22a)
$$a\left(1-\frac{r}{b}\right)r = crf$$
$$af = crf$$
$$\Rightarrow a\left(1-\frac{r}{b}\right)r = af$$
$$\Rightarrow f = \left(1-\frac{r}{b}\right)r$$
$$\Rightarrow a\left(1-\frac{r}{b}\right)r = car^2\left(1-\frac{r}{b}\right)$$
$$\Rightarrow a = cr \Leftrightarrow r = \frac{a}{c} = \frac{10}{.1} = 100$$
$$f = \left(1-\frac{a}{cb}\right)\left(\frac{a}{c}\right) = 10^2\left(1-\frac{1}{10^4}\right) = 100 - 10^{-2} \approx 100$$

So, 100 rabbits and 100 foxes is the steady-state

22b) when the program is started near steady-state it is highly erratic and noisy.

25) The distance increases with time at an exponential rate.

2)
$$\frac{dx}{dt} = \sigma(y-x) = 0 \qquad \Rightarrow \text{Clearly } x^* = y^* = z^* = 0 \text{ is a soln.}$$
$$\frac{dy}{dt} = rx - y - xz = 0 \qquad \Rightarrow \text{IF } \sigma \neq 0 \Rightarrow x = y$$
$$\frac{dz}{dt} = xy - bz = 0 \qquad \Rightarrow x^2 = bz \Rightarrow x = \sqrt{bz} \Rightarrow r\sqrt{bz} - \sqrt{bz} - z\sqrt{bz} = 0$$
$$\Rightarrow r - 1 = z \Rightarrow x = y = \pm\sqrt{b(r-1)}$$

IF $\sigma = 0$, we loose an equation, system becomes underdetermined, parameterize with x

$$y = rx - xz \Rightarrow x(rx - xz) - bz = 0 \Rightarrow rx^2 - z(x^2 - b) = 0$$
$$\Rightarrow \frac{rx^2}{x^2-b} = z \Rightarrow y = rx - \frac{rx^3}{x^2-b} \quad \square$$

19a) ① $\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_1}\right) = \frac{dL}{d\theta_1}$

① $\Rightarrow$ $LHS = (m_1+m_2)L_1^2\ddot{\theta}_1 + m_2L_1L_2\ddot{\theta}_2\cos(\theta_1-\theta_2) - m_2L_1L_2\dot{\theta}_2\sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2)$

$RHS = -m_2L_1L_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2) - (m_1+m_2)gL_1\sin\theta_1$

② $\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_2}\right) = \frac{dL}{d\theta_2}$

② $\Rightarrow$ $LHS = m_2L_2^2\ddot{\theta}_2 + m_2L_1L_2\ddot{\theta}_1\cos(\theta_1-\theta_2) - m_2L_1L_2\dot{\theta}_1\sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2)$

$RHS = -m_2gL_2\sin\theta_2 + m_2L_1L_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)$

① $\Rightarrow$ $(m_1+m_2)L_1\ddot{\theta}_1 + m_2L_2\ddot{\theta}_2\cos(\theta_1-\theta_2) - m_2L_2\dot{\theta}_2\sin(\theta_1-\theta_2)(\cancel{\dot{\theta}_1}-\dot{\theta}_2)$

$= \cancel{-m_2L_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)} - (m_1+m_2)g\sin\theta_1$

② $\Rightarrow$ $L_2\ddot{\theta}_2 + L_1\ddot{\theta}_1\cos(\theta_1-\theta_2) - L_1(\dot{\theta}_1 - \cancel{\dot{\theta}_2})\dot{\theta}_1\sin(\theta_1-\theta_2)$

$= -g\sin\theta_2 + \cancel{L_1\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)}$

① $\Rightarrow$ $(m_1+m_2)L_1\ddot{\theta}_1 + m_2L_2\ddot{\theta}_2\cos(\theta_1-\theta_2) + m_2L_2\dot{\theta}_2^2\sin(\theta_1-\theta_2)$

$= -(m_1+m_2)g\sin\theta_1$

② $\Rightarrow$ $L_2\ddot{\theta}_2 + L_1\ddot{\theta}_1\cos(\theta_1-\theta_2) - L_1\dot{\theta}_1^2\sin(\theta_1-\theta_2) = -g\sin\theta_2$

Let $\alpha_1(\theta_1,\theta_2) = \frac{L_2}{L_1}\left(\frac{m_2}{m_1+m_2}\right)\cos(\theta_1-\theta_2)$

$\alpha_2(\theta_1,\theta_2) = \frac{L_1}{L_2}\cos(\theta_1-\theta_2)$

$F_1(\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2) = -\frac{L_2}{L_1}\left(\frac{m_2}{m_1+m_2}\right)\dot{\theta}_2^2\sin(\theta_1-\theta_2) - \frac{g}{L_1}\sin\theta_1$

$F_2(\qquad) = \frac{L_1}{L_2}\dot{\theta}_1^2\sin(\theta_1-\theta_2) - \frac{g}{L_2}\sin\theta_2$

$\Rightarrow$ $\ddot{\theta}_1 + \alpha_1\ddot{\theta}_2 = F_1$

$\ddot{\theta}_2 + \alpha_2\ddot{\theta}_1 = F_2$, Let $A = \begin{bmatrix} 1 & \alpha_1 \\ \alpha_2 & 1 \end{bmatrix}$

Note: $\det(A) \neq 0$ since $1-\alpha_1\alpha_2 > 0$

$\Rightarrow \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \frac{1}{1-\alpha_1\alpha_2}\begin{bmatrix} F_1 - \alpha_1F_2 \\ -\alpha_2F_1 + F_2 \end{bmatrix} \Rightarrow \frac{d}{dt}\begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \\ \\ \end{bmatrix}$

∎