Ozyegin University
CS 321 Programming Languages
Sample Problems on Lambda Calculus

1. Apply **one** step of beta reduction on the following lambda terms, if possible. If there is no beta reduction possibility, write NORMAL FORM.

   - $(\lambda x\,.\,x\,y\,z)\,a\,b\,c \;\Rightarrow$ ________________________
   - $(\lambda w\,.\,\lambda p\,.\,p\,w) \;\Rightarrow$ ________________________
   - $(\lambda x\,.\,\lambda y\,.\,x\,(y\,x))\,(\lambda w\,.\,\lambda v\,.\,w)\,(\lambda z\,.\,z) \;\Rightarrow$ ____
   - $(\lambda z\,.\,z\,(\lambda k\,.\,k)\,m) \;\Rightarrow$ ________________________
   - $(\lambda u\,.\,(\lambda x\,.\,x\,u)\,(\lambda y\,.\,u)) \;\Rightarrow$ ________________________
   - $(\lambda n\,.\,n\,n)\,(\lambda n\,.\,n\,n) \;\Rightarrow$ ________________________
   - $(\lambda g\,.\,(\lambda h\,.\,h)\,g) \;\Rightarrow$ ________________________

2. Reduce the following lambda term to its normal form. If a normal form does not exist, show that the term does not converge.

$$(\lambda x.\ \lambda y.\ x\ (y\ x))\ (\lambda w.\ \lambda v.\ w)\ (\lambda z.\ z)$$

3. Reduce the following lambda expression to its normal form.

$$(\lambda f.\lambda m.\lambda p.m\,(f\,m\,p))\;(\lambda y.\lambda z.y\,z)\;(\lambda w.w)$$

4. The following are the Church encodings for boolean values and conditional expression. Write down an encoding for logical *and*.

$$\begin{aligned}
\textbf{true} &= \lambda a.\lambda b.\ a \\
\textbf{false} &= \lambda a.\lambda b.\ b \\
\textbf{if} &= \lambda c.\lambda t.\lambda e.\ c\ t\ e \\
\textbf{and} &= \underline{\hspace{6cm}}
\end{aligned}$$

5. Using the encodings below, show that **mult 3 2** is **6**.

$$
\begin{aligned}
\mathbf{0} &= (\lambda f.\lambda x.x) \\
\mathbf{1} &= (\lambda f.\lambda x.fx) \\
\mathbf{2} &= (\lambda f.\lambda x.f(fx)) \\
\mathbf{3} &= (\lambda f.\lambda x.f(f(fx))) \\
\mathbf{mult} &= \lambda m.\lambda n.\lambda f.\lambda x.m(nf)x
\end{aligned}
$$