hw3b_ex19c

March 6, 2020

[1]:
```python
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
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"""
# python 3 version
import numpy as np
import matplotlib.pyplot as plt

def rk4(x,t,tau,derivsRK,param):
    """
    ##  Runge-Kutta integrator (4th order)
    ## Input arguments -
    ##   x = current value of dependent variable
    ##   t = independent variable (usually time)
    ##   tau = step size (usually timestep)
    ##   derivsRK = right hand side of the ODE; derivsRK is the
    ##             name of the function which returns dx/dt
    ##             Calling format derivsRK(x,t).
    ## Output arguments -
    ##   xout = new value of x after a step of size tau
    """
    half_tau = 0.5*tau
    F1 = derivsRK(x,t,param)
    t_half = t + half_tau
    xtemp = x + half_tau*F1
    F2 = derivsRK(xtemp,t_half,param)
    xtemp = x + half_tau*F2
    F3 = derivsRK(xtemp,t_half,param)
    t_full = t + tau
    xtemp = x + tau*F3
    F4 = derivsRK(xtemp,t_full,param)
    xout = x + tau/6.*(F1 + F4 + 2.*(F2+F3))
    return xout
```

```python
def rka(x,t,tau,err,derivsRK,param):
    """
    ## Adaptive Runge-Kutta routine
    ## Inputs
    ##   x          Current value of the dependent variable
    ##   t          Independent variable (usually time)
    ##   tau        Step size (usually time step)
    ##   err        Desired fractional local truncation error
    ##   derivsRK   Right hand side of the ODE; derivsRK is the
    ##              name of the function which returns dx/dt
    ##              Calling format derivsRK(x,t).
    ## Outputs
    ##   xSmall     New value of the dependent variable
    ##   t          New value of the independent variable
    ##   tau        Suggested step size for next call to rka
    """
    # Set initial variables
    tSave = t;  xSave = x    # Save initial values
    safe1 = .9;  safe2 = 4.  # Safety factors
    eps = np.spacing(1) # smallest value

    # Loop over maximum number of attempts to satisfy error bound
    maxTry = 100

    for iTry in range(1,maxTry):

        # Take the two small time steps
        half_tau = 0.5 * tau
        xTemp = rk4(xSave,tSave,half_tau,derivsRK,param)
        t = tSave + half_tau
        xSmall = rk4(xTemp,t,half_tau,derivsRK,param)

        # Take the single big time step
        t = tSave + tau
        xBig = rk4(xSave,tSave,tau,derivsRK,param)

        # Compute the estimated truncation error
        scale = err * (np.abs(xSmall) + np.abs(xBig))/2.
        xDiff = xSmall - xBig
        errorRatio = np.max( [np.abs(xDiff)/(scale + eps)] )

        #print safe1,tau,errorRatio

        # Estimate news tau value (including safety factors)
        tau_old = tau

        tau = safe1*tau_old*errorRatio**(-0.20)
```

```
        tau = np.max([tau,tau_old/safe2])
        tau = np.min([tau,safe2*tau_old])

        # If error is acceptable, return computed values
        if errorRatio < 1 :
            xSmall = xSmall
            return xSmall, t, tau

def double_pend(s,t,param):
    """
    Defines the system of diff eqs needed for the double pend problem

    """
    theta1 = s[0]
    theta2 = s[1]
    theta1_p = s[2]
    theta2_p = s[3]

    L1 = param[0]
    L2 = param[1]
    m2 = 1
    m1 = param[2]
    g = param[3]

    a1 = (L2/L1)*(m2/(m1+m2))*np.cos(theta1-theta2)
    a2 = (L1/L2)*np.cos(theta1-theta2)

    F1 = -(L2/L1)*(m2/(m1+m2))*(theta2_p**2)*np.sin(theta1-theta2) - (g/L1)*np.
 ↪sin(theta1)
    F2 = (L1/L2)*(theta1_p**2)*np.sin(theta1-theta2) - (g/L2)*np.sin(theta2)

    g1 = (F1 - a1*F2) / (1 - a1*a2)
    g2 = (F2 - a2*F1) / (1 - a1*a2)

    deriv = np.zeros(4)
    deriv[0] = theta1_p
    deriv[1] = theta2_p
    deriv[2] = g1
    deriv[3] = g2

    return deriv


def␣
 ↪doublepend_data_gen(init_theta1,init_theta2,init_theta1_p,init_theta2_p,param,plotting=Fals
 ↪
    """
```

```
    Solves the double pendulum problem

    Parameters
    ----------
    init_theta1, init_theta2 : Float
        Inital theta.
    init_theta1_p,init_theta2_p : Float
        Initial theta prime value.
    param : list
        List of model parameter (L1,L2,mr,g).
        Note than we have m1 = mr*m2

    Returns
    -------
    rplot : Numpy array
        Array of r-values used to plot.
    fplot : Numpy array
        Array of f-values used to plot.
    tplot : Numpy array
        Array of time-values used to plot.

    """
    # Set initial state x,y,z and parameters r,sigma,b
    state = np.zeros(4)
    state[0] = float(init_theta1)
    state[1] = float(init_theta2)
    state[2] = float(init_theta1_p)
    state[3] = float(init_theta2_p)

    # Model paramaters
    L1 = param[0]
    L2 = param[1]
    m2 = 1
    m1 = param[2]
    g = param[3]
    tau = .05      # Initial timestep guess
    err = 1.e-6   # Error tolerance

    # Loop over the desired number of steps
    time = 0
    nstep = 50000
    # initialize arrays
    tplot = np.array([])
    tauplot = np.array([])
    th1plot = np.array([])
    th2plot = np.array([])
    th1_p_plot = np.array([])
```

