

CTP 499

Computer Graphics for CT



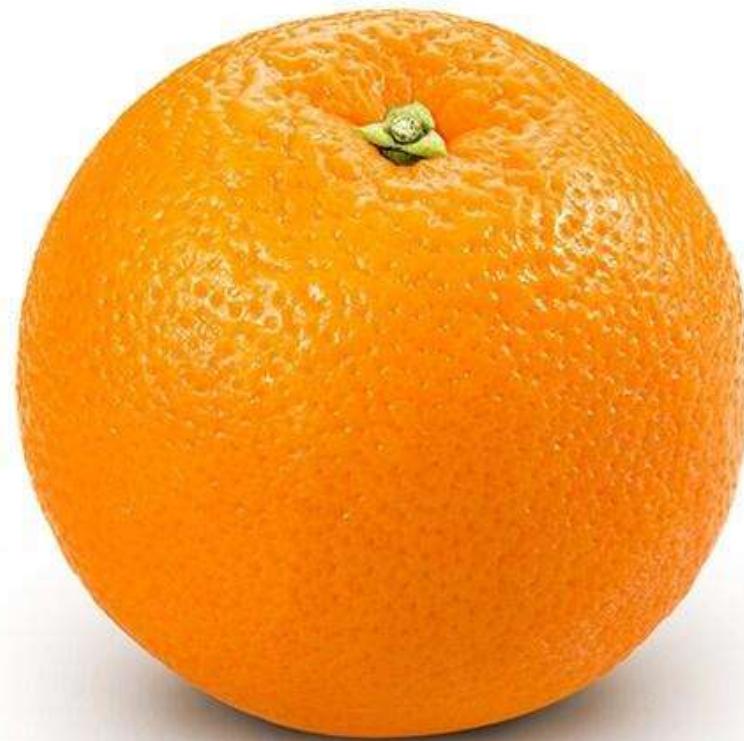
Texture Mapping



November 5, 2025

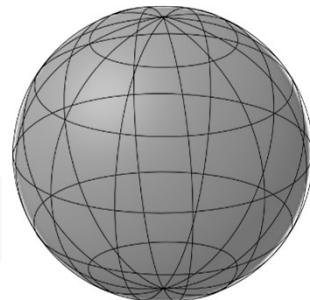
Professor Junyong Noh

Modeling an Orange



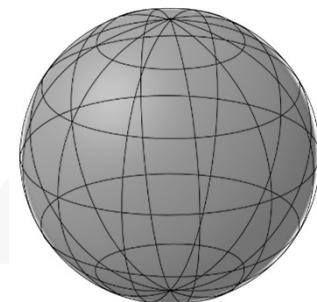
Modeling an Orange

- Geometric orange shape - sphere



Modeling an Orange

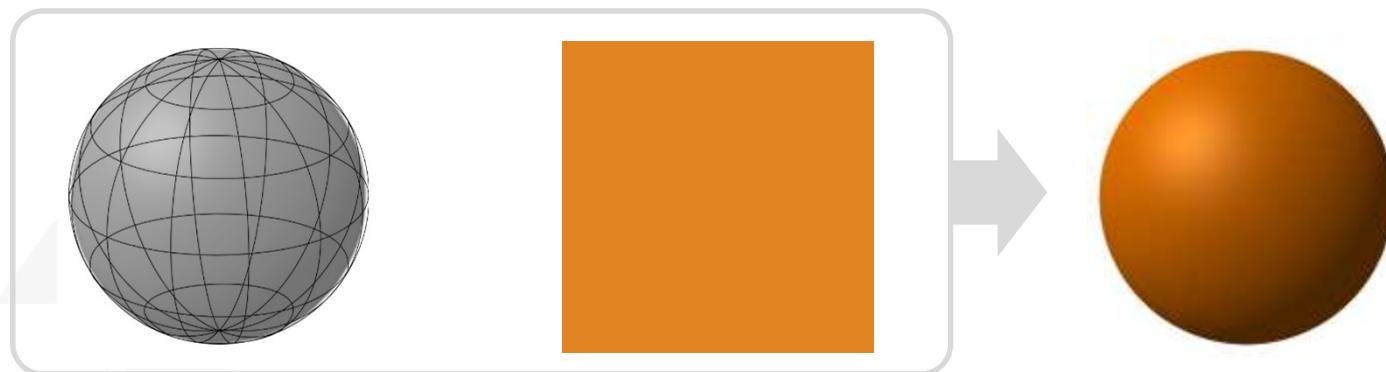
- Orange-colored sphere



Modeling an Orange

- **Orange-colored sphere**

- Too simple
- Surface with uniform reflectance
- Okay for walls or solid balls, but not for most objects



Modeling an Orange

- **Orange-colored sphere**
 - Too simple
 - Surface with uniform reflectance
 - Okay for walls or solid balls, but not most objects
- **Replace sphere with a more complex shape**
 - Takes too many polygons to model all the dimples



Modeling an Orange

■ Texture mapping

- Take a picture of a real orange, scan it, and “paste” onto simple geometric model



Example



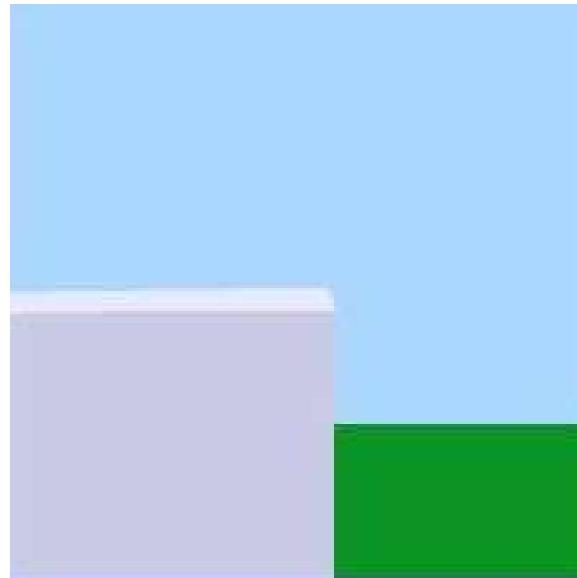
Texture Mapping

- Goal: adding visual detail without increasing geometric detail



Texture Mapping

- Goal: adding visual detail without adding geometric detail

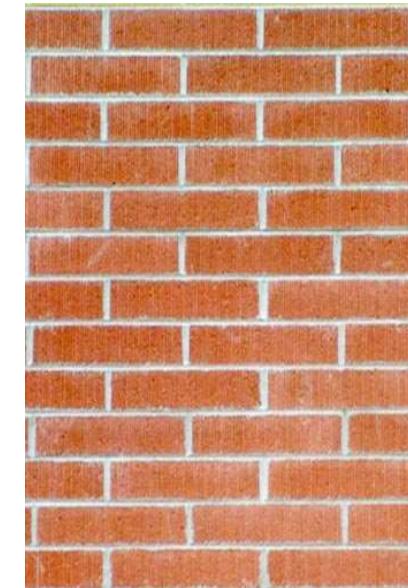
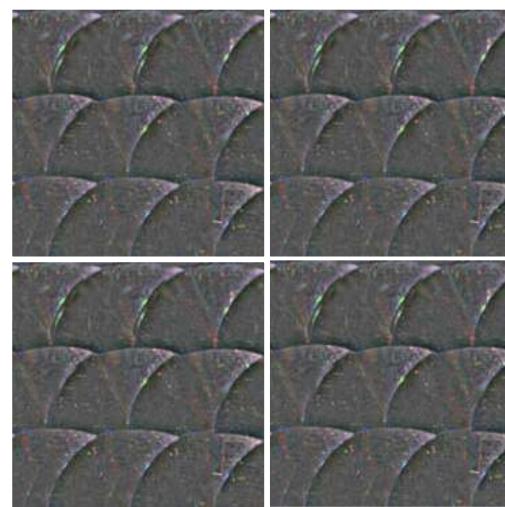
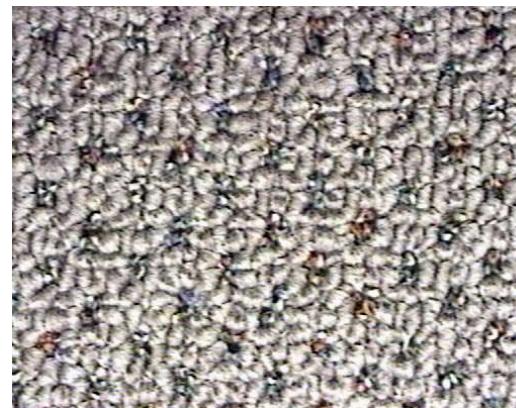
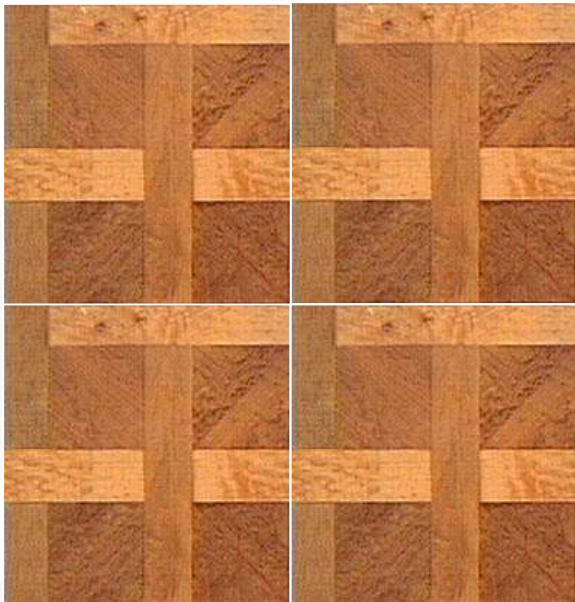


8 polygons



8 polygons

Typical Textures



Textures

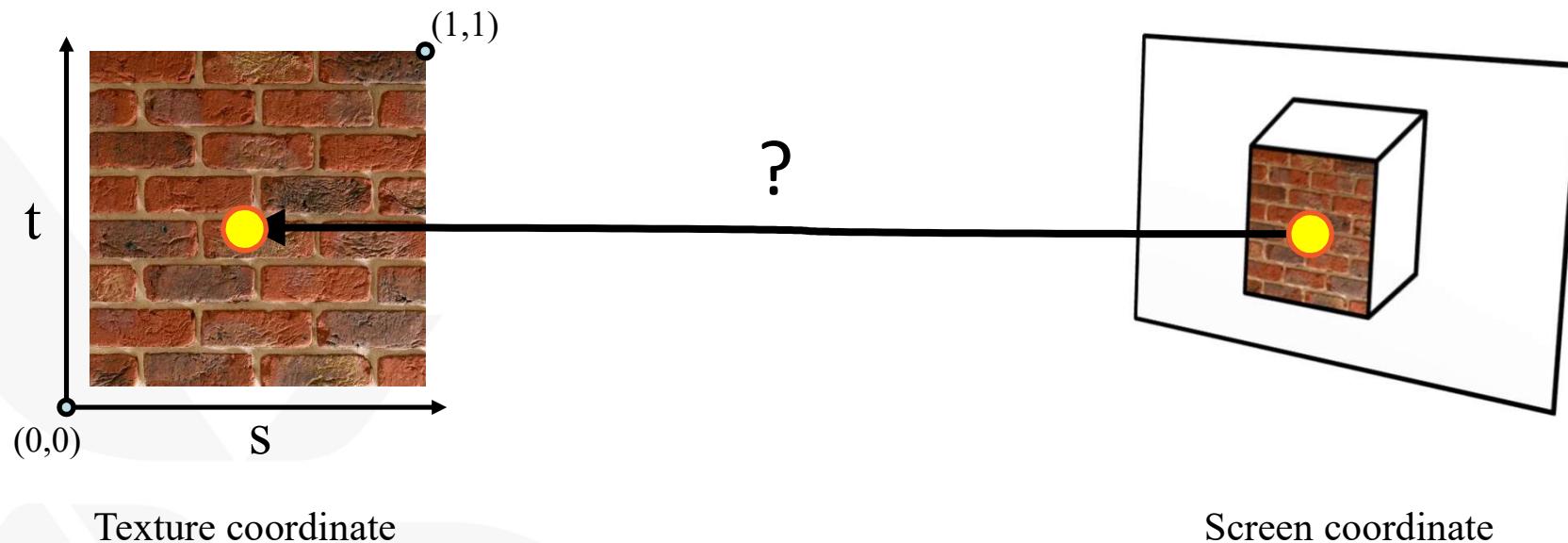
- Any image can be used as a texture map



Texture Mapping Process

■ Mapping Pixels to Texels

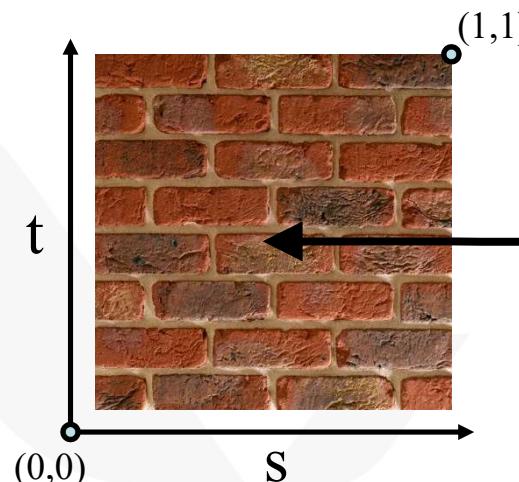
- Basic renderer request: “given pixel (x,y) , what is its color?”
 - “given pixel (x,y) , what is corresponding texel value?”



Texture Mapping Process

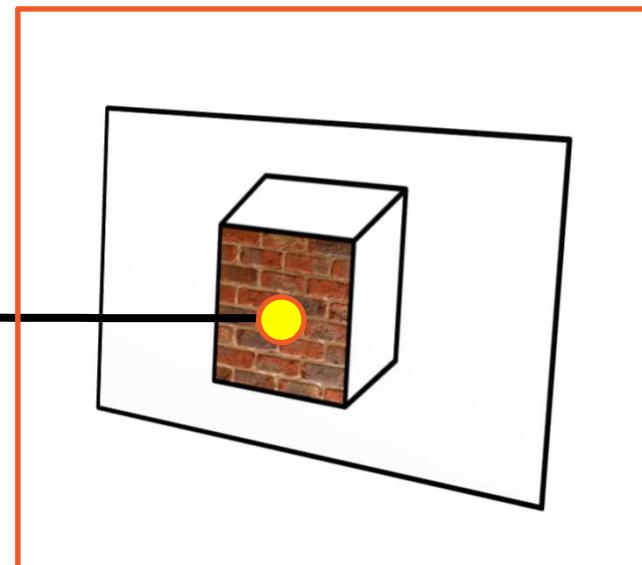
■ Mapping Pixels to Texels

- Basic renderer request: “given pixel (x,y) , what is its color?”
 - “given pixel (x,y) , what is corresponding texel value?”



Texture coordinate

?



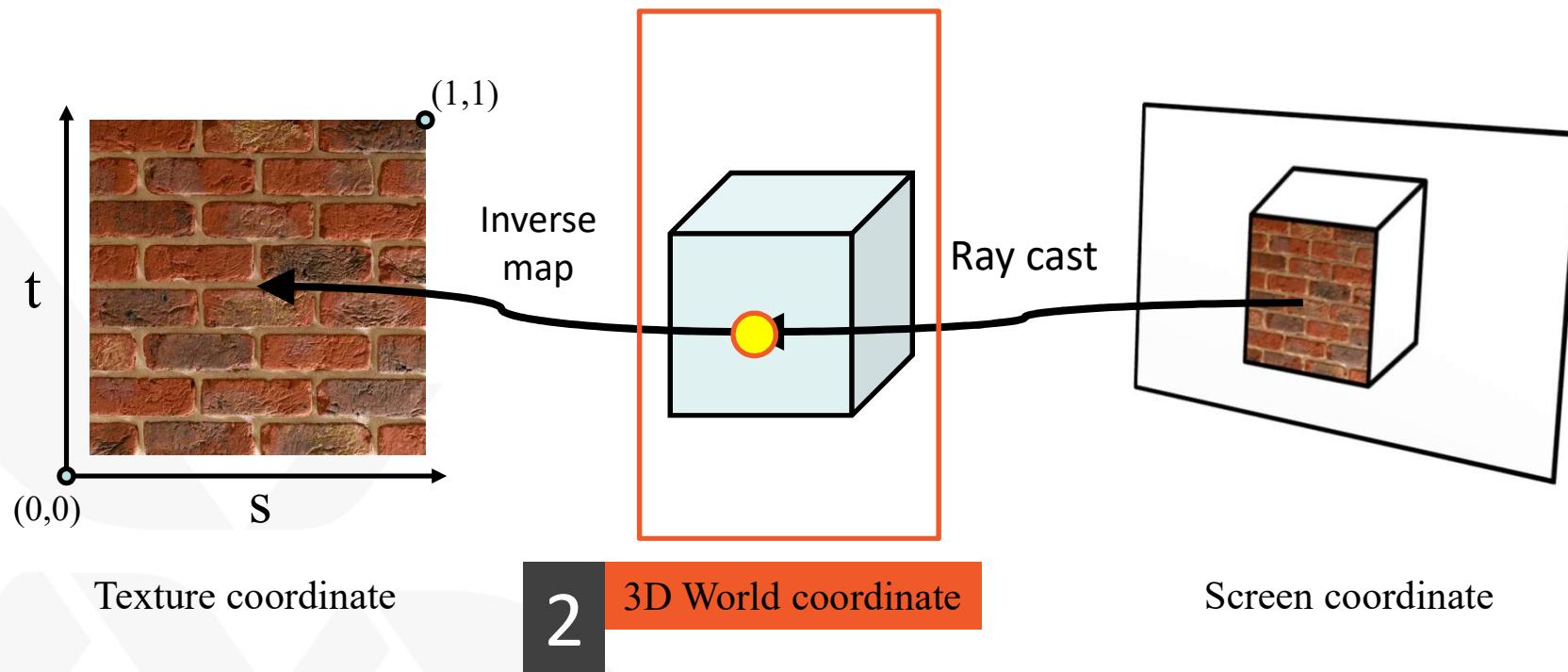
1

Screen coordinate

Texture Mapping Process

■ Mapping Pixels to Texels

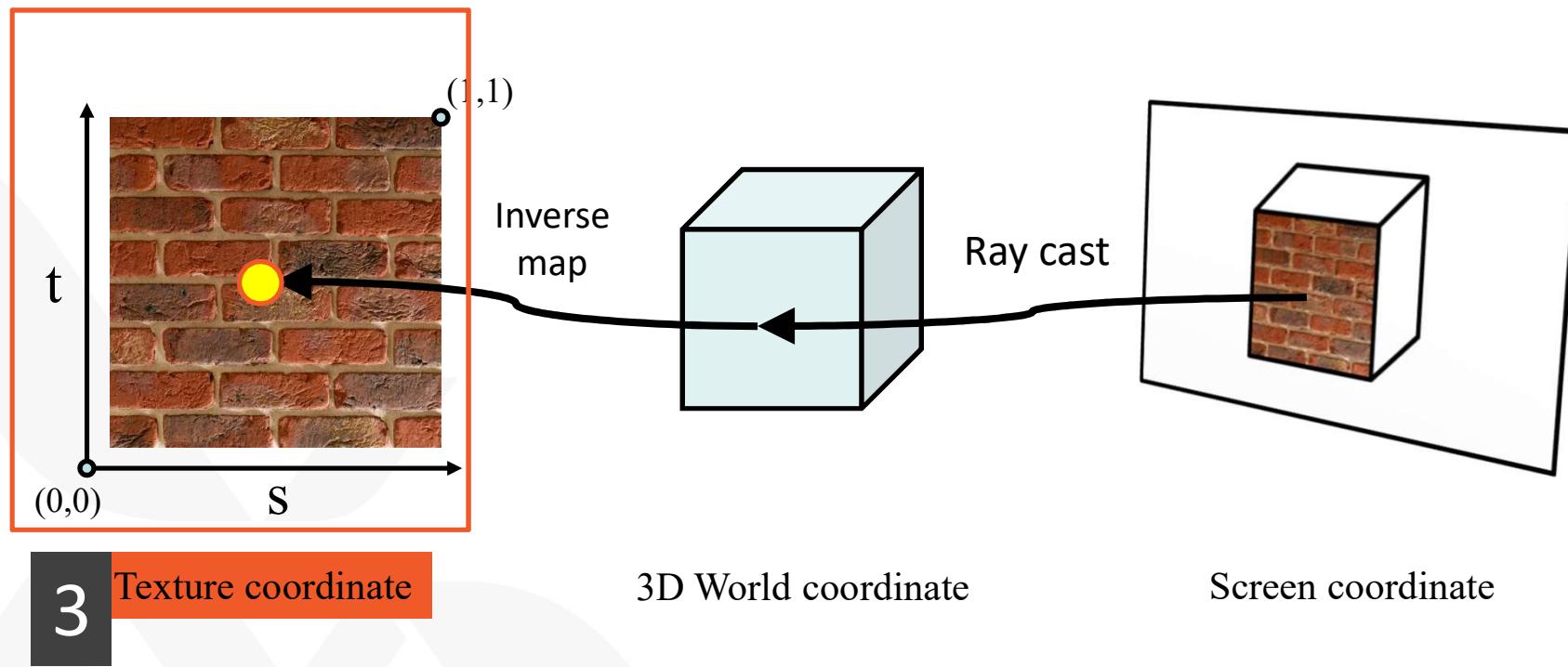
- Basic renderer request: “given pixel (x,y) , what is its color?”
 - “given pixel (x,y) , what is corresponding texel value?”



