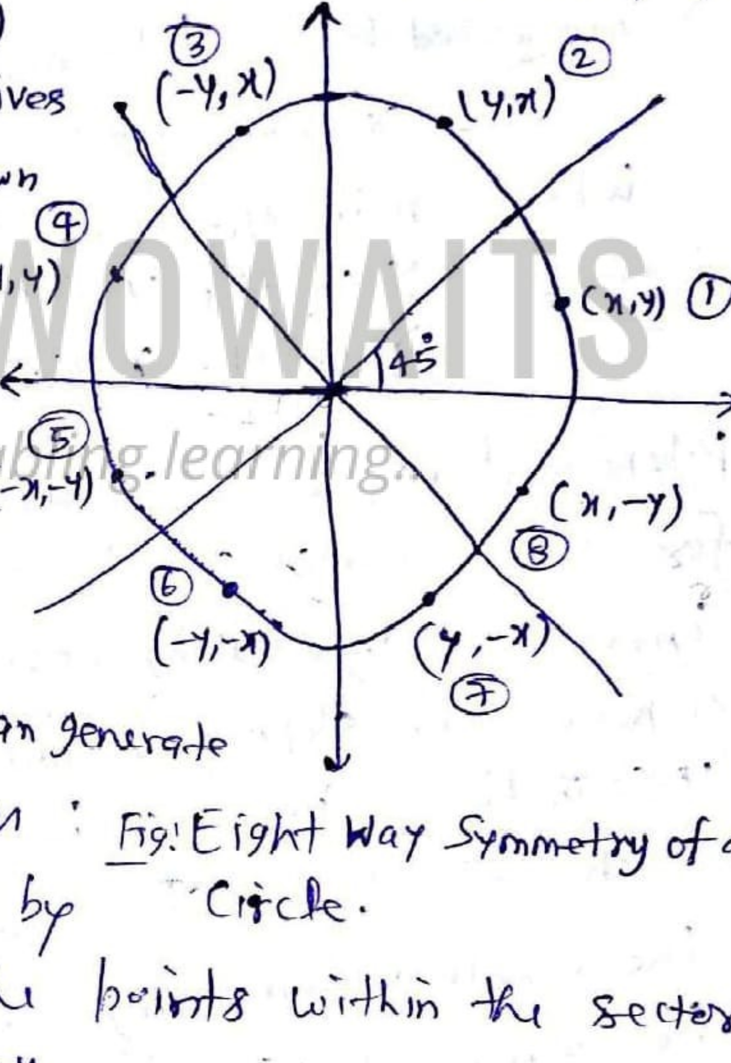


Fig: Eight Way Symmetry of a Circle.

Representation of a Circle:

There are two standard Methods of mathematically representing a circle centred at the origin.

(a) Polynomial Method

(b) Trigonometric Method

(a) Polynomial Method: In this method a circle is represented by a polynomial equation

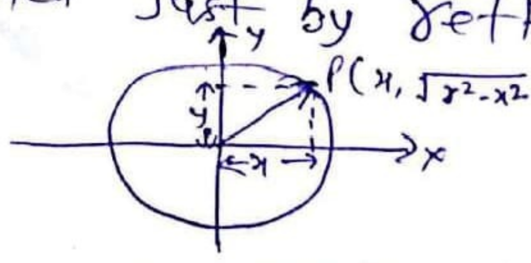
$$x^2+y^2=r^2$$

Where x is the x coordinate

y is the y coordinate

r is the radius of a circle.

Polynomial equation can be used to find y coordinate for the known x coordinate. Therefore, the scan converting circle using polynomial method is achieved by stepping x from 0 to $r/\sqrt{2}$ and each y coordinate is found by evaluating $\sqrt{r^2-x^2}$ for each step of x. This generates 1/8 portion of the circle. Remaining part of the circle can be generated just by reflection.



Midpoint Circle Algorithm

1. Input radius r and circle centre (x_c, y_c) and obtain the first point on the circumference of a circle centered on the origin as

$$(x_0, y_0) = (0, r)$$

2. Calculate the initial value of the decision parameter as

$$p_0 = \frac{5}{4} - r$$

3. At each x_k position, starting at $k=0$, perform the following test: if $p_k < 0$, the next point along the circle centered on $(0,0)$ is (x_{k+1}, y_k) and

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

Otherwise if $p_k > 0$ the next point along the circle is $(x_k + 1, y_k - 1)$ and $p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$

where $2x_{k+1} = 2x_k + 2$ and $2y_{k+1} = 2y_k - 2$

4. Determine the symmetry point in the other seven octants.

5. Move each calculated pixel position (x, y) onto the circular path centred on (x_c, y_c) and plot the coordinate values:

$$x = x + x_c$$
$$y = y + y_c$$

6. Repeat steps 3 through 5 until $x \geq y$.

(8)

Q1. Using Midpoint Circle algorithm plot a circle whose radius = 10 units. which is centred at origin.

