Ozyegin University
CS 321 Programming Languages
Sample Problems on Interpretation

1. (From PLC, Exercise 1.1) Given the definition of the simple ArithLang below, extend this language with conditional expressions (i.e. "if") corresponding to Java's expression e_1 `?` e_2 `:` e_3, or OCaml's `if` e_1 `then` e_2 `else` e_3. Evaluation of a conditional expression should evaluate e_1 first. If it yields a non-zero value, evaluate e_2, otherwise evaluate e_3.

```
type exp = CstI of int
         | Var of string
         | Add of exp * exp
         | Mult of exp * exp
         | Subt of exp * exp
         | Div of exp * exp
         | LetIn of string * exp * exp

(* lookup: string -> (string * int) list -> int *)
let rec lookup x env =
  match env with
  | [] -> failwith ("Unbound name " ^ x)
  | (y,i)::rest -> if x = y then i
                   else lookup x rest

(* eval: exp -> (string * int) list -> int *)
let rec eval e env =
  match e with
  | CstI i -> i
  | Var x -> lookup x env
  | Add(e1, e2)  -> eval e1 env + eval e2 env
  | Mult(e1, e2) -> eval e1 env * eval e2 env
  | Subt(e1, e2) -> eval e1 env - eval e2 env
  | Div(e1, e2)  -> eval e1 env / eval e2 env
  | LetIn(x, e1, e2) -> let v = eval e1 env
                        in let env' = (x, v)::env
                           in eval e2 env'
```

Solution: Here is the diff:

```
diff --git a/sampleProblems/interpretation/arith.ml b/sampleProblems/interpretation/arith.ml
index 17db0c0..bbc4556 100644
--- a/sampleProblems/interpretation/arith.ml
+++ b/sampleProblems/interpretation/arith.ml
@@ -5,6 +5,7 @@ type exp = CstI of int
           | Subt of exp * exp
           | Div of exp * exp
           | LetIn of string * exp * exp
+          | If of exp * exp * exp

 (* lookup: string -> (string * int) list -> int *)
 let rec lookup x env =
@@ -25,3 +26,6 @@ let rec eval e env =
    | LetIn(x, e1, e2) -> let v = eval e1 env
                          in let env' = (x, v)::env
                             in eval e2 env'
+   | If(e1, e2, e3) -> if (eval e1 env) = 0
+                       then eval e3 env
+                       else eval e2 env
```

2. (From PLC, Exercise 1.1) Extend ArithLang to handle three additional operators: "max", "min", and "=". Like the existing binary operators, they take two argument expressions. The equals operator should return 1 when true and 0 when false.

Solution: Here is the diff:

```
diff --git a/sampleProblems/interpretation/arith.ml b/sampleProblems/interpretation/arith.ml
index 17db0c0..921d4de 100644
--- a/sampleProblems/interpretation/arith.ml
+++ b/sampleProblems/interpretation/arith.ml
@@ -4,6 +4,9 @@ type exp = CstI of int
          | Mult of exp * exp
          | Subt of exp * exp
          | Div of exp * exp
+         | Min of exp * exp
+         | Max of exp * exp
+         | Eq of exp * exp
          | LetIn of string * exp * exp

 (* lookup: string -> (string * int) list -> int *)
@@ -22,6 +25,15 @@ let rec eval e env =
   | Mult(e1, e2) -> eval e1 env * eval e2 env
   | Subt(e1, e2) -> eval e1 env - eval e2 env
   | Div(e1, e2)  -> eval e1 env / eval e2 env
+  | Min(e1, e2)  -> let v1 = eval e1 env
+                    in let v2 = eval e2 env
+                       in if v1 < v2 then v1 else v2
+  | Max(e1, e2)  -> let v1 = eval e1 env
+                    in let v2 = eval e2 env
+                       in if v1 > v2 then v1 else v2
+  | Eq(e1, e2)   -> let v1 = eval e1 env
+                    in let v2 = eval e2 env
+                       in if v1 = v2 then 1 else 0
   | LetIn(x, e1, e2) -> let v = eval e1 env
                         in let env' = (x, v)::env
                            in eval e2 env'
```