Texture Mapping Process

■ Mapping Pixels to Texels

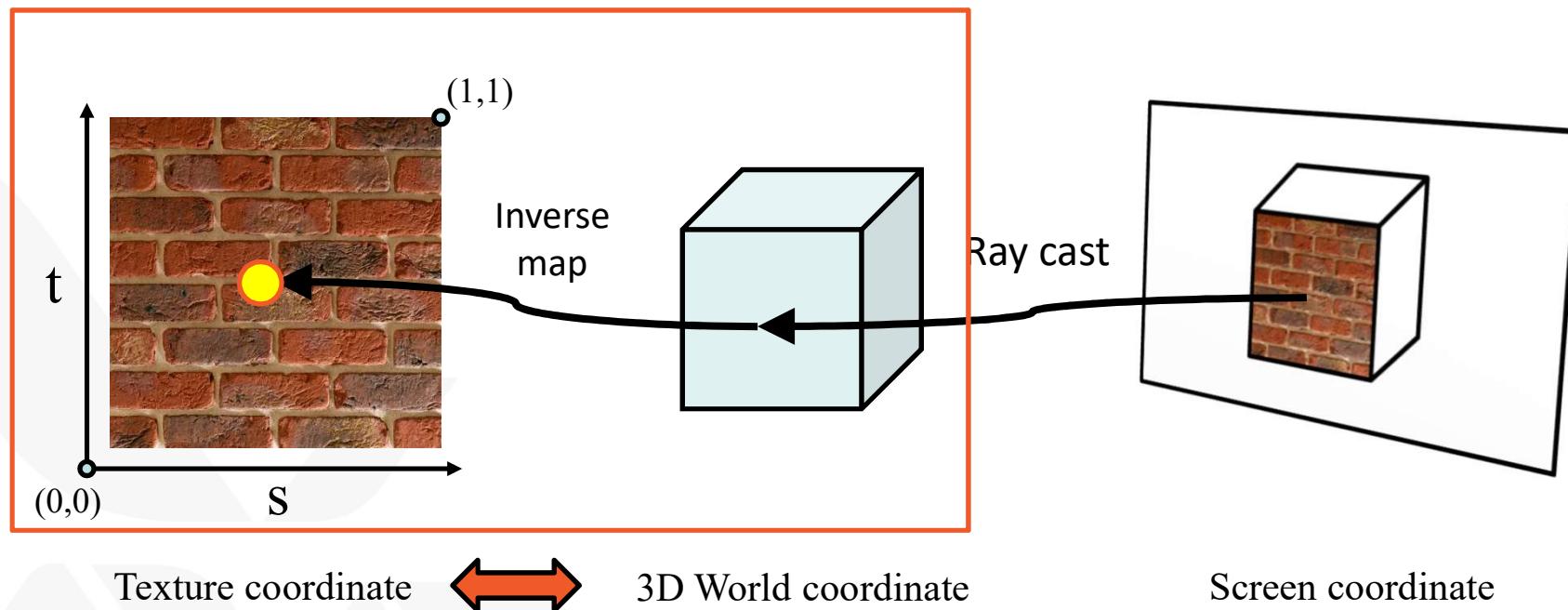
- Basic renderer request: “given pixel (x,y) , what is its color?”
 - “given pixel (x,y) , what is corresponding texel value?”



Texture Mapping Process

■ Mapping Pixels to Texels

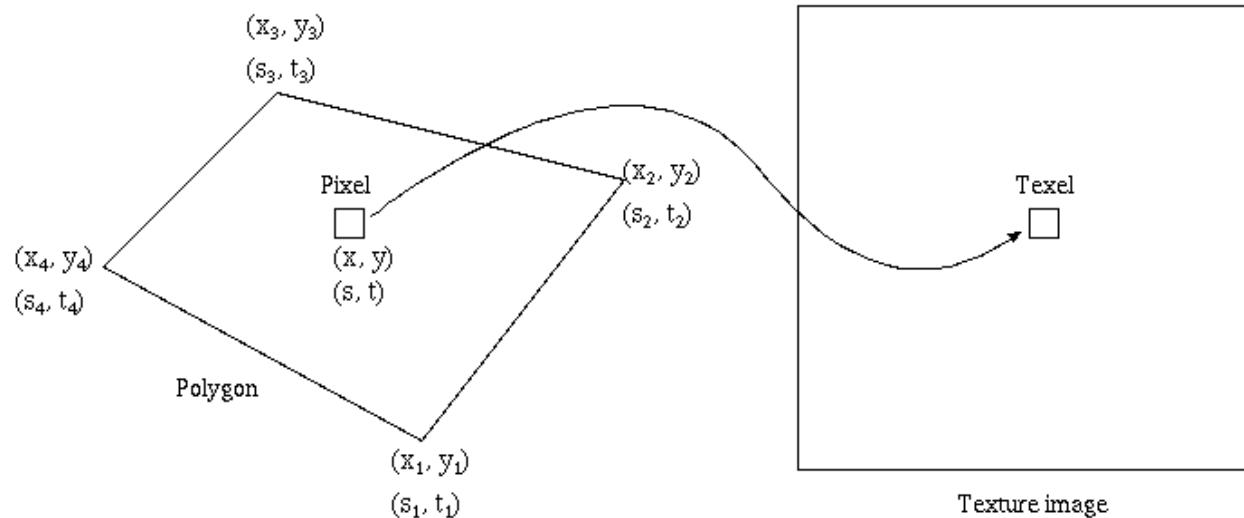
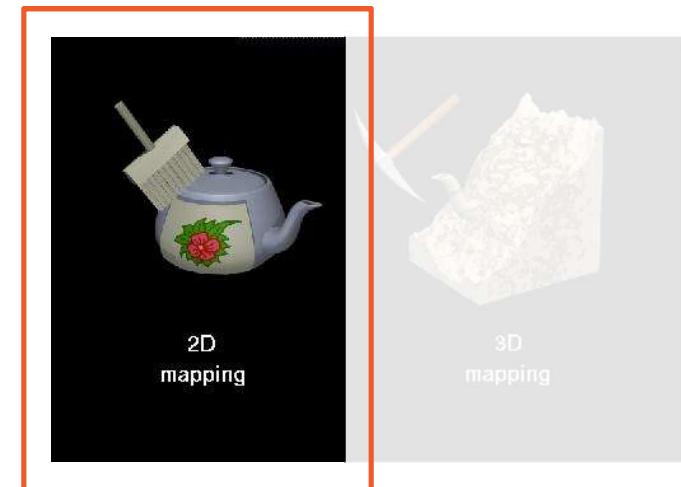
- Basic renderer request: “given pixel (x,y) , what is its color?”
 - “given pixel (x,y) , what is corresponding texel value?”



Texture Mapping

■ 2D Texture

- ‘Wallpaper’ 2D image onto an object



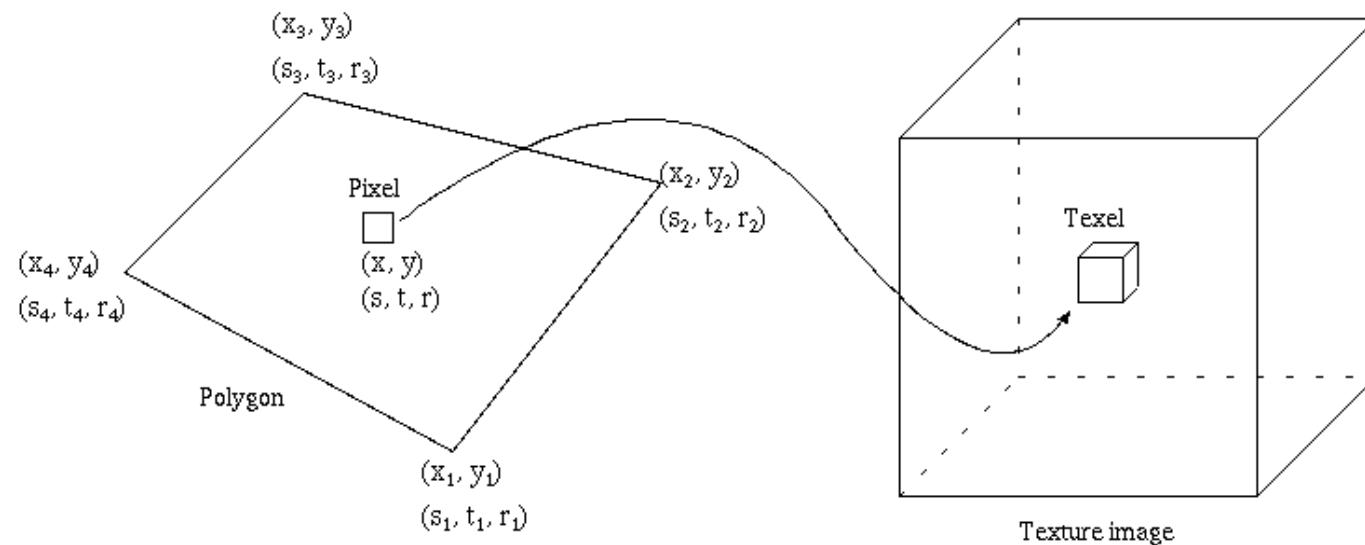
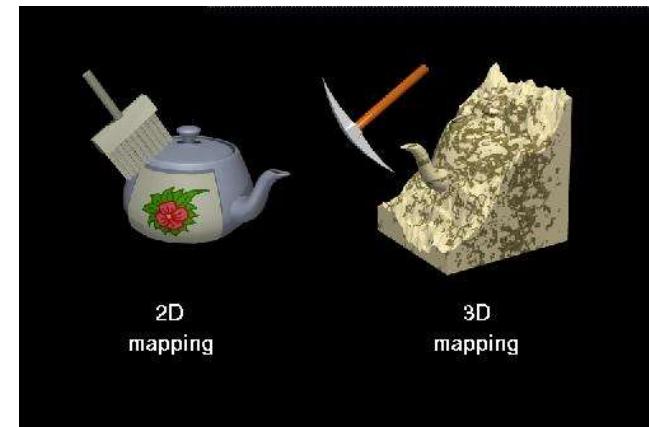
Texture Mapping

■ 2D Texture

- ‘Wallpaper’ 2D image onto an object

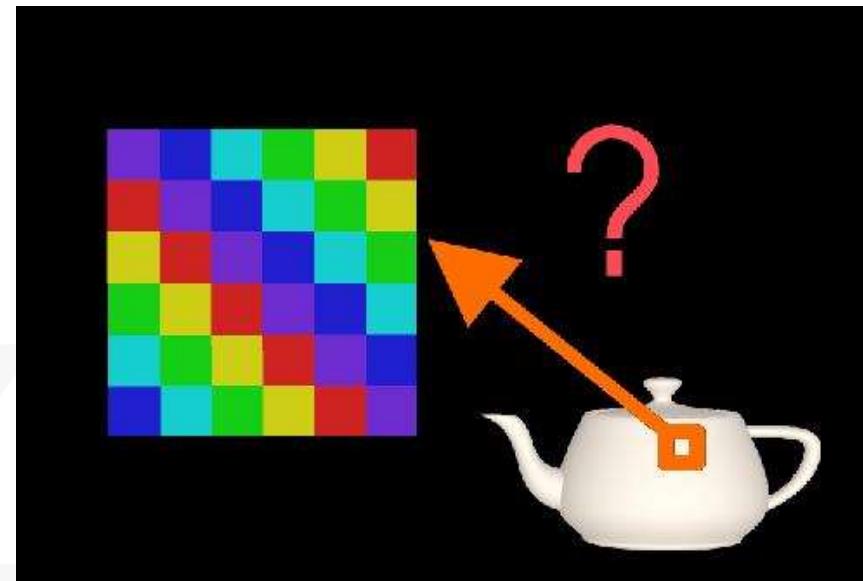
■ 3D Texture

- ‘Carve’ 3D object out of a block



2D Texture Mapping

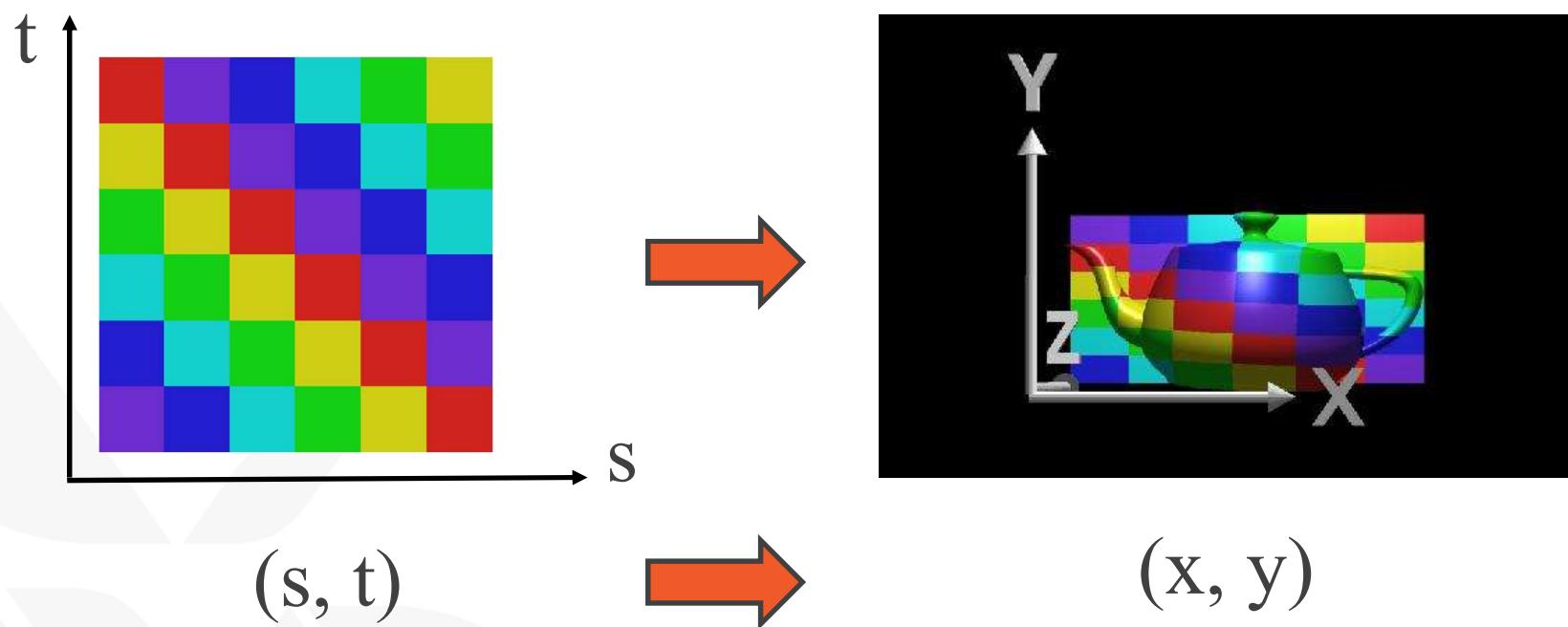
- Given a 2D texture (image) and a 3D object, map the texture onto the object
 - Where does each point on the object map into the texture?



Texture Map Shapes

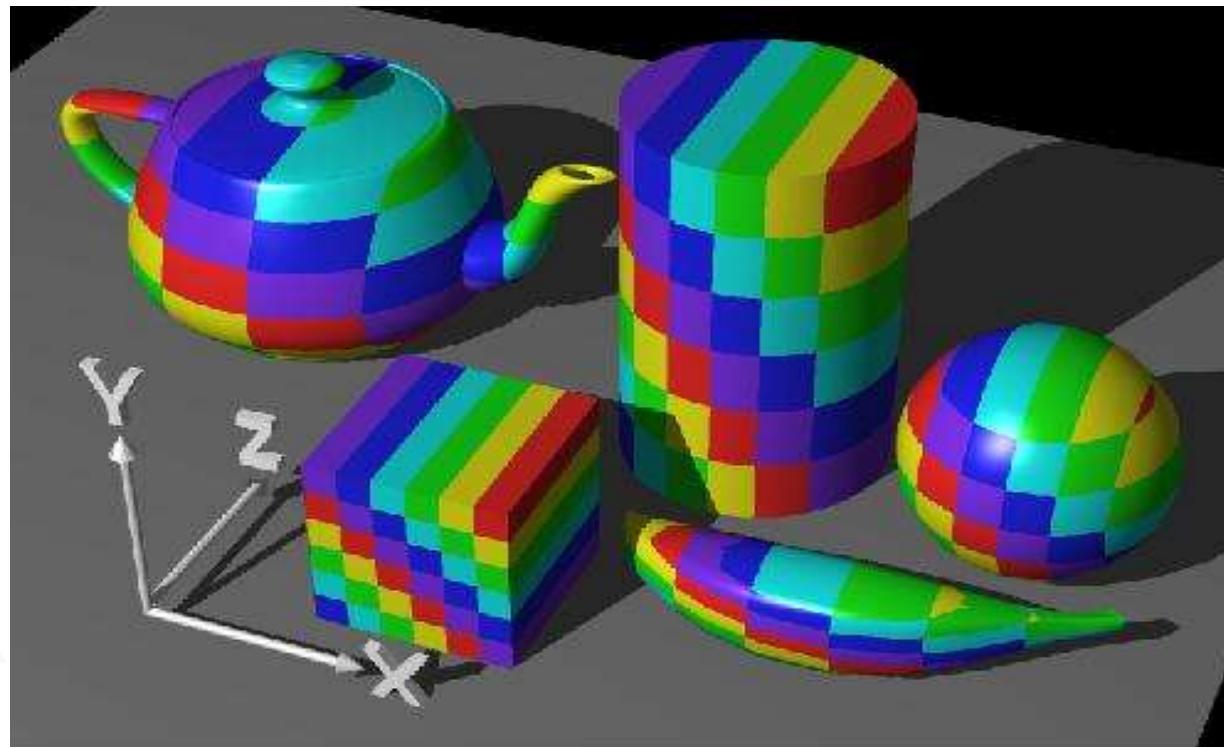
■ Planar Map

- Simply remove one of the object's coordinates (project onto that coordinate plane)



