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A review of vehicle routing with simultaneous pickup and delivery

Çağrı Koç [a,*], Gilbert Laporte [b,d], İlknur Tükenmez [c]

[a] Department of Business Administration, Social Sciences University of Ankara, Ankara, Turkey
[b] CIRRELT, Canada Research Chair in Distribution Management and HEC Montréal, Montréal, Canada
[c] Department of Industrial Engineering, Eskişehir Osmangazi University, Eskişehir, Turkey
[d] School of Management, University of Bath, United Kingdom

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ABSTRACT

In the vehicle routing problem with simultaneous pickup and delivery (VRPSPD), goods have to be transported from different origins to different destinations, and each customer has both a delivery and a pickup demand to be satisfied simultaneously. The VRPSPD has been around for about 30 years, and significant progress has since been made on this problem and its variants. This paper aims to comprehensively review the existing work on the VRPSPD. It surveys mathematical formulations, algorithms, variants, case studies, and industrial applications. It also provides an overview of trends in the literature and identifies several interesting promising future research perspectives.

1. Introduction

The classical vehicle routing problem (VRP) introduced around sixty years ago by Dantzig and Ramser (1959), as well as its variants, have been intensively studied. The VRP aims to determine a routing plan to serve a set of customers for a fleet of identical vehicles, such that each customer is visited once by one vehicle, each route starts and ends at the depot, and several side constraints are satisfied. Many heuristic and exact algorithms have been developed for the VRP and its variants. The interested reader is referred to the book by Toth and Vigo (2014), and to the reviews by Laporte (2009), Koç et al. (2016), Koç and Laporte (2018), Vidal et al. (2019).

One important variant of the VRP arises in pickup and delivery problems (PDPs). Several types of PDPs have been studied. Battarra et al. (2014) presented an overview of studies for PDPs arising in the transportation of goods, without providing detailed computational comparisons of solution methods. Table 1 presents a classification of PDPs based on Berbeglia et al. (2007) and Battarra et al. (2014). It consists of three main categories. The first category includes many-to-many problems where each commodity may have more than one start node and more than one end node, and any node may be the origin and destination node of a number of commodities. In the second category, one-to-many-to-one, some commodities are carried from a depot to many customers, while other commodities are collected at customers and delivered to the depot. The third category contains one-to-one problems in which each commodity has a single start node and a single end node. The most studied and general variant of the second category or one-to-many-to-one, is the vehicle routing problem with simultaneous pickup and delivery (VRPSPD). This problem is also known as the multiple-vehicle Hamiltonian one-to-many-to-one PDP with combined demands. In the VRPSPD some customers have a delivery demand, some have a pickup demand, and at least one customer has both a pickup and a delivery demand. VRPSPDs are rooted in the seminal paper of Min (1989) and have since evolved into a rich and active research field. Typical applications of VRPSPDs arise in the distribution of beverages and the collection of empty cans and bottles.

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* Corresponding author.
E-mail addresses: cagri.koc@asbu.edu.tr (Ç. Koç), gilbert.laporte@cirrelt.ca (G. Laporte), itukenmez@ogu.edu.tr (İ. Tükenmez).

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Table 1
Classification of pickup and delivery problems.

1. Many-to-many problems
1.1. The swapping problem
1.2. The one-commodity pickup and delivery traveling salesman problem
1.3. The Q-delivery traveling salesman problem
2. One-to-many-to-one problems pickup and delivery problems (1-M-1-PDP)
2.1. The 1-M-1-PDP with combined demands
2.1.1. General and lasso solutions of the 1-M-1-PDP with combined demands
2.1.2. The multi-vehicle Hamiltonian 1-M-1-PDP with combined demands
2.2. The 1-M-1-PDP with single demands
2.2.1. The single vehicle 1-M-1-PDP with single demands and backhauls
2.2.2. The single vehicle 1-M-1-PDP with single demands and mixed solutions
2.2.3. The multi-vehicle 1-M-1-PDP with single demands and backhauls
2.2.4. The multi-vehicle 1-M-1-PDP with single demands and mixed solutions
3. One-to-one problems
3.1. The stacker crane problem
3.2. The vehicle routing problem with pickups and deliveries
3.2.1. The single vehicle routing problem with pickups and deliveries
3.2.2. The multi-vehicle routing problem with pickups and deliveries
3.3. The dial-a-ride problem
3.3.1. The single vehicle dial-a-ride problem
3.3.2. The multi-vehicle dial-a-ride problem
3.4. The vehicle routing problem with pickups, deliveries and transshipments

combined demands. In the VRPSPD some customers have a delivery demand, some have a pickup demand, and at least one customer has both a pickup and a delivery demand. VRPSPDs are rooted in the seminal paper of Min (1989) and have since evolved into a rich and active research field. Typical applications of VRPSPDs arise in the distribution of beverages and the collection of empty cans and bottles.

The problem has been extensively studied in recent years because of its practical importance for distribution companies. Parragh et al. (2008a,b) surveyed the PDP literature until 2007. Berbeglia et al. (2007, 2010) reviewed the static and dynamic PDP, respectively. Survey papers of Caceres-Cruz et al. (2014), Braekers et al. (2016), and Gansterer and Hartl (2018) have briefly reviewed VRPSPDs. The main focus of all of these surveys are not VRPSPDs. They briefly discussed VRPSPD, and did not provide detailed analyses on the main problem and its variants. We therefore believe that there exists merit to specifically review VRPSPDs.

Our review methodology can be summarised as follows. We mainly focus on articles and book chapters about the VRPSPD. We carried out the literature search within well-known databases such as ISI Web of Knowledge and SCOPUS with keywords "vehicle routing problem with simultaneous pickup and delivery", and followed by reference and citation analyses to find related contributions. We summarized the resultant studies by several descriptive statistics to provide an overall view of the research area.

The contribution of this review paper is fourfold. First, we present a detailed review of the existing studies on the standard VRPSPD, including mathematical formulations. Second, we provide a performance comparison of heuristics developed for the standard VRPSPD. Third, we describe several VRPSPD variants, case studies and industrial applications, and we provide synthetic tables. Fourth, we give an overview of the main trends observed in the literature and identify several interesting promising future research perspectives.

The remainder of this paper is structured as follows. Mathematical models and exact algorithms for the VRPSPD are presented in Section 2. We survey the heuristics developed for the standard VRPSPD in Section 3, miscellaneous variants in Section 4, and case studies in Section 5. We provide a summary and comparison of recent metaheuristics in Section 6. We finally present our conclusions and future research perspectives in Section 7.

2. Mathematical models and exact algorithms

The VRPSPD is defined on a complete directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ where $\mathcal{V}$ is the node set and $\mathcal{A}$ is the arc set. Node 0 represents the depot, which is the starting node of the delivery commodities and the end node of the pickup commodities. The other nodes of $\mathcal{V}$ are the customers. Let $\mathcal{V}' = \mathcal{V} \setminus \{0\}$. A homogeneous fleet of vehicles is available and each vehicle has a capacity Q. The cost of traveling on arc (i,j) is denoted by c_{ij}. For delivery and pickup commodities, each customer i has a non-negative demand d_i and p_i, respectively. Let y_{ij} and z_{ij} be the amount of picked up commodity and delivery commodity on arc $(i,j) \in \mathcal{A}$, respectively. These are common notations for each of the mathematical formulations. Specific notations of each formulation are defined within the related subsections.

The VRPSPD is to construct a set of vehicle routes such that (i) each customer is visited by exactly one vehicle; (ii) each vehicle performs at most one route; (iii) each vehicle route starts and ends at the depot; (iv) the pickup and delivery requests of each customer in a single visit must be satisfied; (v) the vehicle capacity is not exceeded by the total demand of a vehicle route; and (vi) the total travel cost is minimized. The VRPSPD is NP-hard since it extends the VRP.

Several exact algorithms are available for the standard VRPSPD. We now survey four exact algorithms that have been developed for the standard VRPSPD and present their standard mathematical models. The models use two- or three-index variables. Vehicle-flow models only specify vehicle routes, whereas commodity-flow models specify the amount of pickup and delivery commodity on each arc. These four models are widely used by researchers to model the VRPSPD and its rich variants.

2.1. Two-index commodity-flow model

We first present the two-index commodity-flow model of Dell'Amico et al. (2006). The authors developed an exact branch-and-price algorithm for their two-index commodity-flow model. To solve the pricing subproblem, dynamic programming and space relaxation procedures are used. Several methods were combined such as bidirectional search, an upper bound on the number of customers visited in a route, and branching strategies. The method yielded optimal solutions on instances with up to 40 customers.

A homogeneous fixed fleet of K vehicles is available. Let x_{ij} be equal to 1 if and only if arc $(i,j) \in \mathcal{A}$ is used. The model is then:

$$\text{Minimize} \sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ij} \quad (1)$$

subject to

$$\sum\nolimits_{j\in\mathcal{V}} x_{ij} = 1 \quad i \in \mathcal{V}' \quad (2)$$

$$\sum\nolimits_{j\in\mathcal{V}'} x_{0j} \leqslant K \quad (3)$$

$$\sum\nolimits_{j\in\mathcal{V}} x_{ij} = \sum_{j\in\mathcal{V}} x_{ji} \quad i \in \mathcal{V} \quad (4)$$

$$\sum\nolimits_{j\in\mathcal{V}} y_{ij} - \sum_{j\in\mathcal{V}} y_{ji} = p_i \quad i \in \mathcal{V}' \quad (5)$$

$$\sum\nolimits_{j\in\mathcal{V}} z_{ji} - \sum_{j\in\mathcal{V}} z_{ij} = d_i \quad i \in \mathcal{V}' \quad (6)$$

$$y_{ij} + z_{ij} \leqslant Qx_{ij} \quad (i,j) \in \mathcal{A} \quad (7)$$

$$y_{ij}, z_{ij} \geqslant 0 \quad (i,j) \in \mathcal{A} \quad (8)$$

$$x_{ij} \in \{0,1\} \quad (i,j) \in \mathcal{A}. \quad (9)$$

The objective function (1) minimizes the total routing cost. Constraints (2) guarantee that each customer must be visited once. Constraints (3) ensure that maximum K vehicles can be used. Constraints (4)–(6) are flow constraints. Constraints (7) imply that the capacity of the vehicle is not exceeded. Finally, constraints (8) and (9) enforce the restrictions on the variables.

