DESIGN & ANALYSIS OF ALGORITHM

(CS 617-01) (SP18)

Homework 2

Submitted by

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Page 61, 3-3 (a). You could skip functions containing log* if you have trouble with it.

3-3 a. Ordering by asymptotic growth rates

Rank the following functions by order of growth; that is, find an arrangement g_1, g_2, ... ,g_{30} of the functions satisfying $g_1 = \Omega(g_2)$, $g_2 = \Omega(g_3)$, ..., $g_{29} = \Omega(g_{30})$. Partition your list into equivalence classes such that functions f (n) and g(n) are in the same class if and only if f(n) = Θ(g(n)).

$\lg(\lg^* n)$	$2^{\lg^* n}$	$(\sqrt{2})^{\lg n}$	n^2	$n!$
$(\lg n)!$	$(3/2)^n$	n^3	$\lg^2 n$	$\lg(n!)$
2^{2^n}	$n^{1/\lg n}$	$\ln \ln n$	$\lg^* n$	$n . 2^n$
$n^{\lg \lg n}$	$\ln n$	1	$2^{\lg n}$	$(\lg n)^{\lg n}$
e^n	$4^{\lg n}$	$(n+1)!$	$\sqrt{(\lg n)}$	$\lg^*(\lg n)$
$2^{\sqrt{(2 \lg n)}}$	n	2^n	$n \lg n$	$2^{2^{(n+1)}}$

Answer:

<u>Order of Growth:</u>

$2^{2^{(n+1)}}$,	2^{2^n},	$(n+1)!$,	$n!$,	e^n ,
$n . 2^n$,	2^n ,	$(3/2)^n$,	$(\lg n)^{\lg n}$, $n^{\lg \lg n}$	$(\lg n)!$,
n^3 ,	n^2 , $4^{\lg n}$	$n \lg n$, $\lg (n!)$	n , $2^{\lg n}$	$(\sqrt{2})^{\lg n}$,
$2^{\sqrt{(2 \lg n)}}$,	$\lg^2 n$,	$\ln n$,	$\sqrt{(\lg n)}$,	$\ln \ln n$,

$2^{\lg^* n}$, $\lg^*(\lg n)$, $\lg(\lg^* n)$, $n^{1/\lg n}$

$\lg^* n$ 1

Some simplifications:

1. $n^{1/\lg n} = n^{\lg 2/\lg n} = 1$
2. $\lg^*(\lg n) = \lg^* n - 1$
3. $\lg^2 n = (\lg n)^2 = 2 \lg n$
4. $(\sqrt{2})^{\lg n} = n^{1/2} = \sqrt{n}$
5. $2^{\lg n} = n$
6. Using Stirling's Approximation:

   $n! = \sqrt{(2\pi n)}(n/e)^n(1 + \Theta(1/n)) = \Theta(n^{n+½} e^{-n})$

   $\lg(n!) = \Theta(n \lg n)$

   $(\lg n)! = \Theta(\lg n^{\lg n+½} e^{-\lg n})$
7. $(\lg n)^{\lg n} = n^{\lg \lg n}$