hw3_ex2

March 6, 2020

[1]:
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# python 3 version
import numpy as np
import matplotlib.pyplot as plt
import mpl_toolkits.mplot3d.axes3d as p3

def rk4(x,t,tau,derivsRK,param):
    """
    ##  Runge-Kutta integrator (4th order)
    ## Input arguments -
    ##   x = current value of dependent variable
    ##   t = independent variable (usually time)
    ##   tau = step size (usually timestep)
    ##   derivsRK = right hand side of the ODE; derivsRK is the
    ##             name of the function which returns dx/dt
    ##             Calling format derivsRK(x,t).
    ## Output arguments -
    ##   xout = new value of x after a step of size tau
    """
    half_tau = 0.5*tau
    F1 = derivsRK(x,t,param)
    t_half = t + half_tau
    xtemp = x + half_tau*F1
    F2 = derivsRK(xtemp,t_half,param)
    xtemp = x + half_tau*F2
    F3 = derivsRK(xtemp,t_half,param)
    t_full = t + tau
    xtemp = x + tau*F3
    F4 = derivsRK(xtemp,t_full,param)
    xout = x + tau/6.*(F1 + F4 + 2.*(F2+F3))
    return xout


def rka(x,t,tau,err,derivsRK,param):
    """
    ## Adaptive Runge-Kutta routine
    ## Inputs
    ##   x          Current value of the dependent variable
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##   t          Independent variable (usually time)
##   tau        Step size (usually time step)
##   err        Desired fractional local truncation error
##   derivsRK   Right hand side of the ODE; derivsRK is the
##              name of the function which returns dx/dt
##              Calling format derivsRK(x,t).
## Outputs
##   xSmall     New value of the dependent variable
##   t          New value of the independent variable
##   tau        Suggested step size for next call to rka
"""
# Set initial variables
tSave = t;  xSave = x    # Save initial values
safe1 = .9;  safe2 = 4.  # Safety factors
eps = np.spacing(1) # smallest value

# Loop over maximum number of attempts to satisfy error bound
maxTry = 100

for iTry in range(1,maxTry):

    # Take the two small time steps
    half_tau = 0.5 * tau
    xTemp = rk4(xSave,tSave,half_tau,derivsRK,param)
    t = tSave + half_tau
    xSmall = rk4(xTemp,t,half_tau,derivsRK,param)

    # Take the single big time step
    t = tSave + tau
    xBig = rk4(xSave,tSave,tau,derivsRK,param)

    # Compute the estimated truncation error
    scale = err * (np.abs(xSmall) + np.abs(xBig))/2.
    xDiff = xSmall - xBig
    errorRatio = np.max( [np.abs(xDiff)/(scale + eps)] )

    #print safe1,tau,errorRatio

    # Estimate news tau value (including safety factors)
    tau_old = tau

    tau = safe1*tau_old*errorRatio**(-0.20)
    tau = np.max([tau,tau_old/safe2])
    tau = np.min([tau,safe2*tau_old])

    # If error is acceptable, return computed values
    if errorRatio < 1 :
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            xSmall = xSmall
            return xSmall, t, tau
    # Issue error message if error bound never satisfied
    print ('ERROR: Adaptive Runge-Kutta routine failed')
    return



def lorzrk(s,t,param):
    """
    #  Returns right-hand side of Lorenz model ODEs
    #  Inputs
    #    s      State vector [x y z]
    #    t      Time (not used)
    #    param  Parameters [r sigma b]
    #  Output
    #    deriv  Derivatives [dx/dt dy/dt dz/dt]
    """
    r = param[0]
    sigma = param[1]
    b = param[2]
    #* For clarity, unravel input vectors
    x = s[0]; y = s[1]; z = s[2]
    #* Return the derivatives [dx/dt dy/dt dz/dt]
    deriv = np.zeros(3)
    deriv[0] = sigma*(y-x)
    deriv[1] = r*x - y - x*z
    deriv[2] = x*y - b*z
    return deriv



def lorenz_data_gen(init_x,init_y,init_z,init_r):
    """
    Generates data needed to plot the results
    of lorentz.py using rk4 as in Ch3 ex 25