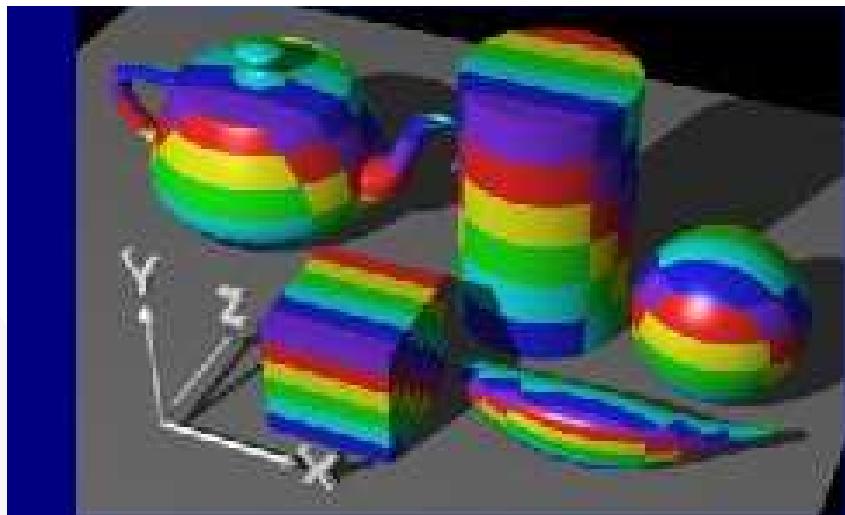
Planar Map

- The texture is constant in one direction.
 - Often not what you want

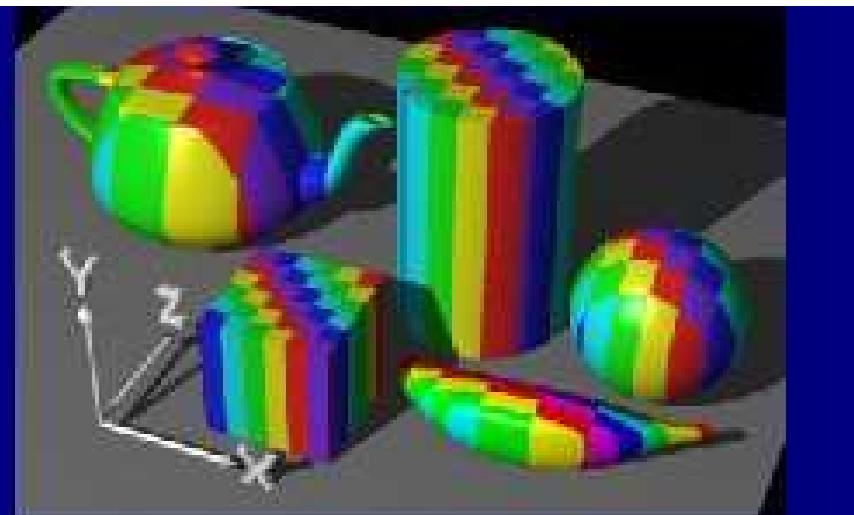


Planar Map

- Similarly, using other directions:



X axis removed



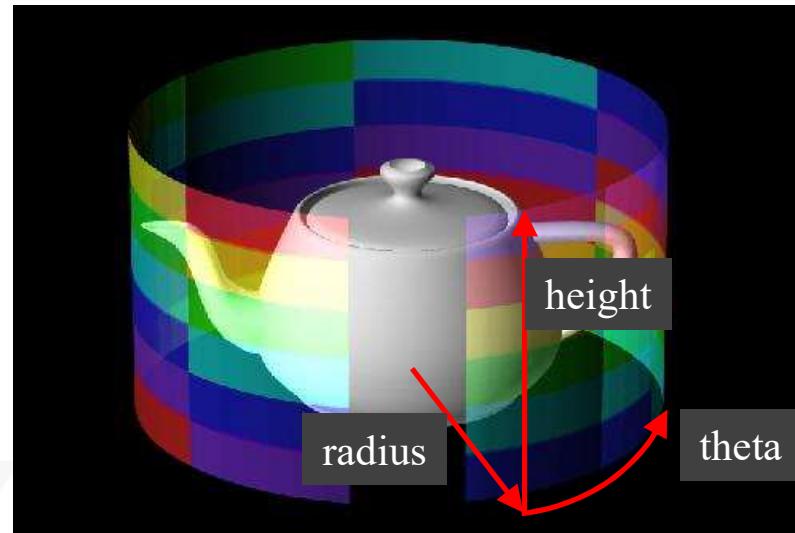
Y axis removed

Cylindrical Map



Cylindrical Map

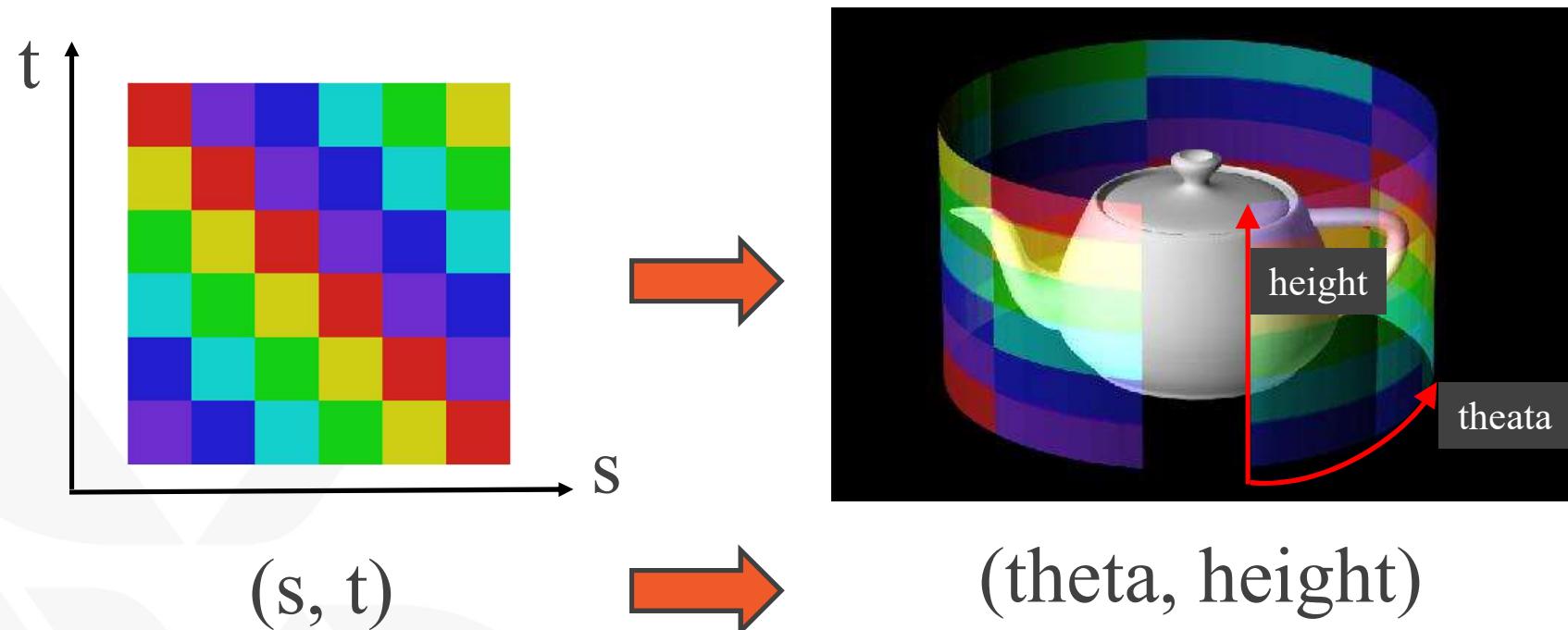
- (x,y,z) is converted to $(r, \theta, \text{height})$.



Cylindrical Map

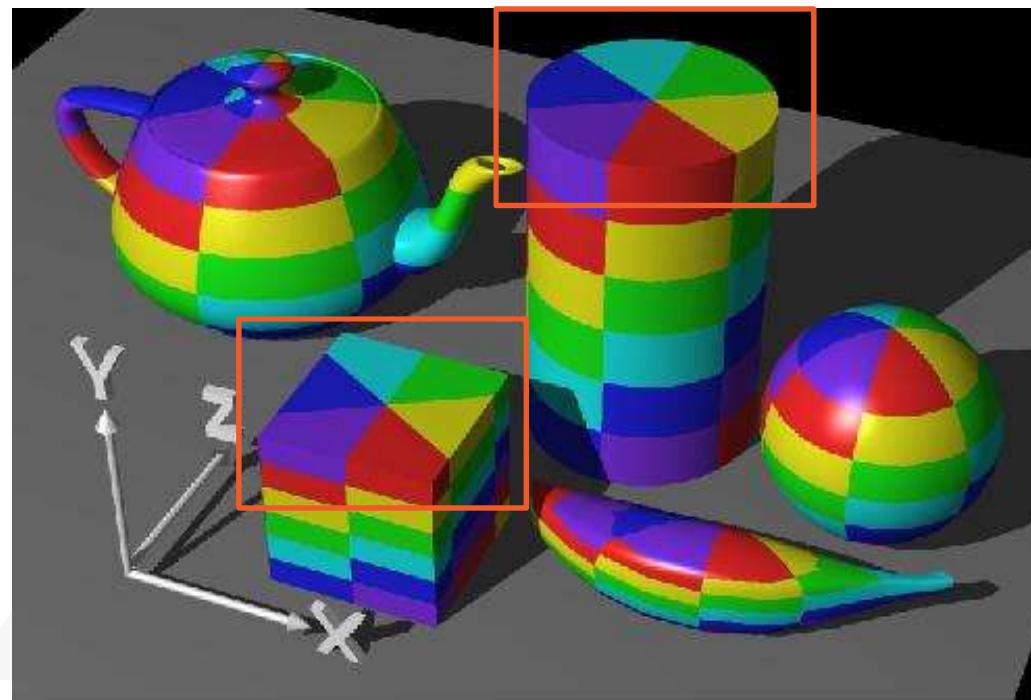
- (x, y, z) is converted to $(r, \theta, \text{height})$.

- For texture mapping, θ is converted into a s -coordinate and height is converted into a t -coordinate.



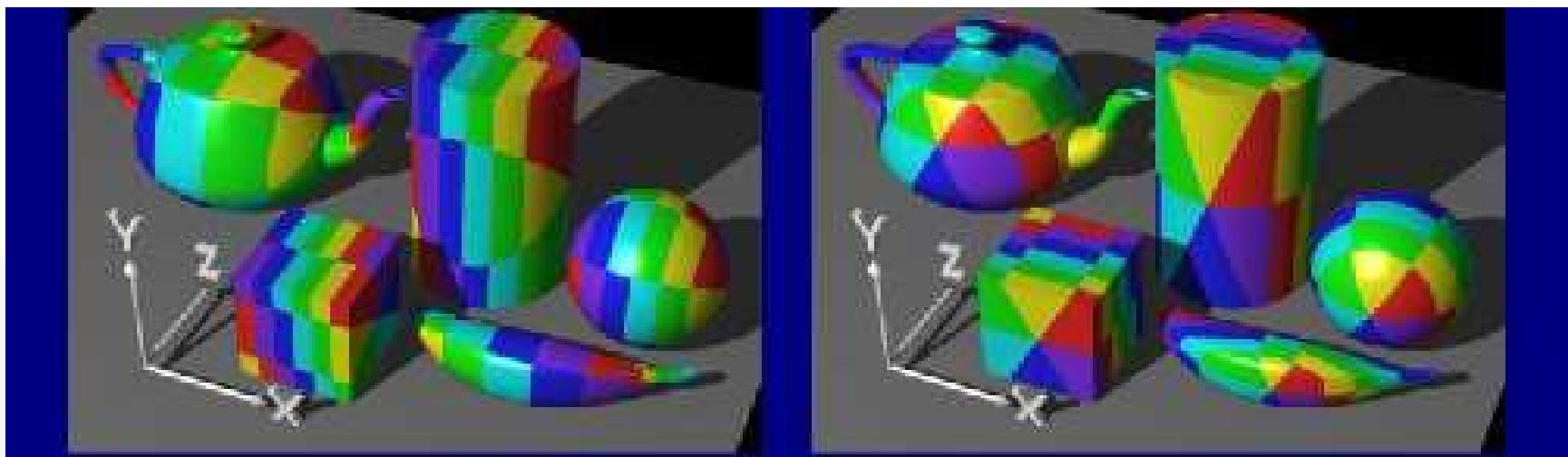
Cylindrical Map

- At minimum and maximum extents of the cylinder, the texture is pinched together.



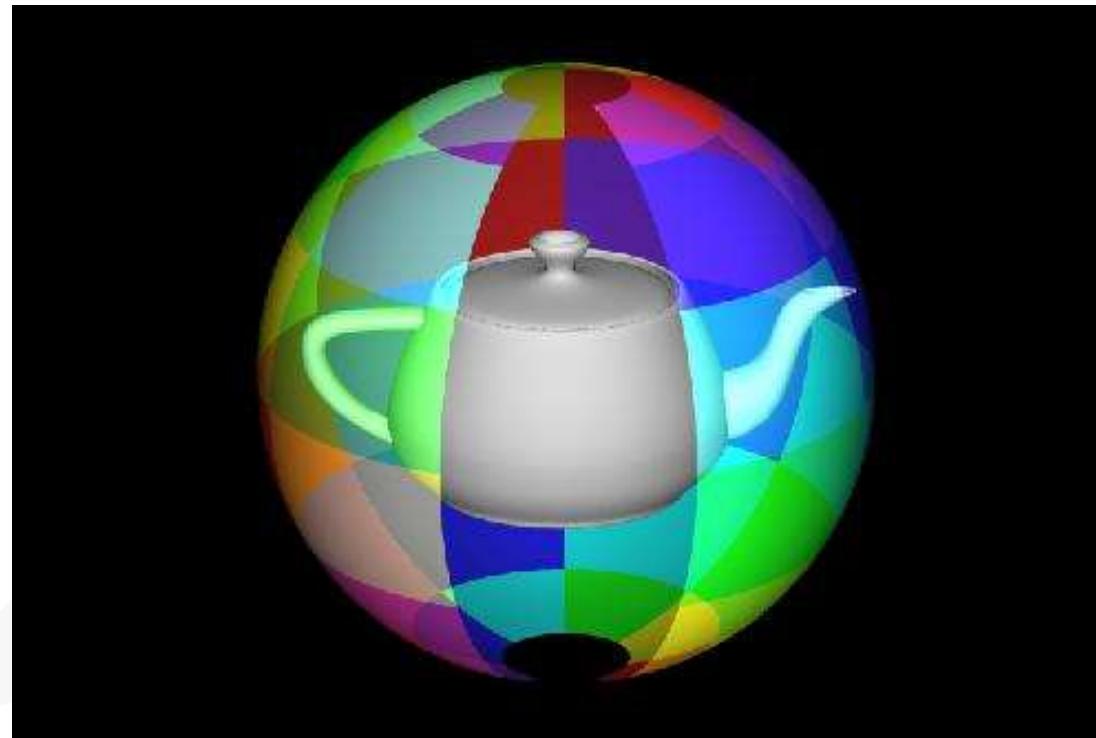
Cylindrical Map

- Similarly, using other directions:



Spherical Map

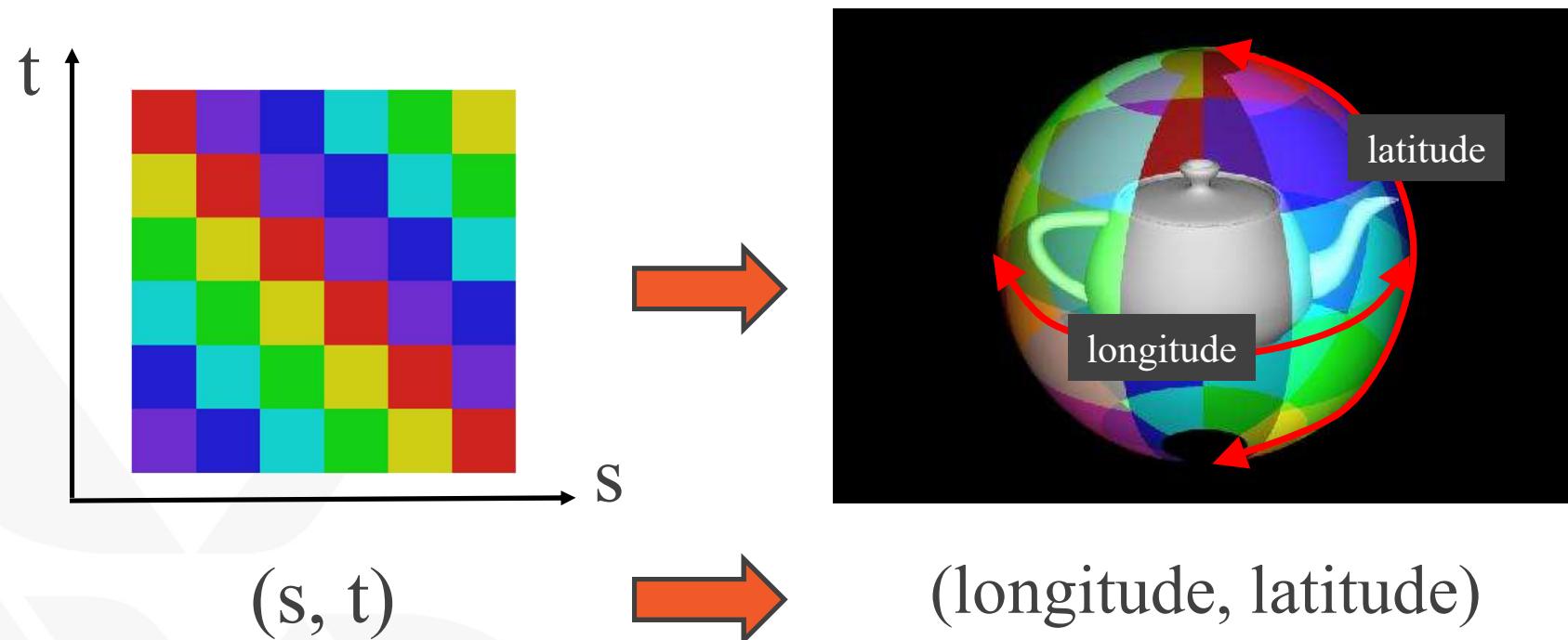
-
- Convert from (x, y, z) to spherical coordinates.



Spherical Map

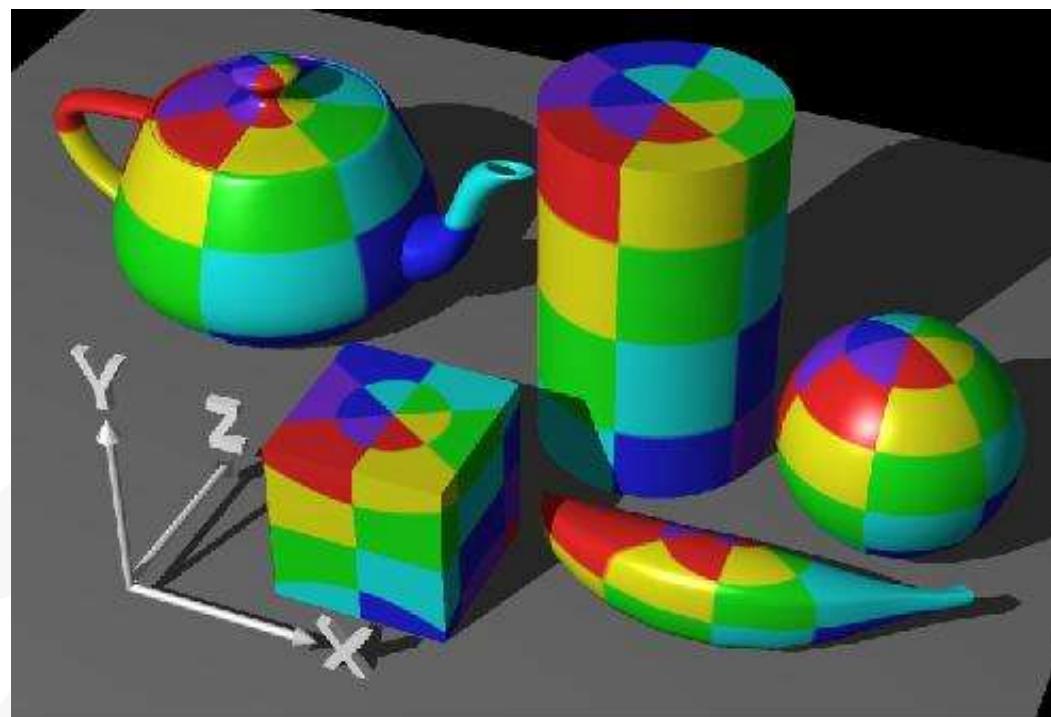
▪ Convert from (x, y, z) to spherical coordinates.

