PHYS 416/517 – Chapter 2 - Exercises

Due: Thursday Feb. 6, 2020, 1 PM

Projectile Motion

3. The **balle** program overestimates the range and time of flight (Figure 2.4 – see below).

Fix this bit of sloppy programming. Compute a corrected maximum range <u>and</u> time of flight by interpolating between the last three values of **r** using the `intrpf` function from the last assignment.

Check your answer with the analytic solution (for both range and time of flight).

Modify your program to compute and plot the potential, kinetic and total energy as a function of time in a separate figure.

Take an initial height of 1 m, initial speed of 50 m/s, angle of $\theta = 45^\circ$ with no air resistance, and try a variety of values for the time step τ to determine which timestep is optimal.

[Computer: Turn in a sample output from you program.]
[For this problem, use the Euler method. Make sure you include the `intrpf` function when you turn it all in.]

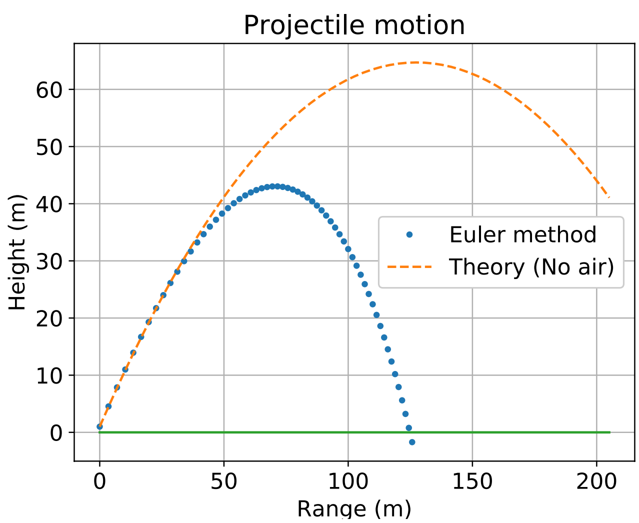

Figure 2.4 from Garcia

5. Suppose that a batter hits a ball and gives it an initial velocity of 50 m/s (take $y_1 = 1$ m). Modify your previous program to make a plot of the range as a function of angle for $10^\circ < \theta \leq 50^\circ$. Determine the angle to within 1° at which the maximum range is achieved.

[*Computer - Turn in the program and the plot*]

Hints:

- Modify the **balle** program to become a function that returns the range for a given initial height, speed and angle, call this function for a set of input angles and then determine the maximum range that you get.
- When you modify your program, make sure you remove the original plotting of the trajectories, otherwise you will get a lot plots.
- Use the midpoint method for this problem.
- You may find use in the max in MATLAB and the numpy.amax in PYTHON/numpy.
- Assume that air resistance is non-zero.

6. In Two New Sciences, Galileo claims that if a 100-lb iron ball and a 1-lb ball are dropped from a height of 100 braccia (about 50 meters), then "when the larger one strikes the ground, the other is 2 inches behind it."

(a) Modify the **balle** program to simultaneously compute the motion of two objects, and show that this statement is inaccurate. Plot the height versus time for the 2 balls. Note: the density of iron is 7.8×10^3 kg/m^3, assume C_d=0.5.

(b) For a falling ball under constant gravity g, starting at height $y(0)$ show that

$$y(t) = y(0) - \frac{1}{b} \log\left[\cosh\left(\sqrt{(bg)}\, t\right)\right]$$

Where $b = \frac{C_d \rho A}{2m}$, where ρ is the density of air, A is the cross-sectional area of the ball, m is the mass of the ball and g is the acceleration due to gravity. Use this analytic result to check your program. (In other words, add to the plot in part (a) the theoretically computed trajectories.)

(c) *Approximately* what would C_d need to be for Galileo's statement to be correct?

[Computer - Turn in a program, calculations for (b), plots and the answer to part (c).]

Hints:

- Be careful about units (use MKS).
- An approximate answer to (c) will suffice.
- Use the midpoint method.
- Note: $\rho_{iron} = 7.8 \times 10^3 kg / m^3$

The effects of Spin and the Physics of Golf

In this section, we will consider the effects of spin on a moving golf ball. In general, it can be shown that that force on a ball due to spin, which is known as the Magnus force, is best described by

$$\vec{F}_M = S_o \vec{\omega} \times \vec{v}$$

where $\vec{\omega}$ is the spin of the ball and $\vec{v}$ is its velocity. The constant S_0 is usually determined from wind tunnel measurements. Note that this force is a vector.