    Parameters
    ----------
    init_x : Float
        Inital x value.
    init_y : Float
        Initial y value.
    init_z : Float
        Inital z value.
    r : float
        Lorenz model parameter.
```

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    Returns
    -------
    xplot : Numpy array
        Array of x-values used to plot.
    yplot : Numpy array
        Array of y-values used to plot.
    zplot : Numpy array
        Array of z-values used to plot.
    tplot : Numpy array
        Array of time-values used to plot.

    """
    # Set initial state x,y,z and parameters r,sigma,b
    sxin,syin,szin = init_x,init_y,init_z
    state = np.zeros(3)
    state[0] = float(sxin); state[1]  = float(syin); state[2]  = float(szin)


    r = init_r
    sigma = 10   # Parameter sigma
    b = 8./3.     # Parameter b
    param = np.array([r, sigma, b])  # Vector of parameters passed to rka
    tau = .02       # Timestep from lorenz with n=500
    #err = 1.e-3   # Error tolerance

    # Loop over the desired number of steps
    time = 0
    nstep = 500
    # initialize arrays
    tplot=np.array([]); tauplot=np.array([])
    xplot=np.array([]); yplot=np.array([]); zplot=np.array([])

    for istep in range(0,nstep):
        # Record values for plotting
        x = state[0]
        y = state[1]
        z = state[2]
        tplot = np.append(tplot,time)
        tauplot = np.append(tauplot,tau)
        xplot = np.append(xplot,x)
        yplot = np.append(yplot,y)
        zplot = np.append(zplot,z)
        #if( istep%50 ==0 ):
          #print('Finished %d steps out of %d '%(istep,nstep))
        # Find new state using Runge-Kutta4
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        state = rk4(state,time,tau,lorzrk,param)
        time += tau
    """
    # Graph the time series x(t)
    plt.figure(1)
    plt.clf()  # Clear figure 1 window and bring forward
    plt.plot(tplot,xplot,'-')
    plt.xlabel('Time');  plt.ylabel('x(t)')
    plt.title('Lorenz model time series')
    # plt.show()

    # Graph the x,y,z phase space trajectory

    """
    fig=plt.figure(2)
    ax=p3.Axes3D(fig)

    ax.plot3D(xplot,yplot,zplot)
    if init_y == 0:
        print("1")
        ax.scatter3D(0,0,0,color='red')
    elif init_y == np.sqrt(b*(r-1)):
        print('2')
        ax.scatter3D(np.sqrt(b*(r-1)),np.sqrt(b*(r-1)),r-1,color='red')
    ax.set_xlabel('x')
    ax.set_ylabel('y')
    ax.set_zlabel('z')
    ax.grid(True)
    # title('Lorenz model phase space')
    #ax.set_aspect('equal')
    plt.show()
    #"""
    return xplot,yplot,zplot,tplot
```
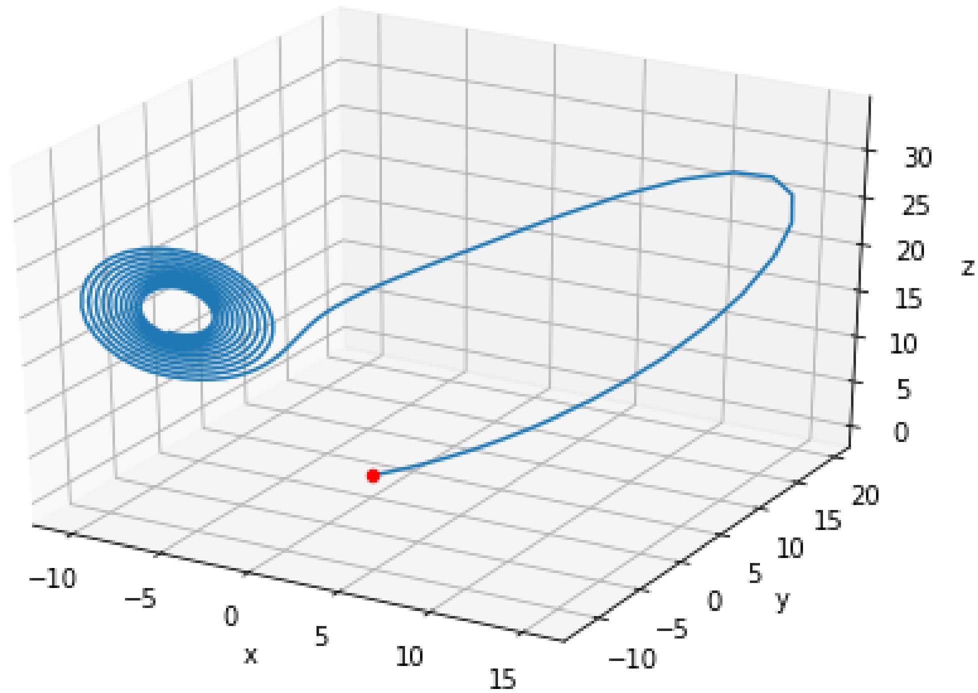

