Lagranges Equations

This brief note provides a prescription of taking the Lagrangian and determining the equations of motion. The actual derivation of Lagranges equations can be found in Classical mechanics textbooks. Here I will simply outline the procedure of starting with a Lagrangian L and getting Newton's laws of motion. For example the Lagrangian for the double pendulum is

$$L=\frac{m_1+m_2}{2}L_1^2\dot{\theta}_1^2+\frac{m_2}{2}L_2^2\dot{\theta}_2^2+m_2gL_2\cos\theta_2+m_2L_1L_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)+(m_1+m_2)gL_1\cos\theta_1$$

which is a function of 4 variables ($(\theta_1,\theta_{2,}\dot{\theta}_1,\dot{\theta}_2)$ as well as time. To get Newton's laws you perform the following operation.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_1}\right)=\frac{\partial L}{\partial\theta_1} \tag{1}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_2}\right)=\frac{\partial L}{\partial\theta_2} \tag{2}$$

For example, the of the first equation above

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_1}\right)=\frac{\partial L}{\partial\theta_1}$$

The partial derivative on LHS of (1) is

$$\frac{\partial L}{\partial\dot{\theta}_1}=(m_1+m_2)L_1^2\dot{\theta}_1+m_2L_1L_2\dot{\theta}_2\cos(\theta_1-\theta_2)$$

taking the full derivative, using the chain rule gives

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_1}\right)=(m_1+m_2)L_1^2\ddot{\theta}_1+m_2L_1L_2\ddot{\theta}_2\cos(\theta_1-\theta_2)-m_2L_1L_2\dot{\theta}_2\sin(\theta_1-\theta_2)\left(\dot{\theta}_1-\dot{\theta}_2\right)$$

this gets equated to the RHS of (1) and will yield Newton's law of motion. A second equation will come from equation (2). What you will get is 2 equations for the 2 unknowns $(\ddot{\theta}_1,\ddot{\theta}_2)$ that need to be solved.