hw3_ex14

February 20, 2020

[1]:
```python
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
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"""

# python 3 version 2/15
import numpy as np
import matplotlib.pyplot as plt

def rk4(x,t,tau,derivsRK):
#%  Runge-Kutta integrator (4th order)
#% Input arguments -
#%   x = current value of dependent variable
#%   t = independent variable (usually time)
#%   tau = step size (usually timestep)
#%   derivsRK = right hand side of the ODE; derivsRK is the
#%             name of the function which returns dx/dt
#%             Calling format derivsRK(x,t).
#% Output arguments -
#%   xout = new value of x after a step of size tau
    half_tau = 0.5*tau
    F1 = derivsRK(x,t)
    t_half = t + half_tau
    xtemp = x + half_tau*F1
    F2 = derivsRK(xtemp,t_half)
    xtemp = x + half_tau*F2
    F3 = derivsRK(xtemp,t_half)
    t_full = t + tau
    xtemp = x + tau*F3
    F4 = derivsRK(xtemp,t_full)
    xout = x + tau/6.*(F1 + F4 + 2.*(F2+F3))
    return xout
def rka(x,t,tau,err,derivsRK):
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#% Adaptive Runge-Kutta routine
#% Inputs
#%   x          Current value of the dependent variable
#%   t          Independent variable (usually time)
#%   tau        Step size (usually time step)
#%   err        Desired fractional local truncation error
#%   derivsRK   Right hand side of the ODE; derivsRK is the
#%              name of the function which returns dx/dt
#%              Calling format derivsRK(x,t).
#% Outputs
#%   xSmall     New value of the dependent variable
#%   t          New value of the independent variable
#%   tau        Suggested step size for next call to rka

#%* Set initial variables
    tSave = t;  xSave = x    # Save initial values
    safe1 = .9;  safe2 = 4.  # Safety factors
    eps = np.spacing(1) # smallest value

#%* Loop over maximum number of attempts to satisfy error bound
    maxTry = 100

    for iTry in range(1,maxTry):

#%* Take the two small time steps
        half_tau = 0.5 * tau
        xTemp = rk4(xSave,tSave,half_tau,derivsRK)
        t = tSave + half_tau
        xSmall = rk4(xTemp,t,half_tau,derivsRK)

  #%* Take the single big time step
        t = tSave + tau
        xBig = rk4(xSave,tSave,tau,derivsRK)

  #%* Compute the estimated truncation error
        scale = err * (np.abs(xSmall) + np.abs(xBig))/2.
        xDiff = xSmall - xBig
        errorRatio = np.max( [np.abs(xDiff)/(scale + eps)] )

        #print safe1,tau,errorRatio

  #%* Estimate news tau value (including safety factors)
        tau_old = tau

        tau = safe1*tau_old*errorRatio**(-0.20)
        tau = np.max([tau,tau_old/safe2])
        tau = np.min([tau,safe2*tau_old])
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  #%* If error is acceptable, return computed values
        if errorRatio < 1 :
          # xSmall = xSmall #% +  (xDiff)/15
          #   xSmall = (16.*xSmall - xBig)/15. # correction
            return xSmall, t, tau

#%* Issue error message if error bound never satisfied
    print ('ERROR: Adaptive Runge-Kutta routine failed')
    return
def gravrk(s,t):
#%  Returns right-hand side of Kepler ODE; used by Runge-Kutta routines
#%  Inputs
#%    s      State vector [r(1) r(2) v(1) v(2)]
#%    t      Time (not used)
#%  Output
#%    deriv  Derivatives [dr(1)/dt dr(2)/dt dv(1)/dt dv(2)/dt]

    GM = 4*np.pi**2

#%* Compute acceleration
    r = np.array([s[0], s[1]])  # Unravel the vector s into position and␣
 ↪velocity
    v = np.array([s[2] ,s[3]])
    accel = -GM*r/np.linalg.norm(r)**3  # Gravitational acceleration

#%* Return derivatives [dr(1)/dt dr(2)/dt dv(1)/dt dv(2)/dt]
    derivs = np.array([v[0], v[1], accel[0], accel[1]])
    return derivs

def gravrk_ex14(s,t):
#%  Returns right-hand side of Kepler ODE; used by Runge-Kutta routines
#%  Inputs
#%    s      State vector [r(1) r(2) v(1) v(2)]
#%    t      Time (not used)
#%  Output
#%    deriv  Derivatives [dr(1)/dt dr(2)/dt dv(1)/dt dv(2)/dt]

    GM = 4*np.pi**2
    # Hardcoded in vars since
    # Im not sure on an elegant way to add them to function
    alpha = .1

#%* Compute acceleration
    r = np.array([s[0], s[1]])  # Unravel the vector s into position and␣
 ↪velocity
    v = np.array([s[2] ,s[3]])
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    # Gravitational acceleration
    Grav_mult = 1-(alpha/np.linalg.norm(r))
    accel = (-GM*r/np.linalg.norm(r)**3)*Grav_mult

#%* Return derivatives [dr(1)/dt dr(2)/dt dv(1)/dt dv(2)/dt]
    derivs = np.array([v[0], v[1], accel[0], accel[1]])
    return derivs