Ans: The initial point $(x_0, y_0) = (0, 10)$

i.e. $x = 0, y = 10$

initial decision parameter $p_0 = 1 - r = 1 - 10 = -9$

Here $p_k < 0 (-9)$ So the next point is $(x_{k+1}, y_k) = (1, 10)$

and $p_{k+1} = p_k + 2x_{k+1} + 1 = -9 + 2 \times 1 + 1 = -6$

Now again $p_k < 0 (-6)$ So next point is $(2, 10)$

again $p_{k+1} = -6 + 2 \times 2 + 1 = -1$

Here $p_k < 0 (-1)$

So next point is $(3, 10)$ Now

$p_{k+1} = -1 + 2 \times 3 + 1 = 6 > 0$

So next point is $(x_{k+1}, y_{k-1}) = (4, 9)$

and $p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$

$p_{k+1} = 6 + 2 \times 4 + 1 - 2 \times 9$

$= 6 + 8 + 1 - 18 = -3 < 0$

So next point is $(5, 9)$ and

$p_{k+1} = -3 + 2 \times 5 + 1 = 8 > 0$

So next point is $(6, 8)$ and

$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$

$= 8 + 2 \times 6 + 1 - 2 \times 8$

$= 8 + 12 + 1 - 16 = 5$

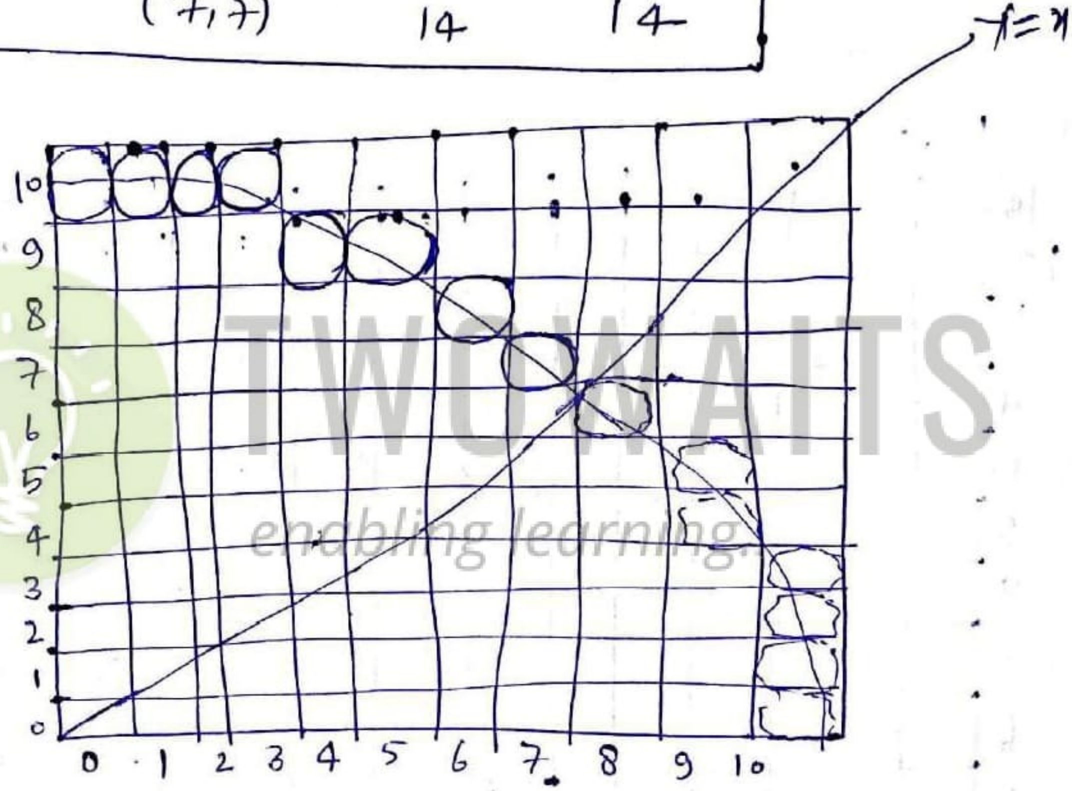
So next point is $(7, 7)$

Since $x \geq y$ so stop calculation.

Since all increments are ...

