Calculate equivalence classes of list, and return indices grouped by equivalence class

```
In[10]:= EquivalenceIndices[list_, f_] := Module[{matchingPairs, graph, components},
            matchingPairs = Select[Tuples[Range[Length[list]], 2],
              f[list⟦#⟦1⟧⟧, list⟦#⟦2⟧⟧] &];
            graph = Graph[Apply[UndirectedEdge, matchingPairs, {1}]];
            ConnectedComponents[graph]
          ];
```

## Example sequences

```
In[41]:= genSeq = StringJoin[RandomChoice[{"112", "102", "012", "002"}, 8000]];
```

```
In[12]:= seqGen[{a_, b_}] := RandomChoice[{Mod[a + b, 2], Mod[a + b + 1, 2]}]
         seqLen = 2000;
         init = {1, 1};
         genSeq =
           StringJoin[ToString /@ Transpose[NestList[{#⟦2⟧, seqGen[#]} &, init, seqLen]]⟦1⟧];
```

Generate binary sequence from binning the logistic map

```
In[16]:= logistic[x_, r_] := r x (1 - x);
         seqLen = 6000;
         genSeq =
           StringJoin[If[# > 0.5, "1", "0"] & /@ NestList[logistic[#, 3.6] &, 0.6, seqLen]];
```

Generate sequence from Fibonacci sequence modulo n

```
In[∘]:= seqLen = 6000;
         genSeq = StringJoin [ToString /@ (Mod[Fibonacci[#], 4] & /@ Range[seqLen])];
         StringTake[genSeq, 50]

Out[∘]= 11231011231011231011231011231011231011231011231011
```

## Build tree from sequence

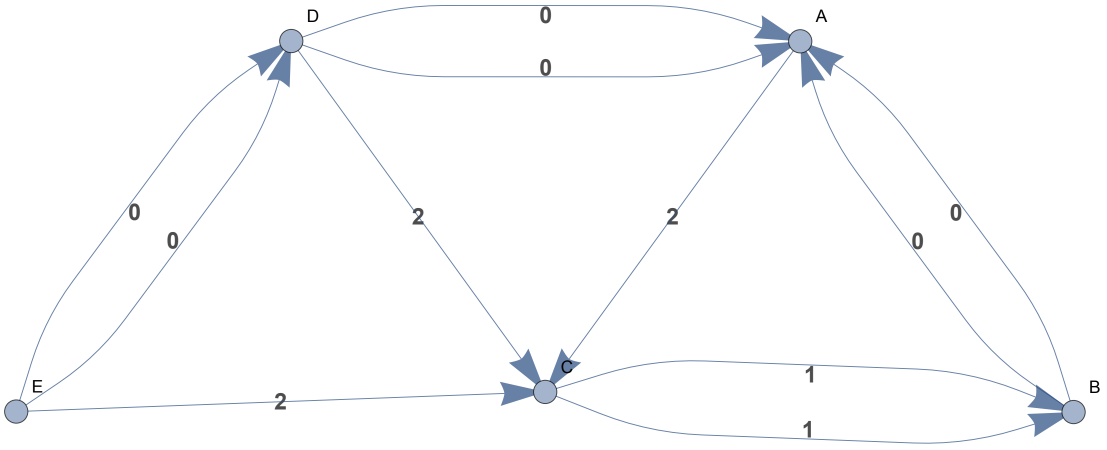
```
In[42]:= (*Build tree from sequence*)
seq = genSeq;
seqLen = StringLength[seq];
L = 8;
subSeqs = Table[StringTake[seq, {i, i + L - 1}], {i, seqLen - L + 1}];
treeData = Function[{subSeq},
     subStrings = StringTake[subSeq, #] & /@ Range[0, L];
     {Take[subStrings, {1, -2}], Take[subStrings, {2, -1}]}
   ] /@ subSeqs;
tree = TreeGraph[
   (#⟦1⟧ → #⟦2⟧) & /@ DeleteDuplicates[Transpose[(Flatten /@ Transpose[treeData])]]
   , VertexLabels → "Name"];
Print[VertexCount[tree]]
```

```
332
```

