Tasks

- Derive a linearized state space model of a two-dimensional overheard gantry crane.
- Create a function to evaluate the state space model for given parameters.

Framework provided:

myCraneODE.m	Template for submission
SSmodelParams.mat	Physical parameters for the linear model
ODE_derivation_start.mn	MuPad (Matlab Symbolic Toolbox) script
UsefulCode.m	Some simple code that will prove useful now and in the future
Params_Simscape.mat	Physical parameters for nonlinear Simulink model
SimscapeCrane_StepInput.slx	Step response of the Simulink model

Approach

- Read and understand the derivation of the Euler-Lagrange equations for the pendulum on a cart system provided in the MuPad notebook **ODE_derivation_start.mn**
- Use the MuPad notebook (or otherwise) to derive the matrices (A, B, C, D) of the linearized continuous-time state-space representation. In other words, if the nonlinear system is in the form

  $$\frac{\partial}{\partial t}x = \frac{\partial}{\partial t}\begin{bmatrix} X \\ \dot{X} \\ Y \\ \dot{Y} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix} = F(x, u)$$

  then linearize about the equilibrium $(x, u) = (0, 0)$, i.e. where $F(0, 0) = 0$, so that

  $$\frac{\partial}{\partial t}x \approx Ax + Bu.$$

  Here $u := [u_x\ u_y]'$ is the input vector, where u_x and u_y are the signals to the motors for the X and Y directions, respectively.

  We will assume full state measurements with no direct term, i.e. the measurement vector is given by $y = Cx + Du = x$.

  Please try not to confuse the notation for the state vector x and measurements y with the notation for the displacements X and Y in the horizontal X and Y directions. θ and ψ are the angles from the vertical in the X and Y directions, respectively.

  Hint: In order to successfully derive A and B for a linearization about zero, you might find it useful to recall that $\sin(\varepsilon) \approx \varepsilon$ and $\cos(\varepsilon) \approx 1$ for $\varepsilon \approx 0$. You might also want to revise material on Jacobian linearization, which is based on Taylor series expansions.

- Use the template **myCraneODE.m** to write a function that evaluates the matrices of the continuous-time or discrete-time model of a gantry crane for given parameters. See the template for the function protocol and conventions.

- You should also do the following, as it will help with the final assignment:
  - Familiarize yourself with **usefulCode.m**. See if you can plot step responses for your linear gantry crane model.
  - Load the parameters from **Params_StepInput.mat**, set the simulation time `T` and sample period `Ts` and run the Simulink model **SimscapeCrane_StepInput.slx**. Observe the animation in the Mechanics Explorer. Observe the outputs using the Simulink Data Inspector. Familiarize yourself with the Simulink environment.
  - Use **lsim** plot the system behaviour of your linear gantry crane model when the input is $\sin(t)$ or similar.

Submission

filename	Description.
myCraneODE.m	Evaluates continuous-time or discrete-time model matrices for given parameters

Details of submission: Edit or upload the Matlab template for the above file into Cody Coursework. You should have received an email invitation to join the Cody Coursework page; if not, please check that you are registered on Blackboard for the course and email me so that we can enrol you.