3. Write the representation of the following ArithLang expressions using the `exp` data type.

(a) `v * 5 - k + 6`

Solution: `Add(Subt(Mult(Var "v", CstI 5), Var "k"), CstI 6)`

(b) `x + y + z + p`

Solution: `Add(Add(Add(Var "x", Var "y"), Var "z"), Var "p")`

(c) `5 - (y - 3) * (g + 1)`

Solution: `Subt(CstI 5, Mult(Subt(Var "y", CstI 3), Add(Var "g", CstI 1)))`

(d)
```
let x =
  let a = 5
  in let b = 8
     in a + b
in x * (let y = x + 2 in y)
```

Solution:
```
LetIn("x",
      LetIn("a", CstI 5,
            LetIn("b", CstI 8,
                  Add(Var "a", Var "b"))),
      Mult(Var "x",
           LetIn("y", Add(Var "x", CstI 2),
                 Var "y")))
```

4. Write an OCaml function named `simplify` that takes an `exp` and returns its simplified form based on the rules below:

$$
\begin{array}{c}
\hline
0 + e \rightarrow e \\
e + 0 \rightarrow e \\
e - 0 \rightarrow e \\
1 \times e \rightarrow e \\
e \times 1 \rightarrow e \\
0 \times e \rightarrow 0 \\
e \times 0 \rightarrow 0 \\
e - e \rightarrow 0 \\
\hline
\end{array}
$$

Remark: This problem is harder than it seems, because simplification of expressions may enable other simplifications, and I want to you to handle those cases, too. See the test cases.

```
# simplify (Mult(CstI 1,
                Mult(Add(Add(CstI 1,
                             Subt(Var "x", Var "x")),
                         Add(CstI 4, CstI 6)),
                     CstI 1)));;
- : exp = Add(CstI 1, Add(CstI 4, CstI 6))

# simplify (Subt(CstI 0, Mult(Add(Var "x", CstI 0), CstI 0)));;
- : exp = CstI 0

# simplify (LetIn("a", CstI 4,
                 Subt(CstI 0,
                      Mult(Add(Var "x", CstI 0),
                           CstI 0))));;
- : exp = LetIn("a", CstI 4, CstI 0)

# simplify (Subt(Add(CstI 7, CstI 0),
                Mult(Add(Var "x", CstI 0), CstI 0)));;
- : exp = CstI 7

# simplify (Div(Subt(CstI 0,
                     Mult(Add(Var "x", CstI 0), CstI 0)),
```

```
                    CstI 7));;
- : exp = Div(CstI 0, CstI 7)
```

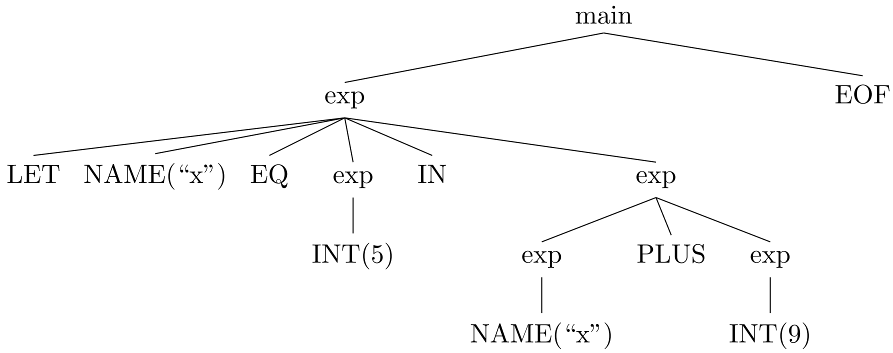
Solution:

```
let rec simplify e =
  match e with
  | CstI i -> e
  | Var x -> e
  | Add(e1, e2) ->
     (match (simplify e1, simplify e2) with
      | CstI 0, e2' -> e2'
      | e1', CstI 0 -> e1'
      | e1', e2'    -> Add(e1', e2'))
  | Subt(e1, e2) ->
     (match (simplify e1, simplify e2) with
      | e1', CstI 0 -> e1'
      | e1', e2'    -> if e1' = e2' then CstI 0
                       else Subt(e1', e2'))
  | Mult(e1, e2) ->
     (match (simplify e1, simplify e2) with
      | CstI 1, e2' -> e2'
      | e1', CstI 1 -> e1'
      | CstI 0, e2' -> CstI 0
      | e1', CstI 0 -> CstI 0
      | e1', e2'    -> Mult(e1', e2'))
  | Div(e1, e2) -> Div(simplify e1, simplify e2)
  | LetIn(x, e1, e2) -> LetIn(x, simplify e1, simplify e2)
```