2.2. Three-index commodity-flow model

We now present the adapted version of the three-index commodity-flow model of Montané and Galvao (2006), which is the extended version of that of Mosheiov (1998). Montané and Galvao (2006) do not seem to have solved their formulation fully. Its linear relaxation was solved in order to compute a lower bound on the optimal solution value, which was then used to evaluate the computational quality of a heuristic developed by the authors. This formulation considers the classical constraints of the VRPSPD but no time window (Angelelli and Mansini, 2001) or maximum route duration constraints (Ai and Kachitvichyanukul, 2009).

A homogeneous unlimited fleet of vehicles is available. Let the binary variable x_{ijk} be equal to 1 if and only if arc (i,j) is traversed by vehicle k. The model is then:

$$\text{Minimize} \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}}\sum_{k\in\mathcal{K}} c_{ij}x_{ijk} \quad (10)$$

subject to

$$\sum\nolimits_{j\in\mathcal{V}}\sum_{k\in\mathcal{K}} x_{ijk} = 1 \quad i \in \mathcal{V}' \quad (11)$$

$$\sum\nolimits_{j\in\mathcal{V}} x_{ijk} - \sum_{j\in\mathcal{V}} x_{jik} = 0 \quad i \in \mathcal{V}, k \in \mathcal{K} \quad (12)$$

$$\sum\nolimits_{i\in\mathcal{V}} x_{0ik} \leqslant 1 \quad k \in \mathcal{K} \quad (13)$$

$$y_{ij} + z_{ij} \leqslant Q\sum_{k\in\mathcal{K}} x_{ijk} \quad (i,j) \in \mathcal{A} \quad (14)$$

$$\sum\nolimits_{i\in\mathcal{V}} y_{ij} - \sum_{i\in\mathcal{V}} y_{ji} = p_i \quad i \in \mathcal{V}' \quad (15)$$

$$\sum\nolimits_{i\in\mathcal{V}} z_{ji} - \sum_{i\in\mathcal{V}} z_{ij} = d_i \quad i \in \mathcal{V}' \quad (16)$$

$$x_{ijk} \in \{0,1\} \quad (i,j) \in \mathcal{V}, k \in \mathcal{K} \quad (17)$$

$$y_{ij}, z_{ij} \geqslant \quad (i,j) \in \mathcal{V}. \quad (18)$$

The objective function (10) minimizes the total routing cost. Constraints (11) and (12) guarantee that each customer is visited exactly by one vehicle. Constraints (13) state that each vehicle performs at most one route. Constraints (14) impose the capacity requirements for each vehicle. Constraints (15) and (16) define the flow for the pickups and deliveries, respectively. Finally, constraints (17) and (18) enforce the restrictions on the variables.

2.3. Two-index vehicle-flow model #1

We now present the two-index vehicle-flow model of Subramanian et al. (2011, 2013a). Subramanian et al. (2011) developed a branch-and-cut algorithm for the two-index model that uses the lazy separation of the capacity inequalities. The method improved some known lower bounds and obtained several new optimal results. It also yielded better solutions than the previous exact algorithms based on directed or undirected two-commodity, and single-commodity formulations. Subramanian et al. (2013a) later proposed an exact algorithm based on branch-cut-and-price for the same model that uses non-elementary paths. Both delivery and pickup demands were taken into account to identify capacity violations. Dynamic programming was used to solve the pricing problem, but at early stages of the method scaling and sparsification were adopted to speed up the computations. The method yielded several best-known lower bounds with up to 200 customers, and obtained several optimal solutions with up to 100 customers.

A homogeneous unlimited fleet of vehicles is available. Let x_{ij} be an integer variable denoting the number of times arc $(i,j) \in A$ appears in a vehicle route. If a vehicle route contains only a single customer, then x_{ij} is equal to 2. Let $S \subseteq V'$. Let $p(S)$ and $d(S)$ be the sum of the pickup and delivery demands, respectively. Let $e(S) = \lceil d(S)/Q \rceil$ and $q(S) = \lceil p(S)/Q \rceil$. Let m be an integer variable denoting the number of used vehicles. Let $\widehat{S} = \mathcal{V}' \setminus S$. Let $\mathcal{R}$ be the set composed by the arc-sets of infeasible routes which exclude the arcs adjacent to the depot. The model is then:

$$\text{Minimize} \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V},j>i} c_{ij}x_{ij} \quad (19)$$

subject to

$$\sum\nolimits_{i\in\mathcal{V},i<k} x_{ik} + \sum_{j\in\mathcal{V},j>k} x_{kj} = 2 \quad k \in \mathcal{V}' \quad (20)$$

$$\sum\nolimits_{j\in\mathcal{V}'} x_{0j} = m \quad (21)$$

$$\sum\nolimits_{i\in\mathcal{S}} \sum_{j\in\widehat{\mathcal{S}},i<j} x_{ij} + \sum_{i\in\widehat{\mathcal{S}}}\sum_{j\in\mathcal{S},i<j} x_{ij} \geqslant 2e(S) \quad S \subseteq \mathcal{V}' \quad (22)$$

$$\sum\nolimits_{i\in\mathcal{S}} \sum_{j\in\widehat{\mathcal{S}},i<j} x_{ij} + \sum_{i\in\widehat{\mathcal{S}}}\sum_{j\in\mathcal{S},i<j} x_{ij} \geqslant 2q(S) \quad S \subseteq \mathcal{V}' \quad (23)$$

$$\sum\nolimits_{i,j\in R} x_{ij} \leqslant |R| - 1 \quad R \in \mathcal{R} \quad (24)$$

$$m \in \mathbb{Z}_+ \quad (25)$$

$$x_{ij} \in \{0,1\} \quad (i,j) \in \mathcal{A}, i > 0 \quad (26)$$

$$x_{ij} \in \{0,1\} \quad (0,j) \in \mathcal{A}. \quad (27)$$

The objective function (19) minimizes the total routing cost. The basic classical VRP constraints are ensured by (20) and (21). Constraints (22) and (23) compute the rounded capacity cuts by considering the delivery and pickup demands, respectively. These two constraints ensure that the capacity of the vehicle is not exceeded in the middle of a route. Constraints (24) guarantee that the capacity of the vehicle is not exceeded anywhere on a route. Finally, constraints (25)–(27) enforce the restrictions on the variables.

2.4. Two-index vehicle-flow model #2

We finally present the two-index vehicle-flow model of Rieck and Zimmermann (2013). The authors developed several preprocessing steps and derived valid inequalities for their vehicle-flow model and the commodity-flow model of Dell'Amico et al. (2006). They solved these two models by branch-and-cut. On a series of tests conducted on instances with up to 60 customers, they demonstrated the positive impact of the valid inequalities, and they confirmed the superiority of the commodity-flow formulation.

A homogeneous fixed fleet of K vehicles is available. Let D_i be the amount of delivery commodity that has to be delivered to node $i \in \mathcal{V}$ and to consecutive nodes of the route. Let L_i be the vehicle load after visiting customer $i \in \mathcal{V}'$. The model is then:

$$\text{Minimize} \sum_{(i,j)\in\mathcal{V}} c_{ij}x_{ij} \tag{28}$$

subject to

$$\sum\nolimits_{i\in\mathcal{V}} x_{ij} = 1 \quad j \in \mathcal{V}' \tag{29}$$

$$\sum\nolimits_{i\in\mathcal{V}} x_{ji} = 1 \quad j \in \mathcal{V}' \tag{30}$$

$$\sum\nolimits_{i\in\mathcal{V}'} x_{0i} \leqslant K \tag{31}$$

$$D_i \geqslant D_j + d_i - M_1(1 - x_{ij}) \quad i \in \mathcal{V}, j \in \mathcal{V}' \tag{32}$$

$$L_j \geqslant D_j - d_j + p_j \quad j \in \mathcal{V}' \tag{33}$$

$$L_j \geqslant L_i - d_j + p_j - M_2(1 - x_{ij}) \quad (i,j) \in \mathcal{V}' \tag{34}$$

$$d_i \leqslant D_i \leqslant Q \quad i \in \mathcal{V} \tag{35}$$

$$p_i \leqslant L_i \leqslant Q \quad i \in \mathcal{V}' \tag{36}$$

$$x_{ij} \in \{0,1\} \quad (i,j) \in \mathcal{V} \tag{37}$$

$$D_i, L_i \geqslant 0 \quad i \in \mathcal{V}. \tag{38}$$

The objective function (28) minimizes the total routing cost. Constraints (29) and (30) guarantee that each customer is visited exactly by one vehicle. Constraints (31) state that maximum K routes can be constructed. Constraints (32) ensure the consistency of the delivery load along the route. Constraints (33) and (34) compute the vehicle load after the visit of the first customer and of the other customers on the route, respectively. M_1 and M_2 are large numbers which can be set equal to Q. Constraints (35) and (36) guarantee that the vehicle capacity is not exceeded. Finally, constraints (37) and (38) enforce the restrictions on the variables.

3. Heuristics for the standard VRPSPD

We now survey the available heuristics for the standard VRPSPD. Classical construction and improvement heuristics in Section 3.1, local search metaheuristics in Section 3.2, followed by population search heuristics in Section 3.3, and ant colony heuristics in Section 3.4.

3.1. Classical construction and improvement heuristics

A number of classical construction and improvement heuristics were proposed to solve the standard VRPSPD. Montané and Galvao (2002) used a tour partitioning heuristic and a modified version of the Gillett and Miller (1974) heuristic to solve the VRPSPD. They adapted the test bed of Augerat (1995) to this problem and tested their method on 27 instances with 32 to 80 nodes.

Gajpal and Abad (2010) described a parallel savings heuristic which is an extension of that of Altinkemer and Gavish (1991). It generates a new route by combining two routes. To check feasibility when two routes are merged, it uses a cumulative net-pickup method. Experiments on benchmark instances showed that the method yielded better solutions than the previous ones.

