Programming assignment no. 2

Write a program (any language) that solves an initial boundary value problem for the two dimensional wave equation

$$\begin{cases} u_{tt} = c^2(u_{xx} + u_{yy}) + S(x,y,t), & 0 < x < a,\ 0 < y < b \\ u(0,y,t) = u(a,y,t) = 0, & 0 < t < \infty \\ u(x,0,t) = u(x,b,t) = 0, & 0 < t < \infty \\ u(x,y,0) = f(x,y), & 0 < x < a,\ 0 < y < b \\ u_t(x,y,0) = g(x,y), & 0 < x < a,\ 0 < y < b \end{cases},$$

using a finite difference method as deduced in class.

We consider a grid with step size $\Delta x = \Delta y$ for the spatial coordinates and Δt for the time component. We define the values of the solution u at the grid points as

$$u(i,j,k) = u(i \cdot \Delta x, j\Delta y, k\Delta t).$$

The boundary conditions imply $u = 0$ on the boundary of the rectangle.
At $t = 0$ the first initial condition gives $u(i,j,k) = f(i,j) =_{notation} f(i\Delta x, j\Delta y)$.
The first time iteration is given by a forward difference formula

$$u(i,j,1) = f(i,j) + g(i,j) \cdot \Delta t.$$

All subsequent iterations will be given as

$$u(i,j,k+1) = r^2[u(i+1,j,k) + u(i,j+1,k) + u(i-1,j,k) + u(i,j-1,k) - 4u(j,k)]$$

$$+2u(i,j,k) - u(i,j,k-1) + S(i,j,k)\Delta t^2,$$

with $r = \frac{c\Delta t}{\Delta x}$ and $i,j,k \geq 1$.

As initial conditions you can consider variations of functions of the type

$$f(x,y) = 3 * (1 - abs(sin(x))) * (1 - abs(sin(2 * y))) + 2 * (1 - abs(sin(5 * x))) * (1 - abs(sin(7 * y)))$$

You can use the scilab framework provided together with exporting data into text files then importing the text files that contain the data and visualizing.