- Longitude is converted to a s-coordinate, latitude is converted to a t-coordinate.



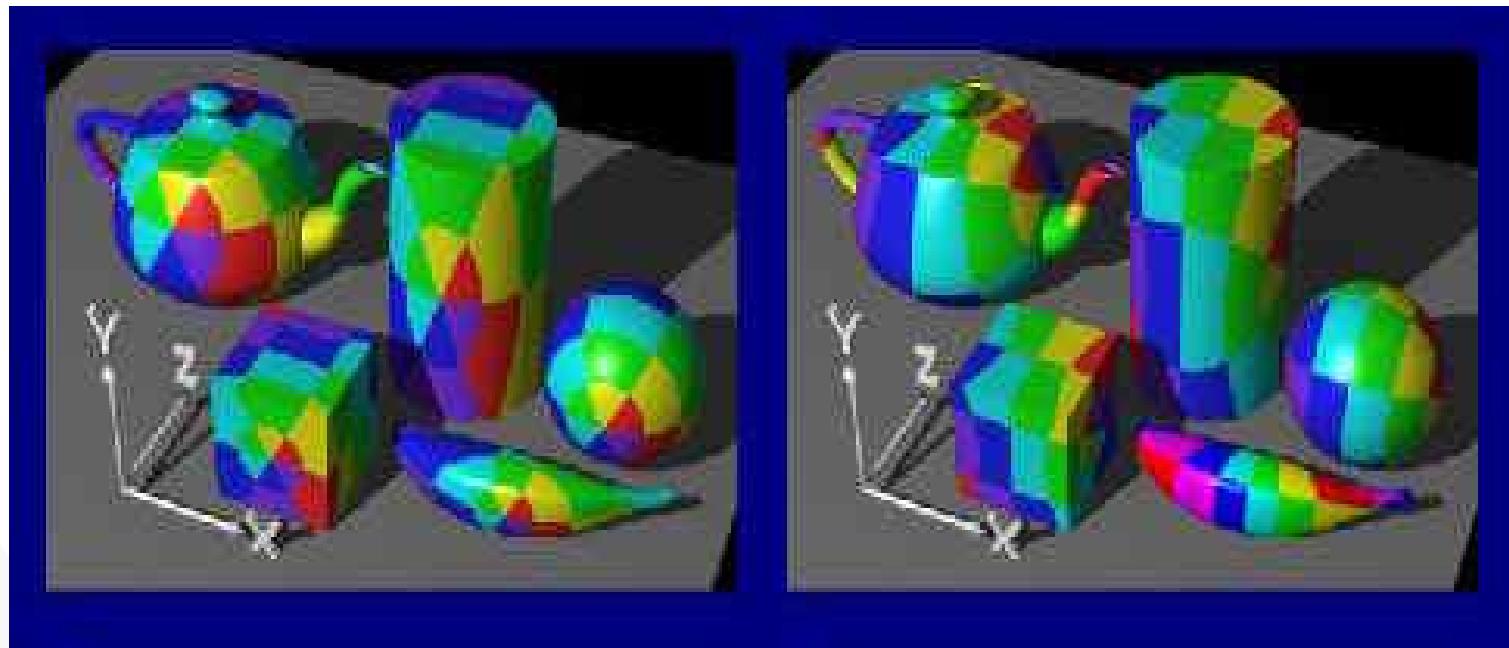
Spherical Map

- This still pinches the texture at the poles, but it's different from using a cylindrical map.



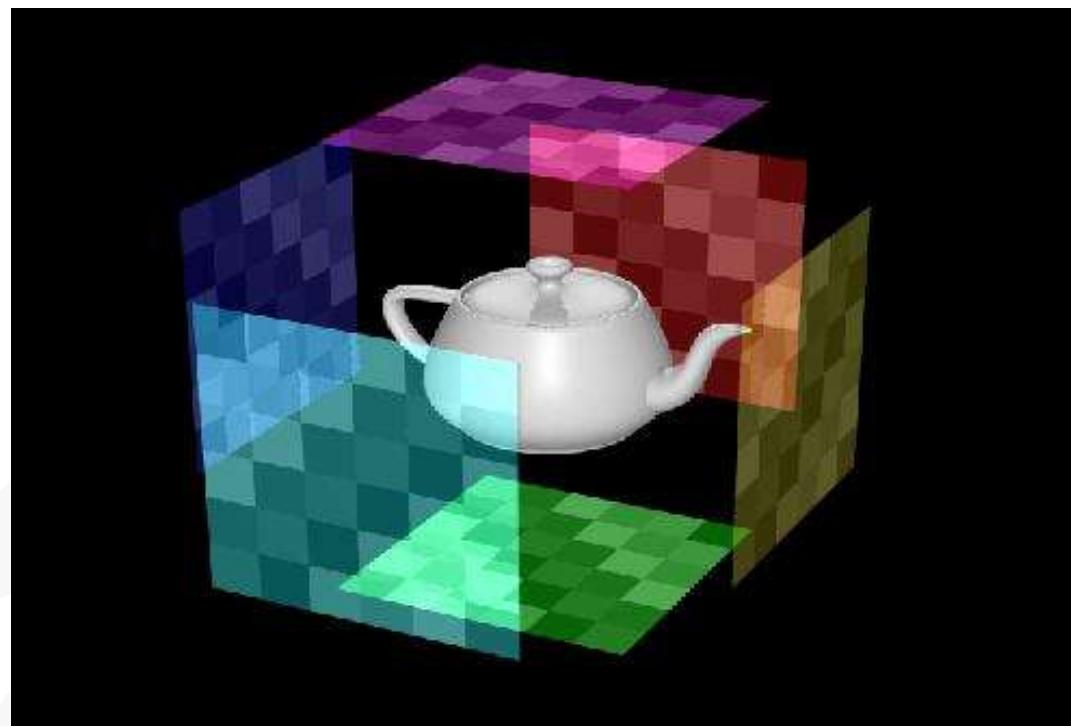
Spherical Map

- Similarly, using other directions:



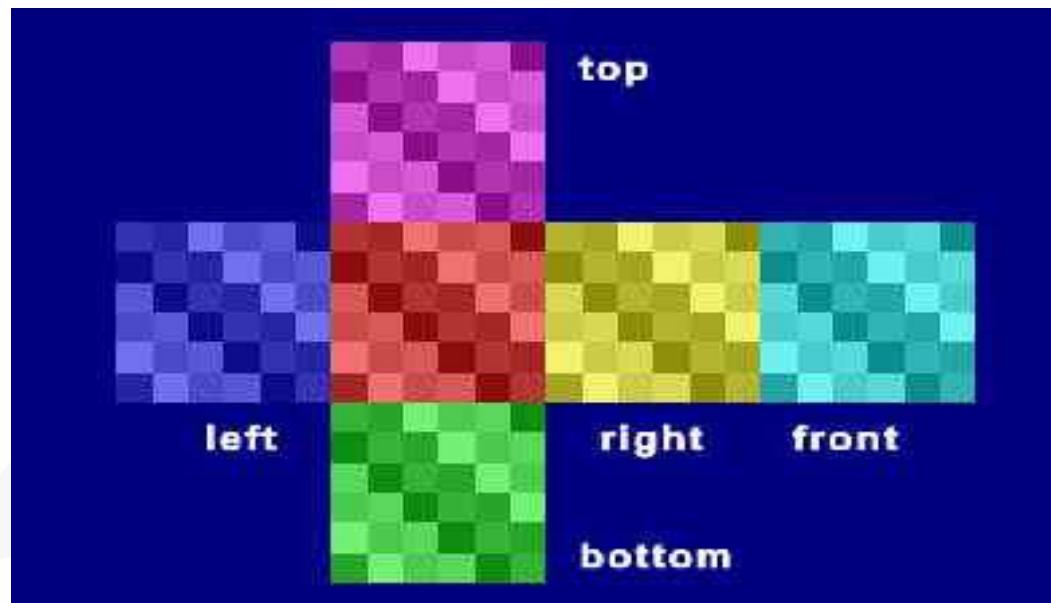
Box Map

- Use of a collection of planar maps to provide better coverage than using a single planar map:

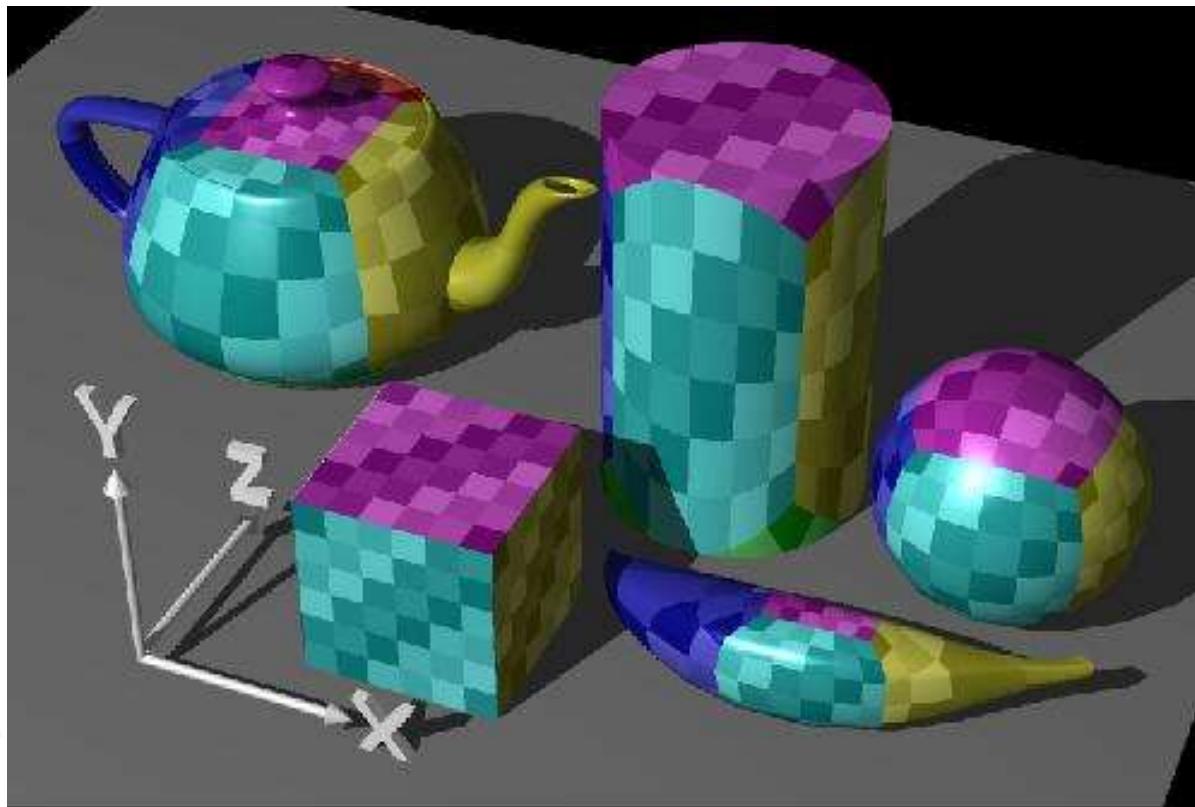


Box Map

- Projecting each plane onto its portion of the object
- Use of the object's normal to determine which texture to use

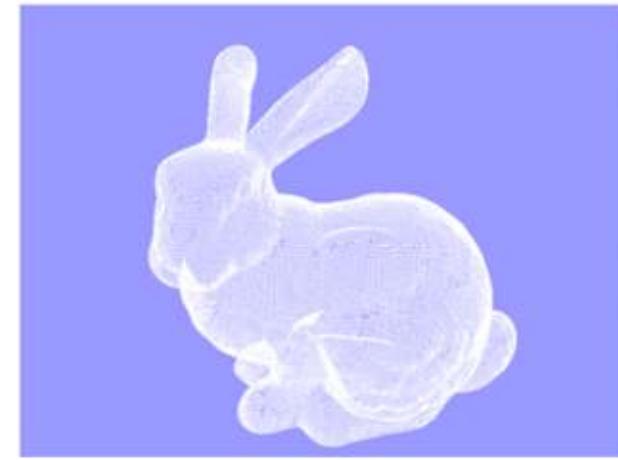
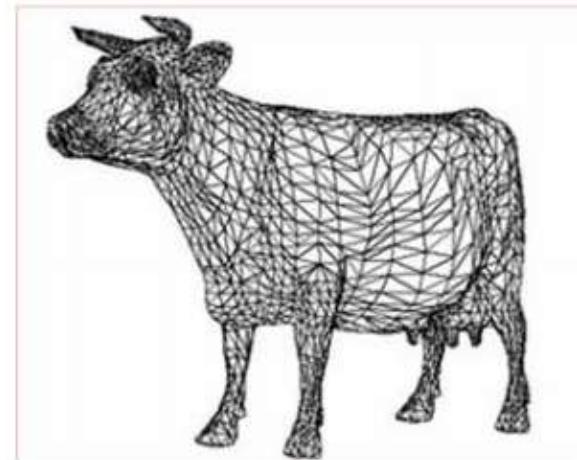
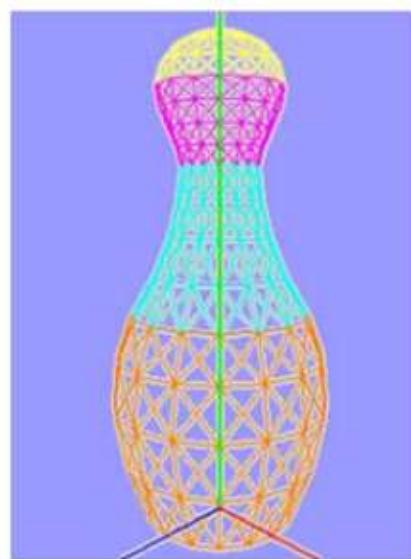


Box Map



Texturing Polygon Meshes

- How to assign texture coordinates to things like?



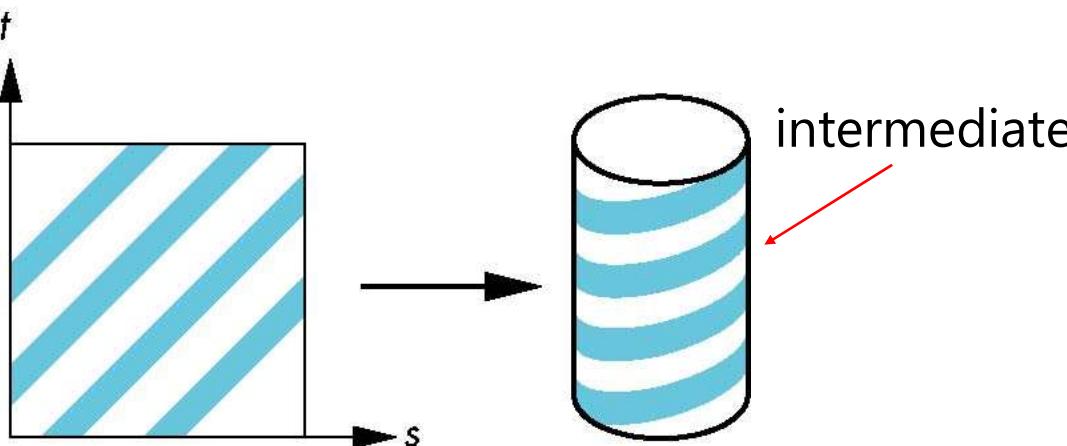
Two-part Mapping

- One solution to the mapping problem is to first map the texture to a simple intermediate surface



Two-part Mapping

- One solution to the mapping problem is to first map the texture to a simple intermediate surface
 - Cylinders and sphere are most common
 - (planes or cubes also used)



Two-part Mapping

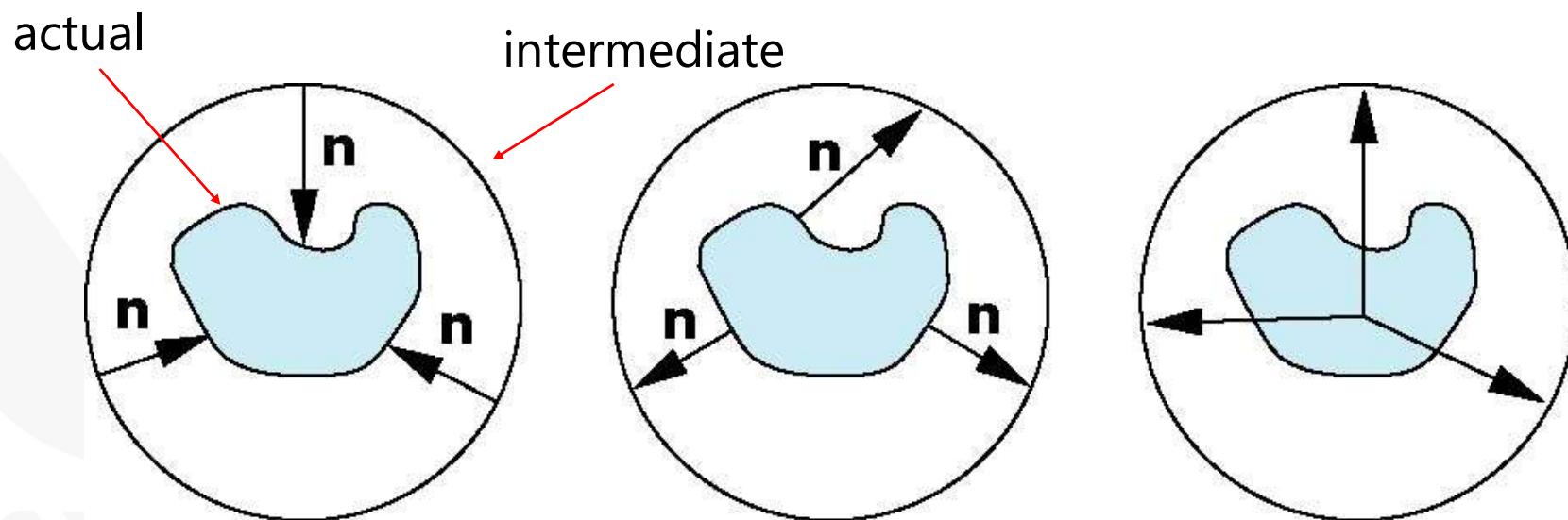
- Map from intermediate object to actual object



Two-part Mapping

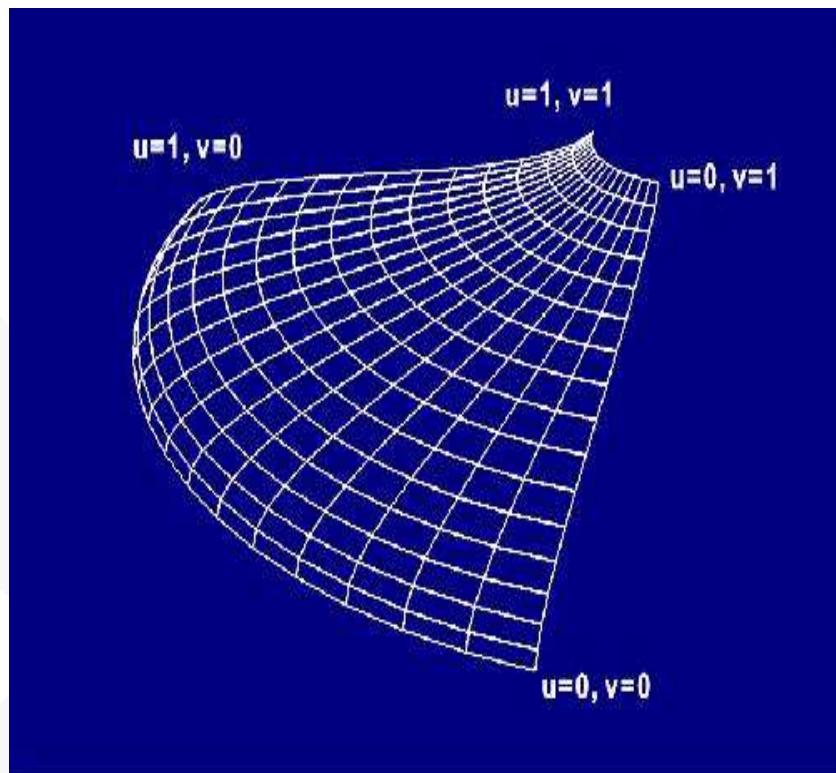
- Map from intermediate object to actual object