Jun and Kim (2012) defined an heuristic which includes a sweep-based route construction phase, an improvement phase, and a solution perturbation procedure. Several improvement algorithms, such as cross-exchange, Or-opt, and intra-route 2-opt are also applied. In the perturbation phase, a solution is perturbed by removing and reinserting several vehicle routes. On experiments performed on the Nagy (1996) instances, the method obtained 53 new best-known solutions.

3.2. Local search metaheuristics

Several studies describe tabu search heuristics for the standard VRPSPD. Chen and Wu (2006) developed a record-to-record algorithm which combines a tabu list procedure and classical local search mechanisms such as an insertion based procedure and route improvement procedures. Experiments were conducted on the instances of Salhi and Nagy (1999) with up to 199 nodes, and optimal solutions were obtained for small-size instances. Montané and Galvao (2006) proposed a tabu search while considering maximum route duration constraints. The method applies three types of moves (relocations, interchanges and crossovers), four types of neighborhoods, as well as intensification and diversification procedures. The solutions yielded by this heuristic were 8.82%, 13.43%, and 27.76% better than those of Dethloff (2001), Salhi and Nagy (1999) (without distance constraints), and Salhi and Nagy (1999), respectively. Wassan et al. (2008) later developed a tabu search heuristic which checks feasibility of tabu moves quickly and reacts to repetitions to guide the search. The heuristic generates an initial solution by using sweep method of Gillett and Miller (1974), which is later improved by shift, swap and reverse procedures, where the latter is specifically designed for the problem. Experiments conducted on Nagy (1996) instances ranging between 50 and 199 customers yielded 10 new best-known solutions.

Bianchessi and Righini (2007) described several heuristics combining local search procedures based on a variable neighborhood scheme. Neighborhoods based on arc-exchange and node-exchange were integrated and made use of several and interacting tabu lists. These heuristics were applied to the Dethloff (2001) instances with up to 50 customers and yielded competitive solutions. Zachariadis et al. (2009) later developed a hybrid algorithm integrating guided local search and tabu search. The method generates an initial solution by using a construction heuristic based on the cost savings of Paessens (1988), which is later improved by inter-route and intra-route moves, and guided local search. Experiments were conducted on various instances ranging from 50 to 400 nodes. The authors obtained quite good results: 21 new best-known solutions were found on the instances of Dethloff (2001), and 18 new best-known solutions were found on those of Montané and Galvao (2006).

Chen (2006) presented a parallel-insertion algorithm which hybridizes simulated annealing, tabu list and route improvement methods. The author used 2-opt, Or-opt, shift and swap moves and a 2-exchange procedure to improve the quality of the routes. Their method provided better solutions than those of Salhi and Nagy (1999). Mu et al. (2016) later proposed a parallel heuristic based on the traditional sequential simulated annealing algorithm, integrated within a master–slave structure, and asynchronous and synchronous multiple Markov chains methods. The algorithm includes several strategies such as move acceleration, parallel moves, and speculative computation. Experiments were conducted on the Dethloff (2001) and Salhi and Nagy (1999) medium-size instances, and on the Montané and Galvao (2006) large-size instances. The method obtained better results than those previously reported within reasonable computation times.

Subramanian et al. (2010) introduced a parallel heuristic which was integrated within a multi-start scheme. It uses the variable neighborhood descent procedure of Mladenović and Hansen (1997) as a local search procedure which applied a random neighborhood ordering. The parallel algorithm is combined with an iterated local search procedure. The heuristic was able to yield high-quality solutions, including three new best-known solutions on the instances of Salhi and Nagy (1999), 12 new best-known solutions on the Montané and Galvao (2006) instances, and matched the best results found on the Dethloff (2001) instances. Subramanian et al. (2013b) later developed a unified algorithm based on a new strategy that dynamically controls the dimension of the model, thus allowing the solution of very large instances. It combines an exact method based on set partitioning, iterated local search framework, and variable neighborhood search with random ordering. This method yielded quite good results on the Salhi and Nagy (1999) instances, including five new best-known solutions, and matched 21 others.

Zachariadis and Kiranoudis (2011) developed a local search algorithm including a strategy for efficiently exploring solution neighborhoods and the use of a concept based on the aspiration criterion of tabu search. They described a constant-time feasibility checking method for two local search operators which are variable length bone exchange operators based on every tentative move exchanging the positions of customer sequences, and the 2-opt operator. Their algorithm improved upon ten best-known solutions on the Montané and Galvao (2006) instances.

Cruz et al. (2012) developed a hybrid heuristic method which combines tabu search, variable neighborhood search, and path relinking. The method generates an initial solution by using heuristic cheapest insertions with multiple routes, and the GENIUS implementation of Souza et al. (2011). Several moves and perturbations are used to improve the solutions. Good quality solutions were obtained within reasonable computation times.

Fard and Akbari (2013) described a hybrid tabu search algorithm which uses greedy and nearest neighborhood constructive heuristics to generate the initial solution and applied shift, interchange, mutation, swap, as well as local shift improvement procedures. The method was applied to the Salhi and Nagy (1999) instances and yielded optimal solutions on the small-size instances. Yousefikhoshbakht et al. (2014) later proposed a hybrid algorithm which makes use of tabu search and ant colony systems. They obtained an initial solution by using a nearest neighborhood insertion algorithm which is later then improved by 2-opt, relocation, and exchange moves and classical ant colony procedures. Experiments on Salhi and Nagy (1999) and Dethloff (2001) instances yielded competitive results.

Souza et al. (2011) described an adaptation of the cheapest insertion hybrid algorithm method which starts with three initial construction procedures, improves the solutions by variable neighborhood descent and iterated local search, and diversifies solutions with perturbation procedures. All best-known results were obtained on the Dethloff (2001) instances, four new best results were obtained on the Salhi and Nagy (1999) instances without route duration constraints, and nine new best-known results were obtained on the Montané and Galvao (2006) instances. Avci and Topaloglu (2015) described an adaptive local search algorithm integrating a simulated annealing and a variable neighborhood descent algorithm. While simulated annealing explores different regions in the search space, variable neighborhood descent improves the obtained solutions with extra refinement procedures and a perturbation mechanism. The method yielded competitive solutions on the Dethloff (2001) and Salhi and Nagy (1999) data sets.

3.3. Population search heuristics

Several population search heuristics were implemented for the VRPSPD.

Ai and Kachitvichyanukul (2009) designed a particle swarm optimization which includes a decoding procedure. The solution representation is based on random keys, and the decoding procedure transforms the particle to a priority list of customers to be inserted in the routes, and a priority matrix of vehicles to serve each customer. High-quality solutions were obtained on the Dell'Amico et al. (2006), Dethloff (2001), and Nagy (1996) instances. Zachariadis et al. (2010) developed an evolutionary method based on the adaptive memory scheme of Rochat and Taillard (1995). The method stores and combines promising sequences of nodes to generate good quality solutions. It uses an additional memory mechanism which records the extraction frequency for each node in the adaptive memory. Obtained solutions were improved by a tabu search mechanism. The authors tested the algorithm on various benchmark instances where it performed quite well and obtained many new best-known solutions. Goksal et al. (2013) later described a hybrid heuristic algorithm which makes use of particle swarm optimization, and applied a variable neighborhood descent heuristic to improve the solutions. The swarm diversity is preserved by using an annealing-like strategy. To represent a solution, a permutation encoding is defined which is a giant tour without trip delimiters. The heuristic performed quite well on the Salhi and Nagy (1999), Dethloff (2001), and Wassan et al. (2008) instances by yielding an improvement around 2% on average, as well as 104 new best-known solutions.

Tasan and Gen (2012) used a classical genetic algorithm based on the idea of Gen et al. (2008). They did not present any comparison on classical benchmark instances, but used 24 new medium-sized instances generated based on those of Augerat (1995). The results of the genetic algorithm were compared with the exact solution of a mathematical formulation obtained by a truncated application of CPLEX, and the former obtained better solutions than the latter. Maquera et al. (2011) developed a scatter search algorithm, which is a variant of an evolutionary algorithm based on Martí et al. (2006). Their method includes a diversification generation method based on the multi-start greedy randomized adaptive search procedure of Feo and Resende (1995). To improve the solutions, three intra-route moves (2-opt, relocation, and 1-interchange), and three inter-route moves (relocation, interchange, and crossover) were used. Good solutions were obtained on the classical instances. Vidal et al. (2014) proposed a unified genetic algorithm to solve several VRP versions, including the VRPSPD. Effective procedures were proposed to intensify and diversify solutions which are problem-independent. A unified route evaluation methodology was developed where the moves are considered as a concatenation of known subsequences. The method yielded several new best-known solutions on the Salhi and Nagy (1999) and on the Montané and Galvao (2006) instances.

3.4. Ant colony heuristics

Five ant colony optimization heuristics were proposed for the VRPSPD. Gajpal and Abad (2009) developed an ant colony system integrating a construction rule, a 2-opt procedure, an interchange multi-route scheme, customer insertions, and a subpath exchange multi-route scheme. The method achieved a good performance and outperformed the previous studies. In particular, experiments on the Dethloff (2001) and Salhi and Nagy (1999) instances yielded nine and 22 new best-known solutions, respectively. Çatay

(2010) later proposed an ant colony system based on savings and pheromone updating procedures. A nearest neighbor heuristic generated the initial solution which was improved by local search. Intra-move, intra-swap, inter-move, and inter-swap procedures are used. The algorithm obtained a successful performance by yielding nine new best-known solutions on the Dethloff (2001) instances, and 11 new best-known solutions on those of Salhi and Nagy (1999). Johnson et al. (2015) used a classical ant colony systems which applies a state transition rule, the global updating rule, and a local updating rule to intensify and diversify solutions. Experiments were conducted on the Dethloff (2001) instances on which it obtained better solutions than Dethloff (2001). Kalayci and Kaya (2016) developed an ant colony systems enhanced with variable neighborhood search which releases pheromones on the edges. For diversification purposes, the ants provide a perturbation mechanism using the pheromone information. The method performed quite well, yielding the same best values as in Subramanian et al. (2011) on the Dethloff (2001) instances, and five new best-known solutions on the instances of Salhi and Nagy (1999). Sayyah et al. (2016) described an ant colony optimization heuristic using several new mechanisms such as a transition rule and a pheromone updating rule. It uses insert, swap and 2-opt moves as local search methods. Experiments were conducted on the benchmark instances of Dethloff (2001) and of Salhi and Nagy (1999) with up to 199 customers. The method obtained the best solutions on 30 out of 40 of the Dethloff (2001) instances, and on 18 of the 28 Salhi and Nagy (1999) instances within reasonable computation times.