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    th2_p_plot = np.array([])
    poincare = np.array([])

    for istep in range(0,nstep):
        # Record values for plotting
        theta1 = state[0]
        theta2 = state[1]
        theta1_p = state[2]
        theta2_p = state[3]

        tplot = np.append(tplot,time)
        tauplot = np.append(tauplot,tau)

        th1plot = np.append(th1plot,theta1)
        th2plot = np.append(th2plot,theta2)
        th1_p_plot = np.append(th1_p_plot,theta1_p)
        th2_p_plot = np.append(th2_p_plot,theta2_p)

        if istep >= 2 and th2plot[-1]*th2plot[-2] < 0:
            poincare = np.append(poincare,[theta1,theta1_p])

        #if( istep%50 ==0 ):
          #print('Finished %d steps out of %d '%(istep,nstep))
        # Find new state using Runge-Kutta4
        #state = rk4(state,time,tau,lotka_volterra,param)
        #time += tau
        [state, time, tau] = rka(state,time,tau,err,double_pend,param)

    p = poincare.reshape((int(poincare.shape[0]/2),2))
    #print(poincare)
    # Print max and min time step returned by rka
    #print('Adaptive time step: Max = %f,  Min = %f '%(max(tauplot[1:]),␣
↪min(tauplot[1:])));