- Normals from intermediate to actual
- Normals from actual to intermediate
- Vectors from center of intermediate



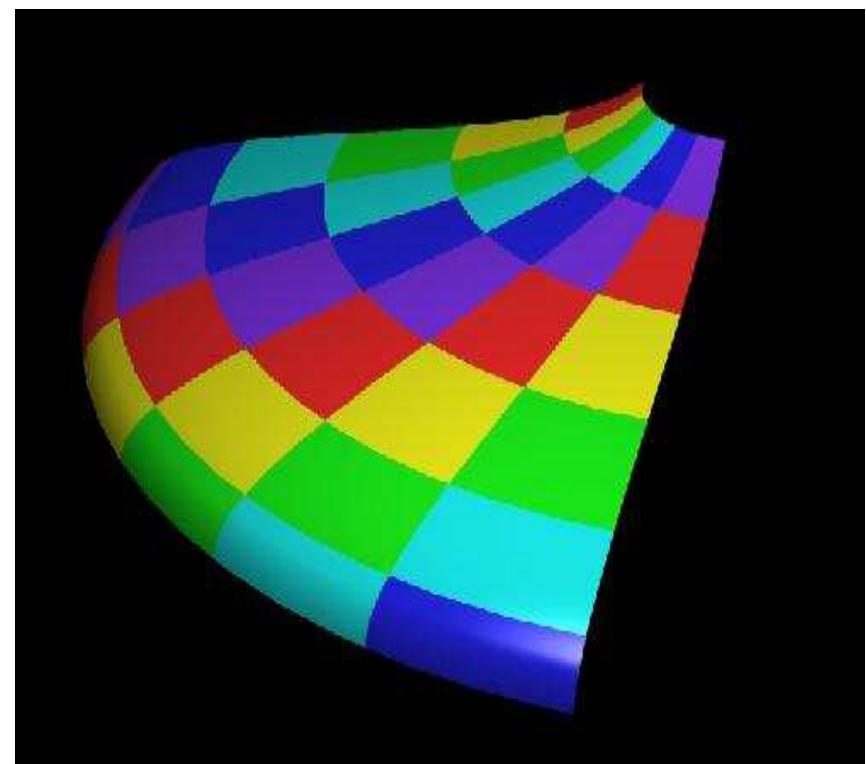
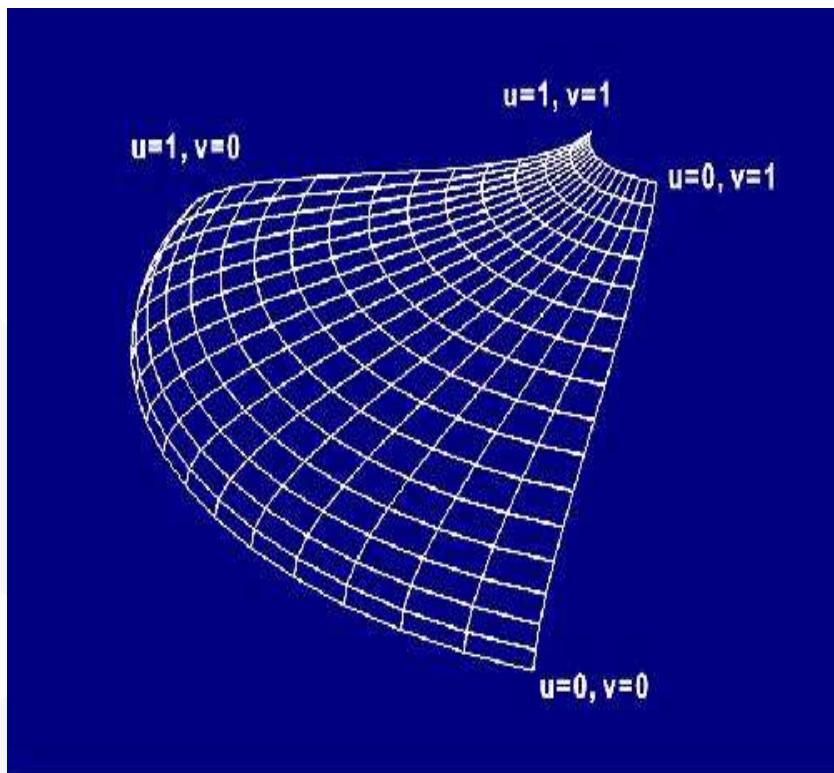
Mapping Parametric Surfaces

- Parametric surfaces are already parameterized by (u, v) .
- Use the (u, v) parameters as the (s, t) texture parameters



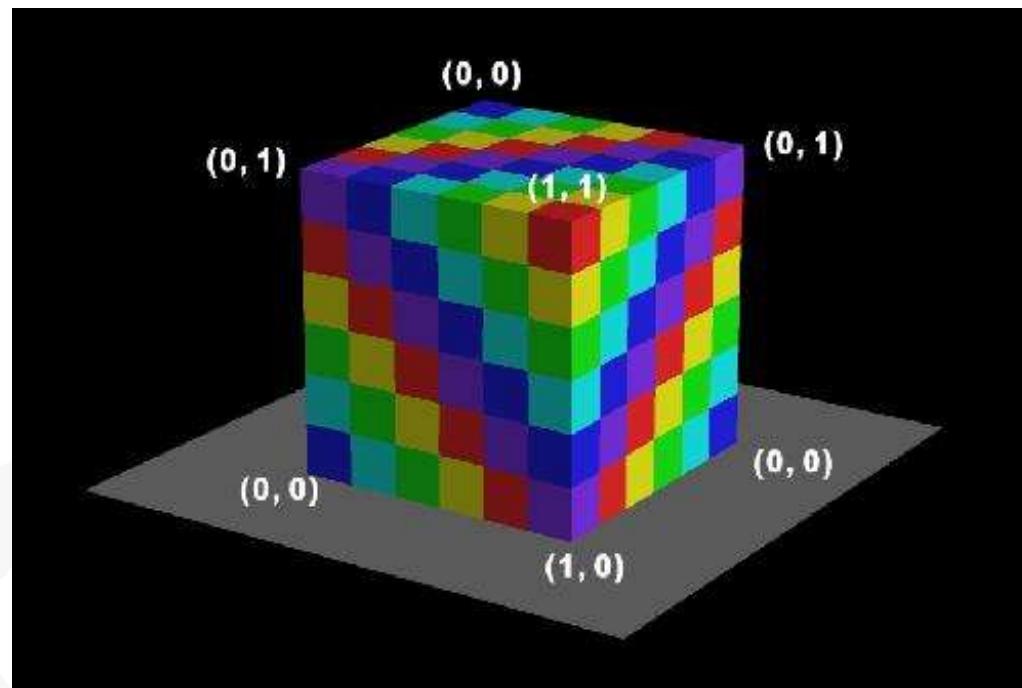
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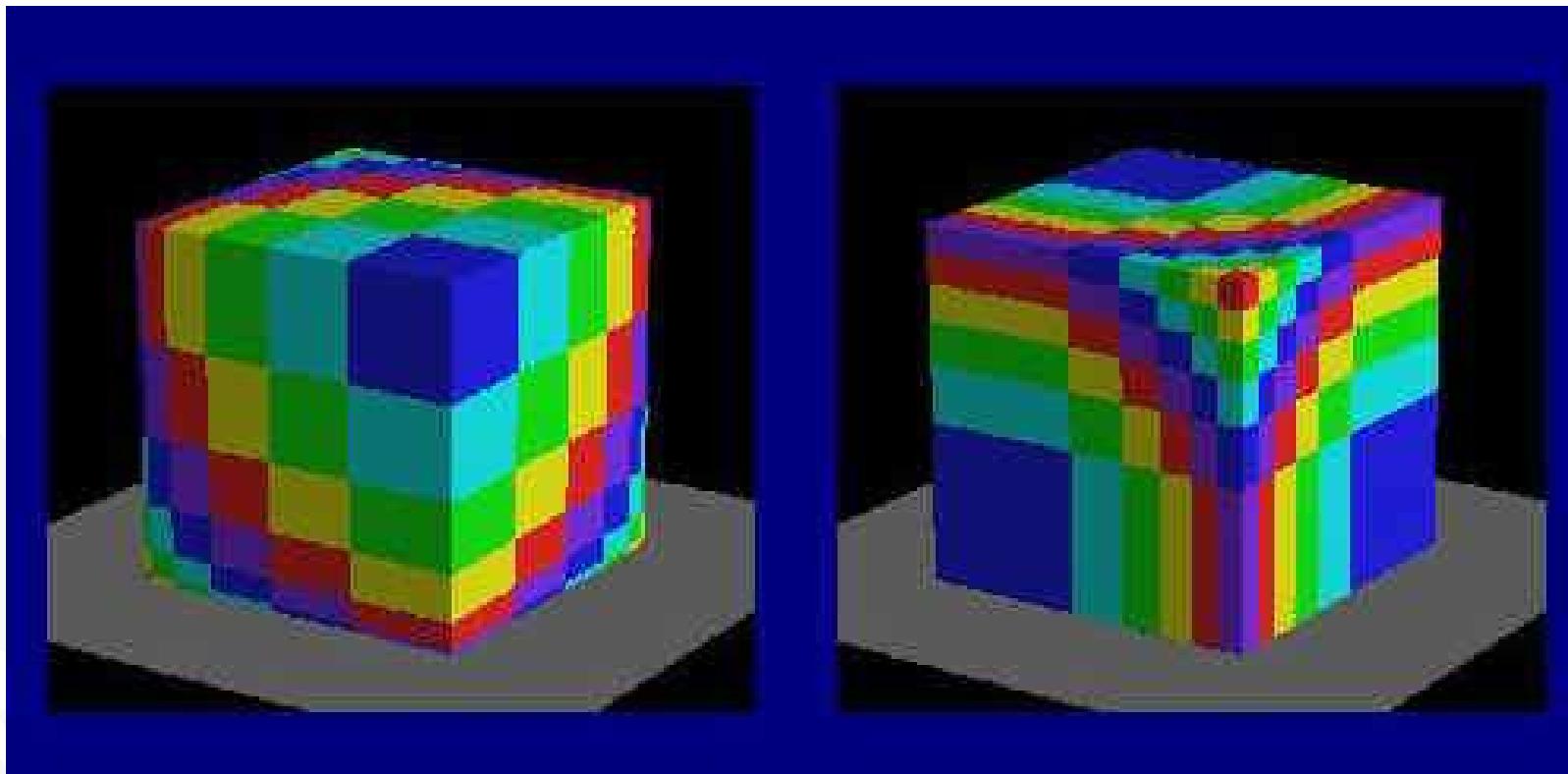
Non-Parametric Surfaces

- If we assign values to the vertices in the range $(0,1)$, we can use the same texture mapping approach.

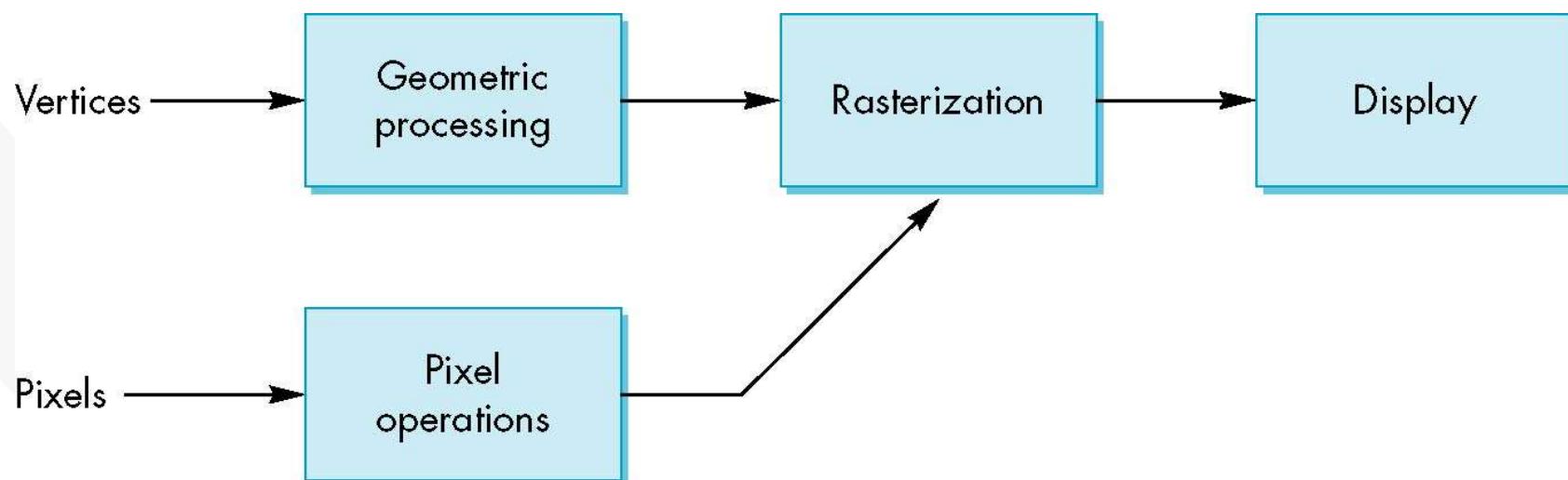


Non-linear Mapping

- We can distort the texture using a non-linear mapping:

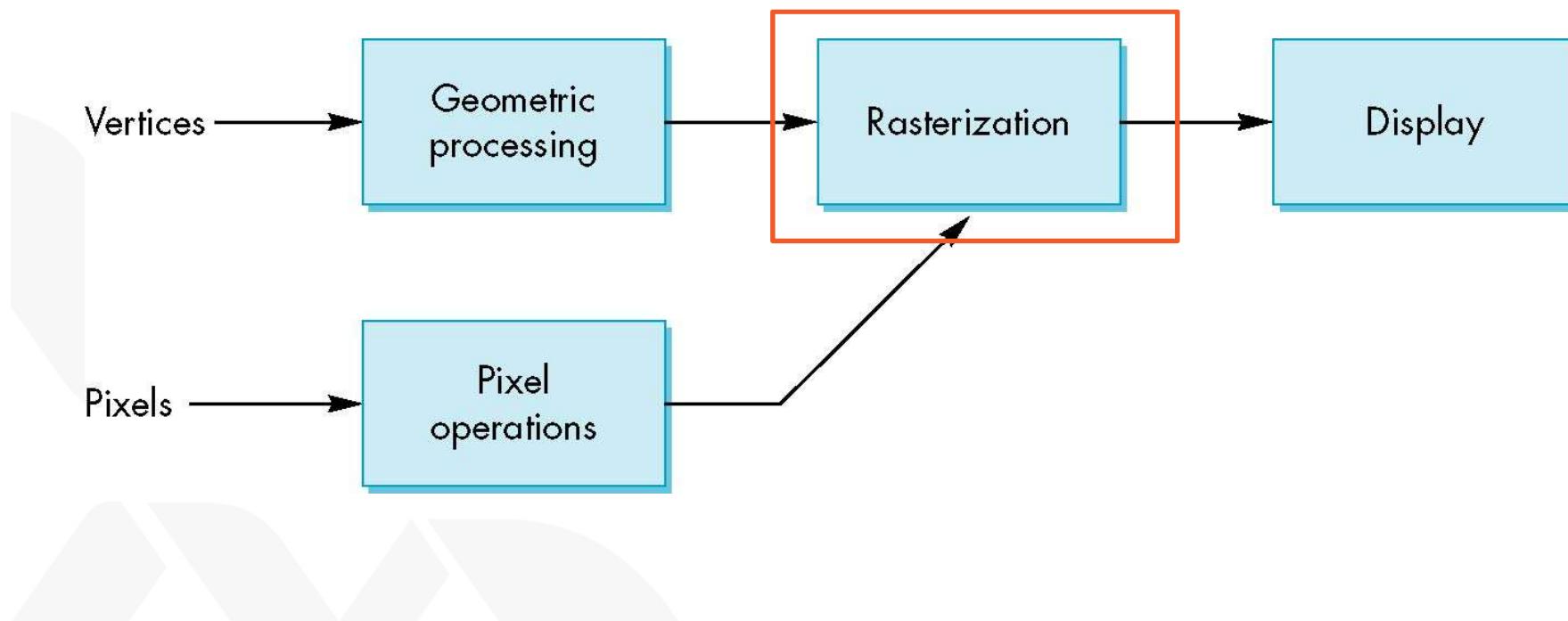


Where Does Mapping Take Place?



Where Does Mapping Take Place?

- **Mapping techniques are implemented at the end of the rendering pipeline**
 - Very efficient because few polygons pass down the geometric pipeline