4. Variants and extensions

Many variants of the VRPSPD have been studied. We now review them in this section. We first review the VRPSPD with time windows in Section 4.1, the heterogeneous VRPSPD in Section 4.2, the multi-depot VRPSPD in Section 4.3, the green VRPSPD in Section 4.4, the stochastic VRPSPD in Section 4.5, and finally miscellaneous VRPSPDs in Section 4.6.

4.1. The VRPSPD with time windows

A predefined time interval is assigned to each customer for pickup and delivery in the VRPSPD with time windows. The vehicle has to wait until the time window of a customer opens if it arrives early. Angelelli and Mansini (2001) introduced this problem, and proposed a branch-and-cut-and-price algorithm for it. The master problem is formulated as a set covering problem. A relaxation of the elementary shortest path problem is applied for the pricing problem. Optimal solutions were obtained on small-size instances with up to 20 customers.

Mingyong and Erbao (2010) developed an evolutionary algorithm for the VRPSPD with time windows, using a decimal coding to construct an initial population. They also implemented several new problem-specific operators, and used an integer order criterion. The authors defined a self-adapting crossover probability and allowed infeasible solutions for diversification. They conducted experiments on eight-customer instances and on 40-customer instances on which competitive results were obtained. Fan (2011) used a classical tabu search and generated the initial solution by means of cheapest insertions. The objective was to maximize customer satisfaction and minimize the total cost, which is inversely proportional to the waiting time for the vehicle from the lower bound of the time window. Wang and Chen (2012) developed a genetic algorithm based on co-evolution to solve the VRPSPD with time windows, and integrated several simple heuristics based on cheapest insertions. The method includes two populations for diversification and intensification. They modified the Solomon (1987) benchmark instances and generated 10-, 25-, 50-, and 100-customer instances. The method was compared with CPLEX and yielded better solutions within less computing time for medium- and large-size instances. Kassem and Chen (2013) later developed a heuristic which uses the sequential route construction algorithm of Chen (2006) to generate an initial solution. Simulated annealing and neighborhood searches were applied in the improvement phase. The authors modified the Solomon (1987) instances and conducted experiments on 10-, 15-, and 50-customer instances. For 10- and 15-customer instances optimal solutions were obtained, and better solutions were obtained for 50-customer instances with a heuristic method.

Liu et al. (2012) studied the VRPSPD with time windows in the context of home health care logistics in France. The patients receive medicines at a hospital, and some materials are picked up from the patient's home to be returned to a laboratory or to a depot. Time windows are imposed for the laboratory, the hospital and the patients. The authors first presented a formulation, and described a hybrid genetic algorithm and a tabu search heuristic. A test bed was generated from the Solomon (1987) instances and the two heuristics were compared. The same authors later studied a version of the VRPSPD with time windows where drugs are delivered hospital to patients, and medical devices and unused drugs are picked up from patients (Liu et al., 2013). The problem has four types of demands: deliveries from hospital to patients, deliveries from depot to patients, pickups from a patient to the laboratory or from a patient to the depot. They described two formulations, and proposed a genetic algorithm and a tabu search heuristic. Permutation chromosomes, a split mechanism and local search were used in the genetic algorithm. Several mechanisms, such as route assignment attributes of patients and attribute based aspiration levels, were also embedded in the tabu search. Modified Gehring and Homberger (1999) and Solomon (1987) data were used for the experiments. The algorithms were also tested on the VRPTW with mixed backhauls on which several best-known results were obtained.

Wang et al. (2013a) developed a simulated annealing heuristic for the VRPSPD with time windows. The method applies a mutation probability, obtains a solution with a slow cooling schedule, and randomizes the local search mechanism. On average, the algorithm obtained 0.22% better solutions on the Wang and Chen (2012) instances with up to 100 customers when compared with the genetic algorithm. Wang et al. (2015) later considered the hard time windows variant of the VRPSPD and defined a parallel simulated annealing algorithm which generates an initial solution by using an insertion method based on radial surcharge and residual capacity. Four local search methods were used for the intensification phase. The aim is to minimize the total traveled distance and number of vehicles. Competitive results on the Wang and Chen (2012) instances were obtained, with 28 new best-known solutions for different sizes in total.

Wang and Chen (2013) studied the flexible VRPSPD with time windows and considered mixed pickups and deliveries. There is no strict rule about the sequence on a route for linehauls and backhauls. The authors developed an evolutionary algorithm that applies a modified cheapest insertion procedure. The algorithm simultaneously makes use of two separate populations, one of which is used for diversification and the second one for intensification. The authors presented a mathematical model and applied their method to generated instances based on those of Wang and Chen (2012) with 10 to 100 nodes.

Wang et al. (2013b) considered split loads within the VRPSPD with time windows and described a hybrid heuristic that generates travel and customer waiting times based on an initial solution, as well as on the quantity of the demand. It then uses a local search method which includes six operators: relocate, relocate split, 2-exchange, swap, 2-opt, and Or-opt. Experiments on modified Solomon (1987) instances showed that the average number of vehicles decreases by 20%, the total cost by 10%, but the loading rate per vehicle increases by 10% when compared with the non-split case.

4.2. The heterogeneous VRPSPD

Qu and Bard (2013) studied the VRPSPD with heterogeneous fleet with a configurable vehicle capacity and modeled the problem as a mixed integer formulation, and developed an adaptive large neighborhood search (ALNS). Several randomized procedures were first used to generate feasible solutions, and the solutions were later improved by problem-specific removal and insertion operators. Experiments were conducted on the Parragh (2011) dial-a-ride problem instances, on real instances provided by the program for the elderly organization in Wichita, Kansas, and on randomly generated instances. The method obtained significant cost reductions of up to 40%.

Another variant of the heterogeneous VRPSPD with time windows was considered by Qu and Bard (2014) who developed a mathematical formulation and a branch-and-cut-and-price algorithm. An elementary shortest path problem-based labeling algorithm was used for the pricing subproblem, and several dominance conditions were applied. Furthermore, to strengthen the lower bound, the authors used subset-row inequalities. Their method yielded several optimal results on instances with up to 50 customers.

Avci and Topaloglu (2016) integrated threshold adjusting mechanism within tabu search for the heterogeneous VRPSPD. Threshold adjusting is a deterministic version of simulated annealing. It also contains an adaptive self-tuning strategy. The authors combined an adaptive cooling mechanism within a short-term tabu list. Random instances were generated and optimal results were obtained for some of the small-size instances by solving a mathematical model. Heuristic results were presented on instances with up to 550-customers on which the method yielded better results than the previous ones within a shorter computing time.

4.3. The multi-depot VRPSPD

Nagy and Salhi (2005) introduced the multi-depot VRPSPD and proposed a number of heuristics for it. The authors allowed intermediate infeasible solutions and applied several improvement techniques. The authors generated two types of instance sets, the first of which contains 14 instances with up to 199 customers, and the second 11 instances with up to 249 customers. Competitive results were obtained.

Li et al. (2015) proposed an iterated local search integrated within an adaptive neighborhood selection scheme to solve the multi-depot VRPSPD. Multiple perturbation methods with adaptive selection mechanisms and allowing worse solutions were used. Experiments on the Salhi and Nagy (1999) data sets yielded 0.26% and 0.31% average gaps with respect to best-known solution values for the first set and second set, respectively.

Koulaeian et al. (2015) considered a version of the multi-depot VRPSPD with heterogeneous fleet. The problem aims to minimize the total cost of routing, penalty cost of drivers who violate the traveling distance limits, and the fixed costs of employing drivers. The authors proposed two heuristics: a variant of evolutionary algorithm and a classical genetic algorithm. The former one is motivated by the concept of imperialist competition in the real world. Experiments on instances with up to 150 customers produced better solutions than a classical genetic algorithm.

4.4. The green VRPSPD

In the last decade, several researchers have modeled and solved green VRPs. Lin et al. (2014) proposed a genetic algorithm to solve the VRPSPD in the context of plastic carboys in order to minimize both economic and environmental criteria, i.e., cost and CO_2 emissions. The problem considers the delivery of the filled plastic carboys and the pickup of empty carboys, and also uses two different pickup models: partial and full. The findings revealed that the full pickup strategy yields a lower cost than the partial pickup strategy on instances with up to 100 customers.

Majidi et al. (2017) introduced the simultaneous pickup and delivery version of the pollution-routing problem of Bektaş and Laporte (2011). A nonlinear mathematical formulation was presented and an ALNS heuristic was proposed. To generate an initial solution, a parallel insertion-based construction heuristic was applied. The authors also developed some problem-specific removal and insertion operators. Competitive solutions were obtained on the 100-customer instances of Demir et al. (2012), and on the VRPSPD instances of Wang and Chen (2012) with up to 100 customers.

A fuzzy green variant of the VRPSPD was considered by Majidi et al. (2017) who formulated it as a mixed integer nonlinear model and considered uncertainty in both pickup and delivery demands. The authors used a fuzzy algorithm with a credibility measure to tackle uncertainty. A classical version of the ALNS heuristic was used to solve the problem. Their algorithm computes fuel consumption, and emissions through a comprehensive emission model. New results on 100-customer instances and validation tests on instances of Demir et al. (2012) were presented.