Build ϵ-machine from tree using depth subL subtrees

```
In[124]:= vertices = VertexList[tree];
subL = 4;
(*To determine the depth of the subtree,
we compare the subtree of depth subL to the subtree of depth subL-
 1. If the tree depth is in fact smaller than subL, the vertex list will be
 identical. VertexOutComponent does not behave correctly for depth zero,
so the conditional returns the root node if the depth is 0*)
{subTreeRoots, subTreeVertices } = Transpose[Select[
     Transpose[
      {vertices, VertexOutComponent[tree, #, subL] & /@ vertices}],
     (#⟦2⟧ ≠ If[subL - 1 == 0, {#⟦1⟧}, VertexOutComponent[tree, #⟦1⟧, subL - 1]]) &]];
(*Remove name of root node from the name of each node,
to compare tree structure*)
treeStructures = Function[{vertexList},
     If[Length[vertexList] < subL + 1, {},
      (Function[{string},
          StringDrop[string, StringLength[vertexList⟦1⟧]]] /@ vertexList)]
   ] /@ subTreeVertices;
(*Comparisons should not depend on order of vertices*)
treeStructures = Sort /@ treeStructures;
equivalenceIndices = EquivalenceIndices[treeStructures, Equal];
classNames = FromCharacterCode[64 + #] & /@ Range[Length[equivalenceIndices]];
edgeList = {};
labelList = {};
Do[
    sourceVertexIndexList = equivalenceIndices⟦sourceIndex⟧;
    Function[{sourceVertexIndex},
      targetVertexIndexList = equivalenceIndices⟦targetIndex⟧;
      Function[{targetVertexIndex},
        isEdge = EdgeQ[tree,
          subTreeRoots⟦sourceVertexIndex⟧ ⟶ subTreeRoots⟦targetVertexIndex⟧];
        edgeList = If[isEdge,
          Append[edgeList,
           classNames⟦sourceIndex⟧ ⟶ classNames⟦targetIndex⟧],
          edgeList
         ];
        labelList = If[isEdge,
          Append[labelList, StringTake[subTreeRoots⟦targetVertexIndex⟧, -1]],
          labelList
         ];
       ] /@ targetVertexIndexList
     ] /@ sourceVertexIndexList,
    {sourceIndex, Length[classNames]}, {targetIndex, Length[classNames]}];
  {edgeList, labelList} =
    Transpose[DeleteDuplicates[Transpose[{edgeList, labelList}]]];
  EdgeLabeler[edgePoints_, edge_] :=
    {Text[Style[labelList⟦Position[edgeList, edge]⟦1, 1⟧⟧, Medium, Bold, Black],
      Mean@edgePoints], Arrow@edgePoints};
  machine = Graph[edgeList, VertexLabels → "Name", EdgeShapeFunction → EdgeLabeler];
  Graph[machine]
  Print[MatrixForm[{edgeList, labelList}]]
```

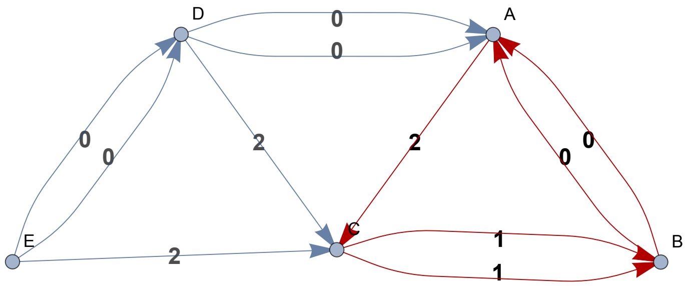
Out[137]=



```
( A ⟶ C  B ⟶ A  B ⟶ A  C ⟶ B  C ⟶ B  D ⟶ A  D ⟶ A  D ⟶ C  E ⟶ C  E ⟶ D  E ⟶ D )
(   2      0      1      1      0      0      1      2      2      0      1   )
```

```
In[139]:= HighlightGraph[machine, FindCycle[machine]]
```