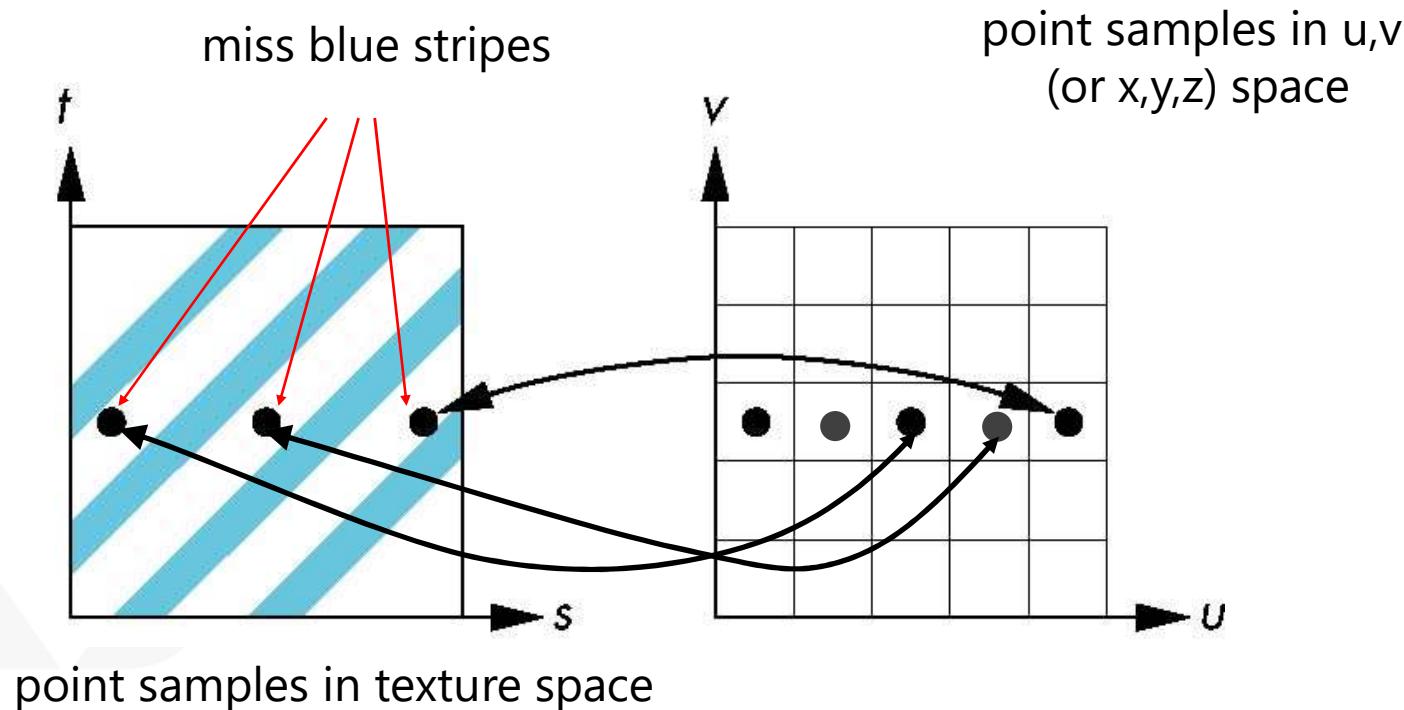
Aliasing

- Point sampling of the texture can lead to aliasing errors



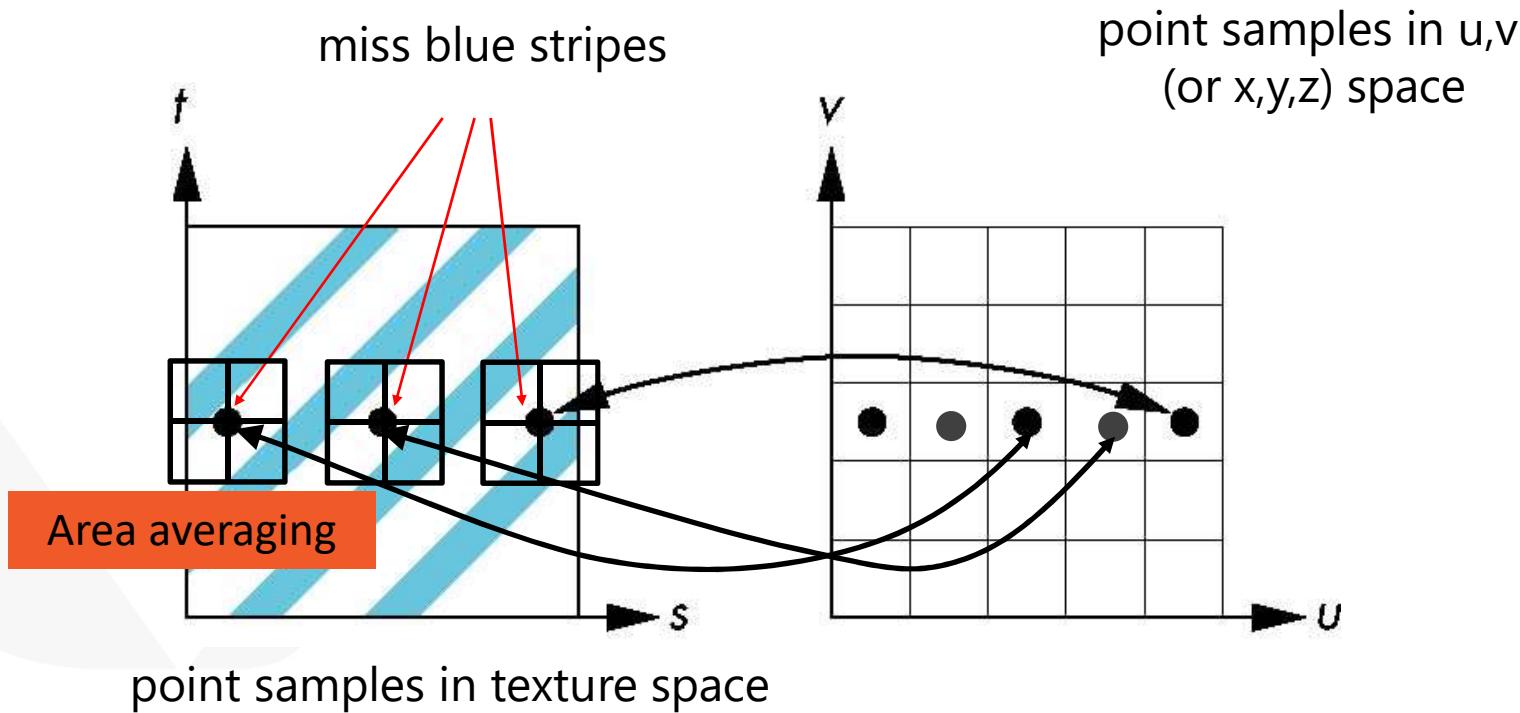
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Aliasing

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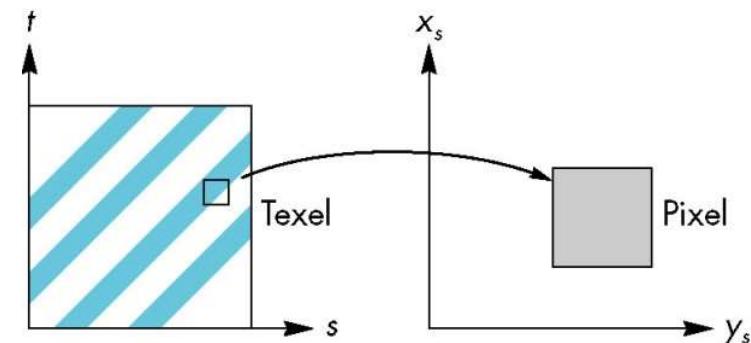
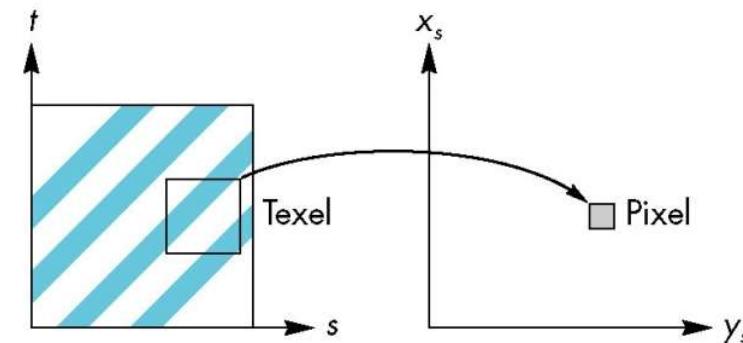
Sampling Issues

- Works fine as long as the size of the rendered image is approximately the same size as the texture source



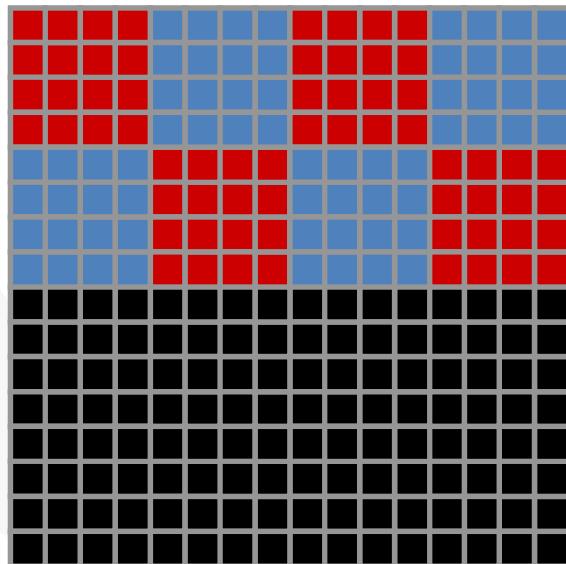
Sampling Issues

- Works fine as long as the size of the rendered image is approximately the same size as the texture source
 - What if much smaller?
 - What if much bigger?

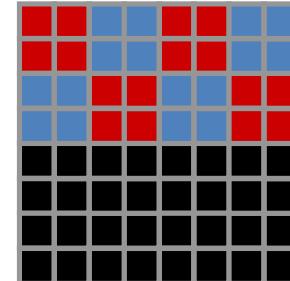


Mip-mapping

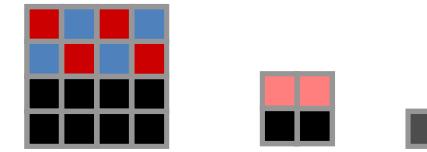
- Mip-mapping is a technique that creates multiple resolutions of an image
 - i.e. Takes a 512x512 image and filters it to create 256x256, 128x128, 64x64, ..., 1x1 versions of it



Original Texture

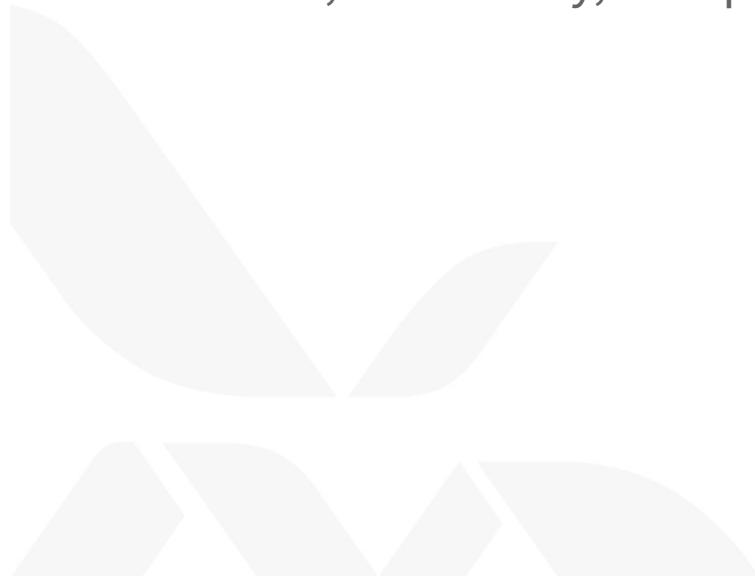


Lower Resolution Versions



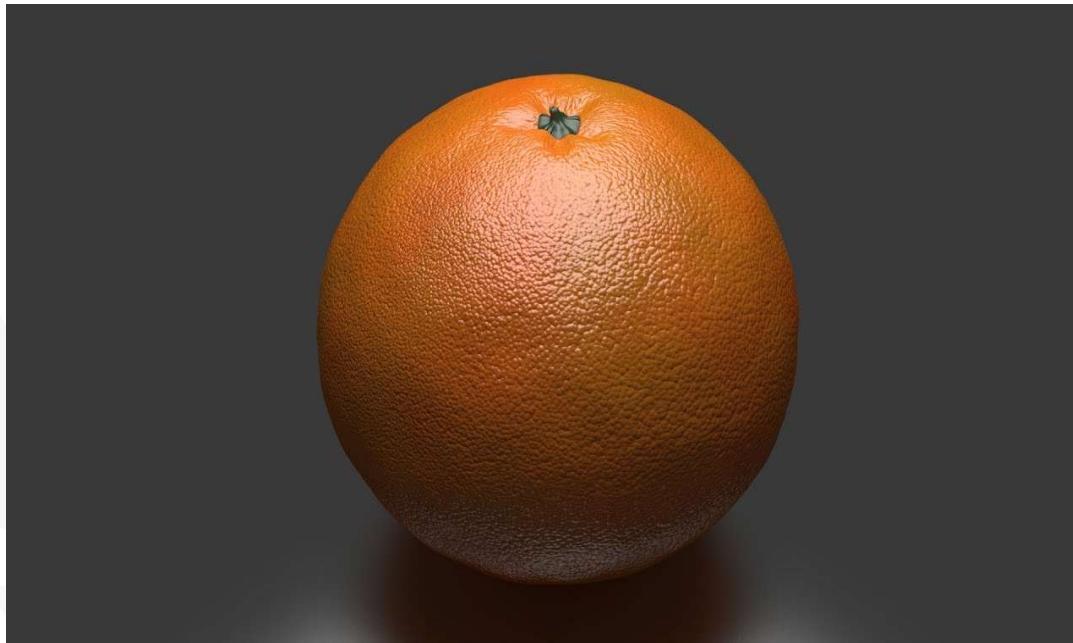
Mip-mapping

- Mip-mapping is a technique that creates multiple resolutions of an image
 - i.e. Takes a 512x512 image and filters it to create 256x256, 128x128, 64x64, ..., 1x1 versions of it
- When texture coordinates are looked up, the most appropriate mip-map level is used.
 - Or, more likely, interpolates between the two closest



Modeling an Orange

- **Texture map a photo of an orange onto a surface**
 - Captures dimples
 - Not correct if we move viewer or light



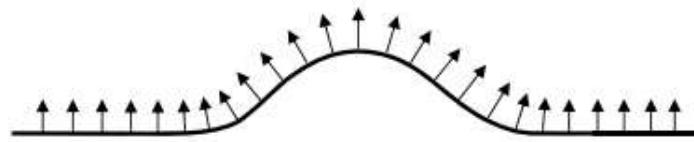
Bump Mapping by Jim Blinn

-
- Perturbed Normals



Bump Mapping by Jim Blinn

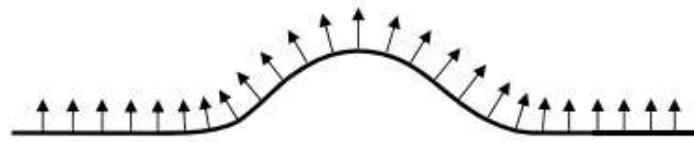
- Perturbed Normals



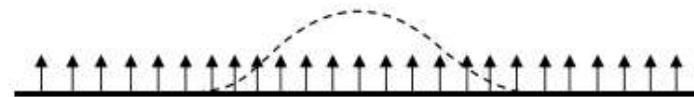
Surface normals on a real “bump”

Bump Mapping by Jim Blinn

■ Perturbed Normals



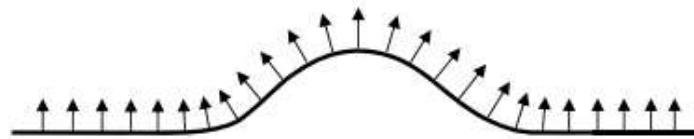
Surface normals on a real “bump”



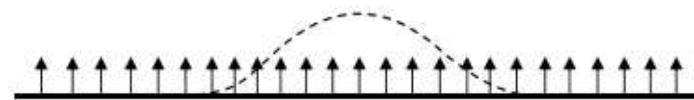
Surface normals on a flat polygon

Bump Mapping by Jim Blinn

■ Perturbed Normals



Surface normals on a real “bump”



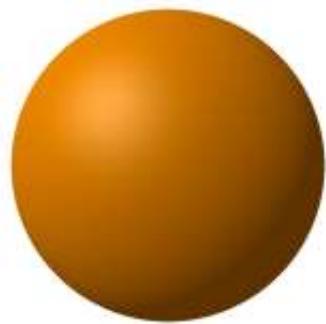
Surface normals on a flat polygon



Modified (“perturbed”) normals

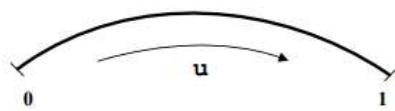
Bump Mapping

- Requires per-pixel (Phong) shading
 - Just interpolating from the vertex normals gives a smooth-looking surface



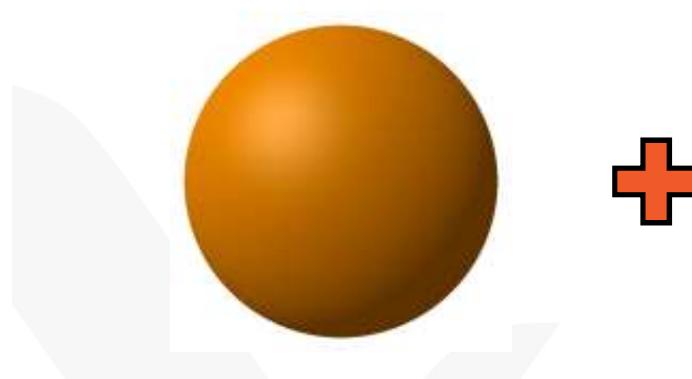
Phong shading

Original parametric surface



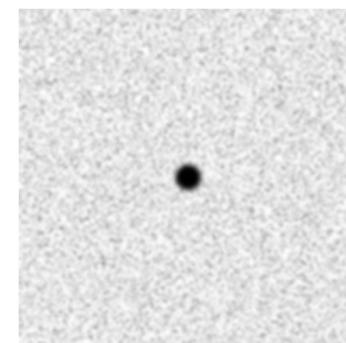
Bump Mapping

- Requires per-pixel (Phong) shading
 - Just interpolating from the vertex normals gives a smooth-looking surface
- Bump mapping uses a “texture” to define how much to perturb the normal at that point



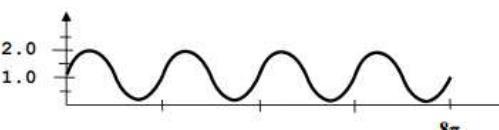
Phong shading

Original parametric surface



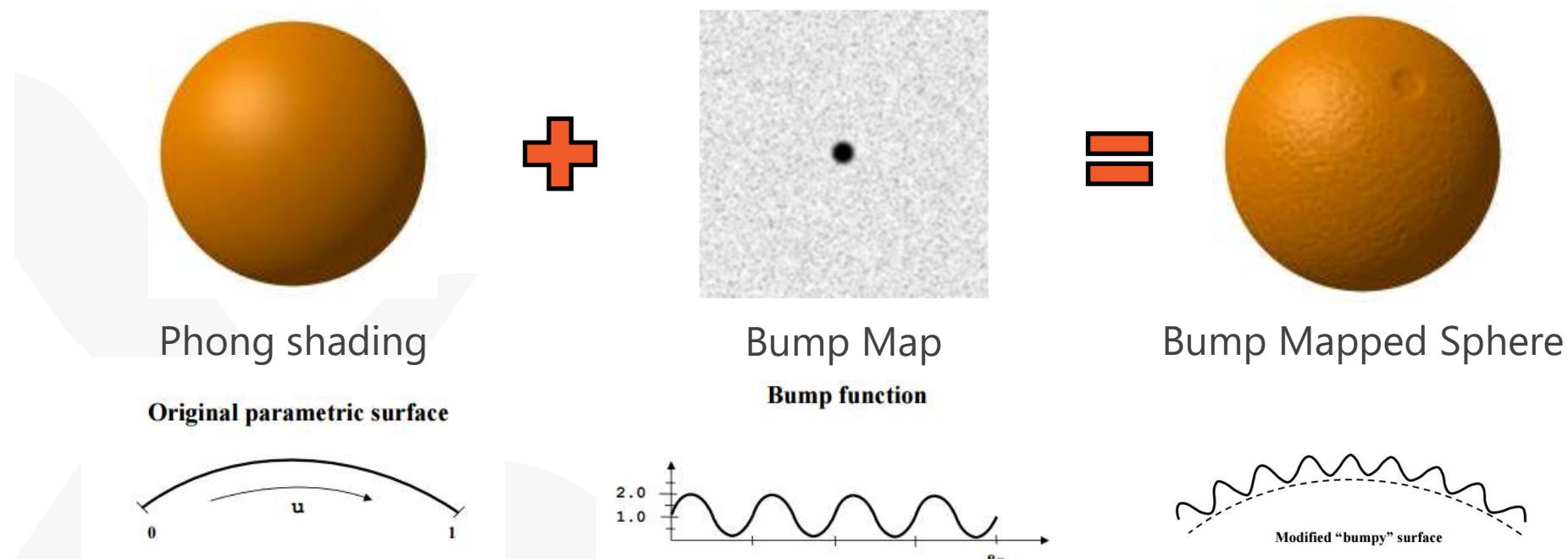
Bump Map

Bump function



Bump Mapping

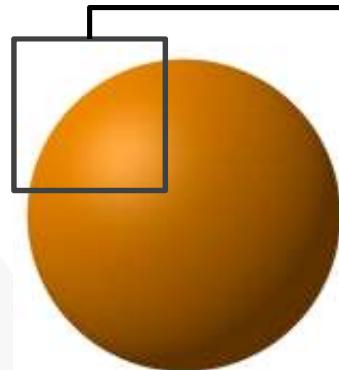
- Requires per-pixel (Phong) shading
 - Just interpolating from the vertex normals gives a smooth-looking surface
- Bump mapping uses a “texture” to define how much to perturb the normal at that point
 - Results in a “bumpy” surface



Bump Mapping

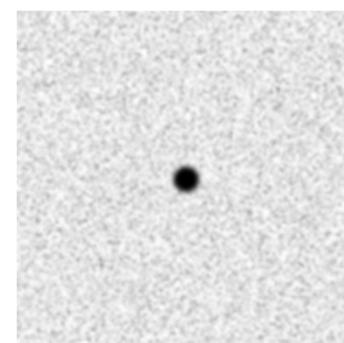
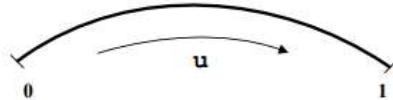
■ Limitation