4.5. The stochastic VRPSPD

Two stochastic variants of the VRPSPD have been studied. Wang and Qiu (2011) considered a stochastic version of the VRPSPD in which demand is uncertain. The authors developed an adaptive cross-entropy heuristic which uses complex stochastic networks of Chepuri and Homem-de-Mello (2005) to estimate the probabilities of rare events. Several numerical analyses were conducted to assess the quality of the proposed method. Zhang et al. (2012) later considered a VRPSPD with stochastic travel time which follows a normal distribution on each arc. The aim was to minimize the costs of vehicles and the expected costs of routes. A scatter search and a classical genetic algorithm were described. The former method uses an evolution-based global search mechanism, and contains five main tools: improvement, solution combination, reference setup date, diversification generation, and subset generation. Experiments on generated instances based on those of Dethloff (2001), and involving between 200 and 400 customers showed that scatter search yields better solutions than genetic search.

4.6. Miscellaneous VRPSPDs

This section gathers various VRPSPDs that do not fit well in the previous sections.

Time-dependent travel time have been considered by many authors in the context of the VRP. It is assumed that travel times are not constant between each node pair and may change because of traffic density, vehicle speed, and weather conditions. Only one paper considered this kind of travel times in the VRPSPD. Zhang et al. (2014) studied the time-dependent version where a time step function is used to calculate the travel time. The model minimizes the total cost of waiting, travel and departure. The authors described a hybrid method integrating an ant colony system and tabu search, and developed two pheromone update procedures: an ant colony heuristic factor improvement strategy, and a selection rule for pseudo-random probability. Modified Solomon (1987) 100-customer instances were used for the experiments.

Polat et al. (2015) defined a time-limit restriction variant of the VPRSPD in which a maximum allowed duration is imposed on each vehicle route. The authors developed a perturbation based neighborhood search algorithm for this problem. An initial solution is generated by the Clarke and Wright (1964) savings heuristic, and is later improved by a variable neighborhood search algorithm. The authors used swap, insertion, 2-opt and 3-opt intra-route neighborhood structures, and used five perturbation structures for diversification. On the instances of Salhi and Nagy (1999), the method obtained several best-known solutions.

Zachariadis et al. (2015) introduced a variant of the VRPSPD where the payload is considered. The problem aims to minimize the carried gross weight and the travelled distance. A local search algorithm was developed which runs in two phases. It first generates an initial solution through a simple constructive heuristic, and then applies a local search procedure which calculates the weighted-distance objective. The method uses three local search operators: customer relocation, customer swaps, and 2-opt moves. The authors described a branch-and-cut method for the validation of the proposed heuristic. The algorithm yielded quite competitive solutions on new generated instances with 100 nodes.

Zachariadis et al. (2016) introduced the VRPSPD with two-dimensional loading constraints where pick-up and delivery requests are rectangular items which are non-stackable. Overlapping and exceeding the loading surface of these items are not allowed, and the items must be packed orthogonally. The authors developed a two-stage heuristic. A fast constructive heuristic generated the initial solution which is then educated by a local search method based on Zachariadis et al. (2013). Three local search operators, relocations, exchanges, and 2-opt are applied. The heuristic employs three examination levels to obtain the loading feasibility status of a local search move. The solution method yielded 12 best-known results on the two-dimensional loading VRP instances of Gendreau et al. (2008), and five best-known results on the Montané and Galvao (2006) instances.

Chen et al. (2016) studied the VRPSPD with multiple products and multiple cross-docks and proposed a particle swarm optimization. In this problem, distribution centers are cross-docks and each has a set of allotted delivery and pickup vehicles. The aim is to minimize the material handling, transportation, and operational costs of all vehicles. The solution method extends the classical particle swarm optimization scheme by using a self-learning mechanism to obtain better solutions quickly and to diversify the solutions. The learning procedure of new solutions is obtained by modifying the diversity of swarms, and the best solution does not deteriorate after updating the particle at each iteration. The method was compared with the classical genetic algorithm coded by the authors for the same problem on the modified 50-customer instances of Lee et al. (2009). Experiments revealed that particle swarm optimization yields better solutions than the classical genetic algorithm.

Osaba et al. (2017) considered a newspaper distribution problem with a recycling policy in the context of the asymmetric and clustered VRPSPD with variable costs and forbidden paths. Customers are grouped in different clusters, each of which representing a city. Two time periods are considered: peak hours and off-peak hours. The problem also contains certain arcs that the vehicles cannot use, such as pedestrian streets. The authors developed a discrete firefly algorithm inspired from the idealized behaviour of fireflies. Each firefly represents a feasible solution which is initialized randomly. Fireflies move using evolution strategies to improve the solutions. Experiments were conducted on 15 generated instances based on real geographical positions of a the province of Bizkaia, Spain, with up to 100 customers. The method was compared with the evolutionary and simulated annealing algorithms, and yielded superior solutions.

5. Case studies

Several authors have solved real-life VRPSPDs.

In the context of long-haul transportation, a heterogeneous fleet variant of the VRPSPD with time windows was considered by Drexl et al. (2013). The problem allows truck and driver changes at relay stations which are geographically dispersed, and considers driver shuttles between stations. The EU legislation for driving and working times were enforced. The authors developed a two-stage large neighborhood search heuristic. The problem was motivated by an advanced truckload business model in Germany and other parts of Europe. The authors used the real data of a major German freight forwarder which has 2,800 delivery and pickup demands between 1,975 locations. The time horizon was six days and 1,645 vehicles located at 43 depots which were considered as stations, and 157 additional stations were considered. The heuristic yielded competitive results, and it is indicated that a fixed truck-driver assignment is the right plan.

Yin et al. (2013) studied the split-load VRPSPD and solved it by a tabu search. An initial solution is generated by a nearest neighborhood insertion procedure. The heuristic uses four nearest neighbor structures: relocation, exchange, 2-opt, and split point reposition operators. The method determines a parameter of tabu length by a randomly selecting method, and applies a self-adjusting penalty function. The authors conducted experiments on real-world data of a subsidiary company of China Railway Express. The data include 10 vehicles, a five-ton vehicle load capacity, a 60 km maximal vehicle travel distance, and 24 luggage and package distribution sites. The heuristic obtained savings with up to 4%.

Wang et al. (2016) considered a multi-objective heterogeneous VRPSPD with time windows, where the objectives minimize the total traveled distance, the number of vehicles, the makespan, the total waiting time due to early arrivals, and the total delay time due to late arrivals. The authors described two heuristics: a multi-objective local search and a multiobjective memetic algorithm. They considered a distribution company based in Tenerife, Spain and generated 45 real-world instances with 50, 150 and 250 customers, and generated the time windows in the same way as the delivery company faces every day. The effectiveness of the proposed algorithms was confirmed through experiments in which multiobjective local search outperformed a multiobjective memetic algorithm. To solve the same problem, Li et al. (2018) later proposed a chemical reaction optimization algorithm. A chemical molecule represents a decomposed subproblem, and each molecule includes several neighboring molecules. For diversification purposes, the authors proposed mechanisms such as intermolecular ineffective collision and on-wall ineffective collision. Experiments on the Wang et al. (2016) real-world instances showed that the algorithm yielded 23 new best-known results.

Zhang et al. (2019) described a multi-commodity many-to-many variant of the VRPSPD arising at a fast fashion retailer in Singapore. The company operates a two-echelon inventory system including a central warehouse and about 30 retail outlets, and focuses on women's footwear and accessories. The number of commodities can be up to 10,000, and picked up products from several customers are encouraged to be reallocated to satisfy demands of other customers. The authors presented a mathematical formulation and several strengthening mechanisms, and developed an adaptive memory programming integrating several procedures such as a regret insertion method to generate initial solutions, the segment-based evaluation scheme, and an advanced pool management strategy. The algorithm was applied to 66 real-world instances containing 30 outlets. The number of commodities ranged between 3,822 and 11,142. The algorithm was applied to 96 generated instances based on those of Salhi and Nagy (1999) and of Montané and Galvao (2006). For small-size instances, optimal solutions were achieved, and competitive solutions were obtained on generated instances. The authors investigated several problem features and stated that the relocation of commodities increases their utilization.

Finally, Belgin et al. (2018) studied a two-echelon VRPSPD in which the pickup and delivery activities are performed from depot to satellites in the first echelon, and from satellites to customers in the second echelon. A node-based mathematical formulation and valid inequalities were presented. The authors developed a hybrid metaheuristic based on variable neighborhood descent and local search. Experiments were conducted on 504 instances in which the number of satellites ranged between two and five, and the number of customers ranged between 12 and 50. Valid inequalities improved the lower bounds, and a hybrid metaheuristic performed quite well within short computation times. The authors considered a distribution system of a Turkish chain that operates 25 supermarkets and uses a single-echelon system. Due to the fact that customers are located far from the main depot, the distribution system is transformed into a two-echelon system. The hybrid metaheuristic yielded much better solutions than the current one, with a 57% reduction in travelled distance.

6. Summary and metaheuristic computational comparison

This section first provides a summary of studies on VRPSPDs, and then presents a comparison of recent metaheuristics developed to solve the standard VRPSPD.

6.1. Summary

We present a summary of publications on the standard VRPSPD in Table 2, and for VRPSPD variants and case studies in Table 3. These tables provide the solution method either exact or heuristic, problem types, benchmark instances, and whether a case study and a mathematical model are included. The abbreviations for problem types, solution methods, and benchmark instances are presented in Tables 4–6, respectively.

For the classical VRPSPD a large majority of the publications used heuristic solution methods (87%), while only 13% used exact methods. Similarly, for the VRPSPD variants, a large majority of the publications used heuristic solution methods (94%), while only 6% used exact methods.

This is valid for most of the VRP variants since the VRPSPD is an extension of the classical VRP which is NP-hard. Despite the development of complex and sophisticated mathematical formulations based algorithms in the last decades, only several benchmark instances up to 100 customers can be solved optimally which requires high computing time. Most of the researchers developed effective and flexible heuristic algorithms to obtain good quality solutions within a short amount of time due to practical interest.

Fig. 1 provides a summary of heuristic methods used to solve the classical VRPSPD. Most publications focus on tabu search (29%) and other types of local search (26%) methods. Furthermore, variable neighborhood search (23%) and ant colony systems (19%) are also often applied. These are followed by genetic algorithm, iterated local search, multi-start, particle swarm optimization and scatter search.