# orbit - Program to compute the orbit of a comet.
#clear all;  help orbit;  % Clear memory and print header

def orbit(input_dict = {},calc_info = False, plot_momentum = False,
            inter_input = False, plot_traj = True, plot_energy = False):
    if inter_input:
        # Set initial position and velocity of the comet.
        r0 = float(input("Enter initial radial distance (AU): "))
        # v0 = float(input("Enter initial tangential velocity (AU/yr): "))
        vstr = str(input("Enter initial tangential velocity (AU/yr) as a number␣
 ↪or multiple of Pi (e.g.,2*pi): "))

        # modify input to allow 'pi'
        vinp = vstr.split('*')
        if (vinp[-1].lower() == 'pi'):
          v0 = float(vinp[0])*np.pi
        else:
          v0 = float(vinp[0])

        nStep = int(input("Enter number of steps: "))
        tau =  float(input("Enter time step (yr): "))
        NumericalMethod=0
        while(NumericalMethod not in np.array([1,2,3,4,5,6,7])):
            NumericalMethod = int(input("Choose a number for a numerical method:
 ↪\n\
         1-Euler, 2-Euler-Cromer, 3-Runge-Kutta 4-Adaptive R-K: "))
    elif input_dict:
        r0 = input_dict['r0']
        v0 = input_dict['v0']
        nStep = input_dict['nStep']
        tau = input_dict['tau']
        NumericalMethod = input_dict['NumericalMethod']
    else:
        r0 = 1
        v0 = 2*np.pi
        nStep = 1000
        tau = .01
        NumericalMethod = 3
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r = np.array([r0, 0.])
v = np.array([0., v0])
state = np.array([ r[0], r[1], v[0], v[1] ])   # Used by R-K routines

#Set physical parameters (mass, G*M)
GM = 4*np.pi**2      # Grav. const. * Mass of Sun (au^3/yr^2)
mass = 1.        # Mass of comet
adaptErr = 1.e-4 # Error parameter used by adaptive Runge-Kutta
time = 0.0

#%* Loop over desired number of steps using specified
#%  numerical method.
for istep in range(0,nStep):

  #%* Record position and energy for plotting.
  # Initially set the arrays for the first step
  if istep == 0:
      rplot = np.linalg.norm(r)
      thplot = np.arctan2(r[1],r[0])
      tplot = time
      kinetic = .5*mass*np.linalg.norm(v)**2
      potential= - GM*mass/np.linalg.norm(r)
      momentum = [np.linalg.norm(np.cross(r,mass*v))]
      perihelion = []
      aphelion = []
  else:
      rplot = np.append(rplot,np.linalg.norm(r))           #Record position for polar plot
      thplot = np.append(thplot,np.arctan2(r[1],r[0]))
      tplot = np.append(tplot,time)
      kinetic = np.append(kinetic,0.5*mass*np.linalg.norm(v)**2)   # Record energies
      potential= np.append(potential,- GM*mass/np.linalg.norm(r))
      momentum.append(np.linalg.norm(np.cross(r, mass*v)))

  #%* Calculate new position and velocity using desired method.
