Ozyegin University CS 321 Programming Languages Sample Problems on Lambda Calculus

1. Using the encodings below, show that **mult 3 2** is **6**.

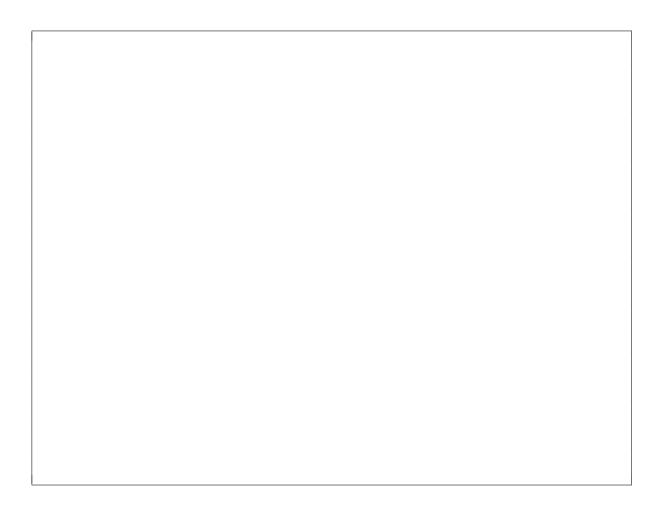
$$\mathbf{0} = (\lambda f.\lambda x.x)$$

$$\mathbf{1} = (\lambda f.\lambda x.fx)$$

$$\mathbf{2} = (\lambda f.\lambda x.f(fx))$$

$$\mathbf{3} = (\lambda f.\lambda x.f(f(fx)))$$

$$\mathbf{mult} = \lambda m.\lambda n.\lambda f.\lambda x.m(nf)x$$



2. Reduce the following lambda expression to its normal form.

$$(\lambda f.\lambda m.\lambda p.m \, (f\, m\, p)) \, (\lambda y.\lambda z.y\, z) \, (\lambda w.w)$$

3. The following are the Church encodings for boolean values and conditional expression. Write down an encoding for logical *and*.

$$\mathbf{true} = \lambda a.\lambda b. \ a$$

$$\mathbf{false} = \lambda a.\lambda b. \ b$$

$$\mathbf{if} = \lambda c.\lambda t.\lambda e. \ c \ t \ e$$

$$\mathbf{and} = \underline{\hspace{1cm}}$$

4. Reduce the following lambda term to its normal form. If a normal form does not exist, show that the term does not converge.

$$(\lambda x. \ \lambda y. \ x \ (y \ x)) \ (\lambda w. \ \lambda v. \ w) \ (\lambda z. \ z)$$

5. Apply **one** step of beta reduction on the following lambda terms, if possible. If there is no beta reduction possibility, write NORMAL FORM.

•
$$(\lambda x \cdot x y z) a b c \Rightarrow$$

• $(\lambda w . \lambda p . p w) \Rightarrow$

 $\bullet \ (\lambda x . \lambda y . x (y x)) (\lambda w . \lambda v . w) (\lambda z . z) \Rightarrow \underline{\hspace{1cm}}$

• $(\lambda z . z (\lambda k . k) m) \Rightarrow$

• $(\lambda u . (\lambda x . x u) (\lambda y . u)) \Rightarrow$

- $\bullet \ (\lambda n \, . \, n \, n) \, (\lambda n \, . \, n \, n) \ \Rightarrow \ \underline{\hspace{1cm}}$
- $\bullet \ (\lambda g \, . \, (\lambda h \, . \, h) \, g) \ \Rightarrow \ \underline{\hspace{1cm}}$