Fig. 2 provide a summary of heuristic methods developed for the variants of the VRPSPD. The majority of publications applied generic local search (23%), genetic algorithm (17%), and tabu

Table 2
Literature on the standard VRPSPD.

References		Mathematical model	Solution type	Algorithm	Benchmark instance
1	Montané and Galvao (2002)	•	Heuristic	TPH	MG02
2	Chen and Wu (2006)		Heuristic	TS, LS, RRA	SN99
3	Dell'Amico et al. (2006)	•	Exact	DP	DRS06
4	Montané and Galvao (2006)	•	Heuristic	TS	D01, SN99, MG06
5	Bianchessi and Righini (2007)		Heuristic	TS, LS, VNS	D01
6	Wassan et al. (2008)		Heuristic	TS	N96
7	Ai and Kachitvichyanukul (2009)	•	Heuristic	PSO	DRS06, D01, N96
8	Gajpal and Abad (2009)		Heuristic	ACS	D01, SN99
9	Zachariadis et al. (2009)		Heuristic	TS, LS	D01, MG06
10	Çatay (2010)		Heuristic	ACS, LS	D01, SN99
11	Gajpal and Abad (2010)		Heuristic	SS	D01, SN99
12	Subramanian et al. (2010)		Heuristic	VNS, LS, MS	D01, SN99, MG06
13	Zachariadis et al. (2010)		Heuristic	AMP	D01, SN99, MG06
14	Maquera et al. (2011)		Heuristic	SS, MS	D01, SN99, MG06
15	Souza et al. (2011)		Heuristic	VNS, ILS	D01, SN99, MG06
16	Subramanian et al. (2011)	•	Exact	BC	D01, SN99, MG06
17	Zachariadis and Kiranoudis (2011)		Heuristic	TS, LS	MG06
18	Cruz et al. (2012)		Heuristic	VNS, TS	D01, SN99, MG06
19	Jun and Kim (2012)		Heuristic	CH	N96
20	Tasan and Gen (2012)	•	Heuristic	GA	TG12
21	Fard and Akbari (2013)	•	Heuristic	TS	SN99
22	Goksal et al. (2013)		Heuristic	PSO, VNS, LS	D01, SN99, WWN08
23	Rieck and Zimmermann (2013)	•	Exact	CP	D01, SN99, CW06, RZ13, DRS06
24	Subramanian et al. (2013a)	•	Exact	BCP	D01, SN99, MG06
25	Subramanian et al. (2013b)	•	Heuristic	SP, ILS, VNS	SN99
26	Yousefikhoshbakht et al. (2014)		Heuristic	TS, ACS	D01, SN99
27	Vidal et al. (2014)		Heuristic	GA	SN99, MG06
28	Avci and Topaloglu (2015)		Heuristic	LS, VNS	D01, SN99
29	Johnson et al. (2015)		Heuristic	ACS	D01
30	Mu et al. (2016)	•	Heuristic	PH, SA	D01, SN99, MG06
31	Kalayci and Kaya (2016)		Heuristic	ACS	D01, SN99
32	Sayyah et al. (2016)		Heuristic	ACS	D01, SN99

Table 3
Literature on VRPSPD variants and case studies.

References		Problem type	Mathematical model	Solution type	Algorithm	Case study	Benchmark instance
1	Angelelli and Mansini (2001)	TW	•	Exact	BCP, SP		GBI
2	Nagy and Salhi (2005)	MD	•	Heuristic	CH		GBI
3	Mingyong and Erbao (2010)	TW	•	Heuristic	EA		GBI
4	Fan (2011)	TW	•	Heuristic	TS		GBI
5	Wang and Qiu (2011)	ST	•	Heuristic	CE		GBI
6	Liu et al. (2012)	TW	•	Heuristic	GA, TS		GBI
7	Wang and Chen (2012)	TW	•	Heuristic	GA, CH		GBI
8	Zhang et al. (2012)	ST	•	Heuristic	SS, GA		GBI
9	Drexl et al. (2013)	HF, TW		Heuristic	LNS	•	GBI
10	Kassem and Chen (2013)	TW	•	Heuristic	SA, CH, LS		GBI
11	Liu et al. (2013)	TW	•	Heuristic	GA, TS		GBI
12	Qu and Bard (2013)	HF, CVC	•	Heuristic	ALNS		GBI, P11
13	Wang et al. (2013b)	TW, SL	•	Heuristic	LS		GBI
14	Wang et al. (2013a)	TW		Heuristic	SA		WC12
15	Wang and Chen (2013)	FL, MX, TW		Heuristic	CEA		GBI
16	Yin et al. (2013)	SL	•	Heuristic	TS	•	GBI
17	Lin et al. (2014)	GR	•	Heuristic	GA		GBI
18	Qu and Bard (2014)	HF, TW	•	Exact	BCP		GBI
19	Zhang et al. (2014)	TD	•	Heuristic	ACS, TS		GBI
20	Koulaeian et al. (2015)	MD, HF	•	Heuristic	GA		GBI
21	Li et al. (2015)	MD	•	Heuristic	ILS, ANS		GBI, SN99
22	Polat et al. (2015)	TL	•	Heuristic	VNS, NS		GBI, SN99, NWSA15
23	Zachariadis et al. (2015)	LD	•	Heuristic	LS		GBI, MG06, XZKX12
24	Wang et al. (2015)	TW	•	Heuristic	SA, LS		GBI, WC12
25	Avci and Topaloglu (2016)	HF	•	Heuristic	LS, TA, TS		GBI
26	Chen et al. (2016)	MP, MCD	•	Heuristic	PSO		GBI, LCSC09
27	Zachariadis et al. (2016)	TDL		Heuristic	CH, LS		GBI, MG06
28	Wang et al. (2016)	HF, TW, MO	•	Heuristic	MA, LS	•	GBI, S87
29	Majidi et al. (2017)	GR	•	Heuristic	ALNS		DBL12, WC12
30	Majidi et al. (2017)	FZ, GR	•	Heuristic	ALNS		GBI, DBL12
31	Osaba et al. (2017)	CL, FP	•	Heuristic	DFA		GBI
32	Belgin et al. (2018)	TE	•	Heuristic	VND, LS	•	GBI
33	Li et al. (2018)	TW, MO	•	Heuristic	CRO	•	GBI
34	Zhang et al. (2019)	MC, MM	•	Heuristic	AMP	•	GBI

GBI denotes the generated benchmark instance by the authors.

Table 4
Abbreviations for problem types.

Problem type	Abbreviation	Problem type	Abbreviation
Coevolutionary algorithm	CEA	Multi-depot	MD
Configurable vehicle capacity	CVC	Multi-commodity	MC
Clustered	CL	Multi-objective	MO
Green	GR	Multiple product	MP
Flexible	FL	Multiple cross-docks	MCD
Forbidden paths	FP	Split load	SL
Fuzzy	FZ	Stochastic	ST
Hard time windows	HTW	Time-dependent	TD
Heterogeneous fleet	HF	Time limit	TL
Load-dependent	LD	Time windows	TW
Many-to-many	MM	Two dimensional loading	TDL
Mixed	MX	Two-echelon	TE

Table 5
Abbreviations for solution methods.

Solution method	Abbreviation	Solution method	Abbreviation
Adaptive large neighborhood search	ALNS	Large neighborhood search	LNS
Adaptive memory programming	AMP	Memetic algorithm	MA
Ant colony system	ACS	Multi-start	MS
Adaptive neighborhood selection	ANS	Neighborhood search	NS
Branch-and-cut	BC	Particle swarm optimization	PSO
Branch-and-cut-price	BCP	Parallel heuristic	PH
Chemical reaction optimization	CRO	Record-to-record algorithm	RRA
Construction heuristic	CH	Scatter search	SS
Cross entropy method	CE	Set partitioning	SP
Cutting planes	CP	Simulated annealing	SA
Discrete firefly algorithm	DFA	Threshold accepting	TA
Dynamic programming	DP	Tabu search	TS
Evolutionary algorithm	EA	Tour partitioning heuristic	TPH
Genetic algorithm	GA	Variable neighborhood search	VNS
Iterated local search	ILS	Variable neighborhood descent	VND
Generic local search	LS		

search (17%). These are followed by constructive heuristics (11%), ALNS (9%), and simulated annealing (9%). Local search based heuristics proved to be effective and quick to find good quality solutions on VRP variants (Laporte et al., 2014). Figs. 1 and 2 clearly indicate this situation. Because of their effectiveness, most of the authors used local search based heuristics for the classical VRPSPD and its variants.

According to Table 3, the most widely studied variant is the one with time windows (46%). Most of the companies work on flexible time schedules. Since the nature of the VRPSPD requires simultaneous collection and deliveries, many practical problem consider

Table 6
Abbreviations for benchmark instances.

Benchmark instance	Abbreviation	Benchmark instance	Abbreviation
Solomon (1987)	S87	Lee et al. (2009)	LCSC09
Nagy (1996)	N96	Parragh (2011)	P11
Salhi and Nagy (1999)	SN99	Demir et al. (2012)	DBL12
Dethloff (2001)	D01	Tasan and Gen (2012)	TG12
Montané and Galvao (2002)	MG02	Xiao et al. (2012)	XZKX12
Chen and Wu (2006)	CW06	Wang and Chen (2012)	WC12
Dell'Amico et al. (2006)	DRS06	Rieck and Zimmermann (2013)	RZ13
Montané and Galvao (2006)	MG06	Nagy et al. (2013)	NWSA15
Wassan et al. (2008)	WWN08		

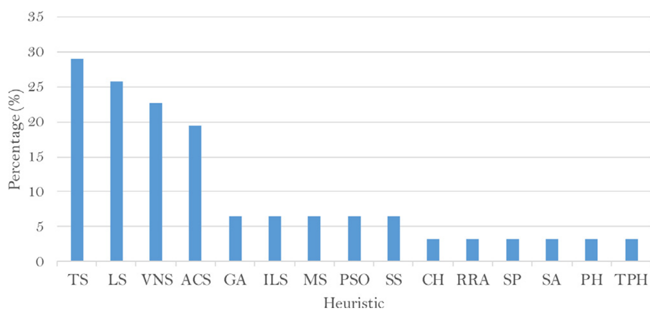

Fig. 1. Heuristics for the classical VRPSPD.

