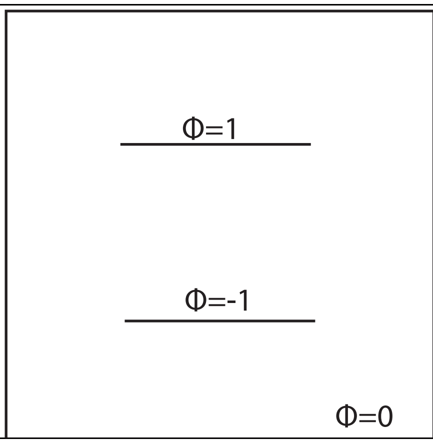
PHYS 416 – Chapter 8 – Elliptic Equations

Due Thursday, April 23 at PM, 2020

8A. Modify the relax program to <u>include</u> the option for zero derivative boundary conditions at the sides of the box (i.e., at x=0 and 1, impose$\frac{\partial \Phi}{\partial x}$=0, instead of $\Phi = 0$).

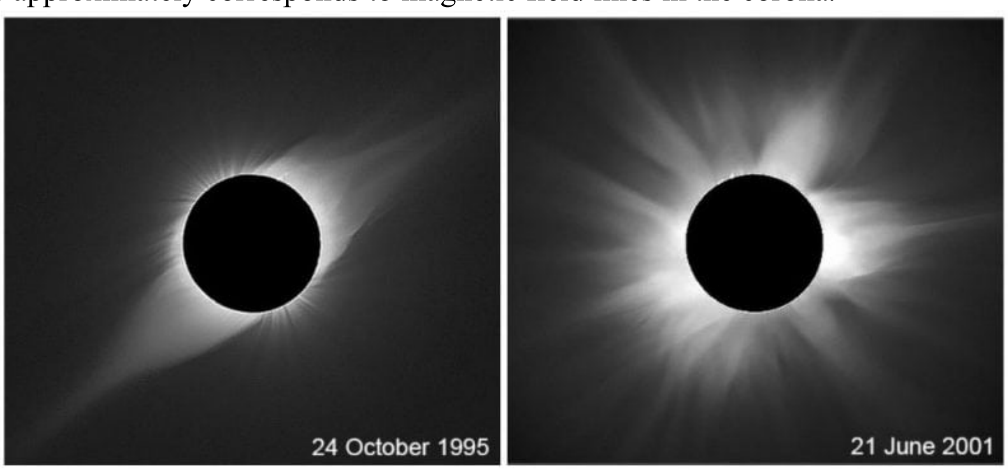
8.B Write a program that uses the SOR method to solves for the potential for the two geometries of a capacitor inside a grounded box of dimension 1x1. The two plates of the capacitor are at y location 0.25 and 0.75, centered on the box and has a width of 0.5. The thickness of the capacitor is the grid spacing. Compute the electric field for this case and qualitatively determine where the magnitude of the field is largest.



Hint: A relatively easy way to do this problem is to define an integer mask array, that has values either 0 or 1. Where you want the solution to be computed, you set the mask value at that location to 1, and where you want the geometry/potential to be fixed, set the mask value to 0. Your code will then check to see if the mask array is set to 1 to update the potential. Changing the mask array allows you to easily change the geometry of the problem.

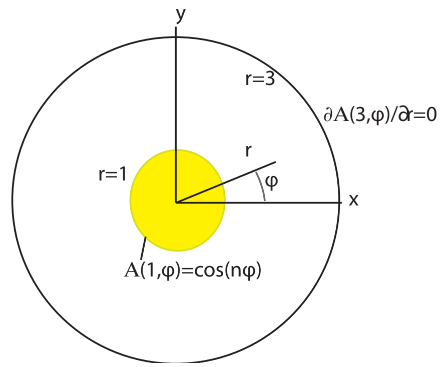
A Simple Solar Coronal Magnetic Field Model

8.C. The solar corona is a basically the atmosphere of the sun, and is a hot, tenuous environment that is controlled by the Sun's magnetic field. The image below is taken during a solar eclipse, the left during solar minimum and the right during solar maximum. The structures that you see in the image approximately corresponds to magnetic field lines in the corona.



A simple way to model the coronal magnetic field is to assume that it is a current-free system that is described by the solution to Laplace's equation. This is known as the 'potential source field model', and this exercise will attempt to create a simplified version of this model, where contours of constant vector potential correspond to magnetic field lines. The magnetic field can be derived from the equation $\vec{B} = \nabla \times (A(r,\varphi)\hat{z})$. For this problem, we will solve the 2D Laplace's equation in cylindrical coordinates (r, φ, z)in the geometry shown in the figure below. We assume that $\frac{\partial}{\partial z} = 0$, the Laplacian in cylindrical coordinates is:

$$\nabla^2 \mathrm{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \mathrm{A}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \mathrm{A}}{\partial \varphi^2} = 0$$



The inner boundary corresponds to the Sun's surface (the Photosphere), while the outer boundary corresponds to the outer layer of the corona. At the photosphere, we will assume that the vector potential is a given function so we will use Dirichlet boundary conditions, while at the outer boundary we will assume that the magnetic field lines (which corresponds to field lines) are radial. Write a program that solves Laplace's equation for the region between $1 \le r \le 3$ where the boundary conditions are

$$\mathrm{A}(r = 1, \varphi) = \cos n\varphi$$

$$\frac{\partial \mathrm{A}(r = 3, \varphi)}{\partial r} = 0$$

This equation has an analytic solution

$$\mathrm{A}(r, \varphi) = \frac{\left(\frac{3^{2n}}{r^n} + r^n\right)}{3^{2n} + 1} \cos{(n\varphi)}$$

Use the SOR method and compare your result to the analytic solution. Plot the contours of the solution in the geometry of the system. Run the model for cases *n*=1 and *n*=3. The *n*=1 case roughly corresponds to solar minimum, while larger values of n are for more active times. Compare these results to the images above.

Hints:

- You can use the relax code as a starting point, but the discretization is different because of the coordinate system. Also, when you plot the contours, you can plot in the geometry of the system by creating arrays $x = rcos\varphi$, $y = rsin\varphi$
- Use contouring with the command `contour(x,y,phi)`. Be careful about how the grid relates to the spatial coordinates.
- I suggest you use a 61x91 grid (in r and φ), anything finer may take too long to converge. Make sure you compare to the analytic solution above to make sure your code has converged. (The optimal grid resolution that doesn't take too long to converge may depend on how fast your computer is.)