The Definition of The Derivative

Given that you are reading this note, I assume you are already familiar with the concepts of limits and functions. In Limits we know that the computation of the slope of a tangent line, the instantaneous rate of change of a function, and the instantaneous velocity of an object at $x = a$ all required us to compute the following limit.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

We also saw that with a small change of notation this limit could also be written as,

$$\lim_{x \to a} \frac{f(a+h) - f(a)}{h} \tag{1}$$

This is such an important limit and it arises in so many places that we give it a name. We call it a derivative. Here is the official definition of the derivative.

Definition of the Derivative

The derivative of $f(x)$ with respect to x is the function $f'(x)$ and is defined as,

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{2}$$

We replaced all the a's in (1) with x's to acknowledge the fact that the derivative is really a function as well.

Practice Problems

1) $f(x) = 2x^2 - 16x + 35$

2) $f(x) = \sqrt{x}$

3) $f(x) = \dfrac{1}{x}$

4) $f(x) = \dfrac{x^2 + 3x}{x + 1}$

5) A car's position along a straight road is given by the function $s(t) = t^2 - 4t$, where s is the position in meters and t is the time in seconds. Use the definition of the derivative to find the car's velocity at $t = 3$ seconds.