- Silhouette doesn't change



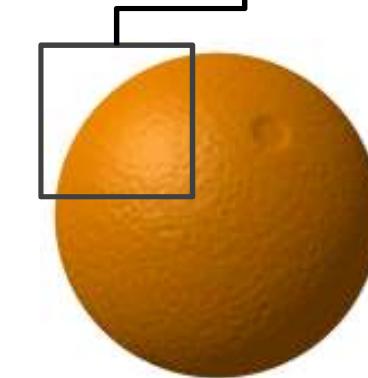
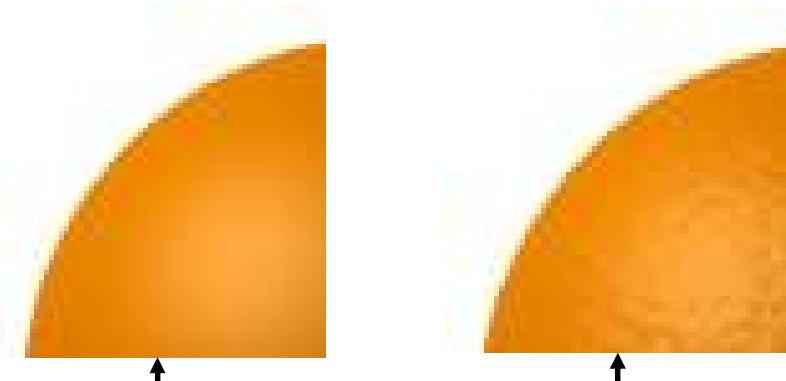
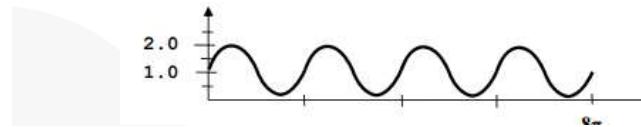
Phong shading

Original parametric surface

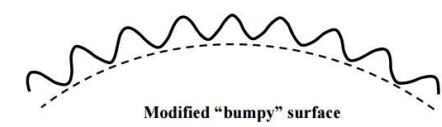


Bump Map

Bump function

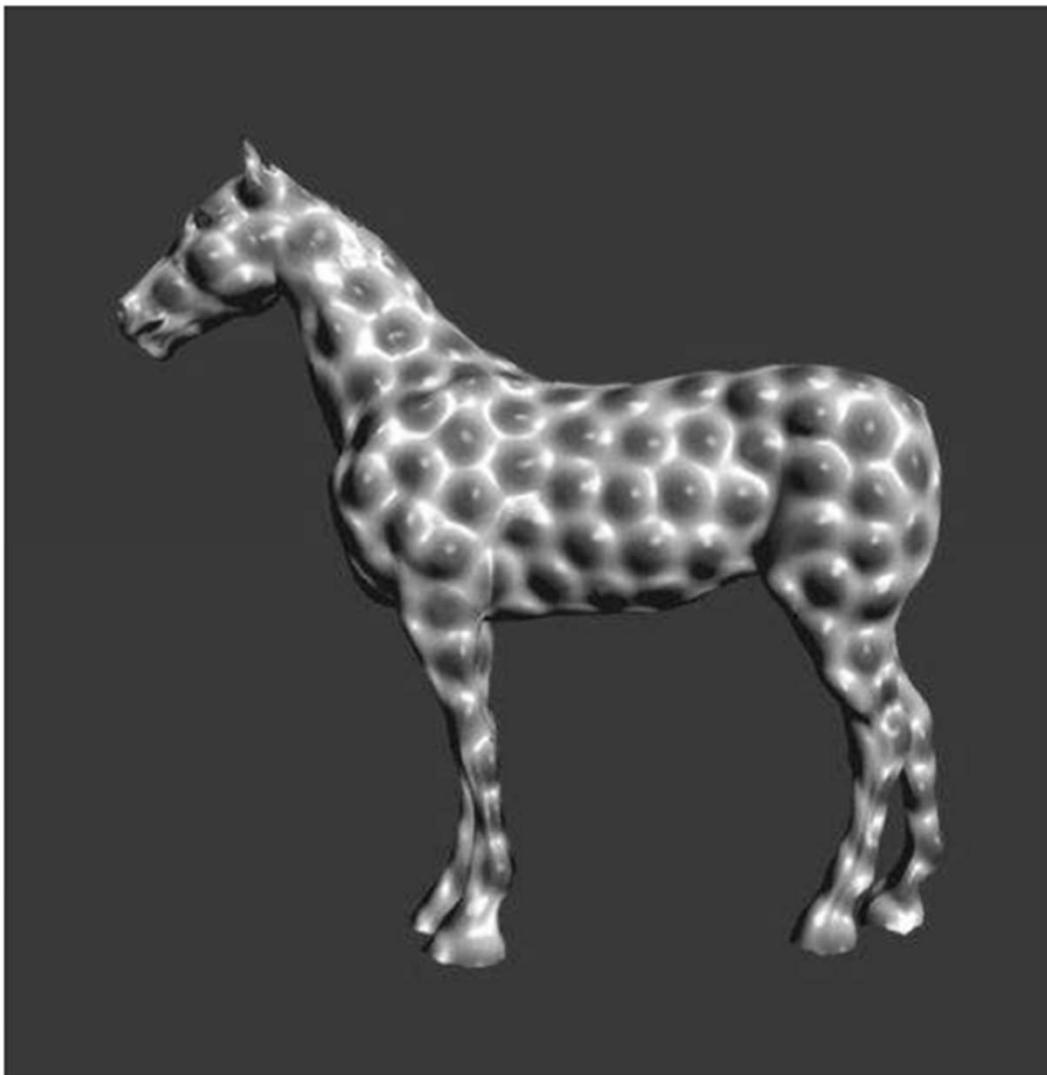


Bump Mapped Sphere



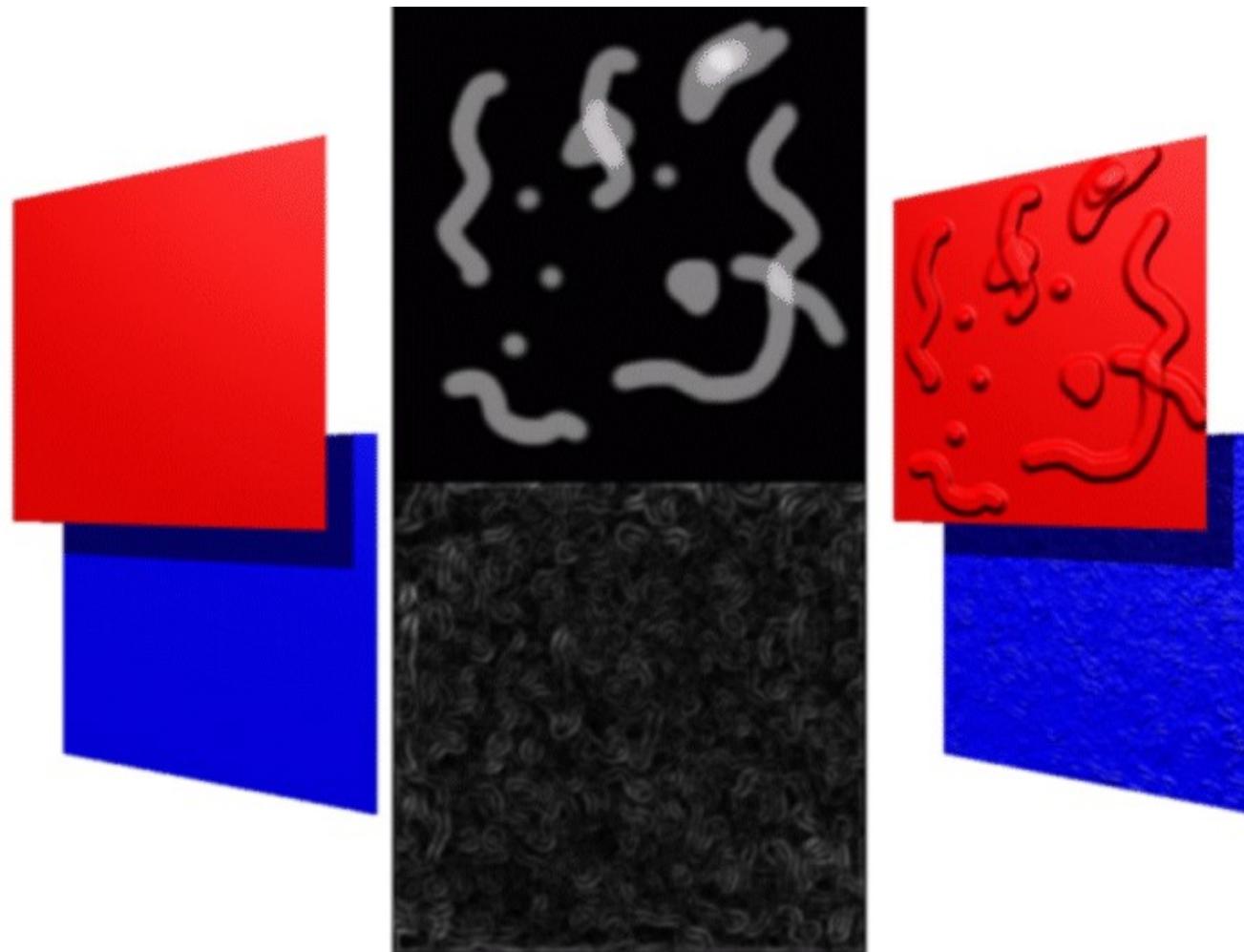
Modified "bumpy" surface

Bump Map Example

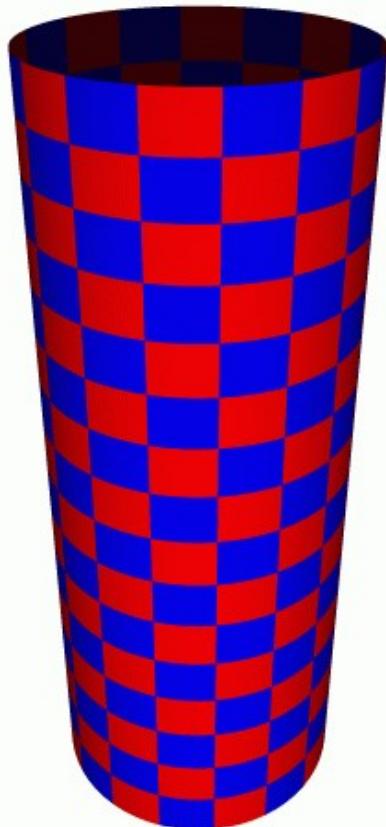


Greg Turk

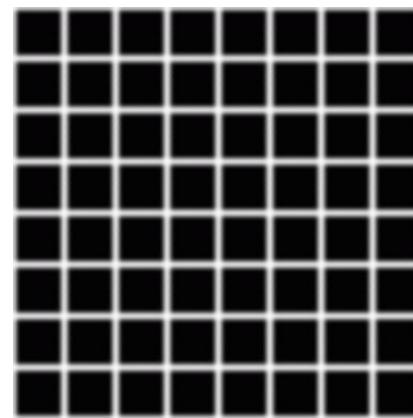
Bump Map Example



Bump Map Example



Cylinder w/Diffuse Texture Map



Bump Map

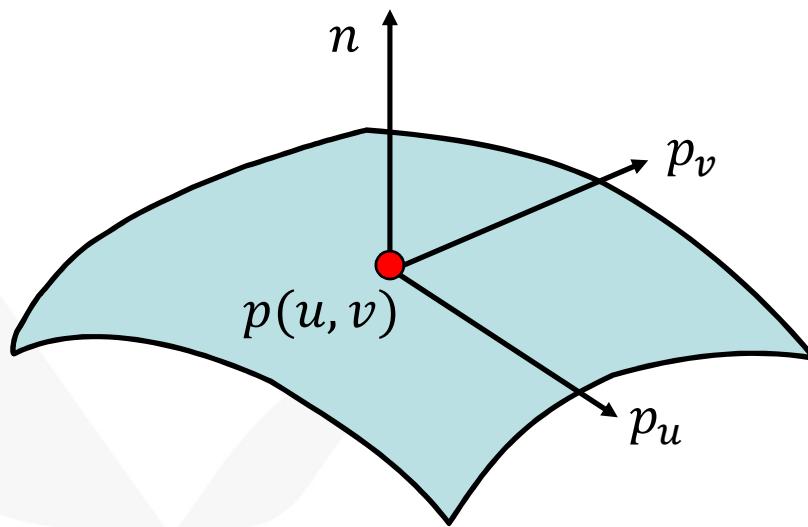


Cylinder w/Texture Map & Bump Map

Equations

- Let $p(u, v)$ be a point on a parametric surface.

$$p(u, v) = [x(u, v), y(u, v), z(u, v)]^T$$



$$p_u = \left[\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right]^T$$

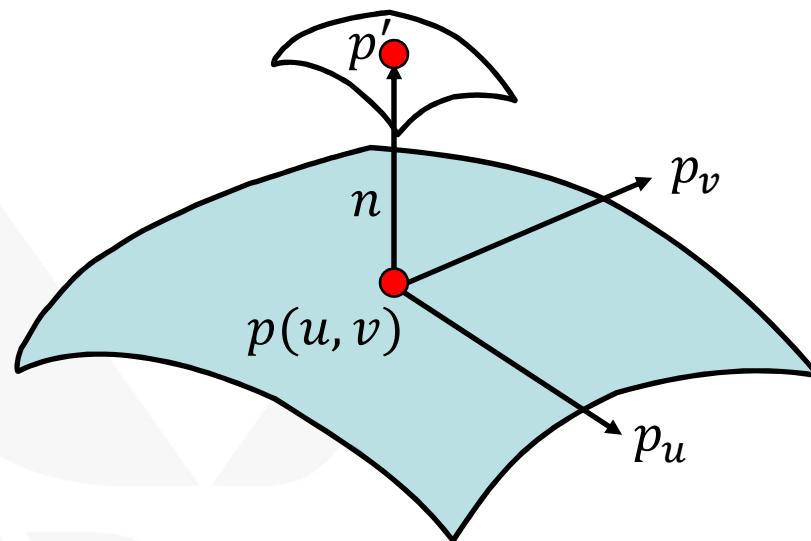
$$p_v = \left[\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v} \right]^T$$

$$n = \frac{(p_u \times p_v)}{|p_u \times p_v|}$$

Equations

- $d(u, v)$ is the bump or displacement function, which we can assume is known and small ($|d(u, v)| \ll 1$)
 - The displaced surface is given by

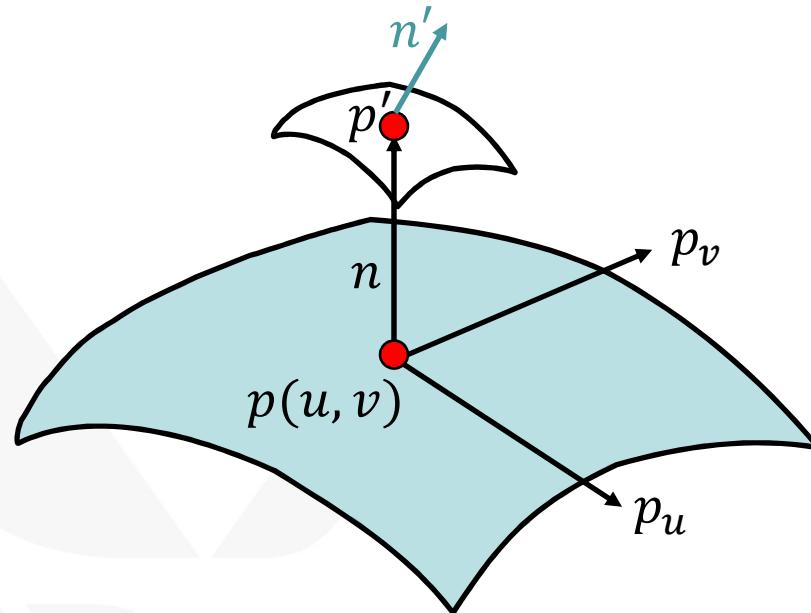
$$p' = p + d(u, v)n$$



Equations

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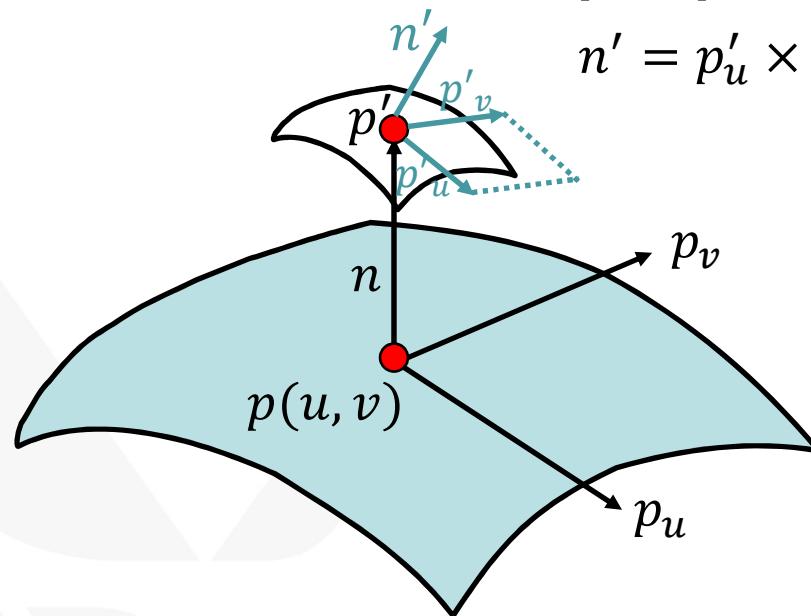


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 - The displaced surface is given by

$$p' = p + d(u, v)n$$

$$n' = p'_u \times p'_v$$



Differentiating p' leads to :

$$p'_u = p_u + \frac{\partial d}{\partial u} n + d(u, v) n_u$$

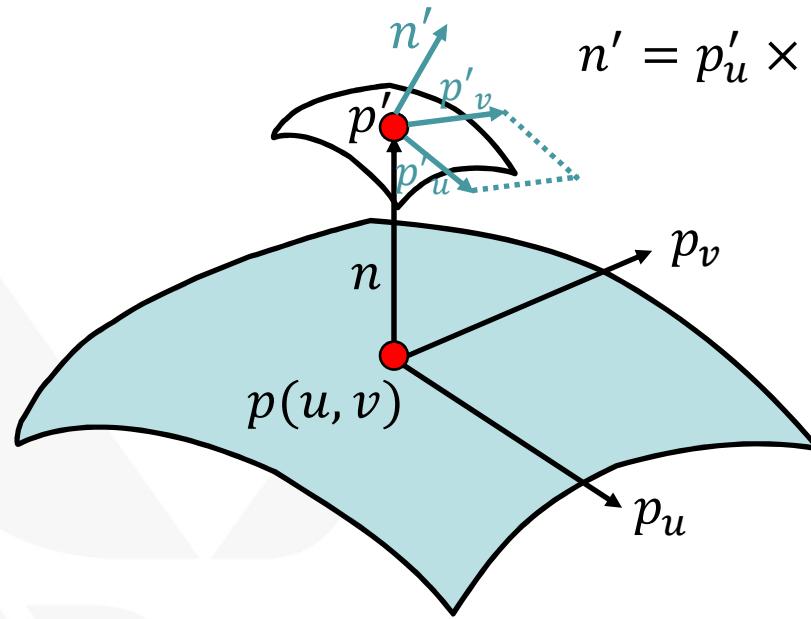
$$p'_v = p_v + \frac{\partial d}{\partial v} n + d(u, v) n_v$$

Equations

- $d(u, v)$ is the bump or displacement function, which we can assume is known and small ($|d(u, v)| \ll 1$)
 - The displaced surface is given by

$$p' = p + d(u, v)n$$

$$n' = p'_u \times p'_v$$



Differentiating of p' leads to :