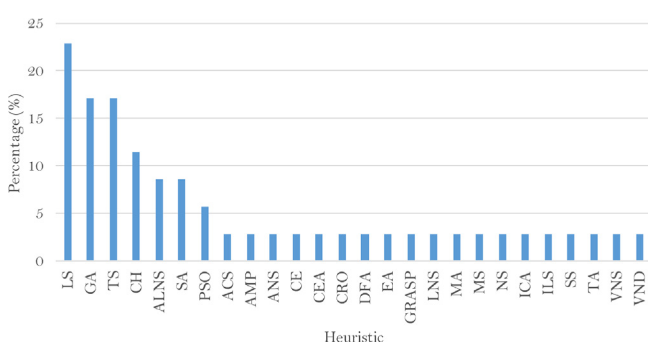

Fig. 2. Heuristics for the variants of the VRPSPD.

Table 7
The journals which most frequently accommodate articles addressing VRPSPDs.

Journal	# of papers
Expert Systems with Applications	9
Computers & Operations Research	7
European Journal of Operational Research	6
Computers & Industrial Engineering	5
Journal of the Operational Research Society	3
Transportation Science	2
Soft Computing	2

time windows. The second widely studied variant of the VRPSPD is the heterogeneous fleets with 17%, followed by multi-depot (11%). As indicated by Koç et al. (2016), heterogeneous fleets have a non-negligible impact on cost reduction. This encouraged researchers to consider heterogeneous fleet in the VRPSPD. The

Table 8
Solution values obtained with the recent metaheuristics on the Salhi and Nagy (1999) instances.

Instance	BKS	References														
		MG06	C06	WWN08	AK09	GA09	ZTK09	C10	SDBOF10	ZTK10	SMSOS11	GKA13	SUO13	YDR14	AT15	PKKG15	KK16
CMT1X	**466.77**	472.00	478.00	468.30	467.00	**466.77**	469.80	470.67	**466.77**	469.80	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**
CMT1Y	**458.96**	470.00	481.00	**458.96**	467.00	466.77	469.80	472.37	466.77	469.80	466.77	466.77	466.77	466.77	466.77	466.77	466.77
CMT2X	**668.77**	695.00	689.00	**668.77**	710.00	684.21	684.21	705.24	684.21	684.21	684.21	684.21	684.21	**668.77**	684.89	684.21	684.21
CMT2Y	**663.25**	700.00	679.00	**663.25**	710.00	684.94	684.21	704.16	684.21	684.21	684.21	684.21	684.21	672.35	684.89	684.21	684.21
CMT3X	**721.00**	**721.00**	745.00	729.63	738.00	721.40	721.27	726.55	721.27	721.27	721.40	721.27	721.27	721.27	721.27	721.27	721.27
CMT3Y	**719.00**	**719.00**	723.00	745.46	740.00	721.40	721.27	729.02	721.27	721.27	721.27	721.27	721.27	721.27	721.27	721.27	721.27
CMT12X	**644.70**	675.00	676.00	**644.70**	691.00	663.01	662.22	681.02	662.22	662.22	662.22	662.94	662.22	662.22	663.94	662.22	662.22
CMT12Y	**659.52**	689.00	673.00	**659.52**	697.00	663.50	662.22	671.32	662.22	662.22	663.50	663.50	662.22	**659.52**	663.69	662.22	662.22
CMT11X	**833.92**	900.00	859.00	861.97	895.00	839.66	838.66	887.36	**833.92**	**833.92**	846.23	**833.92**	846.23	835.26	874.85	**833.92**	**833.92**
CMT11Y	**830.39**	910.00	860.00	**830.39**	900.00	840.19	837.08	874.13	833.92	833.92	836.04	833.92	846.23	833.92	874.85	833.92	833.92
CMT4X	**852.46**	880.00	867.00	876.50	912.00	854.12	**852.46**	893.90	**852.46**	**852.46**	**852.46**	852.83	**852.46**	**852.46**	**852.46**	**852.46**	**852.46**
CMT4Y	**852.00**	878.00	**852.00**	870.44	913.00	855.76	852.46	895.25	852.46	852.46	862.28	852.46	852.46	852.46	852.46	852.46	852.46
CMT5X	**1029.25**	1098.00	1067.00	1044.51	1167.00	1034.87	1030.55	1115.75	**1029.25**	1030.55	1033.51	1033.50	**1029.25**	**1029.25**	1032.89	1030.55	1030.55
CMT5Y	**1029.25**	1083.00	1046.00	1054.46	1142.00	1037.34	1030.55	1112.61	**1029.25**	1030.55	1036.14	1036.00	**1029.25**	1030.55	1032.89	1030.55	1030.55

Table 9
Solution values obtained with the recent metaheuristics on the Dethloff (2001) instances.

Instance	BKS	References												
		RP06	GA09	ZTK09	C10	SDBOF10	ZTK10	MLGS11	SMSOS11	GKA13	YDR14	AT15	KK16	MWZS16
SCA3-0	**635.62**	636.10	**635.62**	636.06	636.10	**635.62**	**635.62**	640.55	**635.62**	**635.62**	**635.62**	**635.62**	**635.62**	636.06
SCA3-1	**697.80**	**697.80**	697.84	697.84	700.10	697.84	697.84	697.84	697.84	697.84	697.84	697.84	697.84	700.50
SCA3-2	**659.30**	**659.30**	659.34	659.34	**659.30**	659.34	659.34	659.34	659.34	659.34	659.34	659.34	659.34	659.34
SCA3-3	**680.00**	680.60	680.04	680.04	**680.00**	680.04	680.04	680.04	680.04	680.04	680.04	680.04	680.34	680.61
SCA3-4	**690.50**	**690.50**	**690.50**	**690.50**	**690.50**	**690.50**	**690.50**	**690.50**	**690.50**	**690.50**	**690.50**	**690.50**	**690.50**	**690.50**
SCA3-5	**659.90**	**659.90**	**659.90**	**659.90**	670.10	**659.90**	**659.90**	**659.90**	**659.90**	**659.90**	659.91	**659.90**	659.91	660.32
SCA3-6	**651.09**	651.10	**651.09**	**651.09**	651.10	**651.09**	**651.09**	653.81	**651.09**	**651.09**	**651.09**	**651.09**	651.11	655.41
SCA3-7	**659.17**	666.10	**659.17**	**659.17**	666.10	**659.17**	**659.17**	**659.17**	**659.17**	**659.17**	**659.17**	**659.17**	**659.17**	659.89
SCA3-8	**719.47**	719.50	**719.47**	**719.47**	719.50	719.48	**719.47**	**719.47**	**719.47**	**719.47**	719.48	**719.47**	719.56	**719.47**
SCA3-9	**681.00**	**681.00**	**681.00**	**681.00**	**681.00**	**681.00**	**681.00**	**681.00**	**681.00**	**681.00**	**681.00**	**681.00**	**681.00**	686.22
SCA8-0	**961.50**	975.10	**961.50**	**961.50**	961.60	**961.50**	**961.50**	981.47	**961.50**	**961.50**	**961.50**	**961.50**	**961.50**	966.27
SCA8-1	**1049.65**	1052.40	**1049.65**	1050.20	1063.00	**1049.65**	**1049.65**	1077.44	**1049.65**	**1049.65**	1052.40	**1049.65**	**1049.65**	1056.28
SCA8-2	**1039.64**	1044.50	1042.69	**1039.64**	1040.60	**1039.64**	**1039.64**	1050.98	**1039.64**	**1039.64**	**1039.64**	**1039.64**	1041.62	1042.48
SCA8-3	**983.34**	999.10	**983.34**	**983.34**	985.90	**983.34**	**983.34**	**983.34**	**983.34**	**983.34**	**983.34**	**983.34**	**983.34**	987.52
SCA8-4	**1065.49**	1065.50	**1065.49**	**1065.49**	1071.00	**1065.49**	**1065.49**	1073.46	**1065.49**	**1065.49**	**1065.49**	**1065.49**	**1065.49**	1074.98
SCA8-5	**1027.08**	1027.10	**1027.08**	**1027.08**	1054.30	**1027.08**	**1027.08**	1047.24	**1027.08**	**1027.08**	**1027.08**	**1027.08**	**1027.08**	1031.70
SCA8-6	**971.82**	977.00	**971.82**	**971.82**	972.50	**971.82**	**971.82**	995.59	**971.82**	**971.82**	**971.82**	**971.82**	**971.82**	975.20
SCA8-7	**1051.28**	1061.00	1052.17	1052.17	1059.70	**1051.28**	**1051.28**	1068.56	**1051.28**	**1051.28**	1061.00	**1051.28**	**1051.28**	1055.33
SCA8-8	**1071.18**	1071.20	**1071.18**	**1071.18**	1082.70	**1071.18**	**1071.18**	1080.58	**1071.18**	**1071.18**	**1071.18**	**1071.18**	1071.22	1074.81
SCA8-9	**1060.50**	**1060.50**	**1060.50**	**1060.50**	1081.40	**1060.50**	**1060.50**	1084.80	**1060.50**	**1060.50**	**1060.50**	**1060.50**	**1060.50**	1064.63
CON3-0	**616.50**	**616.50**	616.52	616.52	**616.50**	616.52	616.52	631.39	616.52	616.52	616.52	616.52	616.52	619.64
CON3-1	**554.47**	554.50	**554.47**	**554.47**	555.60	**554.47**	**554.47**	**554.47**	**554.47**	**554.47**	**554.47**	**554.47**	**554.47**	556.28
CON3-2	**518.00**	521.40	**518.00**	519.26	521.40	**518.00**	**518.00**	522.86	**518.00**	**518.00**	518.01	**518.00**	**518.00**	521.53
CON3-3	**591.19**	591.20	**591.19**	**591.19**	591.20	**591.19**	**591.19**	**591.19**	**591.19**	**591.19**	**591.19**	**591.19**	**591.19**	593.49
CON3-4	**588.79**	588.80	**588.79**	589.32	589.30	**588.79**	**588.79**	591.12	**588.79**	**588.79**	**588.79**	**588.79**	**588.79**	**588.79**
CON3-5	**563.70**	**563.70**	**563.70**	**563.70**	**563.70**	**563.70**	**563.70**	**563.70**	**563.70**	**563.70**	**563.70**	**563.70**	**563.70**	566.24
CON3-6	**499.05**	500.80	**499.05**	500.80	499.20	**499.05**	**499.05**	506.19	**499.05**	**499.05**	500.80	**499.05**	**499.05**	502.18
CON3-7	**576.48**	576.50	**576.48**	**576.48**	577.50	**576.48**	**576.48**	577.68	**576.48**	**576.48**	**576.48**	**576.48**	**576.48**	579.62
CON3-8	**523.00**	523.10	523.05	523.05	523.10	523.05	523.05	**523.00**	523.05	523.05	523.05	523.05	523.05	523.68
CON3-9	**578.20**	586.40	578.25	580.05	**578.20**	578.25	578.25	580.05	578.24	578.25	578.25	578.25	578.25	581.40
CON8-0	**857.17**	857.20	**857.17**	**857.17**	858.90	**857.17**	**857.17**	860.48	**857.17**	**857.17**	**857.17**	**857.17**	**857.17**	860.17
CON8-1	**740.85**	740.90	**740.85**	**740.85**	740.90	**740.85**	**740.85**	**740.85**	**740.85**	**740.85**	**740.85**	**740.85**	**740.85**	743.39
CON8-2	**712.89**	716.00	**712.89**	713.44	714.30	**712.89**	**712.89**	723.32	**712.89**	**712.89**	**712.89**	**712.89**	**712.89**	718.88
CON8-3	**811.07**	811.10	**811.07**	**811.07**	812.30	**811.07**	**811.07**	811.23	**811.07**	**811.07**	**811.07**	**811.07**	**811.07**	821.26
CON8-4	**770.10**	772.30	772.25	772.25	**770.10**	772.25	772.25	772.25	772.25	772.25	772.25	772.25	772.25	774.12
CON8-5	**754.88**	755.70	**754.88**	756.91	766.60	**754.88**	**754.88**	756.91	**754.88**	**754.88**	755.70	**754.88**	**754.88**	761.52
CON8-6	**678.92**	693.10	**678.92**	**678.92**	697.20	**678.92**	**678.92**	**678.92**	**678.92**	**678.92**	**678.92**	**678.92**	**678.92**	680.53
CON8-7	**811.96**	814.80	**811.96**	**811.96**	814.80	**811.96**	**811.96**	814.50	**811.96**	**811.96**	814.80	**811.96**	813.96	813.63
CON8-8	**767.53**	774.00	**767.53**	**767.53**	771.30	**767.53**	**767.53**	775.59	**767.53**	**767.53**	**767.53**	**767.53**	**767.53**	769.31
CON8-9	**809.00**	809.30	**809.00**	**809.00**	815.10	**809.00**	**809.00**	**809.00**	**809.00**	**809.00**	**809.00**	**809.00**	**809.00**	811.02