Out[139]=



## Do a random walk on the graph with n steps starting from some vertex

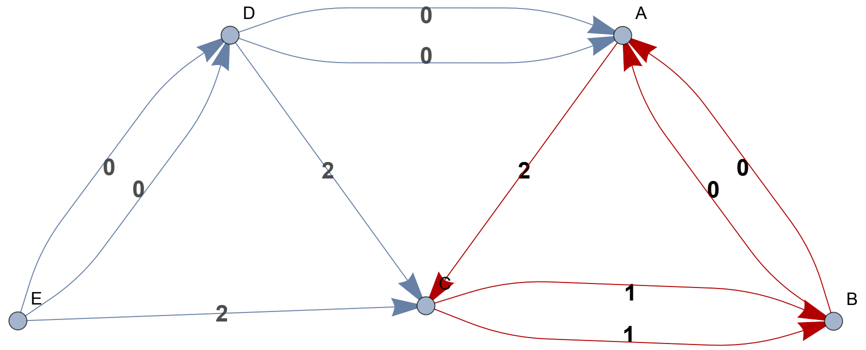
In[140]:=

```
n = 40;
init = "A";
curVertex = init;
pathSeq = "";
pathVertexList = {};
pathEdgeList = {};
With[{machine = machine},
  Do[
   {edge, edgeLabel} = RandomChoice[
     Cases[Transpose[{edgeList, labelList}], {curVertex ⟷ vf_, _}]
    ];
   newVertex = edge /. (a_ ⟷ b_) → b;
   pathSeq = pathSeq <> edgeLabel;
   pathVertexList = Append[pathVertexList, newVertex];
   pathEdgeList = Append[pathEdgeList, edge];
   curVertex = newVertex;
   , {i, n}]
 ];
pathSeq
HighlightGraph[machine, pathEdgeList]
```

Out[147]=

```
2102002012012012112102002012112112012012
```

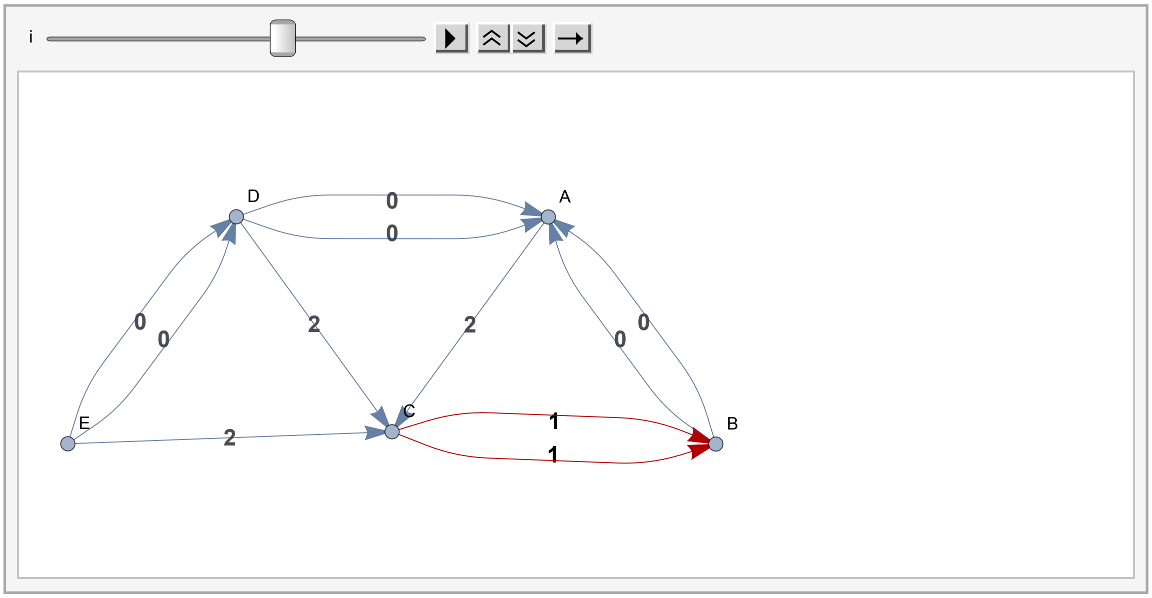
Out[148]=



••• Set: Tag Inherited in Inherited[State] is Protected.

```
In[•]:= Animate[
         HighlightGraph[machine, pathEdgeList⟦i⟧], {i, 1, Length[pathEdgeList], 1}]
```

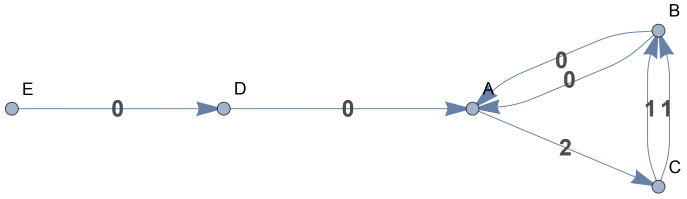
Out[•]=



Delete edges from graph  to remove initial condition states (unfinished). Need to differentiate multi-edges by label, and delete orphan vertices

```
In[149]:= (*Delete vertices from graph, e.g. to remove initial condition states*)
          minusInits = EdgeDelete[machine,
            Complement[edgeList, Union @@ FindCycle[machine, {0, Length[edgeList]}]]]
```

Out[149]=


B
E
D
A
C
0
0
0
0
1 1
2