$(x_0, y_0) = (0, 10)$ 9

k	p_k	(x_{k+1}, y_{k+1})	$2x_{k+1}$	$2y_{k+1}$
0	-9	(1,10)	2	20
1	-6	(2,10)	4	20
2	-1	(3,10)	6	20
3	6	(4,9)	8	18
4	-3	(5,9)	10	18
5	8	(6,8)	12	16
6	5	(7,7)	14	14



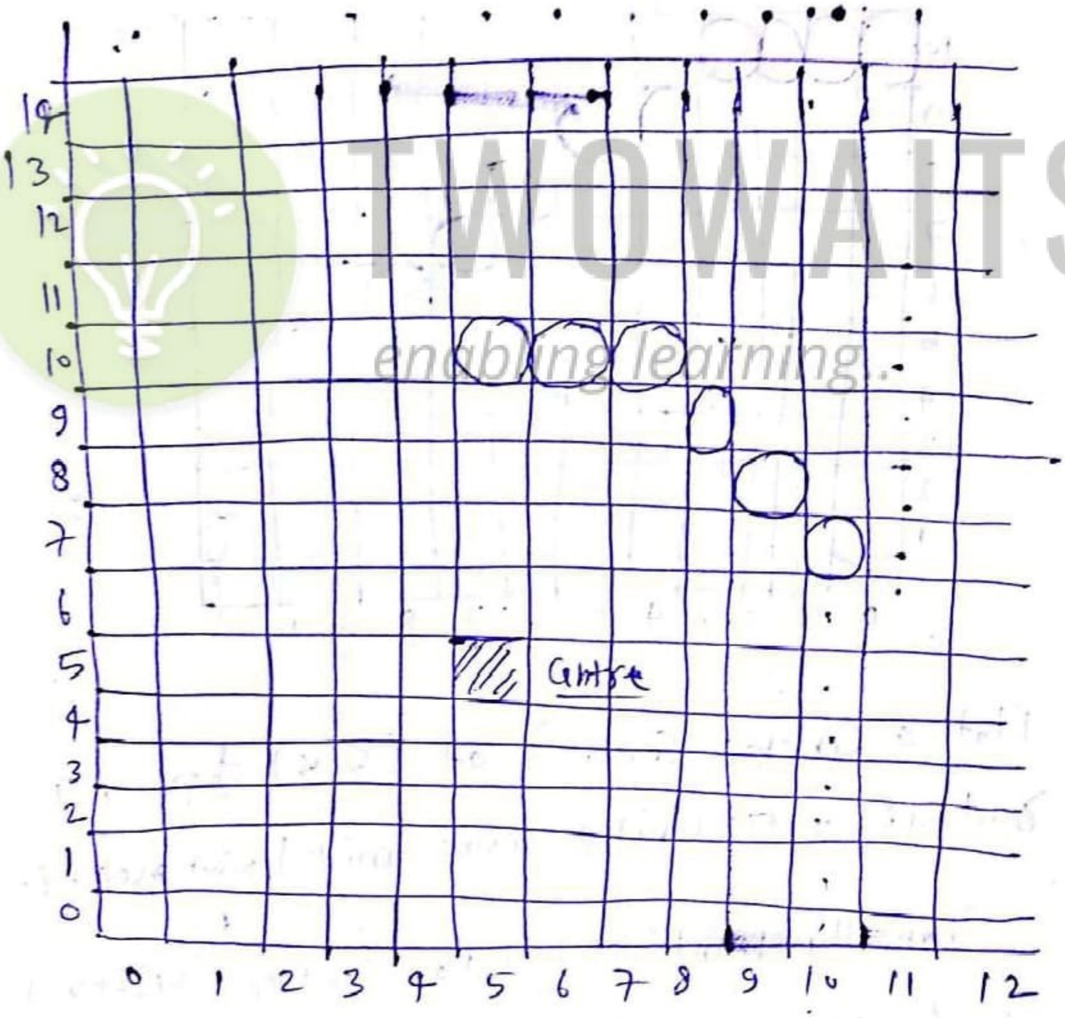
Q2 Plot a Circle Centred at (5,5) having a radius = 5 units using mid point method.

Soln: Then the point to plot is $(x_0 + x_c, y_0 + y_c)$
$= (0+5, 5+5) = (5, 10)$

Initial decision parameter $p_0 = 1 - r = 1 - 5 = -4$

$(x_0, y_0) = (5, 10)$

k	p_k	x_{k+1}, y_{k+1}	Actual points $(x_c + x, y_c + y)$
0	-4	(1, 5)	(6, 10)
1	-1	(2, 5)	(7, 10)
2	4	(3, 4)	(8, 9)
3	3	(4, 3)	(9, 8)
4	6	(5, 2)	(10, 7)



$1/p_0$

Since all increments are integer...