  if NumericalMethod == 1 :
    accel = -GM*r/np.linalg.norm(r)**3
    r = r + tau*v             # Euler step
    v = v + tau*accel
    time = time + tau
  elif NumericalMethod == 2:
    accel = -GM*r/np.linalg.norm(r)**3
    v = v + tau*accel
    r = r + tau*v             # Euler-Cromer step
    time = time + tau
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      elif NumericalMethod == 3:
        state = rk4(state,time,tau,gravrk_ex14)
        r = np.array([state[0], state[1]])   # 4th order Runge-Kutta
        v = np.array([state[2], state[3]])
        time = time + tau
      else:
        [state, time, tau] = rka(state,time,tau,adaptErr,gravrk_ex14)
        r = np.array([state[0], state[1]])   # Adaptive Runge-Kutta
        v = np.array([state[2], state[3]])
      # Find perihelion, aphelion
      if istep >= 2:
          if rplot[istep-1] > rplot[istep-2] and rplot[istep-1] > rplot[istep]:
              #if rplot[istep-1] >= min(rplot)-tau and rplot[istep-1] <= min(rplot)+tau:
              #    perihelion.append( (rplot[istep-1],tplot[istep-1]) )
              if rplot[istep-1] >= max(rplot)-tau and rplot[istep-1] <= max(rplot)+tau:
                  aphelion.append( [rplot[istep-1],thplot[istep-1],tplot[istep-1]] )
          elif rplot[istep-1] < rplot[istep-2] and rplot[istep-1] < rplot[istep]:
              perihelion.append( [rplot[istep-1],thplot[istep-1],tplot[istep-1]] )

    # Given alpha=.1, calculate a
    alpha = .1
    a = np.sqrt(1 + ( GM*mass**2*alpha / (momentum[0]**2) ))
    shift_rad = (360*(1-a)/a)*(np.pi/180)

    # Calculate aphelion, perihelion
    aphelion = np.array(aphelion)
    perihelion = np.array(perihelion)
    ap_thetas = aphelion[:,1]
    peri_thetas = perihelion[:,1]

    a1 = ap_thetas[0:-1]
    a2 = ap_thetas[1:]
    adelta_norm = np.array([i+2*np.pi if i<0 else i for i in (a1-a2)])

    p1 = peri_thetas[0:-1]
    p2 = peri_thetas[1:]
    pdelta_norm = np.array([i+2*np.pi if i<0 else i for i in (p1-p2)])
    print("Difference between predicted and average observed orbit precession in aphelion:")
    print(np.mean(shift_rad+adelta_norm))
```

```python
    if plot_traj:
        #%* Graph the trajectory of the comet.
        plt.figure(1); plt.clf()  #Clear figure 1 window and bring forward
        plt.polar(thplot,rplot,'-')  # Use polar plot for graphing orbit
        plt.xlabel('Distance (AU)')
        plt.grid(True)

    if plot_energy:
        #%* Graph the energy of the comet versus time.