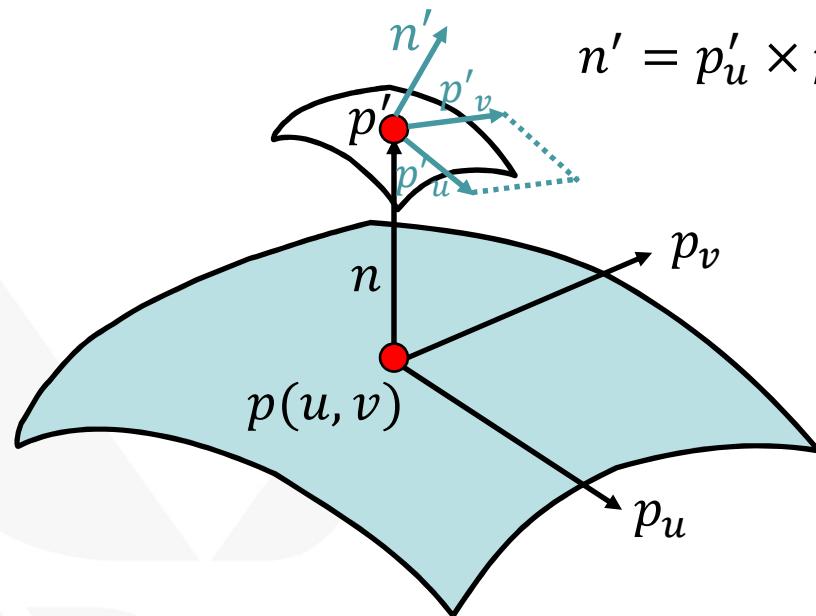
$$p'_u = p_u + \frac{\partial d}{\partial u} n + d(u, v) n_u$$

$$p'_v = p_v + \frac{\partial d}{\partial v} n + d(u, v) n_v$$

If d is small, we can neglect last term

Equations

- $d(u, v)$ is the bump or displacement function, which we can assume is known and small ($|d(u, v)| \ll 1$)
 - The normal n' at the perturbed point p' is given by the cross product



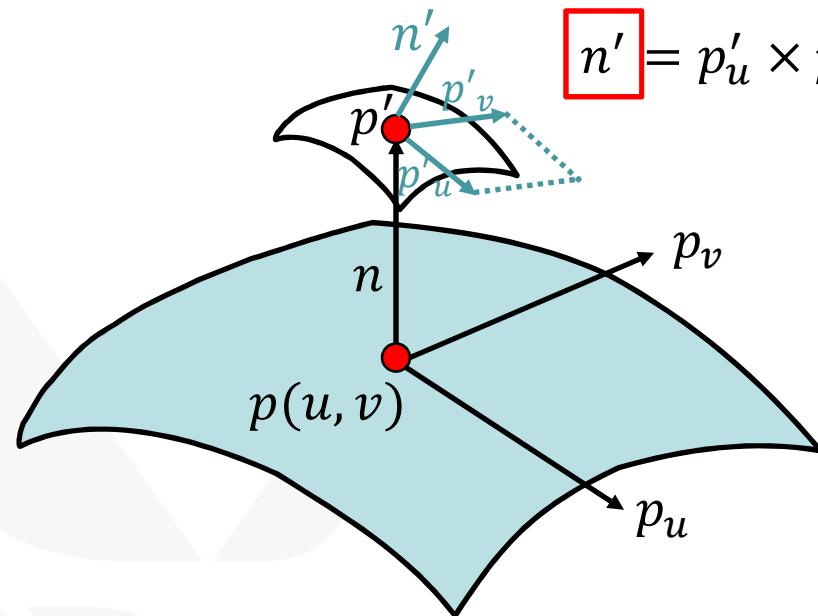
$$\begin{aligned}
 n' &= p'_u \times p'_v = \left(p_u + \frac{\partial d}{\partial u}n \right) \times \left(p_v + \frac{\partial d}{\partial v}n \right) \\
 &\approx (p_u \times p_v) + p_u \times \frac{\partial d}{\partial v}n + p_v \times \frac{\partial d}{\partial u}n \\
 &\approx n + \underbrace{p_u \times \frac{\partial d}{\partial v}n + p_v \times \frac{\partial d}{\partial u}n}_{\text{These are displacement, difference between the original and perturbed normal}}
 \end{aligned}$$

These are displacement, difference between the original and perturbed normal

The vectors $n \times p_v$ and $n \times p_u$ lie in the tangent plane.

Equations

- $d(u, v)$ is the bump or displacement function, which we can assume is known and small ($|d(u, v)| \ll 1$)
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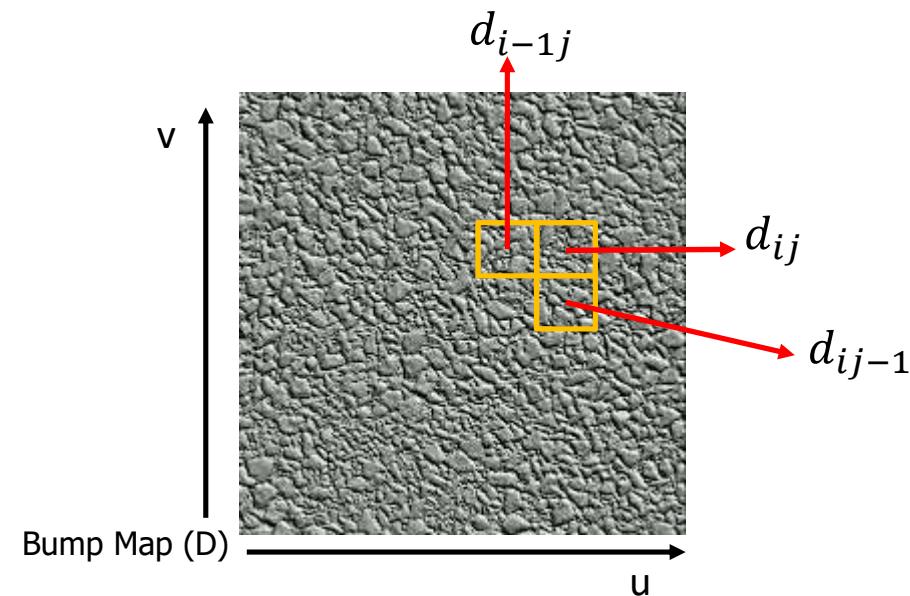
$$\begin{aligned}
 \boxed{n'} &= p'_u \times p'_v = \left(p_u + \frac{\partial d}{\partial u} n \right) \times \left(p_v + \frac{\partial d}{\partial v} n \right) \\
 &\approx (p_u \times p_v) + p_u \times \frac{\partial d}{\partial v} n + p_v \times \frac{\partial d}{\partial u} n \\
 &\approx \underbrace{n}_{\text{original}} + \underbrace{p_u \times \frac{\partial d}{\partial v} n}_{\text{displacement}} + \underbrace{p_v \times \frac{\partial d}{\partial u} n}_{\text{difference}}
 \end{aligned}$$

These are displacement, difference between the original and perturbed normal

The vectors $n \times p_v$ and $n \times p_u$ lie in the tangent plane.

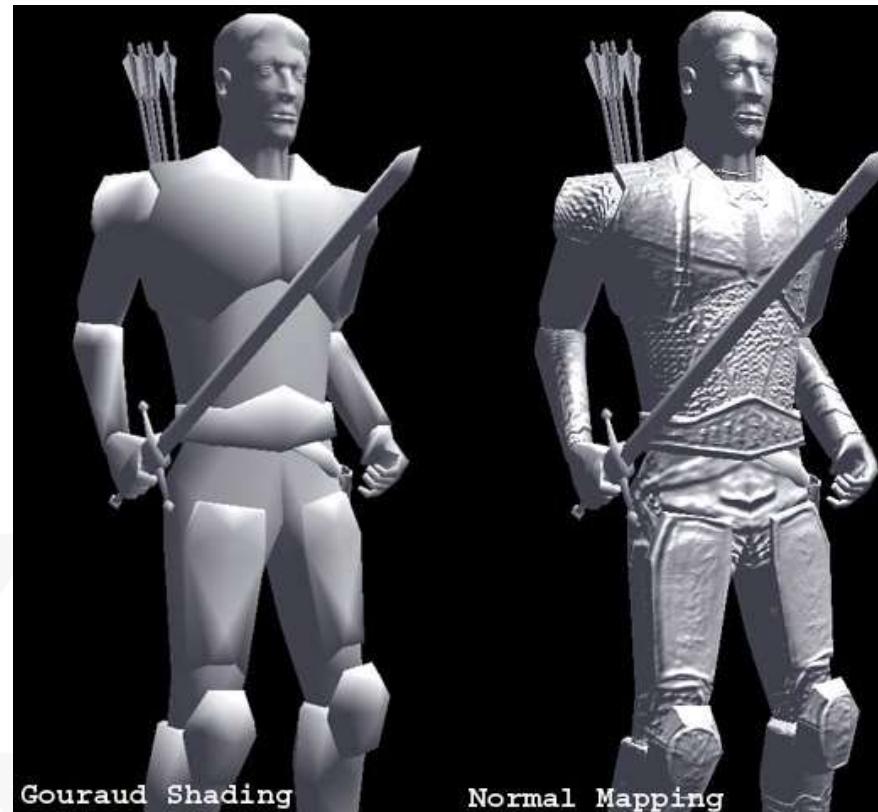
Image Processing

- Suppose that we start with a function $d(u, v)$
- We can sample it to form an array $D = [d_{ij}]$
- Then $\frac{\partial d}{\partial u} \approx d_{ij} - d_{i-1,j}$ and $\frac{\partial d}{\partial v} \approx d_{ij} - d_{i,j-1}$

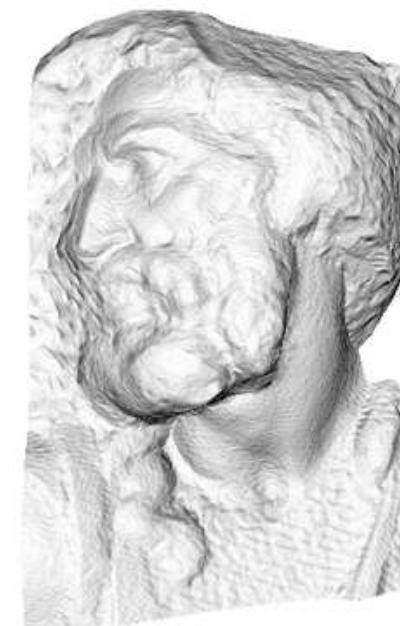
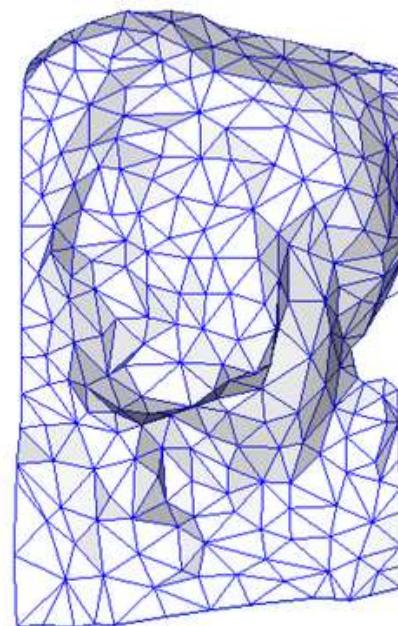


Normal Mapping

- Exploiting a normal map coming from a high resolution model



Normal Mapping

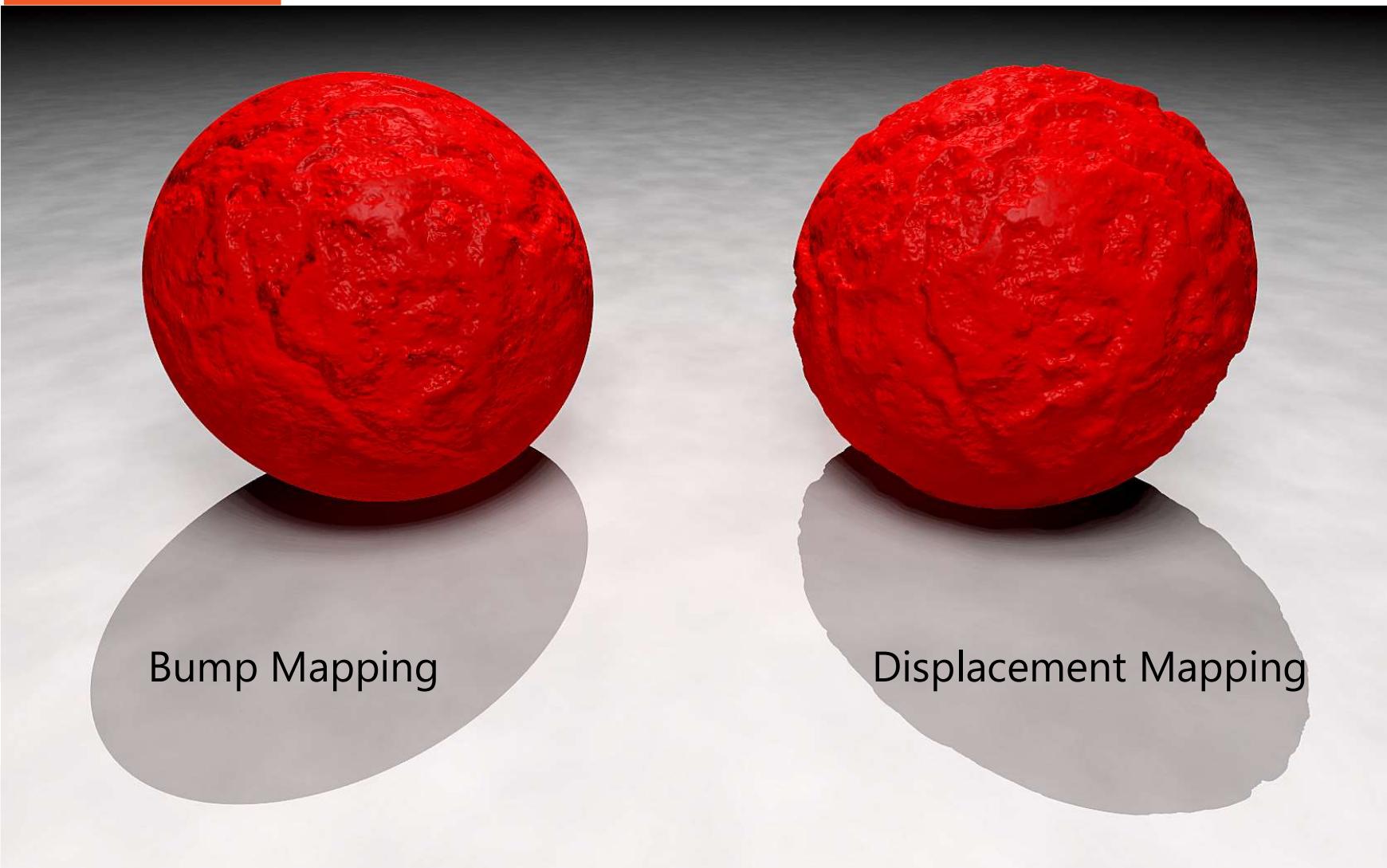


original mesh
4M triangles

simplified mesh
500 triangles

simplified mesh
and normal mapping
500 triangles

Displacement Mapping



Displacement Mapping

- We use the texture map to actually move the surface point. This is called displacement mapping.



ORIGINAL MESH

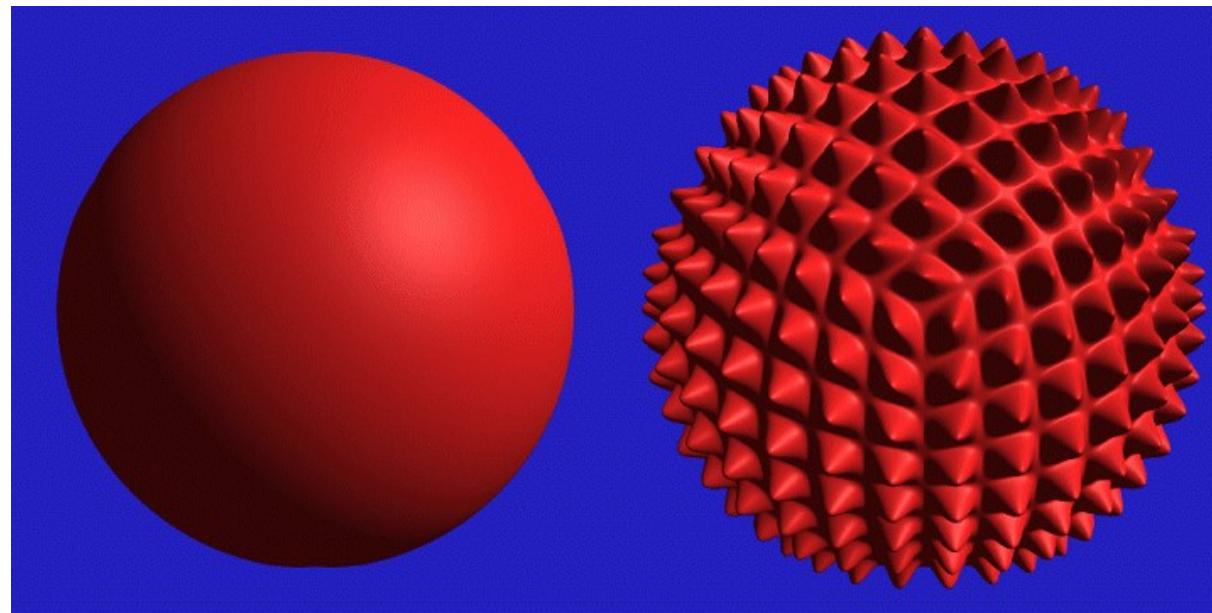


DISPLACEMENT MAP

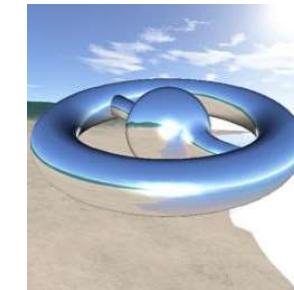


MESH WITH DISPLACEMENT

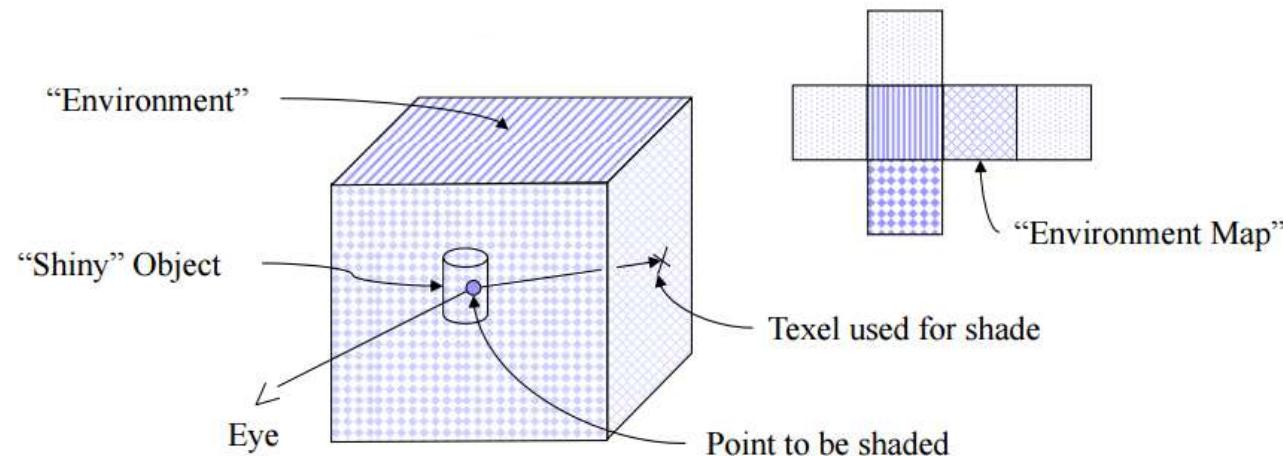
Displacement Mapping



Environment Mapping



- Texture contains an image of the surroundings
- Cube mapping is commonly used to implement environment maps



This allows us to “hack” reflection/refraction

- Render the scene from the center of the cube in each face direction
- Store each of these results into a texture
- Then render the scene from the actual viewpoint, applying the environment textures

Environment Mapping Example

