IC252 Lab 4

1. **Independence and conditional probability.** Let $X1$ and $X2$ be independent random variables, taking values from the set $\{0, 1\}$, such that both outcomes are equally likely. Define $Z = X1 + X2$. Note that Z can take values $\{0, 1, 2\}$. Do the following.

   - Demonstrate by counting, that the $X1$ and $X2$ are independent. Generate $X1$ and $X2$ a large number of times (call this N). Count the number of times $X1$ takes the value 1, $X2$ takes the value 1, and the number of times both $X1$ and $X2$ take the value 1. This should be approximately equal. Compute the probability by hand and compare with the result of the simulation. $X1$ and $X2$ can be stored in two arrays.
   - Now generate Z, using the already computed values of $X1$ and $X2$. Is Z independent of $X1$? Determine using counting, and by hand.
   - Now condition $X1$ and $X2$ on $Z = 1$. Is $X1$ conditioned on Z, independent of $X2$ conditioned on Z? In other words, is $P(X1 = 1, X2 = 1|Z = 1) = P(X1 = 1|Z = 1)P(X2 = 1|Z = 1)$? Demonstrate by counting and calculate by hand.

2. Simulate the number of heads obtained in N independent throws of a coin with $p(H) = p$. Accept N and p form the user. Run the experiment 10,000 times and plot the histograms of the number of heads obtained, for various values of N and p. Each N, p will need a separate histogram.

3. Count the number of times a message needs to be transmitted until it reaches correctly at the destination. The probability for a successful transmission is p. Repeat the experiment 10,000 times and plot the histograms of the count for various values of p. Each p will need a separate histogram.
   Hint: Generate a random bit b with $P(b = 1) = p$. Keep adding to the count until you generate a 1. A successful transmission at the 4th try (ie. count = 4) corresponds to the binary string 0001.