8b) Let $\frac{2\gamma k}{h^2} = \sigma$

$T_j^n = T^n e^{ikhj}$, $T_j^{n+1} = AT^n e^{ikhj}$

$\Rightarrow A = \frac{1}{A} + \sigma(e^{ikh} - 2 + e^{-ikh})$

$\Rightarrow A^2 - A\sigma(2\cos kh - 2) - 1 = 0$

$A = (\sigma(\cos kh - 1)) \pm \frac{1}{2}\sqrt{(\sigma(2\cos kh - 2))^2 + 4}$

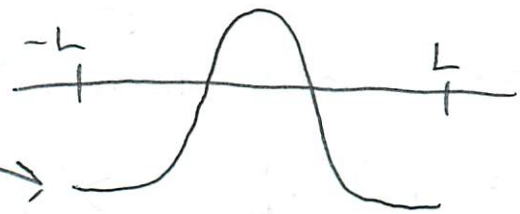
$= \sigma(\cos kh - 1) \pm \sqrt{\sigma^2(\cos kh - 1)^2 + 1}$

$\Rightarrow |A| \leq 1$ iff $\sigma = 0$ o.w. $|A| > 1$ $\square$

6a) From the construction of $T(x,t)$ as in the notes/book it is clear that it delivers a periodic function for which $u_t = ku_{xx}$ holds everywhere and so as $t \to 0$ wht. $T(x,t) \to \sum_{n=-\infty}^{\infty} (-1)^n \delta(x - nL)$

If instead of the interval $[-L/2, L/2]$, we cut out $T(x,t)$ on $[-L, L]$

By symmetry $\partial_x T = 0$ at $x = \pm L$



Note that a soln $u(x,t)$ to $\partial_t u = k\partial_{xx} u$ with neumann conditions isn't unique.

So, let $\int_{-L}^{L} u(x,t)\,dx = 0$, to provide uniqueness

cont. on next pg.

6a cont.

To rephrase it to B.C. at $x = \pm L/2$

let $T(x,t) = \sum_{n=-\infty}^{+\infty} (-1)^n T_0(x + 2nL, t)$ and

this will be a soln with $\partial_x T = 0$ at $x = \pm L/2$

and $\int_{-L/2}^{L/2} T(x,t)\,dx = 0$

12a) We follow just as in the book from eq 6.33 to 6.39. Instead of 6.40, let

$X(x) = b_0 + \sum_{j=1}^{\infty} b_j \cos\left(\frac{j\pi}{L}\left(x + L/2\right)\right)$

This ensures that Neumann b.c. are met.

For $j = 1, \dots \infty$ wht: $-\left(\frac{j\pi}{L}\right)^2 = \frac{\alpha - c}{D}$

For $j = 0$, wht. $\frac{\alpha - c}{c} = 0 \Rightarrow \alpha = c$

Our final soln looks like

$N(x,t) = e^{\alpha_0 T} \cdot b_0 + \sum e^{\alpha_j T} b_j \cos(\dots)$

Since $c > 0 \Rightarrow \alpha_0 > 0$, ~~...~~ and since $b_0 \neq 0$ (even any machine error) and $b_0 = \int_{-L/2}^{L/2} n(x,0)\,dx$ wht. $N(x,t)$ will be driven to exponential growth by $b_0 e^{\alpha_0 T}$ $\square$