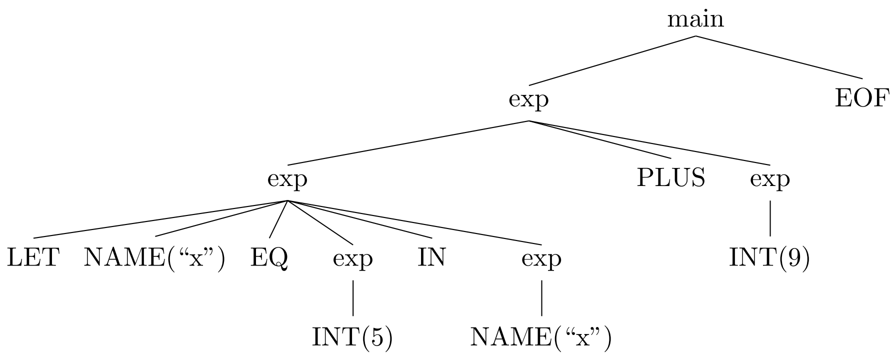
5. Is the grammar shown below ambiguous? If yes, give me an input that at least two different parse trees, and show those trees. If no, prove it.

```
main ::= exp EOF
exp  ::= INT
       | NAME
       | exp PLUS exp
       | exp STAR exp
       | LET NAME EQ exp IN exp
       | IF exp THEN exp ELSE exp
```

Solution: It is ambiguous. Here is an input: `let x = 5 in x + 9`.



And the second tree is

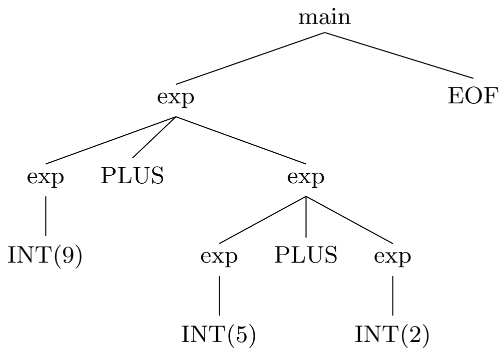


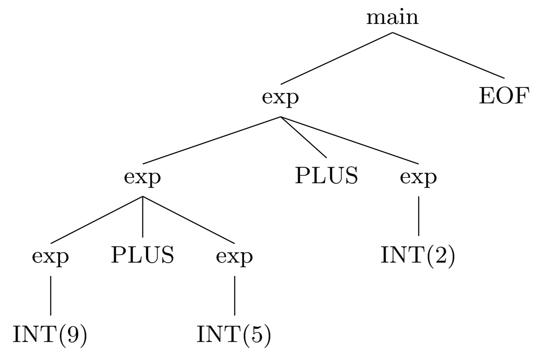
6.
```
main ::= exp EOF
exp  ::= INT
       | NAME
       | exp SLASH exp
       | exp PLUS exp
       | LET NAME EQ exp IN exp
       | IF exp THEN exp ELSE exp
```

Based on the grammar given above, show two different parse trees for the following inputs. For each, also state whether the ambiguity is related to **precedence** or **associativity**.

(a) `9 + 5 + 2`

Solution: This is related to associativity. Does the "+" sign associate to the left or to the right? That's the problem. If "+" associates to the right, we would get the tree on the left; if "+" associates to the left, we would get the tree on the right.





(b) `9 + 5 / 2`

Solution: This is related to precedence. Which operator has higher precedence, "/" or "+"? That is, who wins the fight over the ownership of "5"? That's the problem. If "/" has higher precedence, we would get the tree on the left; if "+" has higher precedence, we would get the tree on the right.