increased environmental consciousness in the last decade motivated OR researchers to consider green issues within the VRP variants. Many papers have been published accordingly (see Bektaş and Laporte, 2011). However, green variants of the VRPSPD have only been studied by three papers comprising 9% of the publications listed in Table 3.

Table 7 presents the number of reviewed articles published in academic journals. It shows often-used journals for presentation of research on VRPSPDs. We see that respected OR journals accommodate articles addressing the topic. The search results show that seven journals contain more than half (34) of the 66 reviewed articles in total.

6.2. Metaheuristic computational comparison

Most of the metaheuristics were tested on benchmark instances of Salhi and Nagy (1999) and Dethloff (2001). Salhi and Nagy (1999) generated a set of 14 instances ranging from 50 to 199 customers derived from Christofides et al. (1979). Dethloff (2001) randomly generated 40 instances which includes two different geographical scenarios. Customers were uniformly distributed in the first scenario. In the second scenario, half of the customers were located as in first one, and the other half were clustered.

We present a detailed comparison of metaheuristics on Salhi and Nagy (1999) instances in Table 8, and on Dethloff (2001) instances in Table 9. The first column shows the best-known solution, while the others show the results for each method. The abbreviations of metaheuristics used in the comparison are as follows: MG06 for Montané and Galvao (2006), C06 for Chen (2006), RP06 for Ropke and Pisinger (2006), WWN08 for Wassan et al. (2008), AK09 for Ai and Kachitvichyanukul (2009), GA09 for Gajpal and Abad (2009), ZTK09 for Zachariadis et al. (2009), C10 for Çatay (2010), SDBOF10 for Subramanian et al. (2010), ZTK10 for Zachariadis et al. (2010), MLGS11 for Maquera et al. (2011), SMSOS11 for Souza et al. (2011), GKA13 for Goksal et al. (2013), SUO13 for Subramanian et al. (2013b), YDR14 for Yousefikhoshbakht et al. (2014), AT15 for Avci and Topaloglu (2015), PKKG15 for Polat et al. (2015), KK16 for Kalayci and Kaya (2016), and MWZS16 for Mu et al. (2016). For each instance, best-known solution values are boldfaced. The computers and programming languages used are not comparable, hence scaled times for one reference computer would not be valid.

Table 8 indicates that the best method is that of Wassan et al. (2008) with six best-known results on the Salhi and Nagy (1999) instances. The method of Wassan et al. (2008) is an effective tabu search heuristic and obtained quited good solutions on the VRPSPD. For the Dethloff (2001) intances, Table 9 indicates that the top performers are Zachariadis et al. (2010), Souza et al. (2011), Goksal et al. (2013), Avci and Topaloglu (2015) who obtained 33 best-known results. Tables 8 and 9 clearly state that local search based heuristics yielded better solutions than the other ones.

7. Conclusions and research perspectives

Over the last three decades, extensive research has been conducted on the vehicle routing problem with simultaneous pickup and delivery (VRPSPD) which was introduced by Min (1989). We have first surveyed and then provided a performance comparison of models and algorithms developed for the standard problem. We have classified the available heuristics as classical construction and improvement heuristics, local search metaheuristics, population search heuristics, and ant colony heuristics. We have then described several problem variations, as well as a number of practical applications and cases. We have classified several extensions under six main dimensions: the VRPSPD with time windows, the heterogeneous VRPSPD, the multi-depot VRPSPD, the green VRPSPD, the stochastic VRPSPD, and miscellaneous VRPSPDs. More than half of the available studies on VRPSPDs, with 34 papers, have focused on VRPSPD extensions.

There still exists an extensive research potential on VRPSPDs since we are witnessing significant innovations in the development of heuristic and exact algorithms, coupled with advances in machine learning and computer technology. This survey helped us identify several interesting promising research perspectives:

- The standard VRPSPD instances of Salhi and Nagy (1999) and Dethloff (2001) have been effectively solved by heuristics, and also many optimal solutions were obtained by exact algorithms. However, these solution methods were evaluated on instances with only a few hundred delivery points. This size could be increased to hundreds or even thousands of customers to reflect more realistic and emerging VRPSPD applications (Arnold et al., 2019, see).
- To effectively solve the VRPSPD and its variants, continuous approximation models should be developed since no study has yet considered these models (Langevin et al., 1996, see; Franceschetti et al., 2017). These models can be used to obtain a first insight into the solution costs at the tactical or strategic level. At the operational level, these models can be complemented by mathematical formulations. To validate the quality of VRPSPD heuristics, approximation models can be used. These models are also helpful in large-size VRPSPD optimization when the exact solution costs computation is almost impossible.
- Researchers have given too little attention to the time-dependent extension of the VRPSPD which was studied in only one paper. To meet the challenges faced in city logistics, time-dependencies should be considered more widely (Gendreau et al., 2015, see).
- The parameter values in most of VRPSPD papers are assumed to be known with certainty. Yet in practical cases, most of the information is often uncertain. For instance, uncertain information related to whether a customer requires service or not, as well as to parameters such as service and travel times, and demands. Such realistic situations can also be addressed within a dynamic environment and they can be embedded within VRPSPDs (Bektaş et al., 2014, see; Gendreau et al., 2016).
- Research in reducing pollution in goods transportation focuses on the design of effective green distribution policies such as the use of electric vehicles. Several variants of the electric VRP have been studied, but not in the context of the VRPSPD. There exists a rich research potential in the study of electric VRPSPDs (Pelletier et al., 2016, see; Koç et al., 2019).
- Some practical situations need to consider multiple planning periods and scenarios (Campbell and Wilson, 2014, see). To our knowledge, no study has yet been performed on multi-period VRPSPDs.
- Increasing interest in using drones in commercial deliveries by distribution companies, inspired several researchers to focus on VRP with drones (Poikonen et al., 2017, see; Otto et al., 2018; Sacramento et al., 2019). We believe that the integration of the VRPSPD with drones will help meet the challenges faced by city transportation networks and supply chains.
- Autonomous vehicles are being introduced as new, potentially disruptive yet beneficial tools for distribution management systems. They hold a potential for travel time reduction, fuel efficiency, safety, and congestion reduction (Fagnant and Kockelman, 2015). The adoption of these up-to-date technological vehicles to satisfy customer demands in VRPSPDs has a tremendous potential to improve routing operations. In VRPSPD operations, the which poses considerable challenges of empty cans or bottles very slowly. This environment is ideal for autonomous vehicles which works quite well at low speeds. It allows vehicles to identify, monitor, and navigate accurately (DHL, 2020).

CRediT authorship contribution statement

Çağrı Koç: Conceptualization, Methodology, Writing - original draft, Writing - review & editing. **Gilbert Laporte:** Supervision, Conceptualization, Writing - review & editing. **İlknur Tükenmez:** Writing - original draft.

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