[2]:

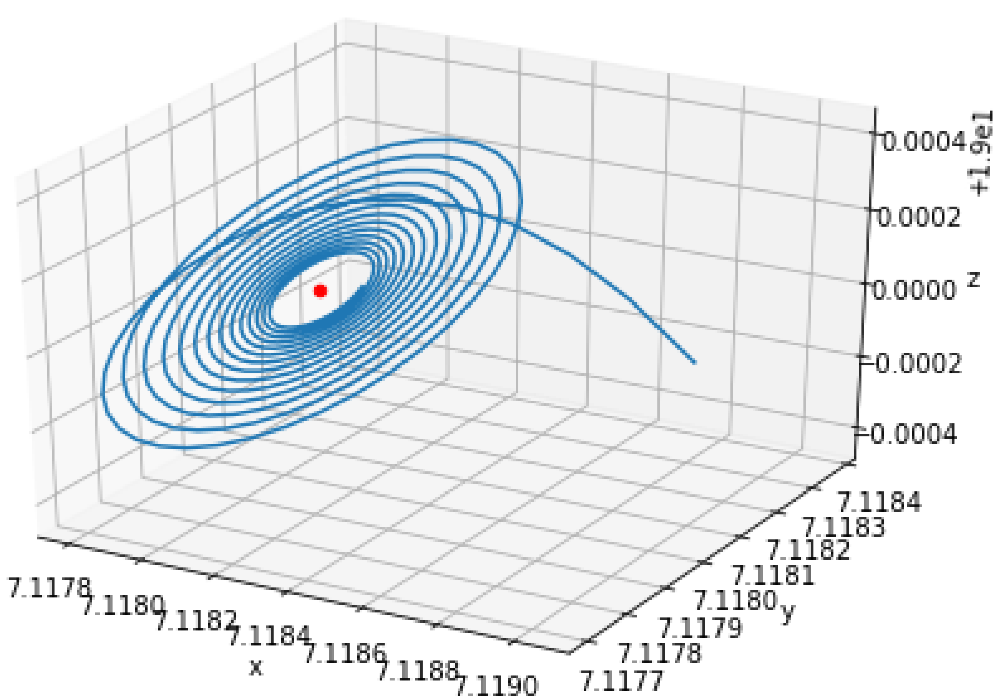
```
init_x,init_y,init_z,init_r = 0.001,0,0,20
xplot,yplot,zplot,tplot = lorenz_data_gen(init_x,init_y,init_z,init_r)

r = 20
sigma = 10
b = 8/3

init_x,init_y,init_z,init_r = np.sqrt(b*(r-1))+.001,np.sqrt(b*(r-1)),r-1,20
xplot,yplot,zplot,tplot = lorenz_data_gen(init_x,init_y,init_z,init_r)
```

1



30
25
20
15
10
5
0
z
−10
−5
0
5
10
15
x
20
15
10
5
0
−5
−10
y

2



0.0004
0.0002
0.0000
−0.0002
−0.0004
+1.9e1
z
7.1178
7.1180
7.1182
7.1184
7.1186
7.1188
7.1190
x
7.1184
7.1183
7.1182
7.1181
7.1180
7.1179
7.1178
7.1177
y

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