A. Investigate the effect of backspin and dimples on the motion of a golf ball. In this case, assume that the initial velocity is 70 m/s. The effect of dimples on a golf ball can have a significant effect on the range of the ball.

For this case assume that C_d = 0.5 for speeds up to 14 m/s and C_d=7.0/v for higher speeds. For the Magnus force, assume $\frac{S\omega_0}{m} \approx 0.2\ s^{-1}$(This value is a high degree of uncertainty and can have a big effect of the trajectory. Feel free to explore the effect of other values if you are interested!) The mass of a golf ball is 0.04593 kg and its radius is 2.213 cm.

a. Plot a few sample trajectories for different launch angles for both smooth and dimpled balls.
b. What is the optimal angle for achieving maximum range for both the smooth and dimpled ball?
c. Compare that to the results for part b for a smooth ball where C_d is constant C_d =0.5.

[*Computer. Turn in the program, results and 2 plots for the smooth versus non-smooth ball. Be sure to label your plots.]*

- In question (A), modify your range program to compute the range for a golf ball.
- To find the correct optimal angle, make sure you sample a large range of angles.

Physics of Pendulums

15. In the small angle approximation, the total energy of a simple pendulum is

$$E = \frac{1}{2}mL^2\omega^2 + \frac{1}{2}mgL\theta^2 - mgL$$

Show analytically that E monotonically increases with time when the Euler method is used to compute the motion. [*Pencil*]

16. Write a version of the **pendul** program that uses:
(a) the Euler-Cromer method
(b) the Leap-Frog method
(c) the mid-point method

Run your program for the cases in figures 2.7 – 2.8 of the text. Compare your results with the Euler and Verlet methods. A table of input parameters is shown below.

Figure number	Initial angle θ_0 (degrees)	Timestep τ (sec.)	Number of steps
2.7	10	0.1	300
2.8	170	0.1	300

[*Computer. Turn in the program and the sample plots.*]

21. Consider the pendulum with a harmonically driven pivot, also known as Kapitza's pendulum, has an equation of motion as

$$\frac{d^2\theta}{dt^2} = -\frac{g + a_d(t)}{L}\sin\theta$$

where $a_d(t) = A_0 \sin\left(\frac{2\pi t}{T_d}\right)$ is the time varying acceleration of the pivot. Write a program that simulates this system; be sure to use a timestep that is appropriate to the driving time period T_d.

Show that when the amplitude of the driving acceleration is sufficiently high ($A_0 \gg g$), the pendulum is stable in the inverted position (i.e., if $\theta(t = 0) \approx 180^\circ$, then the pendulum oscillates about the point $\theta = 180^\circ$. [Computer] [Suggestion: try T_0~0.2, A_0~100g, where g is the gravitational acceleration.]

Optional Exercises (1 point each extra credit)

Follow the bouncing ball

B. In this exercise, we will allow the ball to bounce once it hits the ground. To check to make sure your program is working properly you will need to modify the analytical solution for a trajectory, without air resistance, to allow for a solution that bounces the ball when it hits the ground. Modify the program to allow the numerical solver to bounce the ball. In order to do that, you need to determine when, where and at what velocity the ball hits the ground and bounces. (Hint: Use the **intrpf** function.)

Use the comparison with the analytical solution (for the case without air resistance) to set the timestep τ.

Use the following initial conditions:

Initial height, y_1 = 10 meters

Initial velocity, v_1, speed of 50 m/s launched at an angle of 45˚

Make plots for cases with and without air resistance. Use the midpoint method.
[*Turn in a program and 2 plots, with and without air resistance.]*.

Moving target problem

C. A fielder that is 100 meters away from a batsman starts moving to the right at 4 m/s at the same time a ball is hit by a batsman. The initial fielder location is shown as a green circle in the figure below. Calculate at what angle the ball should be launched at so it lands the same time the fielder arrives. The ball leaves the batsman at a speed of 50 m/s at a height of 1 meter. Assume air resistance (C_d=0.35), but you should check your code for the case of no air resistance which has an analytic solution. Note, you may get 2 answers for the angle.