        plt.figure(2); plt.clf()   # Clear figure 2 window and bring forward
        totalE = kinetic + potential   # Total energy
        plt.plot(tplot,kinetic,'-.',tplot,potential,'--',tplot,totalE,'-')
        #plt.legend('Kinetic','Potential','Total')
        plt.xlabel('Time (yr)'); plt.ylabel('Energy (M AU^2/yr^2)')
        plt.grid(True)
        plt.show()

    if plot_momentum:
        # Plots angular momentum as a function of time
        plt.figure(3)
        plt.plot(tplot,momentum)



    return rplot, thplot

if __name__ == "__main__":
    # elliptical
    input_dict = {
        'r0': 1,
        'v0': 1*np.pi,
        'nStep': 1000,
        'tau': .01,
        'NumericalMethod': 4
        }



    rplot, thplot = orbit(input_dict,plot_energy = True)
```

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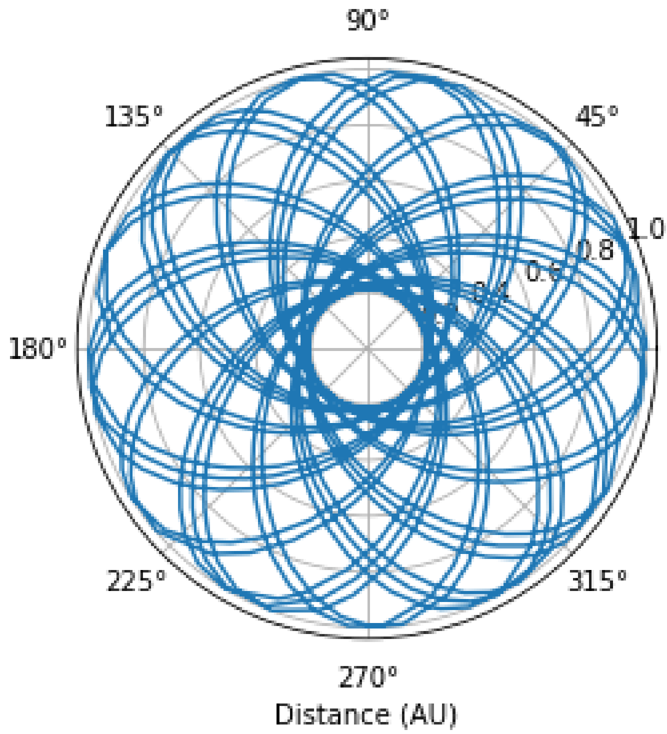
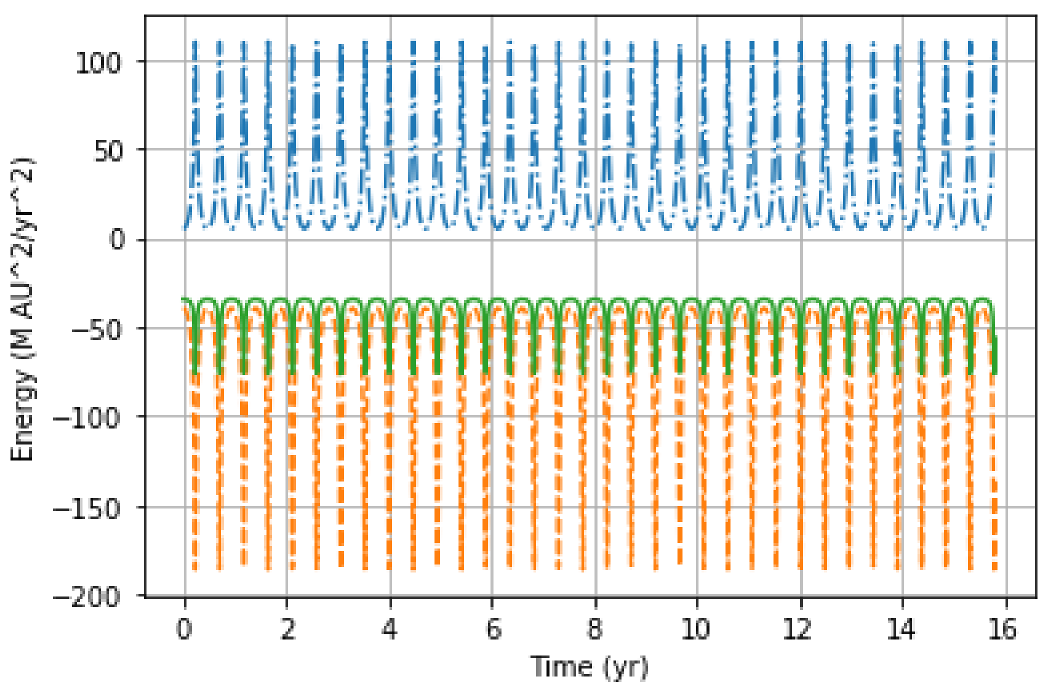
Difference between predicted and average observed orbit precession in aphelion:
-0.002481749038696069
```


90°
135°
45°
1.0
0.8
0.6
0.4
180°
0°
225°
315°
270°
Distance (AU)


Energy (M AU^2/yr^2)
100
50
0
−50
−100
−150
−200
0
2
4
6
8
10
12
14
16
Time (yr)

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