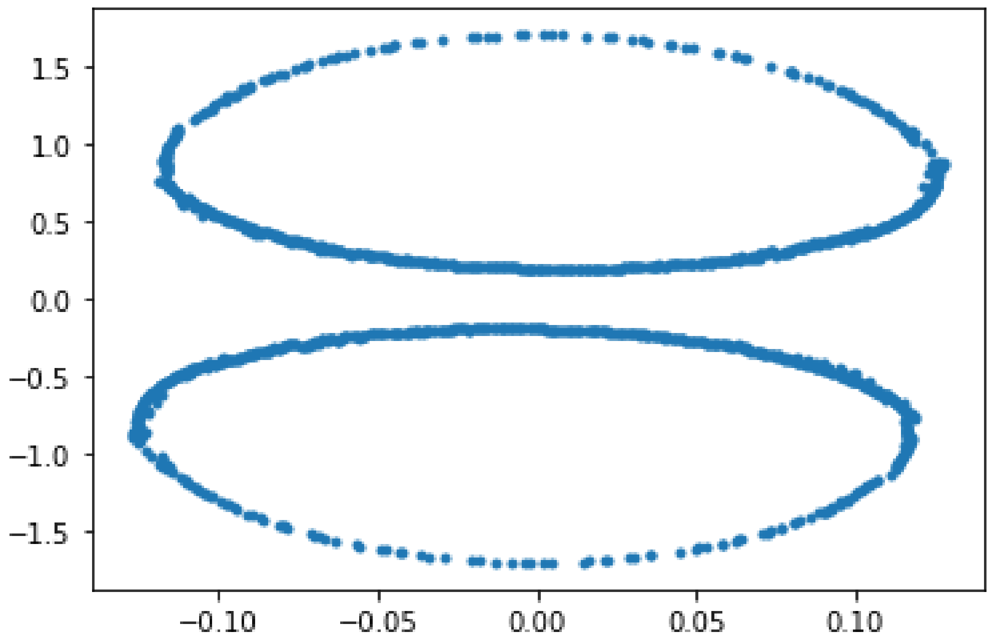

    if plotting:
        # Graph the time series x(t)
        plt.figure()
        plt.scatter(p[:,0],p[:,1],marker='.')
        #plt.plot(tplot,th2plot)

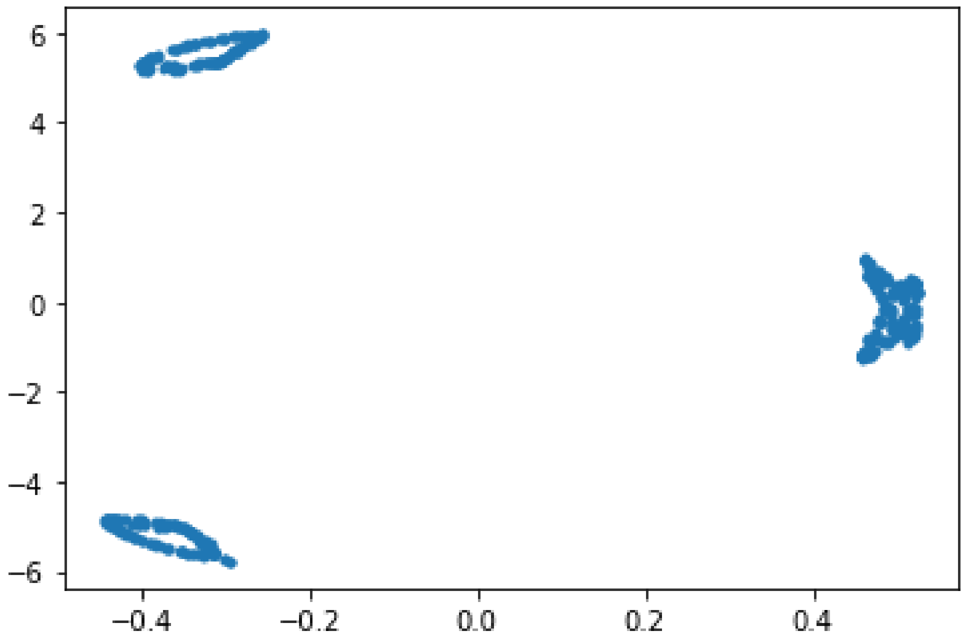
    return th1plot,th2plot,th1_p_plot,th2_p_plot,tplot,p
```

[2]:
```
param = (.1,.1,10,9.81)
init_theta1 = np.radians(10)
init_theta2 = np.radians(10)
```

```
init_theta1_p = np.radians(0)
init_theta2_p = np.radians(0)
th1plot,th2plot,th1_p_plot,th2_p_plot,tplot,p =␣
 ↪doublepend_data_gen(init_theta1,init_theta2,init_theta1_p,init_theta2_p,param,True)

param = (.1,.1,10,9.81)
init_theta1 = np.radians(45)
init_theta2 = np.radians(45)
init_theta1_p = np.radians(0)
init_theta2_p = np.radians(0)
th1plot,th2plot,th1_p_plot,th2_p_plot,tplot,p =␣
 ↪doublepend_data_gen(init_theta1,init_theta2,init_theta1_p,init_theta2_p,param,True)
```




6
4
2
0
−2
−4
−6
−0.4
−0.2
0.0
0.2
0.4

[ ]: