Prob 2, Ch 3

Phys 416
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$v_{n+1} = v_n + a_n \Delta t$

$r_{n+1} = x_n + v_{n+1}\Delta t = x_n + (v_n + a_n \Delta t)\Delta t$

$x_0 = 0$

$y_0 = a(1-\epsilon)$

$L = |v \times r|$

$\vec{v} = v_x \hat{x} + v_y \hat{y}$

$\vec{r} = r_x \hat{x} + r_y \hat{y} = x\hat{x} + y\hat{y}$

or this because I ~~forgot~~ my convention

$\Rightarrow L = v_x r_y - v_y r_x$

$L^{n+1} = v_x^{n+1} r_y^{n+1} - v_y^{n+1} r_x^{n+1}$

$= (v_x^n + a_x^n \Delta t)(y_n + (v_y^n + a_y^n \Delta t)\Delta t)$

$- (v_y^n + a_y^n \Delta t)(x_n + (v_x^n + a_x^n \Delta t)\Delta t)$

$= v_x^n y^n + y^n a_x^n \Delta t + v_x^n (v_y^n + a_y^n \Delta t)\Delta t + a_x^n (v_y^n + a_y^n \Delta t)\Delta t^2$

$- [v_y^n x^n + x^n a_y^n \Delta t + v_y^n (v_x^n + a_x^n \Delta t)\Delta t + a_y^n (v_x^n + a_x^n)\Delta t^3$

Discard h.o.t.

$\Rightarrow v_x^n y^n - v_y^n x^n + \Delta t \underbrace{[y^n a_x^n - x^n a_y^n]}_{0 = \text{ since it is } a \times r = 0 \text{ for central force}} = L^{n+1}$

$\Rightarrow L^{n+1} = L^n$, and therefore, momentum is conserved □